Adaptive Difference Beam with Low Sidelobes at Subarray Level Based on Semidefinite Programming

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Abstract **─** This paper proposes a semidefinite programming (SDP) method to form adaptive difference beam at subarray level. Its performance is investigated via computer simulations. Compared with loaded sample matrix inversion (LSMI) and constrained adaptive beam-pattern synthesis (CAPS). The proposed algorithm not only has manifest lower sidelobes in quiescent pattern control and sidelobe interference suppression, but also produces more accurate and deeper null in look direction when mainbeam interference deforms the pattern.

Index Terms **─** Adaptive at subarray level, difference beam, low sidelobe, and semidefinite programming.

I. INTRODUCTION

In search-track system, antennas are usually required to generate sum and difference beams simultaneously. However, the implementation of two independent excitations for the sum and difference modes of operation is generally unacceptable because of the costs and complexity [1]. Thus, it is necessary to form the two types of beams at subarray level. Since the sum pattern is used in both signal transmission and reception, the most common way to solve the problem is to generate an optimal sum pattern and suboptimal difference pattern [2]. Therefore, when element tapering is in favor of sum beam, sidelobe reduction contributed by element tapering is not effective for difference beam. Nevertheless, it is desirable to obtain low sidelobes in the adaptive beams to aid the performance against clutter [3]. Moreover, difference pattern is required to have deep slope at boresight to improve the radar sensitivity [1].

The minimum variance distortionless response (MVDR) is a popular algorithm used for adaptive beamforming. However, its high sidelobe level is an issue when practical sample covariance matrix is used [4]. The loaded sample matrix inversion (LSMI) [5] is a modification of sample matrix inversion (SMI), which suppresses sidelobes by adding a small value on the diagonal of the covariance matrix. However, there is no closed-form solution for the optimal loading value and it is usually obtained by simulation trials or empirical experience [4]. A new projection based algorithm, constrained adaptive beam-pattern synthesis (CAPS), combines advantages of subspace and penalty function (PF) approaches [3], and its performance is similar to LSMI except for very low sidelobe antennas and severe jamming situations [6]. Another approach based on optimization is realized by second-order cone (SOC) programming [7], but a hard threshold needs to be preset and the proper choice of threshold is difficult [4]. Besides, sidelobe areas have to be delimited previously for optimization. Some research has been done on adaptive difference beam at subarray level in [8-9]. However, in [8], interests go to the influence of subarray configuration on adaptive sum and difference beam. Sidelobe reduction of difference beam in [9] is mainly contributed by element tapering, especially in quiescent pattern control, which rises remarkably when element tapering is designed for sum beam (e.g. Taylor taper). References [1, 2] have investigated the effects of subarray configuration and subarray level weights on optimum sum and difference pattern. Although,

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suboptimum difference beams were achieved, they were aimed at non-adaptive beamforming.

In this paper, we propose semidefinite programming (SDP) method to form adaptive difference beam at subarray level in linear array while Taylor tapering is applied at element level for sum beam. Sidelobe reduction of sum beam can be achieved by element tapering, thus is not considered here. The proposed algorithm is based on the thought of matching reference difference weights with given subarray configuration. Besides, we impose constraints on interferences suppression and null depth in look direction so that deep null depth and adaption are achieved. The feasibility and advantages of the novel algorithm are verified via numerical simulations.

II. PROBLEM FORMULATION

Consider a uniform linear array with $N = 2M$ omnidirectional antennas spaced with $d = \lambda / 2$, receiving N narrowband signals $\vec{s}(t) \in C^{N\times 1}$, where λ is the carrier wave length, as shown in Fig. 1. The array is assumed to be symmetrical about the origin. When K signals impinge on the array, the n^{th} snapshot received data vector $\vec{x}_{ele}(n) \in C^{N\times 1}$ is given by [10],

$$\vec{x}_{ele}(n) = A\vec{s}(n) + \vec{v}(n), \qquad (1)$$

where $\vec{v}(n)$ is a noise vector, characterizing additive Gaussian white noise. A is the array manifold matrix, which is the combination of all possible steering vectors [10],

$$A = \left[\vec{a}(\theta_1), \vec{a}(\theta_2), \ldots\ldots, \ \vec{a}(\theta_K)\right], \qquad (2)$$

where $\vec{a}(\theta_k)$ is the steering vector for the k^{th} $(0 \le k \le K)$ signal from θ_k and is defined as,

$$\vec{a}(\theta_k) = [e^{-j\frac{2\pi d}{\lambda}\left(n-\frac{N+1}{2}\right)\sin\theta_k}]^T, \ 1 \le n \le N\,. \qquad (3)$$

Assume an amplitude weight vector $\vec{w} = \left[w_{-M} \ w_{-M+1} \ldots w_M\right]^T$ is applied at element level to control sidelobe for the quiescent sum beam. Besides, the array is divided into L subarrays. Sub-arraying is symmetrical about the array center. The subarray geometry is shown in Fig. 2. The element to subarray transformation matrix can be described by an $N \times L$ matrix [11],

$$T_d = D_{\theta_0} \cdot D_w \cdot T\,, \qquad (4)$$

where $D_{\theta_0} = diag\left(\vec{a}_0\right)$, $D_w = diag\left(\vec{w}\right)$, θ_0 is the look direction. T describes how different the array elements are arranged into L subarrays [11]. Then the interferences plus noise received data and the $L \times L$ disturbance covariance matrix at subarray level can be given by equations (5) and (6), respectively,

$$\vec{x}_{sub} = T_d^H \vec{x}_{ele} \qquad (5)$$

$$R_{sub} = T_d^H E\left[\vec{x}_{ele}\vec{x}_{ele}^H\right]T_d = T_d^H R_{ele} T_d\,. \qquad (6)$$

In practice, the covariance matrix R_{ele} is unknown and we replace it with its maximum likely-hood estimation [6],

$$\hat{R}_{ele} = \frac{1}{N_{sap}} \sum_{n=1}^{N_{sap}} \vec{x}_{ele}(n)\,\vec{x}_{ele}^H(n)\,,$$

where N_{sap} is the sampling rate and $N_{sap} = 2N$ in this paper. If $\vec{w}_{sub}$ indicates weighting at subarray level, then the pattern with subarray configuration is given by,

$$f(\theta) = \left[diag\left(\vec{w}\right)\cdot T \cdot \vec{w}_{sub}\right]^H \left[\vec{a}(\theta)\circ \vec{a}^*(\theta_0)\right] (7)$$

where "$\circ$" and "$*$" denote Hadamard product and conjugate, respectively.

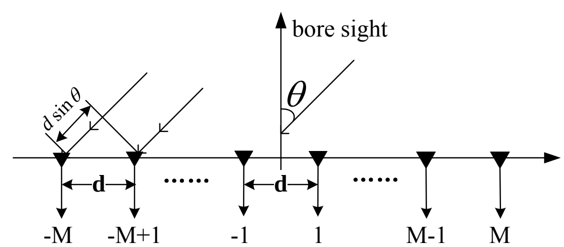

Fig. 1. The uniform linear array of N = 2M sensors.

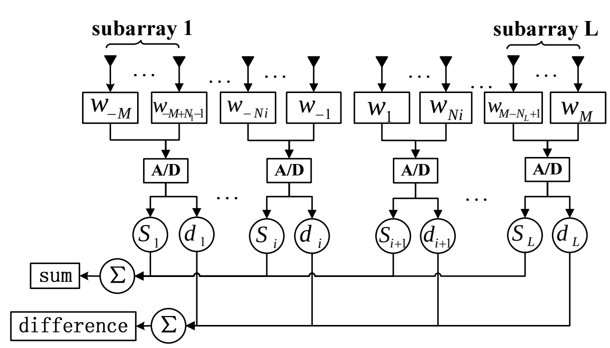

Fig. 2. Linear array with subarray configuration.

It is known that adaptive beamforming is data-dependent [12]. In this paper, we achieve adaption at subarray level. Adaptive weighting is calculated with the output from all subarray channels. A general block diagram of adaptive beam former is shown in Fig. 3.

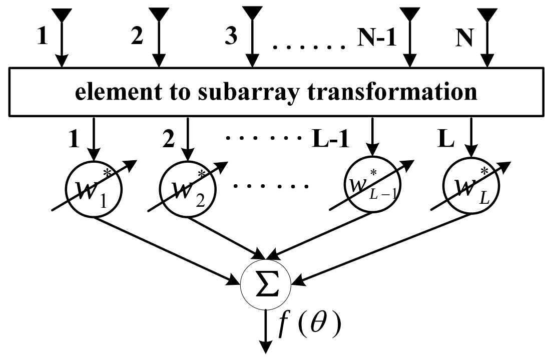

Fig. 3. Adaptive beamforming block.

A. Loaded sample matrix inversion

In reference [9], the steering vector of difference beam at subarray level $\vec{s}_\Delta(\theta_0)$ is given by,

$$\vec{s}_\Delta(\theta_0) = T_d^H\left[\vec{g} \circ \vec{a}(\theta_0)\right], \tag{8}$$

where $\vec{g} = [\underbrace{-1,...,-1}_{M}, \underbrace{1,...,1}_{M}]^T$.

The adaptive weighting at subarray level based on LSMI [6] is,

$$\vec{w}_{LSMI} = \left(\hat{R}_{sub} + \delta I\right)^{-1} \vec{s}_\Delta(\theta_0). \tag{9}$$

I denotes the identity matrix and δ is a positive constant, we choose $\delta = 4\sigma^2$ for simplification [6], where σ^2 is the power of noise.

B. Constrained adaptive beam-pattern synthesis

The adaptive weighting based on CAPS algorithm [6] is,

$$\vec{w}_{CAPS} = \vec{w}_{SMI} - X_\perp\left(X_\perp{}^H C X_\perp\right)^{-1} X_\perp{}^H C\left(\vec{w}_{SMI} - \vec{s}_\Delta\right) \tag{10}$$

where $\vec{w}_{SMI} = \hat{R}_{sub}^{-1}\vec{s}_\Delta \Big/ \left[\vec{s}_\Delta^H \hat{R}_{sub}^{-1}\vec{s}_\Delta\right]$. The columns of the matrix $X_\perp$ span the space orthogonal to $\left[J, \vec{s}_\Delta\right]$. J is a unitary $L \times K$ matrix with columns spanning the interference subspace(ISS). L and K are the number of subarray channels and interferences, respectively. J can be estimated from the received data by eigen decomposition of $\hat{R}_{sub}$. After eigen decomposition of $\hat{R}_{sub}$, we rank its eigenvalues in descending order as $\lambda_1 \ge \lambda_2 \ge ...$ $\ge \lambda_K > \lambda_{K+1} \ge ... \ge \lambda_L$, and their corresponding eigenvectors are $\vec{u}_1,...\vec{u}_K, \vec{u}_{K+1},...\vec{u}_L$, among which $\vec{u}_1,...\vec{u}_K$ span the ISS when interference to noise ratio (INR) is large [13]. To determine the dimension of ISS (i.e., to obtain K), we use the Akaike information criterion (AIC) [14],

$$AIC(k) = -2\ln\left[\frac{\prod_{i=k+1}^{L}\lambda_i^{\frac{1}{L-k}}}{\frac{1}{L-k}\sum_{i=k+1}^{L}\lambda_i}\right]^{(L-k)N_{sap}+2k(2L-k)}.$$
$$\hat{K} = \min\left\{AIC(k), k = 0,1,...,L-1\right\} \tag{11}$$

Overestimation of dimension of ISS causes signal to interference and noise ratio (SINR) loss while underestimation results in insufficient suppression of interferences. Incorrect estimation may occur in complicated scenarios or the situations where noise power of each subarray channel differs considerably. Diagonal loading can improve the robustness of AIC against errors [6]. Therefore, we replace $\hat{R}_{sub}$ with $\hat{R}_{sub} + 4I$ in our simulation. C is a directional weighting function, and $C \approx I$ for no directional weighting, thus the CAPS weight vector used in this paper is [6],

$$w_{CAPS} = \vec{s}_\Delta + P_{[J,\vec{s}_\Delta]}\left(\vec{w}_{SMI} - \vec{s}_\Delta\right), \tag{12}$$

where $P_{[J,\vec{s}_\Delta]}$ denotes the projection onto space $\left[J, \vec{s}_\Delta\right]$.

III. THE PROPOSED METHOD

There are some algorithms such as Taylor tapering, Dolph-Chebyshev synthesizing, for low sidelobe sum pattern synthesis [15]. In terms of difference pattern, Bayliss weighting is used to achieve low side lobes [16]. Despite the fact that Taylor tapering is exploited as element excitation for sum beam, we may try to minimize the difference between $\vec{w}_{ref}$ and $T_w\vec{w}_{sub}$ with constraint of interference suppression, where

$\vec{w}_{ref}$ is the optimum difference excitations for sidelobe reduction. $T_w\vec{w}_{sub}$ is the equivalent element weight vector with subarray configuration, and $T_w = diag(\vec{w})\cdot T$. $\vec{w}_{sub}$ is the weight vector at subarray level, which we are looking for. In addition, we can impose constraints on the null in look direction for difference beam synthesis.

Suppose Taylor tapering $\vec{w}_{Taylor}$ is applied at each element for sumbeam forming. To form adaptive difference beam at subarray level, we design an optimization problem, given as follows,

$$\begin{aligned} &\vec{w}_{sub} = \arg\min_{\vec{w}_{sub}} \left\| T_w\vec{w}_{sub} - \vec{w}_{ref} \right\|_2^2 \\ &s.t.:\ \left(T_d \cdot \vec{w}_{sub}\right)^H \vec{a}_0 = 0 \\ &\quad \left(T_d \cdot \vec{w}_{sub}\right)^H \vec{a}\left(\theta_k\right) = 0, \\ &\qquad k = 1,2,...,K < L \end{aligned} , \tag{13}$$

where $\|\vec{x}\|_2$ is the Frobenius norm of vector $\vec{x}$. θ_k denotes the direction of the k^{th} interference. Interferences are assumed incoherent with each other. However, in general, we have no prior information of interferences, i.e., θ_k is unknown. Nevertheless, in the situation of strong interferences and small signal, the optimal weight vector tends to be orthogonal to the interference subspace [6]. Thus, we can modify equation (13) as,

$$\begin{aligned} &\vec{w}_{sub} = \arg\min_{\vec{w}_{sub}} \left\| T_w\vec{w}_{sub} - \vec{w}_{ref} \right\|_2^2 \\ &s.t.:\ \left(T_d \cdot \vec{w}_{sub}\right)^H \cdot \vec{a}_0 = 0 \\ &\quad \vec{w}_{sub}^H \cdot J = 0 \end{aligned} . \tag{14}$$

Assume $P\left(\vec{v}\right) = \| T_w\vec{v} - \vec{w}_{ref} \|_2^2$, $\vec{v} \in C^{L\times 1}$. It can be easily shown that $P(\vec{v})$ satisfies the following inequality for all $0 \le \alpha \le 1$,

$$P\left(\alpha\vec{\gamma} + (1-\alpha)\vec{v}\right) \le P\left(\alpha\vec{\gamma}\right) + (1-\alpha)P\left(\vec{v}\right) \tag{15}$$

where $\vec{\gamma} \in C^{L\times 1}$. Thus, the objective function in equation (14) is convex [17]. As its constraint functions are affine, equation (14) is a quadratic program [17]. We can introduce a non-negative auxiliary variable t that serves as an upper bound on the objective [18],

$$\begin{aligned} &\min_{t,\vec{w}_{sub}}\ t \\ &s.t.:\left(T_d \cdot \vec{w}_{sub}\right)^H \cdot \vec{s}_0 = 0 \\ &\quad \vec{w}_{sub}^H J = 0 \\ &\quad \left\| T_w\vec{w}_{sub} - \vec{w}_{ref} \right\|_2^2 \le t \end{aligned} . \tag{16}$$

Equation (16) satisfies the standard form of second-order cone programming [17]. When t reaches its minimum, we get the optimal $\vec{w}_{sub}$.

SDP is a subfield of convex optimization concerned with the optimization of a linear objective function over the intersection of the cone of positive semidefinite matrices. The typical form of SDP is given by [18],

$$\begin{aligned} &\min\ \vec{c}^T\vec{x} \\ &s.t.\ \ A\vec{x} = b \\ &\quad F_0 + x_1F_1 + ... + x_pF_p \ge 0 \end{aligned} , \tag{17}$$

where $\vec{x} = [x_1, x_2, ..., x_p]^T$ is the vector to be optimized, and $F_0, F_1, ..., F_p$ are semidefinite matrices with the same order. The inequality sign $F\left(\vec{x}\right) \ge 0$ means that $F\left(\vec{x}\right)$ is positive semidefinite.

We can reformulate the nonlinear convex problem of equation (16) as the semidefinite programming of equation (18) in the variables $\vec{w}_{sub}$ and t [18],

$$\begin{aligned} &\min_{t,\vec{w}_{sub}} t \\ &s.t.:\left(T_d \cdot \vec{w}_{sub}\right)^H \cdot \vec{s}_0 = 0 \\ &\quad \vec{w}_{sub}^H J = 0 \\ &\quad \begin{bmatrix} I_{N\times N} & T_w\vec{w}_{sub} - \vec{w}_{ref} \\ \left(T_w\vec{w}_{sub} - \vec{w}_{ref}\right)^H & t \end{bmatrix} \ge 0. \end{aligned} \tag{18}$$

This semidefinite program has dimensions $m = L + 1$ and $n = N$. The number of iterations required to solve a semidefinite program grows with problem size as $O(\sqrt{n})$ and it requires $O(m^2 n)$ operations per iteration [18]. Several specialized tools are available to solve it such as SeDuMi [19], YALMIP [20], etc. YALMIP is used in this work. However, YALMIP called

SeDuMi as external solver in our simulation. It usually converges after 10~13 iterations.

IV. SIMULATION RESULT

Let us consider a uniform linear array of 102 antennas with half-wavelength spacing. Look direction is set as $\theta_0 = 0^{\circ}$. The subarray configuration is [22, 8, 5, 4, 4, 4, 4, 4, 4, 4, 4, 5, 8, and 22], so that equal noise levels in all subarray channels are achieved approximately. We compare three approaches, LSMI, CAPS and SDP in scenarios of interference free, mainbeam interference and sidelobe interferences. In all simulations, a Taylor tapering with constrained side lobe level (SLL) = -30 dB and $\bar{n} = 8$ is impinged on each element, and difference beam is formed at subarray level digitally. INR is set to 30 dB. In SDP, Bayliss tapering with SLL = -30 dB and $\bar{n} = 8$ is set as the reference weights $\vec{w}_{ref}$. Signal of interest is neglected since it is usually possible to form the interference covariance matrix with signal absent in radar applications [21].

Figure 4 shows the adaptive patterns in the absence of interference obtained by the three algorithms. Although LSMI and CAPS suppress SLL effectively in [11], they deteriorate when we form difference beam at subarray level instead of sum beam. Nevertheless, SLL is remarkably low with SDP. Null depth in look direction, expressed as N_0, and SLL of each pattern are given in Table I.

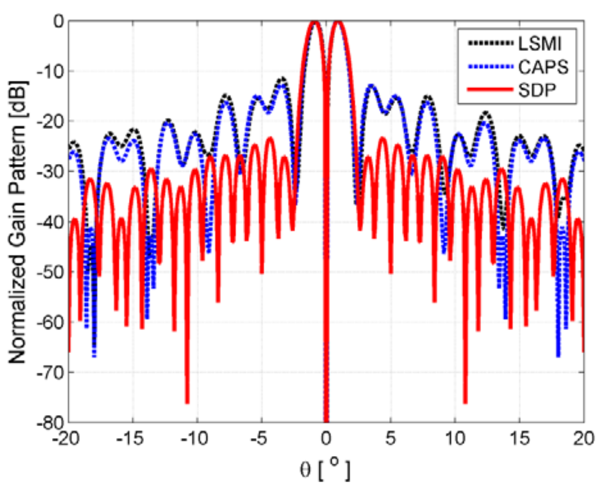

Fig. 4. Normalized pattern in absence of interference (noise only).

Figure 5 demonstrates adaptive patterns in the presence of one main lobe interference from 1.5°. SDP performs slightly better in sidelobe control, but it forms deep null exactly in look direction. Meanwhile, the other two have 0.1° deviation caused by disturbance of the main lobe interference.

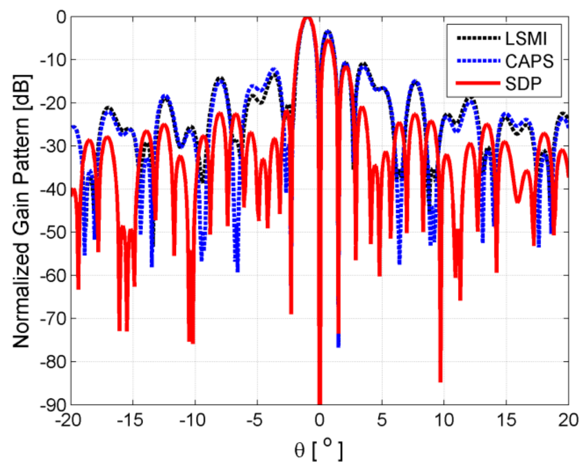

Fig. 5. Normalized pattern with one main lobe interference at 1.5°.

Figure 6 illustrates adaptive patterns in presence of two sidelobe interferences in direction of -5° and 10°. Three algorithms can suppress interference effectively, lower than -67 dB. However, compared with LSMI and CAPS, SDP reduces sidelobe dramatically and has a deeper null in look direction.

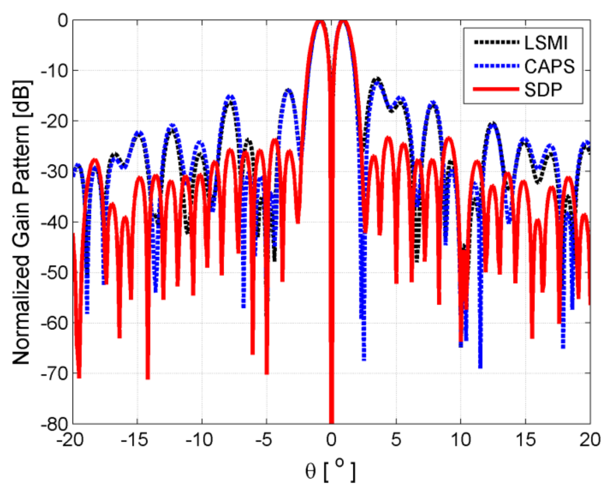

Fig. 6. Normalized pattern with two side lobe interferences at -5° and 10°.

From Table I, we can see that in situations of noise only and sidelobe interferences, SDP outperforms the other two algorithms in side lobe reduction and null depth in look direction manifestly. When the pattern is disturbed by main lobe interference, SDP can still maintain accurate deep null in look direction. This is due to the first equality constraint in equation (18).

Figure 7 and Table II depict the comparison of SINR of each approach versus θ. LSMI has the highest SINR. SDP has a small SINR loss, 0.68 dB compared with LSMI and 0.49 dB compared with CAPS, as shown in Table II. Thus, slight SINR loss is the cost of using SDP.

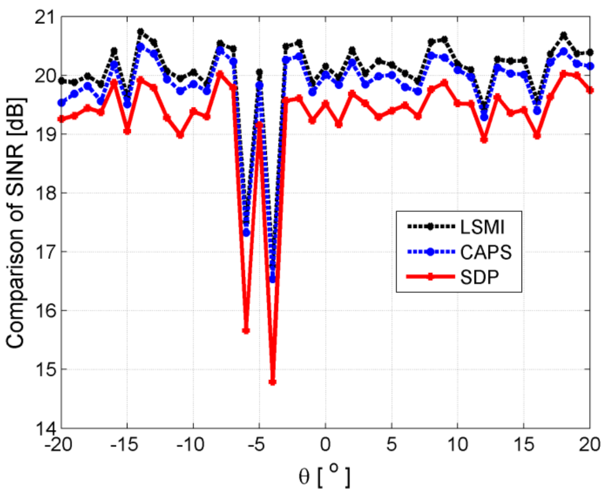

Fig. 7. SINR of three methods, interference at–5^0.

In our simulations, patterns using LSMI and CAPS differ slightly, which agrees with the conclusion in [6]. Although Taylor tapering is used at element level both in [6] and this paper, sum beam is formed at subarray level in [6] while difference beam is formed in this paper. In this case, SDP performs considerably well in suppressing sidelobe and producing accurate deep null in look direction.

As discussed in section III, the computational complexity is closely related to the size of the array and its subarrays. Thus, when we utilize the proposed algorithm to compute adaptive weights, the size of array and the amount of subarrays should be taken into consideration according to the practical requirement of real-time.

Table I: Null depth in θ_0 and SLL for LSMI, CAPS, and SDP [dB].

	absence of interference		**Mainlobe interference**		**sidelobe interference**	
	N_0	SLL	N_0	SLL	N_0	SLL
LSMI	-24	-11.1	-21*	-11.0	-24	-12.1
CAPS	-328	-12.8	-19*	-10.7	-44	-12.5
SDP	-134	-23.4	-135	-11.6	-135	-23.2

*: denoting 0.1^o deviation.

Table II: SINR and SINR Loss for LSMI, CAPS, and SDP [dB].

	SINR	**SINR Loss**
LSMI	20.17	+0.68
CAPS	19.98	+0.49
SDP	19.49	

V. CONCLUSIONS

In this paper, a semidefinite programming method is proposed to form adaptive difference beam at subarray level when element excitations are for optimum sum pattern. The proposed method realizes sidelobe reduction in adaptive difference beamforming via optimization. Compared with LSMI and CAPS, the proposed method has the merits of reducing sidelobe considerably and producing an accurate deep null in look direction. The aforementioned merits of the proposed method have been verified by computer simulations. Meanwhile, it suffers from a small SINR loss, which is the cost of SDP algorithm. Thus, a tradeoff should be considered in practical situations.

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