COMBINED-FIELD FORMULATION FOR CONDUCTING BODIES WITH THIN COATINGS*

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Abstract

A Galerkin method-of-moments (MM) formulation is developed for the problem of electromagnetic scattering from a conducting body with a thin coating. The formulation incorporates both the electric-field integral equation (EFIE) and magnetic-field integral equation (MFIE) formulations on the conductor and the dielectric surfaces. The formulation is developed in terms of generalized Galerkin matrix operators which allow for straightforward implementation into existing computer codes for coated bodies. The analysis allows a surface coating of nonuniform thickness that is characterized by complex permeability and permittivity; the analysis can be easily extended to the case of a thin multilayered body.

Standard MM formulations for coated bodies, based on either the EFIE or the MFIE at the conductor surface, fail as the coating thickness approaches zero. The combined-field integral equation (CFIE) also fails in the limit of zero thickness. The present thin-coating formulation (TCF) will be shown to remain valid as the coating thickness approaches zero. In the limit, the matrix equation for the TCF reduces to a self-consistent set of equations for scattering from a conducting body, independent of the dielectric coating parameters.

The TCF has been implemented for the case of scattering by a conducting body of revolution (BOR) with a thin dielectric layer. Examples are presented comparing the present formulation to other MM formulations. The TCF is demonstrated near internal resonant frequencies, and for some limiting cases for both bistatic and monostatic scattering.

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Introduction

The method-of-moments (MM) has been applied to the problem of electromagnetic (EM) scattering from conducting bodies by numerous investigators. In [1] the electric-field integral equation (EFIE) is solved for an arbitrary conducting body of revolution (BOR), and in [2] the magnetic- and combined-field integral equations (MFIE and CFIE) solutions are obtained. Analogous formulations were developed for coated conducting bodies by applying the PMCHW (after Poggio, Miller, Chu, Harrington, and Wu) formulation [3] at the coating surface and solving either the EFIE [4] or the CFIE [5] at the conducting surface. The formulations are valid for coatings which have nonuniform thickness and are characterized by complex permeability and permittivity. However, in the limit as the coating thickness approaches zero, both the EFIE and MFIE formulations fail. Numerical implementations of the formulations fail for small nonzero thicknesses due to the approximations inherent in the MM solution procedure.

In this paper a new formulation is presented for conducting bodies with thin coatings. The formulation incorporates both the EFIE and MFIE formulations at the conducting surface, and remains valid in the limit of zero thickness. Representative numerical results are presented to validate the present approach.

Formulation

Consider the coated conducting body in Fig. 1. The coating region R_2 is homogeneous with permittivity ϵ_2 and permeability μ_2. Starting with the symmetric form of Maxwell's equations, incorporating electric and magnetic charges and currents for a homogeneous region with electric and magnetic sources, the fields in each of the regions in Fig. 1 can be written as

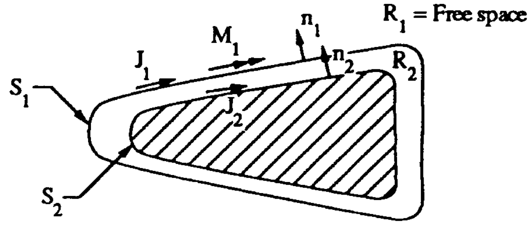

Fig. 1. Coated conducting body geometry.

follows. In region R_1, the total electric and magnetic fields $\vec{E}_1$ and $\vec{H}_1$ can be written as

$$\theta(\vec{r})\vec{E}_1 = \vec{E}^i(\vec{r}) - L_1\vec{J}_1(\vec{r}) + K_1\vec{M}_1(\vec{r}) \tag{1}$$

and

$$\theta(\vec{r})\vec{H}_1 = \vec{H}^i(\vec{r}) - K_1\vec{J}_1(\vec{r}) - \frac{1}{\eta_1^2} L_1\vec{M}_1(\vec{r}) \ , \tag{2}$$

where $\vec{E}^i$ and $\vec{H}^i$ are the incident electric and magnetic fields and the function $\theta(\vec{r})$ is defined as

$$\theta(\vec{r}) = \begin{cases} 1 & \text{for } \vec{r} \in R_1 \\ 1/2 & \text{for } \vec{r} \in \partial R_1 \\ 0 & \text{otherwise} \end{cases} \ , \tag{3}$$

where ∂R_1 is the boundary of region R_1. The electric and magnetic surface currents on the region boundaries in general are

$$\vec{J}_i = \hat{n}_i \times \vec{H}_i\Big|_{S_i}, \tag{4a}$$

$$\vec{M}_i = -\hat{n}_i \times \vec{E}_i\Big|_{S_i}, \tag{4b}$$

and

$$\eta_i = \eta_0 \sqrt{\frac{\mu_{ir}}{\epsilon_{ir}}} \, , \; \epsilon_i = \epsilon_0 \epsilon_{ir}, \; \mu_i = \mu_0 \mu_{ir}, \; \eta_0 = 377 \, \Omega \; .$$

Time variation of $e^{j\omega t}$ is implied and suppressed in this discussion. The integro-differential operators L_i and K_i are defined as

$$L_i \vec{X}(\vec{r}) = j\omega\mu \int_{\partial R_i} \left(\vec{X}(\vec{r}') + \frac{1}{\omega^2 \mu \epsilon} \nabla\nabla' \bullet \vec{X}(\vec{r}') \right) \Phi(\vec{r} - \vec{r}') \, ds' \tag{5}$$

and

$$K_i \vec{X}(\vec{r}) = \int_{\partial R_i} \vec{X}(\vec{r}') \times \nabla\Phi(\vec{r} - \vec{r}') \, ds' \quad , \tag{6}$$

where for $\vec{r} = \vec{r}'$, the operators are interpreted as Cauchy principal-value integrals.

The vector function $\vec{X}$ is in the domain of the operators L_i and K_i and is defined on ∂R_i. The Green's function is Φ, with Φ, ϵ, and μ defined in region R_i.

The fields in region R_2 can be expressed likewise as

$$\theta(\vec{r})\vec{E}_2 = L_2\vec{J}_1(\vec{r}) - K_2\vec{M}_1(\vec{r}) - L_2\vec{J}_2(\vec{r}) \tag{7}$$

and

$$\theta(\vec{r})\vec{H}_2 = K_2\vec{J}_1(\vec{r}) + \frac{1}{\eta_2^2} L_2\vec{M}_1(\vec{r}) - K_2\vec{J}_2(\vec{r}) \; . \tag{8}$$

Applying the boundary conditions at each interface S_i yields a set of coupled integral equations for the unknown electric and magnetic currents. At the dielectric-dielectric interface S_1, continuity of the tangential fields is enforced, i.e.,

$$\vec{E}_1\Big|_{\tan S_1} = \vec{E}_2\Big|_{\tan S_1} \tag{9}$$

and

$$\vec{H}_1\Big|_{\tan S_1} = \vec{H}_2\Big|_{\tan S_1} \quad . \tag{10}$$

Substituting (1), (2) and (7), (8) into the above boundary conditions yields two integral equations on surface S_1

$$\left[(L_1 + L_2)\vec{J}_1 - (K_1 + K_2)\vec{M}_1 - L_2\vec{J}_2\right]_{\tan S_1} = \vec{E}^i\Big|_{\tan S_1} \tag{11}$$

and

$$\left[(K_1 + K_2)\vec{J}_1 + (\frac{1}{\eta_1^2} L_1 + \frac{1}{\eta_2^2} L_2)\vec{M}_1 - K_2 \vec{J}_2\right]_{\tan S_1} = \vec{H}^i\Big|_{\tan S_1} \tag{12}$$

where, for convenience, the field point $\vec{r}$ has been omitted. Similarly on the dielectric-conducting interface S_2, the tangential electric field vanishes. Substituting the boundary condition into Eq. (7) and $\hat{n}$ cross Eq. (8) yields two integral equations on surface S_2:

$$\left[- L_2\vec{J}_1 + K_2\vec{M}_1 + L_2\vec{J}_2\right]_{\tan S_2} = 0 \tag{13}$$

and

$$\hat{n}_2 \times \left[- K_2\vec{J}_1 - \frac{1}{\eta_2^2} L_2\vec{M}_1 + K_2\vec{J}_2\right] + \frac{\vec{J}_2}{2} = 0 \quad . \tag{14}$$

If we apply the MM technique, the four integral equations (11-14) are transformed into a system of linear algebraic equations. Once the form of the current expansion and testing procedure is specified, the MM technique transforms the integral operators L and K to the corresponding matrix operators $\mathscr{L}$ and $\mathscr{K}$. The resulting MM matrix operators are defined as $\mathscr{L}(S,S';R)$ and $\mathscr{K}(S,S';R)$, where S specifies the surface where the testing is performed, S′ specifies the surface on which the current is defined, and R identifies the region where the Green's function is evaluated. Similarly, one can defined $_x\mathscr{L}$ and $_x\mathscr{K}$ as the matrix operators that result from testing the integral operators $\hat{n} \times L$ and $\hat{n} \times K$. The matrix operator $\tilde{\mathscr{K}}(S)$ results from testing the expansion functions directly. Similarly, $\mathscr{E}(S)$ and $\mathscr{H}(S)$ are the column vectors which result from testing the incident electric and magnetic fields, respectively. Using this notation [6], the integral equations (11-14) are transformed into the following matrix equation:

$$
\begin{bmatrix}
\mathscr{L}(S_1,S_1;R_1) + \mathscr{L}(S_1,S_1;R_2) & -\mathscr{K}(S_1,S_1;R_1) - \mathscr{K}(S_1,S_1;R_2) & -\mathscr{L}(S_1,S_2;R_2) \\
\mathscr{K}(S_1,S_1;R_1) + \mathscr{K}(S_1,S_1;R_2) & \frac{\epsilon_{1r}}{\mu_{1r}}\mathscr{L}(S_1,S_1;R_1) + \frac{\epsilon_{2r}}{\mu_{2r}}\mathscr{L}(S_1,S_1;R_2) & -\mathscr{K}(S_1,S_2;R_2) \\
-\mathscr{L}(S_2,S_1;R_2) & \mathscr{K}(S_2,S_1;R_2) & \mathscr{L}(S_2,S_2;R_2)
\end{bmatrix}
\begin{bmatrix} J_1 \\ M_1 \\ J_2 \end{bmatrix}
=
\begin{bmatrix} \mathscr{E}(S_1) \\ \mathscr{H}(S_1) \\ 0 \end{bmatrix}
\begin{matrix} (15) \\ (16) \\ (17) \end{matrix}
$$

$$\begin{bmatrix} -{}_x\mathcal{K}(S_2,S_1;R_2) & -\frac{\epsilon_{2r}}{\mu_{2r}}\,{}_x\mathcal{L}(S_2,S_1;R_2) & {}_x\mathcal{K}(S_2,S_2;R_2) + \tilde{\mathcal{K}}(S_2) \end{bmatrix} \qquad \begin{bmatrix} 0 \end{bmatrix} \tag{18}$$

When the coating thickness is zero (i.e., $S_1 = S_2$), the solution should reduce to the solution for a conducting body (i.e., $\vec{J}_1 = \vec{J}_2$ and $\vec{M}_1 = 0$). The electric current should be a solution of either the EFIE or MFIE:

$$L_1 \vec{J}_1 \Big|_{\tan S_1} = \vec{E}^i \Big|_{\tan S_1} \tag{19}$$

$$\hat{n}_1 \times K_1 \vec{J}_1 + \frac{\vec{J}_1}{2} = \hat{n}_1 \times \vec{H}^i \tag{20}$$

The two E-field Eqs. (11) and (13) are consistent with the above; however, the two H-field Eqs. (12) and (14) are not. Solving any three of the equations (11-14) for the three unknown currents will yield a solution which will fail as the coating thickness approaches zero. Combining Eqs. (13) and (14) on surface S_2 (i.e., the CFIE formulation in [5]) does not circumvent the problem.

A new, thin-coating formulation (TCF) which overcomes these failings is obtained by combining the two H-field matrix equations (16) and (18). First, Eq. (18) is written in a different form by applying the MM technique to Eq. (14) after taking the cross product with the unit normal vector, to yield

$$\mathcal{K}(S_2,S_1;R_2)J_1 + \frac{\epsilon_{2r}}{\mu_{2r}} \mathcal{L}(S_2,S_1;R_2)M_1 - \mathcal{K}(S_2,S_2;R_2)J_2 + {}_x\tilde{\mathcal{K}}(S_2)J_2 = 0 \tag{21}$$

This new equation is then subtracted from Eq. (16) to yield the system of equations shown below:

$$
\left[\begin{array}{c|c|c}
\begin{array}{l}\mathcal{L}(S_1,S_1;R_1)\\ +\\ \mathcal{L}(S_1,S_1;R_2)\end{array} &
\begin{array}{l}-\mathcal{K}(S_1,S_1;R_1)\\ \\ -\mathcal{K}(S_1,S_1;R_2)\end{array} &
-\mathcal{L}(S_1,S_2;R_2) \\
\hline
\begin{array}{l}\mathcal{K}(S_1,S_1;R_1)\\ +\\ \mathcal{K}(S_1,S_1;R_2)\\ -\\ \mathcal{K}(S_2,S_1;R_2)\end{array} &
\begin{array}{l}\frac{\epsilon_{1r}}{\mu_{1r}}\mathcal{L}(S_1,S_1;R_1)\\ +\\ \frac{\epsilon_{2r}}{\mu_{2r}}\mathcal{L}(S_1,S_1;R_2)\\ -\\ \frac{\epsilon_{2r}}{\mu_{2r}}\mathcal{L}(S_2,S_1;R_2)\end{array} &
\begin{array}{l}-\mathcal{K}(S_1,S_2;R_2)\\ +\mathcal{K}(S_2,S_2;R_2)\\ -\tilde{\mathcal{K}}_x(S_2)\end{array} \\
\hline
-\mathcal{L}(S_2,S_1;R_2) & \mathcal{K}(S_2,S_1;R_2) & \mathcal{L}(S_2,S_2;R_2)
\end{array}\right]
\left[\begin{array}{c} J_1 \\ \hline M_1 \\ \hline J_2 \end{array}\right]
=
\left[\begin{array}{c} \mathcal{E}(S_1) \\ \hline \mathcal{H}(S_1) \\ \hline 0 \end{array}\right]
\qquad
\begin{array}{r}(22)\\ (23)\\ (24)\end{array}
$$

Since the TCF, is obtained by combining the H-field equations at the matrix-equation level, the two surfaces S_1 and S_2 must be discretized with the same number of points. This restriction is not severe since the formulation is only intended for conducting bodies with thin coatings. In the limit as $S_1 \to S_2$, the TCF remains valid and is consistent with the solution for a conducting surface, independent of the dielectric properties of the coating.

Results

The present thin-coating formulation was implemented (using the Galerkin method of testing) in a computer code for conducting bodies of revolution with nonuniformly thick coatings. In Fig. 2, the computed backscatter cross-section of an air-coated sphere is compared with the Mie series solution. The EFIE, MFIE, and CFIE formulations all diverge from the Mie solution as the coating thickness decreases, while the present TCF solution remains valid. A similar calculation is shown in Fig. 3 for a dielectrically coated sphere. Again, the TCF solution remains valid and the standard formulations fail as the coating thickness decreases.

In Fig. 4, the EFIE and TCF backscatter calculations are shown for a conducting cylinder when the coating thickness is zero. These results are compared with the computed results for an uncoated cylinder. The EFIE results are grossly in error, while the TCF results nearly overlap those for the uncoated conducting cylinder.

In Fig. 5, the computed bistatic cross section for an air-coated sphere near an internal resonance is given. Using the EFIE formulation, the numerical resonance occurs at ka = 2.768 when the sphere is discretized with

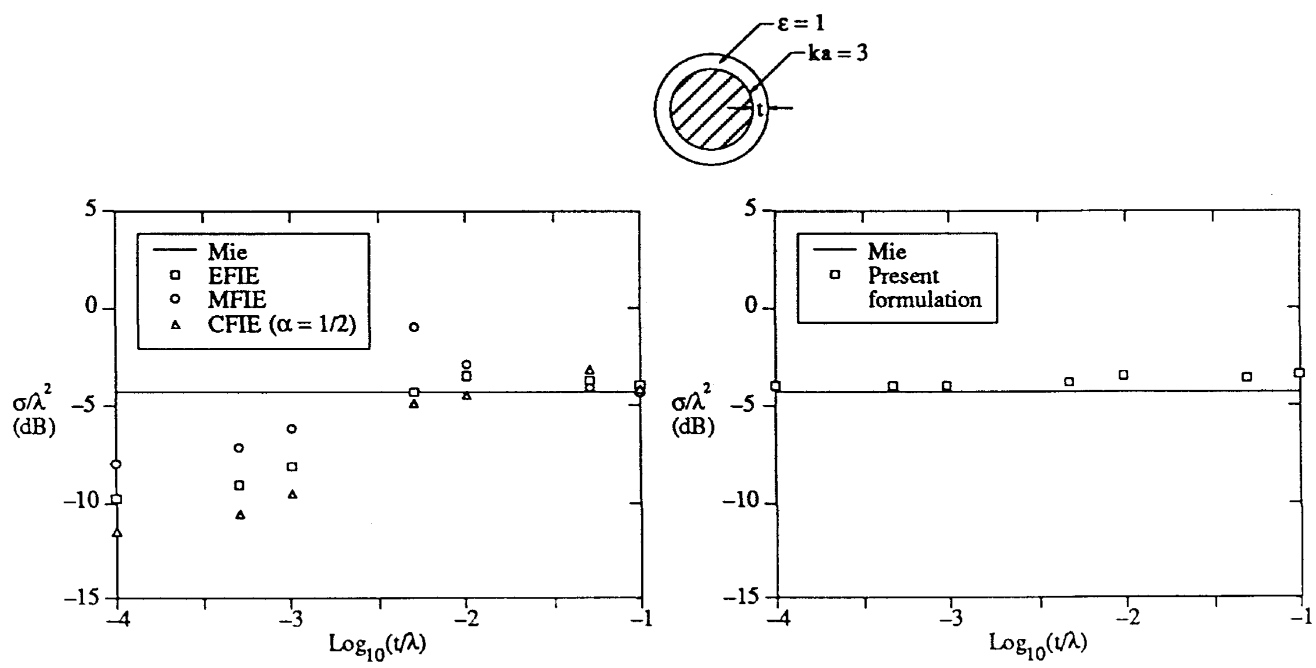

Fig. 2. Backscatter cross-section as a function of coating thickness (t) for an air-coated sphere.

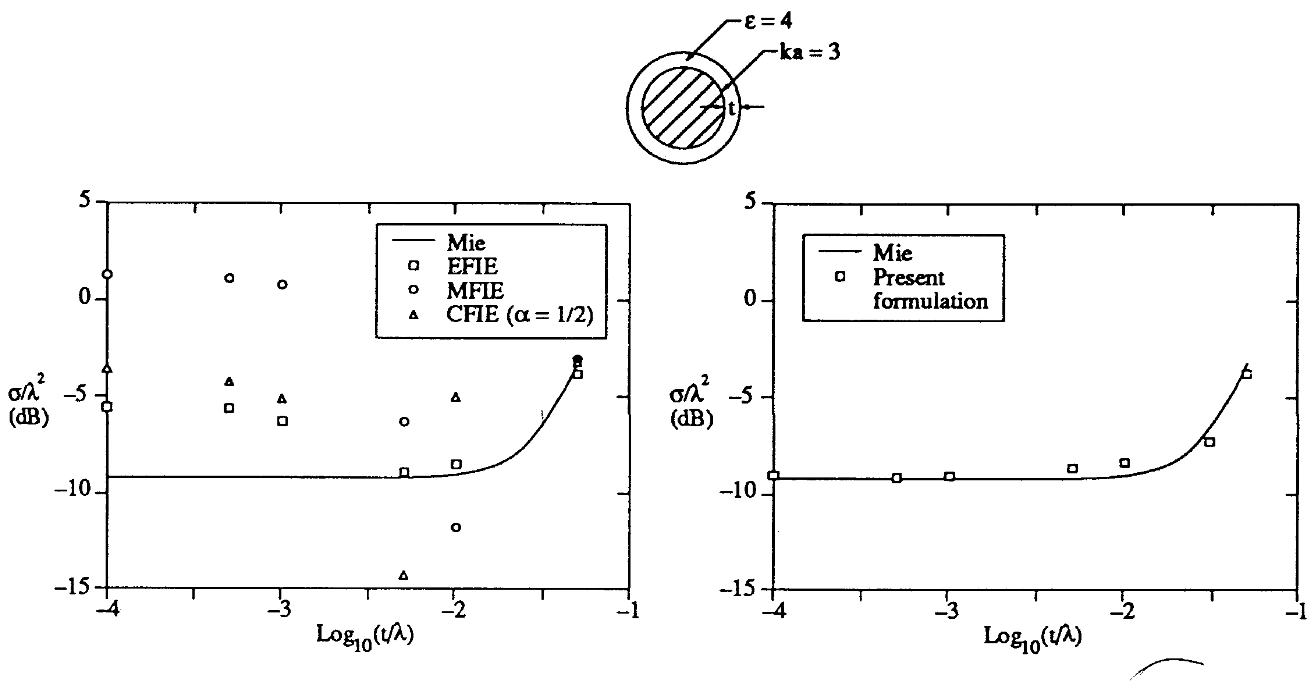

Fig. 3. Backscatter cross-section as a function of coating thickness (t) for a dielectrically coated sphere.

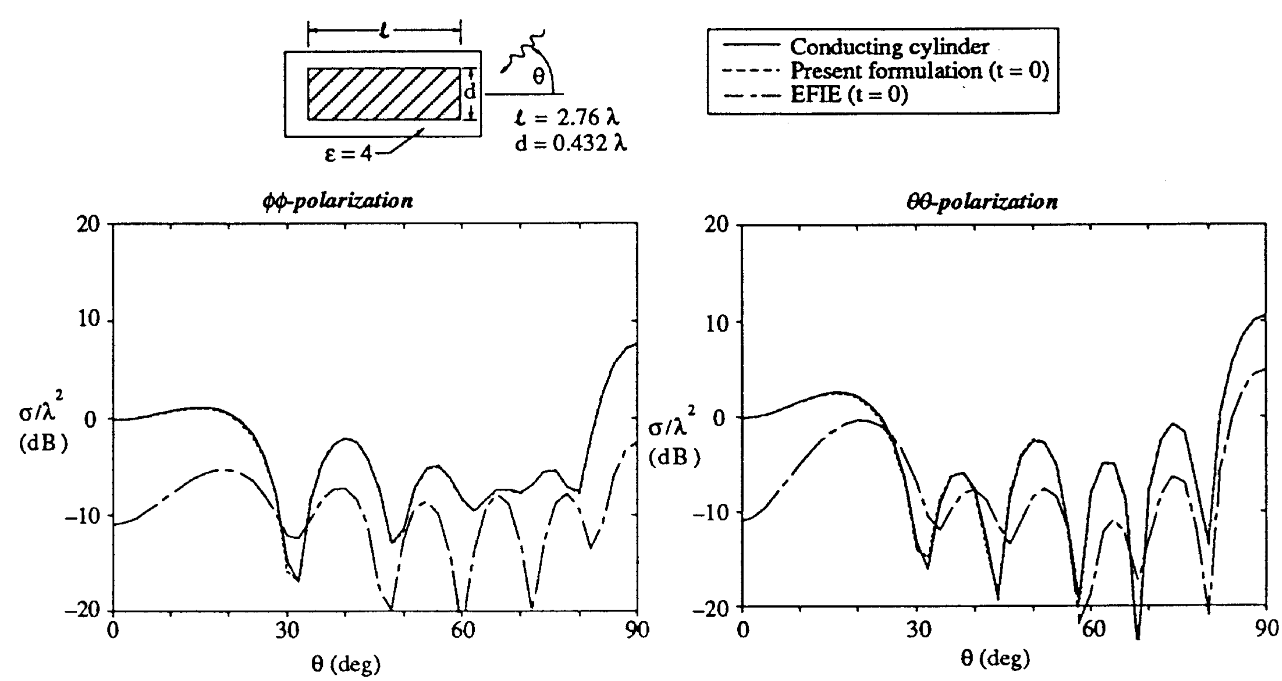

Fig. 4. Backscatter calculations for a coated cylinder when $S_1 = S_2$.

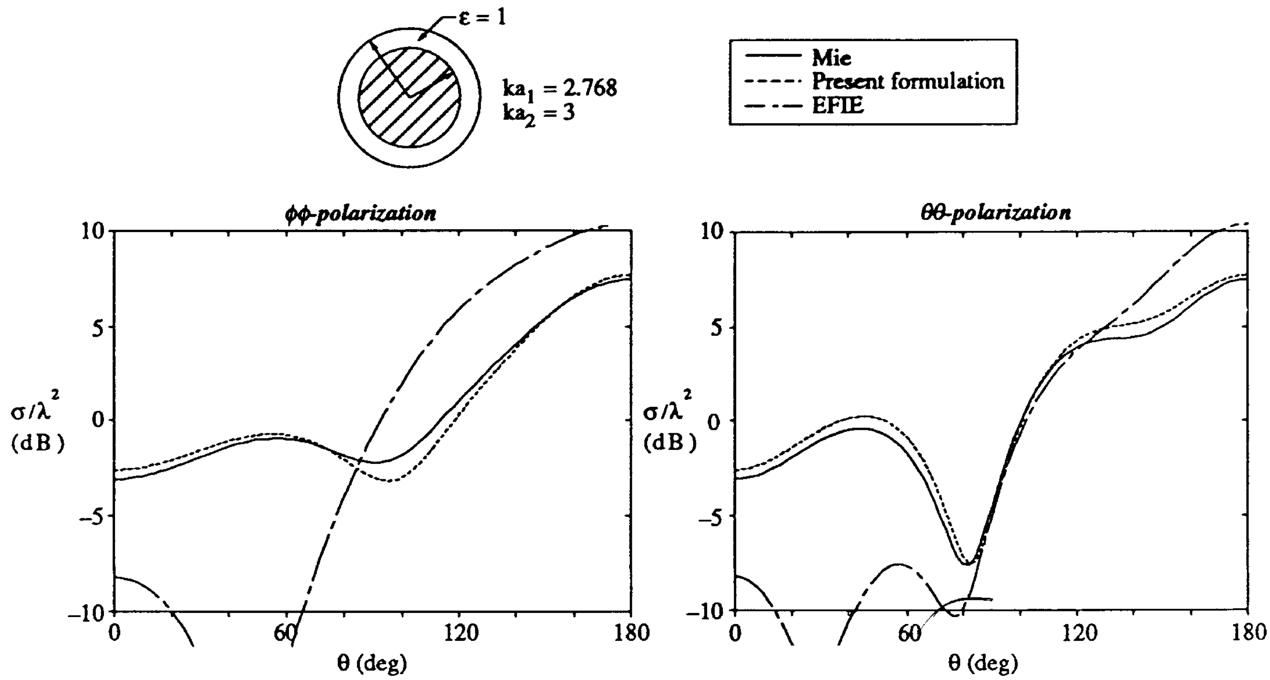

Fig. 5. Bistatic cross section for a resonant coated conducting sphere.

31 points (14 triangle functions). The EFIE solution is in error while the TCF solution is in good agreement with the Mie series solution. The TCF solution was also compared with the Mie series solution as a function of ka in the vicinity of the true resonance (ka=2.744) and remained stable.

Summary

A new thin-coating formulation was presented for scattering from coated conductors. The H-field equations on the conducting and dielectric surfaces are combined at the matrix equation stage of the MM technique. The formulation was shown to remain valid in the limit as the coating thickness goes to zero. Representative numerical examples have confirmed the formulation for conducting bodies of revolution with thin coatings and have shown that the formulation remains valid near resonant frequencies.

References

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