A Shifted SSOR Preconditioner with Low-Rank Compression for Monostatic RCS Calculation

J. Chen [1, 2], Z. Liu [3, 4], B. Yong [5], Z. Jiang [6], and N. Cao [1]

[1] College of Computer and Information Engineering, Hohai University, Nanjing, China
cjq19840130@163.com

[2] Department of Space Microwave Remote Sensing System, Institute of Electronics, Chinese Academy of Sciences, Beijing, China

[3] School of Information Engineering, East China Jiaotong University, Nanchang, China

[4] State Key Laboratory of Millimeter Wave, Southeast University, Nanjing, China

[5] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing, China

[6] Department of Communication Engineering, Nanjing University of Science & Technology, Nanjing, China

Abstract ─ Computation time and memory consumption are two crucial bottlenecks for solving large dense complex linear system arising from electric field integral equations (EFIE) formulation of monostatic scattering problems. The traditional symmetric successive over-relaxation (SSOR) preconditioner, derived from the near-field matrix of the EFIE, is widely used to accelerate the convergence rate of iterative solvers. This technique can be greatly improved by modifying the near-field matrix of the EFIE with the principal value term of the magnetic field integral equation (MFIE) operator. Additionally, the adaptive cross approximation (ACA) algorithm is applied to compress the near-field interaction matrix to save memory. Numerical experiment results indicate that the novel technique can significantly reduce both the computational time and memory significantly with low cost for construction and implementation of preconditioners.

Index Terms - Adaptive cross approximation, iterative methods, low-rank property, monostatic RCS, preconditioning techniques, and shifted technique.

I. INTRODUCTION

For electromagnetic scattering analysis, a classic problem is to compute the current distribution on the surface of an object illuminated by a given incident plane wave [1]. The formulation considered here is electric field integral equations (refer to as EFIE) since it has the most general form and does not require any assumption about the geometry of the object. The EFIE matrix equation can be solved by using iterative solvers, and the required matrix-vector product operation can be efficiently calculated by multi-level fast multi-pole algorithm (MLFMA) [2]. The use of MLFMA could reduce both the memory requirement and the computational complexity to O (NlogN) [3].

It is well-known that EFIE provides a first-kind integral equation, which is ill-conditioned and difficult to solve in a linear system [4]. Therefore, some researchers have been attempting to adopt the preconditioning method to accelerate the solution of linear systems for this problem [5-

Submitted On: Aug. 29, 2012
Accepted On: Aug. 29, 2013

7]. Simple preconditioners such as the diagonal or diagonal blocks of the coefficient matrix might be effective when the matrix has some degree of diagonal dominance. Incomplete LU (ILU) factorizations have been successfully used for nonsysmmetric dense systems [8]. However, the factorization is commonly rather ill-conditioned. Thus, this makes the triangular solvers highly unstable and the use of the ILU preconditioner might be ineffective as a whole [9]. Presently, the sparse approximate inverse (SAI) preconditioning techniques have been successfully integrated with the MLFMA [10, 11]. But the construction cost of SAI is normally higher. Relative to ILU and SAI, the symmetric successive over-relaxation (SSOR) [12, 13] preconditioner has the obvious advantage in construction cost. Furthermore, the SSOR preconditioning technique contains more information of the coefficient matrix when compared with a diagonal/block diagonal matrix, which is perhaps efficient only for very long and narrow structures. However, the conventional SSOR preconditioner is sometimes ineffective for the iterative solution of the symmetric indefinite linear systems arising from the EFIE formulation of electromagnetic scattering problems. As an attempt for a possible remedy, SSOR preconditioner combined with a tri-diagonal shift from the principal value term of MFIE operator is proposed, which is called shifted SSOR (S-SSOR) [14]. Compared to original SSOR method, this shift scheme can significantly improve the performance of the SSOR preconditioner, meanwhile it does not require much more computational and storage costs. Except that the process of monostatic scattering computation could be accelerated by S-SSOR preconditioner, another remaining bottleneck of EFIE solution is the limited memory. Some previous studies have shown that the far-field impedance matrix can be compressed by MLFMA well [2-3], while the near-field self-interaction matrix is full rank, which makes it incompressible. However, in some cases, near-field interaction matrix (excluding the self-interaction matrix) might have characteristics of low rank [15-19], such as dealing with the multi-scale problems [20]. To achieve the purpose of low memory cost, the adaptive cross approximation algorithm (ACA) [21-24] is used in this paper to compress the near-field interaction matrix.

The paper is organized as follows, section II gives a brief introduction to the EFIE formulation and the MLFMA. The shifted SSOR preconditioning technique is depicted for more details in section III. Section IV demonstrates the basic theory of low-rank decomposition strategy for near-field interaction matrix. Numerical experiments with several monostatic scattering problems are presented to verify the efficiency of the proposed method in section V. The conclusions are summarized in section VI.

II. EFIE FORMULATION AND MLFMA

The EFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [1]. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows,

$$\sum_{n=1}^{N} Z_{mn} I_n = V_m, \qquad m = 1, 2, ..., N \tag{1}$$

where

$$Z_{mn} = jk\int_s \mathbf{f}_m(\mathbf{r}) \bullet \int_{s'} (\bar{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\bullet)[G(\mathbf{r},\mathbf{r}')\mathbf{f}_n(\mathbf{r}')]dsds'$$

$$V_m = \frac{1}{\eta}\int_s \mathbf{f}_m(\mathbf{r}) \bullet \mathbf{E}^i(\mathbf{r})ds\,, \quad G(\mathbf{r},\mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}.$$

Here $G(\mathbf{r},\mathbf{r}')$ refers to the Green's function in free space and $\{I_n\}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions $\mathbf{f}_m$. Also, as usual, $\mathbf{r}$ and $\mathbf{r}'$ denote the observation and source point locations. $\mathbf{E}^i(\mathbf{r})$ is the incident excitation plane wave, and η and k denote the free space impendence and wave number, respectively. Once the matrix in equation (1) is solved by numerical matrix equation solvers, the expansion coefficients $\{I_n\}$ can be used to calculate the scattered field and RCS. In the following, we use $\mathbf{Z}$ to denote the coefficient matrix in equation (1), $\mathbf{I} = \{I_n\}$, and $\mathbf{V} = \{V_m\}$ for simplicity. Then, the EFIE matrix in equation (1) can be symbolically rewritten as,

$$\mathbf{ZI} = \mathbf{V}. \tag{2}$$

The basic idea of the fast multipole method (FMM) is to convert the interaction of element-to-element to the interaction of group-to-group. Here a group includes the elements residing in a spatial box. The mathematical foundation of the FMM is the addition theorem for the scalar Green's

function in free space. Using the FMM, the matrix-vector product **ZI** can be written as,

$$\mathbf{ZI} = \mathbf{Z}_N\mathbf{I} + \mathbf{Z}_F\mathbf{I} \tag{3}$$

where $\mathbf{Z}_N$ is the near part of $\mathbf{Z}$ and $\mathbf{Z}_F$ is the far part of $\mathbf{Z}$.

In the FMM, the operation complexity to perform **ZI** is $O(N^{1.5})$. If the FMM is implemented in multilevel, the total cost can be reduced further to $O(NlogN)$ [2]. The calculation of elements in the matrix $\mathbf{Z}_N$ remains the same as in the method of moments (MoM) procedure. However, those elements in $\mathbf{Z}_F$ matrix can not be explicitly computed and stored. Hence, it is impossible to use the matrix $\mathbf{Z}_F$ directly.

III. SHIFTED SSOR PRECONDITIONER WITH LOW-RANK DECOMPOSITION STRATEGY

In the traditional SSOR preconditioning scheme, the preconditioner is chosen as follows,

$$\mathbf{M}_{SSOR} = (\tilde{\mathbf{D}} + \mathbf{L})(\tilde{\mathbf{D}})^{-1}(\tilde{\mathbf{D}} + \mathbf{U}) \tag{4}$$

where $\mathbf{Z}_N = \mathbf{L}+\mathbf{D}+\mathbf{U}$ in equation (3), **L** is the lower triangular matrix, **D** is the positive diagonal matrix, **U** is the upper triangular matrix, and $\tilde{\mathbf{D}} = (1/\omega)\mathbf{D}$, $0 < \omega < 2$ (ω is the relaxation parameter).

Although the SSOR preconditioner performs well in the case of Hermitian positive definite matrices, the performance is often poor when the matrices are indefinite or non-Hermitian, as in the case of the EFIE. The matrix of MFIE has good condition number mainly due to the existence of the principal value term [25]. Accordingly, combining the EFIE and MFIE leads to the well-conditioned combined field integral equations (CFIE). Inspired by CFIE, the principal value term of MFIE is used in order to improve the condition of EFIE matrix. More specifically, we use [14],

$$\mathbf{Z}_\tau = \mathbf{Z}_N + \alpha\mathbf{Z}_{MFIE} \tag{5}$$

to construct the S-SSOR preconditioner $\mathbf{M}_{S\text{-}SSOR}$. The impedance matrix $\mathbf{Z}_{MFIE}$ is the discretized tridiagonal matrix from the principal value term of MFIE operator and α stands for a nonnegative real parameter. It is known that $\mathbf{Z}_{MFIE}$ is a well-conditioned and very sparse real symmetric matrix. As a result, it requires a small amount of computation and storage.

In order to save memory consumption for construction of S-SSOR preconditioner, an ACA based method is proposed and the methodology is discussed in this section. The ACA decomposition is used to the near-field sub-matrices (exclude the self-interaction sub-matrices) [15]. Although the efficiency of ACA filled the near-field sub-matrices is not better than that of the ACA filled the far-field sub-matrices, it is still a little more efficient than that of direct fill.

In the FMM, the near-part matrix $\mathbf{Z}_N$ can be rewritten as,

$$\mathbf{Z}_N = \mathbf{Z}_{NS} + \mathbf{Z}_{NI} \tag{6}$$

where $\mathbf{Z}_{NS}$ is near-field self-interaction matrix and $\mathbf{Z}_{NI}$ is near-field interaction matrix. According to the theory of FMM, the near impedance can not be decomposed. However, if the diagonal part is removed from the near impedance matrix, it can be decomposed by low-rank decomposition method. As $\mathbf{Z}_{NI}$ denotes the near impedance matrix excluding the diagonal part, the matrix $\mathbf{Z}_{NI}$ could be divided into many sub-matrices by a tree data structure in MLFMA. Obviously, each none-zero sub-matrix of $\mathbf{Z}_{NI}$ denotes the near action. Accordingly, the adaptive cross approximation approach is used and the sub-matrix of $\mathbf{Z}_{NI}$ can be approximated by two small sub-matrices $\mathbf{U}_{ACA}$ and $\mathbf{V}_{ACA}$. Denoting the sub-matrix of $\mathbf{Z}_{NI}$ with $\mathbf{Z}'$, we have [21],

$$[\mathbf{Z}']_{QM} \approx [\mathbf{U}_{ACA}]_{Qk} \cdot [\mathbf{V}_{ACA}]^H_{Mk} \tag{7}$$

where M and Q are the dimensions of matrix $\mathbf{Z}'$. The symbol k is the rank of the matrix $\mathbf{Z}'$, which is much smaller than M and Q. According to equation (7), the memory requirement of the matrices $\mathbf{U}_{ACA}$ and $\mathbf{V}_{ACA}$ is much less than that of the direct filling of $\mathbf{Z}'$. The procedure of the ACA algorithm is present as follows [22]:

First, let $\mathbf{U}_{ACA}$ = NULL in order to save the selected columns and $\mathbf{V}_{ACA}$ = NULL in order to save the selected rows.

Step1: Choose the first column $\mathbf{u}_1$ randomly and let $\mathbf{U}_{ACA} = \mathbf{U}_{ACA} \cup \{\mathbf{u}_1\}$. Find the maximum value u_{1k} in $\mathbf{u}_1$. Then choose the first row $\mathbf{v}_1$, which is located at the kth row in the matrix. Let $\mathbf{V}_{ACA} = \mathbf{V}_{ACA} \cup \{\mathbf{v}_1\}$.

Step 2: Find the maximum value v_{ik} in $\mathbf{v}_i$. Then choose the $(i + 1)$th column $\mathbf{u}_{i+1}$, which is located at the kth column in the matrix.

Step 3: Let $\mathbf{U}_{ACA} = \mathbf{U}_{ACA} \cup \{\mathbf{u}_{i+1}\}$.

Step 4: Find the maximum value $\mathbf{u}_{i+1,k}$ in $\mathbf{u}_{i+1}$. Then choose the $(i + 1)$th row $\mathbf{v}_{i+1}$, which is located at the kth row in the matrix.

Step 5: $\mathbf{v}_{i+1} = \mathbf{v}_{i+1} - \sum_{j=1}^{i} u_{j,i}\mathbf{v}_j$, and let $\mathbf{V}_{ACA} = \mathbf{V}_{ACA} \cup \{\mathbf{v}_{i+1}\}$.

Step 6: If $\frac{\|\mathbf{u}_{i+1}\|\|\mathbf{v}_{i+1}\|}{\|\mathbf{u}_1\|\|\mathbf{v}_1\|} < \varepsilon$, the algorithm will stop, otherwise, go to Step 2. The low-rank decomposition form of near-field interaction matrix is $\mathbf{Z}_{NI} \approx \mathbf{U}_{ACA} \cdot \mathbf{V}_{ACA}^{H}$.

This algorithm produces a sequence of decompositions of a matrix into a sum of low-rank matrix and error matrix. Neither the original matrix nor the error matrix will be computed completely. The decision of the tolerance error ε needs a trade-off between accuracy and effectiveness. If the tolerance error is set too high, the solution results will lead to less accurate or even wrong. In contrast, a too-low tolerance error will degrade the compression effect. By classic ACA reference [23], as well as our numerical experiments, $\varepsilon = 10^{-3}$ is appropriate for most cases. Furthermore, single precision is used in the remaining part of this paper.

When ACA technique is used, the near impedance matrix can be compressed to save the memory. Moreover, the computation time to fill the near impedance will be saved. However, the diagonal part of the near impedance can not be compressed due to its full rank. This part should be computed by conventional MoM procedure. Since the ACA technique is only for compression, the SSOR accelerated iterative solver will not be affected by ACA. Accordingly, the SSOR and ACA can be integrated to improve the efficiency.

IV. NUMERICAL EXAMPLES

In this section, numerical results based on on-site experiments will demonstrate the accuracy and efficiency of the proposed method for fast calculation of monostatic RCS. In our experiments, the restarted version of GMRES algorithm [12] is used as the iterative solver, and the dimension of Krylov subspace is set to be 30 in this paper. All experiments are performed on a Core(TM)II E8400 with 3 GHz CPU and 3.24 GB RAM in single precision. Additional details and comments on the implementation are given below:

- zero vector is taken as initial approximate solution for all examples,
- the iteration process is terminated when the normalized backward error is reduced by 10^{-3} for all examples,
- the dimension of Krylov subspace is taken to be 30,
- 1.0 is taken as the relaxation parameter (ω) for building both SSOR and S-SSOR preconditioner mentioned in this paper,
- 3.0 is taken as the shift parameter for building S-SSOR preconditioner.

Although CFIE shows higher efficiency for objects with closed structure than EFIE, it fails for geometries with open structure [26]. As a result, proposed technique in this paper is a suitable choice to alleviate this difficulty due to its fast iteration capability and less memory requirement. The performance of the proposed method is investigated on three examples with open structure for monostatic RCS calculation. As shown in Fig. 1, we consider a cube-plate perfect electrical conductor (PEC) scatterer consisting of a plate of size (1 m × 0.5 m) placed on a 1 m × 1 m × 1 m cube with 4867 unknowns at 400 MHz. The second example, as shown in Fig. 2, is a 1 m × 1 m × 1 m open cavity with 8101 unknowns at 500 MHz and the final structure in Fig. 3, is a disk (radius is 2 m) with 4280 unknowns at 300 MHz.

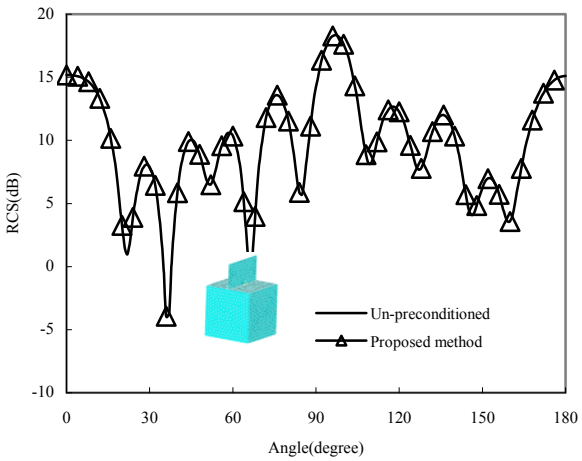

Fig. 1. The monostatic RCS for a cube-plate scatterer using the proposed method.

The sets of angles of interest for the monostatic RCS vary from 0 to 180 degree for the first two examples and 0 to 89 degree for the last

instance in pitch direction when azimuth angle is fixed at 0 degree. The RCS curve computed with repeated solution at each angle are taken as reference values. The accuracy of the compressed S-SSOR preconditioner can be seen from its agreement with the reference values.

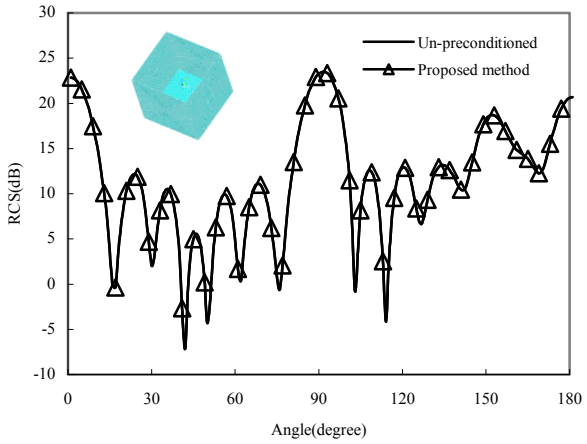

Fig. 2. The monostatic RCS for an open cavity using the proposed method.

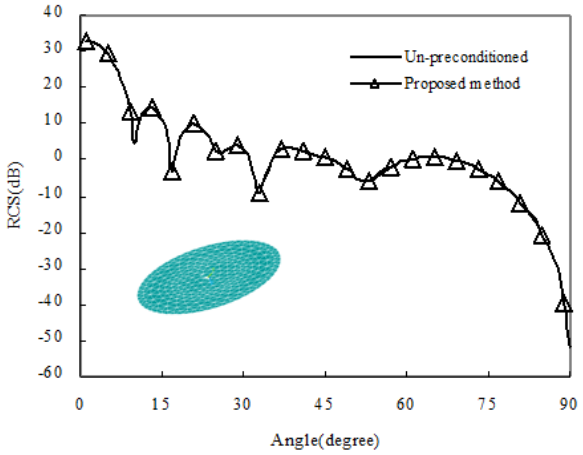

Fig. 3. The monostatic RCS for a disk using the proposed method.

Figures 4, 5, and 6 show number of iteration in each pitch angle of GMRES with unpreconditioned GMRES method, SSOR preconditioner and the proposed preconditioner for three geometries, respectively. It can be observed that the novel operator has the highest convergence rate for each example. Table 1 lists the computation time for all angles to cover the entire monostatic RCS curve on these three examples. Similar improvements can also be found in comparison with the conventional SSOR methods in terms of computational time. Compared with the unpreconditioned GMRES algorithm, the compressed S-SSOR technique decreases the computational time by a factor of 3.11 on the cube-plate scatterer example, 2.78 on the open cavity example, and 3.26 on the disk example, respectively.

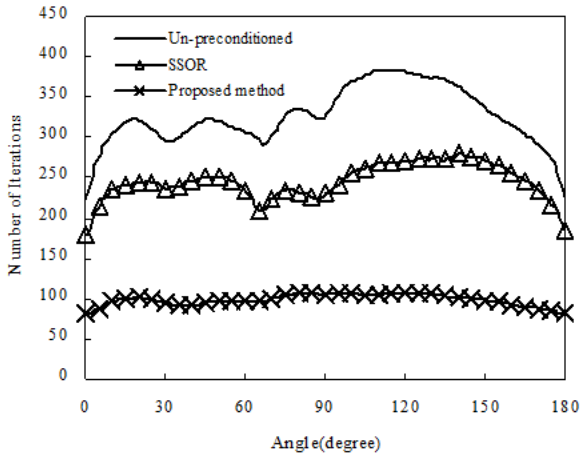

Fig. 4. Iterative number for a cube-plate scatterer with unpreconditioned GMRES method, SSOR preconditioner, and the proposed method.

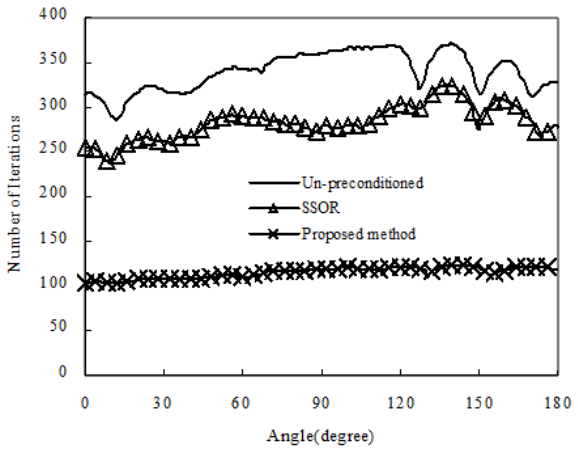

Fig. 5. Iterative number for an open cavity with unpreconditioned GMRES method, SSOR preconditioner, and the proposed method.

Table 2 demonstrates the compression effect of near-field interaction memory of the three examples. The proposed method could effectively reduce the near-field interaction memory consumption by a factor of 2.09 on the cube-plate scatterer example, 2.05 on the open cavity example, and 2.95 on the disk example,

respectively. The time for compressed operation is also listed in Table 2. It is obvious that compression time is much smaller than solution time. Summarizing the discussions we can see that, our proposed technique outperforms the conventional techniques in terms of the efficiency and memory consumption.

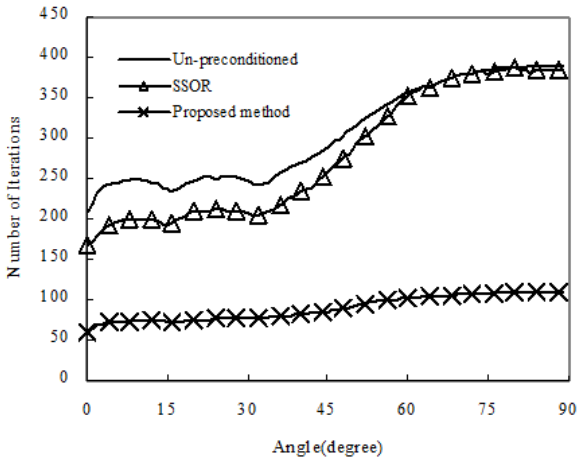

Fig. 6. Iterative number for a disk with unpreconditioned GMRES method, SSOR preconditioner, and the proposed method.

Table 1: Comparison of solution time (in seconds) with unpreconditioned GMRES method, SSOR preconditioner, and the proposed method on three different examples.

	Solution time (s)		
Geometry	**No**	**SSOR**	**S-SSOR**
Cube-plate	7972.92	6383.61	2566.77
Open cavity	12590.05	11123.44	4533.09
Disk	4952.08	4700.38	1516.77

Table 2: Compression effect of near-field interaction memory on three different examples.

Object	**Compression time(s)**	**Near-field interaction memory before compression (MB)**	**Near-field interaction memory after compression (MB)**
Cube-plate	55.13	57.41	27.45
Open cavity	77.30	78.70	38.38
Disk	37.94	55.11	18.69

The SSOR technique can accelerate the convergence of the iterative solver while the GMRES method is used in this paper. Besides, the ACA technique can save memory consumption by the near impedance matrix. Numerical results show that the ACA technique will not affect the accuracy and efficiency of the SSOR preconditioner. An "ACA-only" technique will cost large computation time for iterative solution without using the SSOR preconditioner.

V. CONCLUSION

In this paper, a new compression scheme is developed and used to construct the robust shifted SSOR preconditioners for efficiently solving the electromagnetic scattering problems existed in the non-Hermitian linear systems derived from EFIE formulation. The new method can significantly reduce both calculation time and memory consumption without compromising the accuracy of the final result. Several numerical experiments for validation are performed. Compared to the traditional SSOR preconditioner, the novel compressed shifted SSOR preconditioner is more efficient and robust.

ACKNOWLEDGMENT

The authors would like to thank the assistance and support of the National Natural Science Foundation of China (No: 61301025), Jiangsu Provincial Natural Science Foundation of China (No: BK20130853), China Postdoctoral Science Foundation (No: 2013M541035), Youth Science Foundation of Jiangxi Provincial Department of Science and Technology (No: 20122BAB211018), and Youth Science Foundation of Jiangxi Provincial Department of Education (No: GJJ13321).

REFERENCES

[1] S. Rao, D. Wilton, and A. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 409-418, 1982.

[2] W. Chew, J. Jin, E. Midielssen, and J. Song, *Fast and Efficient Algorithms in Computational Electromagnetics*. Boston, MA: Artech House, 2001.

[3] J. Song, C. Lu, and W. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Transactions on*

Antennas and Propagation, vol. 45, no. 10, pp. 1488-1493, 1997.

[4] Z. Fan, M. Chen, R. Chen, and D. Ding, "An efficient parallel FE-BI algorithm for large-scale scattering problems," *ACES Journal*, vol. 26, no. 10, pp. 831-840, Oct. 2011.

[5] X. Hu, R. Chen, D. Ding, Z. Fan, and Y. Xu, "Two-step preconditioner of multilevel simple sparse method for electromagnetic scattering problems," *ACES Journal*, vol. 27, no. 1, pp. 14-21, Jan. 2012.

[6] Z. Liu, J. Chen, and R. Chen, "An adaptive preconditioning technique using Fuzzy controller for efficient solution of electric field integral equations," *ACES Journal*, vol. 26, no. 6, pp. 512-518, June 2011.

[7] M. Chen, R. Chen, Z. Fan, and D. Ding, "Accelerating the multilevel fast multipole method with parallel preconditioner for large-scale scattering problems," *ACES Journal*, vol. 26, no. 10, pp. 815-822, Oct. 2011.

[8] K. Sertel and J. Volakis, "Incomplete LU preconditioner for FMM implementation," *Microwave and Optical Technology Letters*, vol. 26, no. 7, pp. 265-267, 2000.

[9] E. Chow and Y. Saad, "Experimental study of ILU preconditioners for indefinite matrices," *J. of Comput. Appl. Math*, vol. 86, pp. 387-414, 1997.

[10] J. Lee, J. Zhang, and C.-C. Lu, "Sparse inverse preconditioning of multilevel fast multipole algorithm for hybrid integral equations in electromagnetics," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 9, pp. 2277-2287, Sep. 2004.

[11] B. Carpentieri, "An adaptive approximate inverse-based preconditioner combined with the fast multipole method for solving dense linear systems in electromagnetic scattering," *ACES Journal*, vol. 24, no. 5, pp. 504-510, Oct. 2009.

[12] Y. Saad, *Iterative Methods for Sparse Linear Systems*, PWS Publishing Company, 1996.

[13] R. Chen, K. Edward, N. Yung, C. Chan, and D. Fang, "Application of SSOR preconditioned conjugate gradient algorithm to edge-FEM for 3-dimensional full wave Electromagnetic boundary value problems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 50, no. 4, pp. 1165 -1172, April 2002.

[14] J. Chen, Z. Liu, K. Xu, D. Ding, Z. Fan, and R. Chen, "Shifted SSOR preconditioning technique for electromagnetic wave scattering problems," *Microwave and Optical Technology Letters*, vol. 51, no. 4, pp. 1035-1039, April 2009.

[15] A. Heldring, J. Rius, J. Tamayo, J. Parron, and E. Ubeda, "Multiscale compressed block decomposition for fast direct solution of method of moments linear system," *IEEE Trans. Antennas Propag.*, vol. 59, no. 2, pp. 526-536, Feb. 2011.

[16] A. Heldring, J. Rius, J. Tamayo, J. Parron, and E. Ubeda, "Fast direct solution of method of moments linear system," *IEEE Trans. Antennas Propag.*, vol. 55, no. 11, pp. 3220-3228, Nov. 2007.

[17] A. Heldring, J. Rius, J. Tamayo, and J. Parron, "Compressed block-decomposition algorithm for fast capacitance extraction," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 27, no. 2, pp. 265-271, Feb. 2008.

[18] A. Heldring, J. Rius, J. Tamayo, and J. Parron, "Multilevel MDA-CBI for fast direct solution of large scattering and radiation problems," *Antennas and Propagation Society International Symposium*, 2007.

[19] A. Heldring, J. Tamayo, J. Rius, J. Parron, and E. Ubeda, "Multiscale CBD for fast direct solution of MoM linear system," *Antennas and Propagation Society International Symposium*, 2008.

[20] Z. Jiang, Z. Fan, D. Ding, R. Chen, and K. Leung, "Preconditioned MDA-SVD-MLFMA for analysis of multi-scale problems," *ACES Journal*, vol. 25, no. 11, pp. 914-925, 2010.

[21] M. Bebendorf, "Approximation of boundary element matrices," *Numer. Math.*, vol. 86, no. 4, pp. 565-589, June 2000.

[22] S. Kurz, O. Rain, and S. Rjasanow, "The adaptive cross-approximation technique for the 3-D boundary element method," *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 421-424, March 2002.

[23] K. Zhao, M. Vouvakis, and J. Lee, "The adaptive cross approximation algorithm for accelerated method of moments computations of EMC problems," *IEEE Trans. Electromagnetic Compatibility*, vol. 47, no. 4, pp. 763-773, Nov. 2005.

[24] Z. Liu, R. Chen, J. Chen, and Z. Fan, "Using adaptive cross approximation for efficient calculation of monostatic scattering with multiple incident angles," *ACES Journal*, vol. 26, no. 4, pp. 325-333, April 2011.

[25] J. Hu and Z. Nie, "Improved electric field integral equation (IEFIE) for analysis of scattering from 3-D conducting structures," *IEEE Transactions on Electromagnetic Compatibility*, vol. 49, no. 3, pp. 644-648, August 2007.

[26] P. Rui, R. Chen, Z. Fan, J. Hu, and Z. Nie, "Perturbed incomplete ILU preconditioner for efficient solution of electric field integral equations," *IET Microwave Antennas Propagation*, vol. 5, no. 1, pp. 1059-1063, 2007.

Jiaqi Chen was born in Gansu, China. He received the B.S. and Ph.D. degrees in communication engineering from Nanjing University of Science and Technology (NUST), Nanjing, China, in 2005 and 2011, respectively. He was with the Center for Sensorsystems (ZESS), University of Siegen, Siegen, Germany, as a visiting scholar in 2009. He is currently working at College of Computer and Information Engineering, Hohai University. His research interests include computational electromagnetics, SAR imaging and precipitation radar.

Zhiwei Liu was born in Jiangxi Province, P. R. China in 1982. He received B.S. degree in computer science from Nanjing University of Science & Technology in 2003, M.S. degree in Nanjing Institute of Electronics & Technology in 2006, and Ph.D. degree in Nanjing University of Science & Technology in 2011, respectively. He was with the Department of Electrical Engineering, Iowa State University, as a visiting scholar in 2009. He is currently working at Department of Information Engineering, East China Jiaotong University. His research interests focus on theory of electromagnetic scattering and inverse scattering.

Zhaoneng Jiang was born in Jiangsu Province, the People's Republic of China. He received the B.S. degree in physics from Huaiyin normal college in 2007, and is currently working toward the Ph.D. degree at Nanjing University of Science and Technology (NUST), Nanjing, China. His current research interests include computational electromagnetics, antennas and electromagnetic scattering and propagation, electromagnetic modeling of microwave integrated circuits.

Bin Yong was born in Anhui, China. He received the B.S. degrees in Computer Science and Technology, Hefei University of technology, Hefei, China and the Ph.D. degree in Cartography and Geography Information System, Nanjing University, Nanjing, China. He is currently a Professor with State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing, China. His research areas mainly includes: 1) Remote Sensing Precipitation (radar, satellite, multi-sensor, multi-platform); 2) Application of NASA multi-satellite products; and 3) Surface Water and Hydrological System Analysis. Dr. Yong is currently the American Geophysical Union member.