On the Convergence Properties of the Multiple Sweep Method of Moments

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Abstract–This paper investigates the convergence properties of the Multiple Sweep Method of Moments (MSMM), both analytically and numerically, and presents some numerical results for various 2D scattering geometries, such as a strip, a cylinder, and a rough surface with and without a target on it. The MSMM is an $\mathcal{O}(N^2)$ iterative method for solving the large matrix equations which arise in the method of moments (MM) analysis of electrically large bodies. In the MSMM, the body is split into P sections and the currents on these sections are found in a recursive fashion. Although the MSMM is a frequency domain solution, it has a time domain interpretation. The first sweep includes the dominant scattering mechanisms and each subsequent sweep includes higher order mechanisms. A connection between the MSMM and classical iterative methods is established in this paper. Under certain conditions, the MSMM is shown to be mathematically equivalent to a block Jacobi preconditioned system of equations that results from the moment method, and solved via the method of symmetric successive over-relaxation (SSOR) with relaxation factor $\omega = 1$. Based on this connection, the convergence is analyzed by examining the eigenvalue distribution of the iteration matrix for different classes of 2D geometries, and for electric and magnetic field integral equation formulations and TE_z and TM_z polarizations. In addition, the MSMM is compared with other recently used iterative methods for rough surface scattering problems, namely the Method of Ordered Multiple Interactions (MOMI), or the Forward-Backward (FB) Method. The results show that the MSMM converges for some problems for which the MOMI (and FB) fails to converge, e.g., the rough surface with a target on it, or when the surface becomes multi-valued which causes large off-diagonal elements in the interaction matrix.

Keywords–Iterative methods, Integral equations, Moment methods, Convergence, Numerical methods.

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I. INTRODUCTION

The frequency domain method of moments (MM) has been one of the most reliable and widely used numerical methods for the analysis of radiation and scattering from bodies of simple or complex shape [1]-[3]. In the MM, the scattered or radiated field is generated by a set of equivalent currents which replace the physical structure. The current on a body is expanded in terms of N expansion functions, and the N unknown coefficients in this expansion are obtained as the solution of an order N matrix equation. The standard MM is often limited by the $\mathcal{O}(N^3)$ CPU time required to solve the matrix equation by direct methods, such as LU decomposition. Over the past several years many methods have been developed to improve the computational efficiency of the MM, and thus allow it to be applied to electrically larger bodies [4]. Iterative solution of the MM matrix equation has received considerable attention since the CPU time is reduced to $\mathcal{O}(N^2)$ [5], [6].

A considerable effort has been directed toward the solution of scattering of electromagnetic waves from rough surfaces at low grazing angles since it has important applications in remote sensing of ocean and land profiles [7]. Monte Carlo simulations of rough surface scattering problems using direct numerical solutions have become popular with the growth of modern computers and the development of the fast methods. Iterative techniques developed for solving general systems of linear equations have been applied to systems resulting from electromagnetic rough-surface scattering problems [8]-[18]. One approach, called the Kirchhoff iterative method (also known as the Neumann expansion) [8]-[10], has been shown to be useful, but that the method may fail to converge for surfaces with large slopes or for large incidence angles [9].

Recently, a new iterative technique termed the Forward-Backward (FB) method has been proposed by Holliday *et al.* [14]. A functionally identical approach, the Method of Ordered Multiple Interactions (MOMI), has been developed by Kapp and Brown [15]. These approaches have been shown to be very

effective for the solution of the magnetic field integral equation (MFIE) for the scattering from perfectly conducting (PEC) surfaces that are single valued and rough in one dimension (two-dimensional scattering). The limitation of the MOMI (and the FB method) is that the convergence of the method is very much dependent on the sequential ordering of the current elements along a certain direction. Hence convergence may be a problem, especially for two-dimensional (2D) rough surfaces (three-dimensional scattering). Tran [16] applied MOMI to the scattering from a 2D rough surface and showed that the convergence of the iterative process depends strongly on the order in which the current elements are updated. With some orderings it did not converge, and it was found that in one case it did not converge even though the optimal ordering was used.

The MOMI is shown to be mathematically equivalent to a point Jacobi preconditioned system of equations that results from the moment method, and solved via symmetric successive over-relaxation (SSOR) with relaxation factor $\omega = 1$ (i.e., Forward Backward Gauss-Seidel method) [17]. West and Sturm [17] tested the performance of the method through application to a series of 1D surface profiles (2D scattering) that approximate breaking ocean waves, and 2D perfectly conducting circular cylinder problems. They found that the method diverges for some breaking wave profiles, as well as for the closed circular cylinder. Pino *et al.* [18] applied the FB method to scattering from targets on 1D ocean-like rough surfaces. They also came to the conclusion that the method does not exhibit a convergent behavior if there is a target on the rough surface, and developed a generalization of the FB method to handle this case.

Recently, a new technique termed the *Multiple Sweep Method of Moments (MSMM)* has been introduced for the analysis of the radiation and scattering from electrically large, perfectly conducting bodies [19]-[21]. The MSMM is an extension or modification of the Spatial Decomposition Technique (SDT) developed by Umashankar *et. al.* [22]. In both methods, the electrically large body is split into P sections containing approximately N/P unknowns per section. The currents on the P sections are found in a recursive fashion until they (hopefully) converge to the exact result. The main difference between the MSMM and the SDT is that the MSMM attempts to perform the recursion so that the first sweep accounts for the dominant mechanisms, while subsequent sweeps account for higher order mechanisms. Tapered resistive cards (R-cards) [20], [23], [24], [25] are used on the first sweep to minimize endpoint scattering from the junctions between sections. Subsequent sweeps are performed in the order in which currents would change with time so that the higher order sweeps correspond to higher order interactions.

In this paper, a connection between the MSMM and well-known classical iterative methods in mathematics is established so that the convergence properties of the MSMM can be investigated using matrix theory. Based on this connection, the convergence difficulties (such as for the closed cylinder) can be explained by the inherent limitations of the corresponding stationary iterative method. In addition, the MSMM can be compared with the recently used iterative methods for rough surface scattering problems (e.g., the MOMI, or FB Method) from both theoretical and numerical points of view.

The organization of this paper is as follows. Section II presents the description of the Multiple Sweep Method of Moments (MSMM) for the analysis of scattering from a 2D PEC strip. The formulation of the MSMM in matrix notation as a stationary iterative algorithm is derived in Section III. An *iteration matrix* is derived analytically, and based on that the convergence properties of the MSMM will be discussed. Section IV presents a numerical investigation of the convergence properties of the MSMM for the aforementioned various 2D scattering geometries (i.e., strip, cylinder, and rough surface with and without a target), by extracting and plotting the eigenvalue spectrum of the iteration matrix. The effect of the initial guess vector on the convergence of the method will be investigated numerically by using zero current, the physical optics current, and the MSMM first sweep current (which uses the R-cards), as starting solutions for the current vector. Finally, discussions and conclusions are included in Section V.

II. THE MSMM PROCEDURE

The basics of the original MSMM algorithm will be illustrated briefly on the MSMM solution for the current on a 2D PEC strip. Most of the material presented in this section is drawn from [20] and [21]. Figure 1 shows the MSMM procedure for the problem of TE_z scattering from a PEC strip. For the purposes of the standard MM solution, the strip is split into N segments of width $d = L/N$, and the $\hat{\mathbf{x}}$ directed current is expanded as

$$J = \sum_{n=1}^{N} i_n J_n \tag{1}$$

where the J_n are the known subsectional expansion functions and the i_n are the unknown coefficients. Hence,the standard MM solution can be written as the matrix equation

$$[Z]I = V^i \tag{2}$$

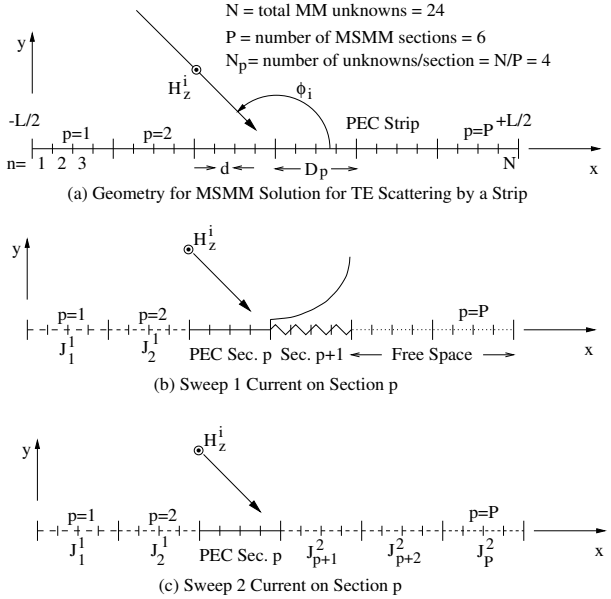


Fig. 1. For the MSMM solution a strip of width L is split into P sections with $N_P = N/P$ unknowns per section.

where [Z] is the $N \times N$ impedance matrix (also called "the interaction matrix"), V^i is the length N voltage vector due to the incident field, and $I = [i_1, i_2, \ldots, i_N]^T$ is the current vector. For the MSMM purposes, the strip is split into P equal sections of $N_P = N/P$ expansion functions per section, and of width D_P. Assuming the incident field propagates from left to right in Fig. 1, the first sweep ($k = 1$) begins by computing the current $J^{k=1}_{p=1}$ on section $p = 1$ excited by the incident field. At this point the remaining sections have no current, thus creating a non-physical truncation of the current at the end of section $p = 1$. To minimize that, the right-hand edge of the section $p = 1$ is terminated in an exponentially tapered R-card, as illustrated in Fig. 1(b). Once the current on section 1 has been determined, the next step is to compute the current $J^{k=1}_{p=2}$ on section $p = 2$ caused by the incident field plus the previously computed current $J^{k=1}_{p=1}$ on section 1. The process is continued for $p = 3, ..., P$ to complete Sweep 1.

Consider the computation of the first sweep current, $J^{k=1}_p$, on an arbitrary section p. As illustrated in Fig. 1(b), sections 1 to $p-1$ contain previously computed currents, section p is a PEC, the next section is tapered R-card, and the remaining sections are free space. Note that for the left hand edge of section p, the previously computed currents on section $p-1$ (approximately) enforce continuity of current at the junction, and thus from an electromagnetic viewpoint there is no edge. The currents on the PEC section p and the R-card section $p+1$ are produced by the superposition of the incident field plus the previously computed currents on sections 1 to $p-1$.

The second sweep is done in reverse order, since this is the natural order in which the currents would change with time. That is, section P is modified first, then section $P-1$, ..., and finally section 1. Hence, the second sweep will include reflections of the wave at the trailing edge. Figure 1(c) illustrates the computation of $J^{k=2}_p$, the Sweep $k = 2$ current on arbitrary section p. In computing the Sweep 2 current on section p, section p is a PEC, while all other sections are represented by their most recently computed currents. At this point in the Sweep 2 computation, sections $p+1$ to P have already been updated to the Sweep 2 currents, while sections 1 to $p-1$ still have the Sweep 1 currents. No R-cards are needed for this and subsequent sweeps because J^1_{p-1} and J^2_{p+1} approximately enforce continuity of current at the left and right edges of section p, respectively. The current on section p is the superposition of that due to the incident field plus the scattered component due to the previously computed currents.

All Sweeps $k > 2$ are identical to Sweep 2, except that odd numbered sweeps proceed from left to right while even numbered sweeps proceed from right to left as the MSMM solution attempts to model multiple interactions across the structure.

III. THE MSMM AS A STATIONARY ITERATIVE METHOD

The MSMM solution procedure has been described in Section II. The formulation of the MSMM in a matrix notation as a classical (or stationary) iterative algorithm is derived in this section. The term *stationary* indicates that the iterative equations do not change as the iterative algorithm is repeated [26], [27].

It is of interest to solve the following matrix equation:

$$\begin{bmatrix} [Z_{11}] & [Z_{12}] & \cdots & [Z_{1P}] \\ [Z_{21}] & [Z_{22}] & \cdots & [Z_{2P}] \\ \vdots & \vdots & \vdots & \vdots \\ [Z_{P1}] & [Z_{P2}] & \cdots & [Z_{PP}] \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_P \end{bmatrix} = \begin{bmatrix} V^i_1 \\ V^i_2 \\ \vdots \\ V^i_P \end{bmatrix} \tag{3}$$

where $[Z_{pq}]$ is the $N_P \times N_P$ block containing the mutual impedances between expansion functions in sections p and q, V^i_p contains the N_P elements of the incident voltage vector V^i for section p, and I_p contains the N_P elements of the solution vector I for section p.

Here we will assume that the first forward sweep of the MSMM, which uses R-cards to isolate sections to obtain the dominant scattering mechanisms, is done to obtain an initial guess and is not part of the iterative method. The second sweep (the backward

sweep) of the MSMM can be written in terms of matrix notation as follows. As seen in Fig. 1(c), the second sweep current on section p is due to the superposition of the incident field and the scattered field due to the previously computed currents, i.e., J_1^1 through J_{p-1}^1 and J_{p+1}^2 through J_P^2, where J_p^k denotes the k^{th} sweep current on section p. Hence, the MSMM uses the most recently updated values of the current on each section as they become available. The current on section p is found as the solution of

$$[Z_{pp}]I_p = V_p^i + V_p^s \tag{4}$$

where V_p^s is the scattered field vector on section p due to the previously computed currents on all other sections, and is given by

$$\left[\ V_p^s\ \right] = -\left[\ [Z_{p1}] \quad [Z_{p2}] \quad \cdots \quad [Z_{pP}]\ \right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_P \end{bmatrix}. \tag{5}$$

In equation (5), I_1 through I_{p-1} includes currents from the previous sweep (the first sweep of the MSMM, or initial guess of the matrix iterative algorithm), $I_p = 0$, and I_{p+1} through I_P includes the currents from the present sweep. From equations (4) and (5), the second sweep can be written as

$$[Z_{pp}]\, I_p^{(2)} = V_p^i - \sum_{j=1}^{p-1} [Z_{pj}]\, I_j^{(1)} - \sum_{j=p+1}^{P} [Z_{pj}]\, I_j^{(2)} \tag{6}$$

where p goes from P to 1, i.e., a backward sweep. Similarly, the third sweep (forward sweep) can be written as

$$[Z_{pp}]\, I_p^{(3)} = V_p^i - \sum_{j=1}^{p-1} [Z_{pj}]\, I_j^{(3)} - \sum_{j=p+1}^{P} [Z_{pj}]\, I_j^{(2)} \tag{7}$$

where in this case p goes from 1 to P.

The remaining sweeps of the MSMM are identical to Sweep 2 except that odd numbered sweeps proceed from left to right while even numbered sweeps proceed from right to left, as the MSMM solution attempts to model the multiple scattering interactions. Hence, the generalization for an arbitrary Sweep $k = 2n$ or $k = 2n + 1$ is

$$[Z_{pp}]\, I_p^{(2n)} = V_p^i - \sum_{j=1}^{p-1} [Z_{pj}]\, I_j^{(2n-1)} - \sum_{j=p+1}^{P} [Z_{pj}]\, I_j^{(2n)}, \text{ for } p = P, ..., 1,$$

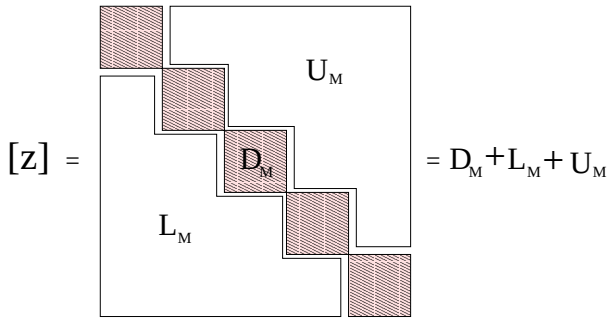


Fig. 2. Decomposition of $[Z]$ into a sum of block matrices.

$$[Z_{pp}]\, I_p^{(2n+1)} = V_p^i - \sum_{j=1}^{p-1} [Z_{pj}]\, I_j^{(2n+1)} - \sum_{j=p+1}^{P} [Z_{pj}]\, I_j^{(2n)}, \text{ for } p = 1, ..., P \tag{8}$$

The operational count of the MSMM is $\mathcal{O}(N^2)$ per iteration, assuming the $[Z_{pp}]$ matrices have been factorized before starting the iteration [19].

Now decompose $[Z]$ in terms of a sum of block matrices

$$[Z] = D_M + U_M + L_M \tag{9}$$

where D_M is a block diagonal matrix, and U_M and L_M are strictly upper and lower triangular block matrices of $[Z]$, respectively, as shown in Fig. 2. Hence, equation (8) can be written in terms of block matrices as,

Backward sweep:

$$D_M I^{(2n)} = V^i - U_M I^{(2n)} - L_M I^{(2n-1)},$$

Forward sweep:

$$D_M I^{(2n+1)} = V^i - L_M I^{(2n+1)} - U_M I^{(2n)}. \tag{10}$$

The form in equation (10) is equivalent to Forward-Backward block Gauss-Seidel (FB-GS, or equivalently, block SSOR with a relaxation factor $\omega = 1$) by considering two half iterations [5], [28]. This iterative algorithm is also equivalent to solving a block Jacobi preconditioned system of equations via forward-backward block GS iteration [29].

Equation (10) can be written in a similar format as that of the FB-GS in [5], [28]

$$\begin{aligned} (D_M + U_M) I^{(k+\frac{1}{2})} &= V^i - L_M I^{(k)} \\ (D_M + L_M) I^{(k+1)} &= V^i - U_M I^{(k+\frac{1}{2})}, \end{aligned} \tag{11}$$

where $I^{(k+\frac{1}{2})}$ is an intermediate solution of the iterate. The *iteration matrix* G [26] of a general stationary iterative system is defined from,

$$I^{(k+1)} \quad = \quad G\, I^{(k)} + V \tag{12}$$

where V is a constant vector. Hence, the iteration matrix of the MSMM can be found as

$$G_M = (D_M + L_M)^{-1} U_M (D_M + U_M)^{-1} L_M. \tag{13}$$

As shown in the numerical results of the next section, the spectral radius of this matrix determines the convergence rate.

One should note that as the MSMM section size reduces to the size of one cell (i.e., the size of a single MM expansion function), then $D_M \rightarrow D$, and the block FB-GS (or block SSOR with $\omega = 1$) becomes point FB-GS (or point SSOR with $\omega = 1$). West *et. al.* [17] showed that MOMI [15] (and hence Forward-Backward method [14]) is mathematically equivalent to a point Jacobi preconditioned system solved via point SSOR with $\omega = 1$, and zero initial guess vector. We showed above that the MSMM is mathematically equivalent to a block Jacobi preconditioned system solved via block SSOR with $\omega = 1$ (or equivalently block FB-GS), and the MSMM first sweep (which uses R-cards) as an initial guess vector. Therefore, we draw the conclusion that the MOMI is equivalent to the MSMM with MSMM section size reduced to one basis element (i.e., block $\rightarrow$ point) and with zero initial guess vector. It is also noted that the MSMM is a more general form of the generalized forward-backward method wherein block sections are only used to encompass the obstacles on the rough surface [18].

IV. NUMERICAL RESULTS

The spectral radius of the iteration matrix $\rho(G)$, which is the magnitude of the largest eigenvalue of G, controls the convergence of the iterative method. The iterative method is convergent if and only if $\rho(G) < 1$ [26]. Smaller values of $\rho(G)$ give higher convergence rates. This section presents a numerical investigation of the convergence properties of the MSMM for various 2D scattering geometries by extracting and plotting the eigenvalue spectrum of the iteration matrix G_M, which was derived in Section III. Examples will include a strip, a closed and open cylinder, and a rough surface with and without a target on it, as shown in Fig. 3. The effect of the initial guess vector on the convergence of the method is also investigated numerically by starting with zero current, the physical optics (PO) current, and the MSMM first sweep current which uses R-cards to isolate the sections. The electric and magnetic field integral equation formulations (EFIE and MFIE) and both polarizations (TE_z or TM_z) of the incident field are considered. The results are compared with the MOMI (or FB Method) which corresponds to the case when the MSMM section size (D_P) equals the cell size. In our results, we use a pulse-basis function cell size (MM segment size) of $\lambda/10$ and a frequency of 300 MHz.

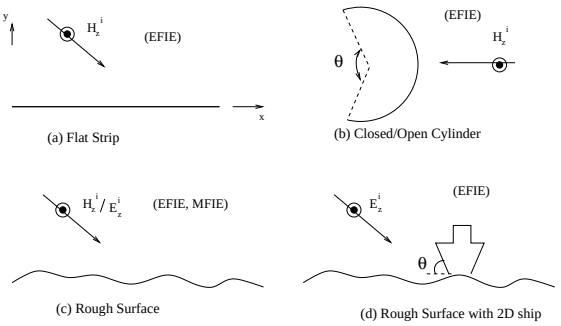


Fig. 3. The 2D problem geometries that are investigated for the convergence analysis.

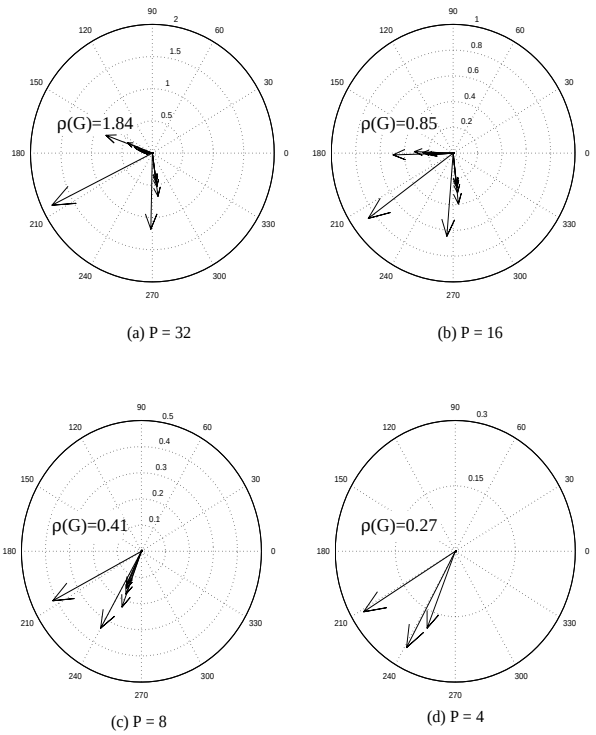
The convergence of the iterative method is monitored by examining the residual error norm

$$R_k = \frac{\|r^{(k)}\|}{\|V^i\|} \tag{14}$$

where $r^{(k)} = V^i - ZI^{(k)}$ is the residual error vector at the k^{th} iteration. R_k is a direct measure of how well the matrix equation is satisfied by the k^{th} solution vector. This is important for integral equation problems because it is a measure of how well the boundary conditions are satisfied for a given set of testing functions. It is noted that the currents are not guaranteed to converge to the exact value [30]; however, for RCS problems one is more interested in the fields radiated by the currents and the enforcement of field boundary conditions.

A. Flat Strip

Figure 4 shows the eigenvalue spectrum of G_M for a 12.8λ PEC strip with various MSMM section sizes (denoted by D_P). The strip is illuminated by a TE_z polarized plane wave with an incidence angle of $\phi_i = 135°$ with respect to the x axis. As seen from the figure, as the section size becomes larger (i.e., the number of sections P of the MSMM gets smaller), the spectral radius of G_M gets smaller and the convergence is expected to be faster. It is also noticed that for $P \geq 32$ (or $D_P \leq 0.4\lambda$), the method diverges, i.e., $\rho(G_M) > 1$ as seen in Fig. 4(a). This is consistent with the result that was reported before in [19]. Hence, the EFIE formulation of TE_z polarized scattering from a strip is divergent if the section size



(a) P = 32 (b) P = 16

(c) P = 8 (d) P = 4

Fig. 4. Eigenvalue spectrum of G_M for various MSMM section sizes for a strip. (a) $D_P = 0.4\lambda$, (b) $D_P = 0.8\lambda$, (c) $D_P = 1.6\lambda$, and (d) $D_P = 3.2\lambda$.

in the MSMM is reduced without limit. This implies that the MOMI (or Forward-Backward method) solution (which corresponds to MSMM with $P = 128$, or $D_P = 0.1\lambda$) also diverges for this case.

The convergence in terms of the residual errors for various initial guess vectors is shown in Figs. 5 and 6 for section sizes of $D_P = 1.6\lambda$ and $D_P = 0.4\lambda$, respectively. As seen from the figures, using the MSMM first sweep current (which uses R-cards) as an initial guess vector for the iterative method results in the lowest error, and using the PO current as an initial guess gives slightly lower error than the zero initial guess vector. The MSMM first sweep has a physical interpretation, i.e., it includes the dominant interactions on the scattering geometry. Hence, it exhibits the lowest error for a given number of iterations compared with PO and zero initial guess vectors. As predicted by the eigenvalue spectrum of G_M for $D_P = 0.4\lambda$ in Fig. 4(a), the method is expected to diverge for any initial start vector, as verified in Fig. 6. As seen in both Figs. 5 and 6, the rate of convergence (or divergence) is relatively independent of the start vectors. So in some cases it may be more efficient to use PO as the start vector and perform a few more iterations than to use the more computationally complex R-card treatment.

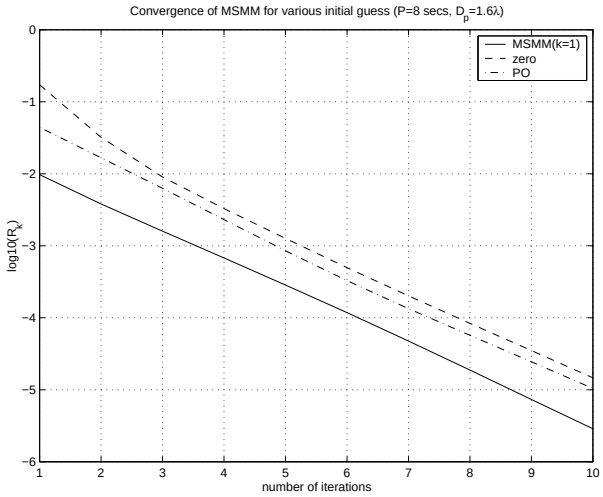


Fig. 5. Convergence of MSMM for various initial guesses for a strip with $P = 8$ ($D_P = 1.6\lambda$).

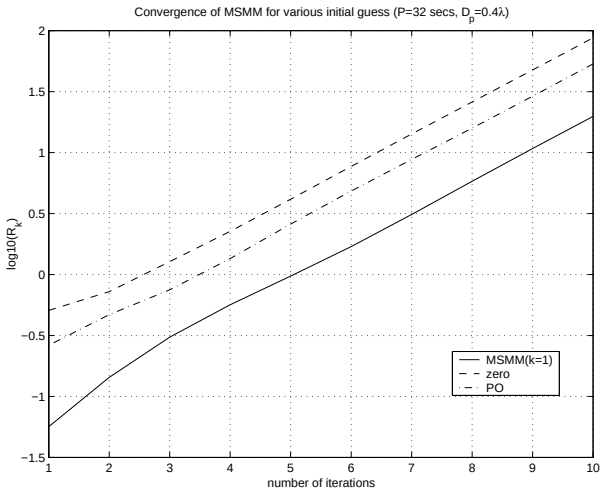


Fig. 6. Convergence of MSMM for various initial guesses for a strip with $P = 32$ ($D_P = 0.4\lambda$).

B. Circular Cylinder

In this section the effect of having various aperture sizes in the circular cylinder of Fig. 3(b) on the eigenvalue spectrum of the iteration matrix G_M is investigated. The cylinder is broken up into MSMM sections starting at the lower lip of the aperture and proceeding counter-clockwise around the cylinder boundary. Figure 7 shows the eigenvalue spectrum of G_M for two MSMM section sizes for a closed cylinder (no aperture) of radius $12\lambda/\pi$. In this case the method diverges for any choice of section size, even with only 2 sections. The reason may be partially due to the EFIE (or MFIE) formulation for closed surfaces which can result in a poorly conditioned or even singular system matrix $[Z]$, due to internal resonances. A combined field integral equation

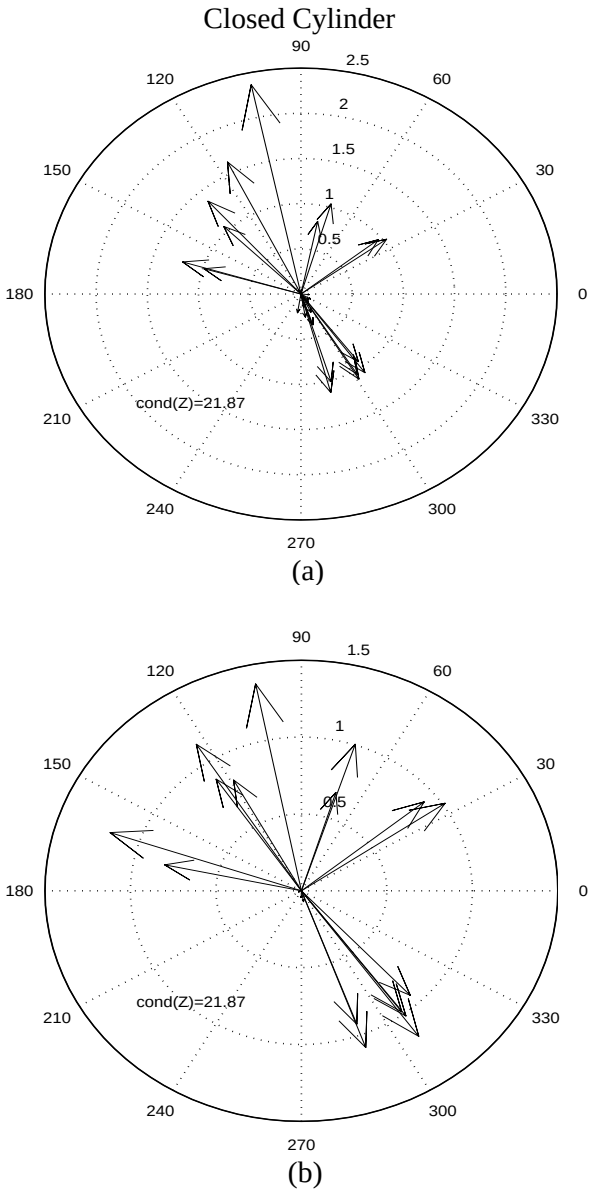


Fig. 7. Eigenvalue spectrum of G_M for two different numbers of MSMM sections for a closed cylinder. (a) $P = 12$, (b) $P = 2$.

(CFIE) formulation would be one way to avoid the internal resonance effects, and hence result in a better conditioned matrix [31]. A summary of various other approaches for overcoming the effect of internal resonances can be found in [32], [33].

The eigenvalue spectrum of G_M for the cylinder with an aperture opening of $\theta = 5°$, using $P = 2$ MSMM sections is shown in Fig. 8(a). The method still exhibits large eigenvalues, hence, the iterations show divergent behavior (i.e. $\rho(G_M) > 1$). Figures 8(b) and (c) show the eigenvalue spectrum of G_M for the aperture sizes of $\theta = 30°$ and $\theta = 60°$, respectively, with $P = 8$ MSMM sections. As the

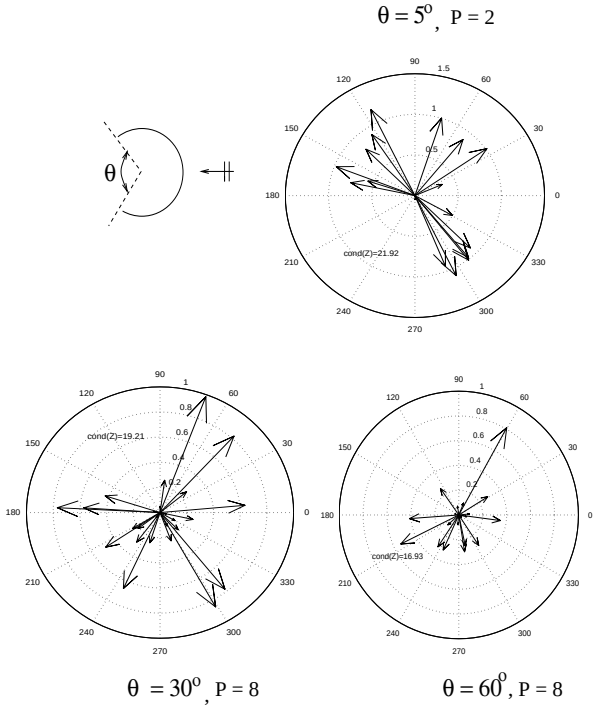


Fig. 8. Eigenvalue spectrum of G_M for open cylinder with (a) $\theta = 5°$, $P = 2$, (b) $\theta = 30°$, $P = 8$, (c) $\theta = 60°$, $P = 8$.

aperture size increases, the rate of convergence of the MSMM also increases, i.e., $\rho(G_M)$ gets smaller. This is due to the fact that increasing the aperture size reduces the interior resonance effects, or "ringing."

C. Rough Surface

The rough sea surface geometry sketched in Fig. 3(c) is a randomly generated realization of a Gaussian random process with a Pierson-Moskowitz ocean spectrum for a given wind speed [34]. The length of the surface is 12.8 m. We consider the following cases in this section:

Case 1: EFIE formulation for TE_z scattering,
Case 2: EFIE formulation for TM_z scattering,
Case 3: MFIE formulation for TE_z scattering.

For Case 1 the wind speed is 10 m/s which gives rise to an RMS surface roughness of 0.54 m. The results are shown in Fig. 9 for various MSMM section sizes. As the number of MSMM sections increases, the convergence rate of the MSMM decreases. Also, notice that if the number of sections is 32 or larger, the method diverges; therefore, MOMI cannot produce a convergent result for this case which would correspond to $P = N = 128$ sections.

A wind speed of 15 m/s (RMS roughness 1.106 m) is used for Cases 2 and 3. The results for Case 2 are shown in Fig. 10. It is seen that the method converges even if the section size is reduced to the cell

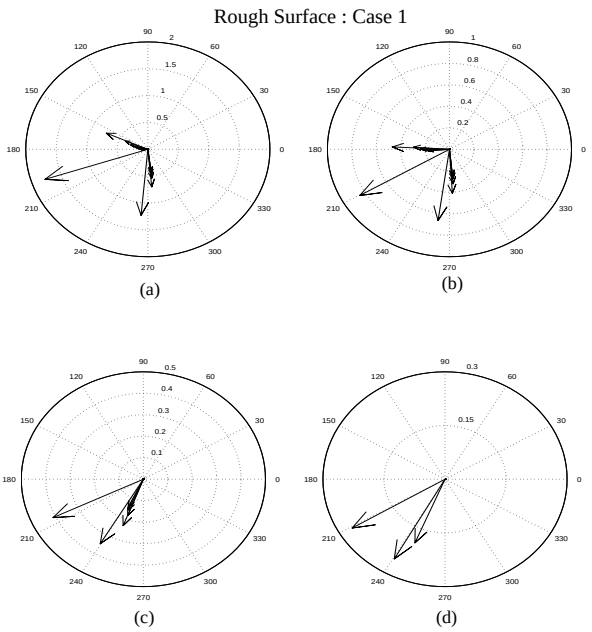


Fig. 9. Eigenvalue spectrum of G_M for the rough surface with 10 m/s wind speed. Case 1: EFIE for TE_z polarization. (a) $P = 32$, $D_P = 0.4\lambda$, (b) $P = 16$, $D_P = 0.8\lambda$, (c) $P = 8$, $D_P = 1.6\lambda$, (d) $P = 4$, $D_P = 3.2\lambda$.

size, i.e. $D_P = 0.1\lambda$, so the MOMI is also expected to converge for this case. It is interesting to note that the eigenvalues are closely grouped, suggesting that a more optimum iterative algorithm could be constructed [26].

The results for Case 3, which makes use of the MFIE formulation, is shown in Fig. 11. The spectral radius is very small compared with the EFIE formulation, so the convergence rate is greatly increased by using the MFIE. Also note that the method converges for $D_P = 0.1\lambda$, i.e., MOMI (or FB) also converges for this case. Hence, the MFIE formulation is well suited for these types of iterative methods. However, one should be aware that the MFIE formulation is technically valid only for closed surfaces, although it is often used for finite rough surface scattering problems.

D. Rough Surface with a Ship−Like Target

The problem geometry considered in this section is sketched in Fig. 3(d), and shown in Fig. 12. The EFIE formulation for TM_z scattering is used to produce the $[Z]$ matrix. The MSMM sections are assumed to have about the same number of modes. No attempt has been made to make the sections of the same length, and have varying numbers of modes on the sections. Therefore, the length of the section which has the target will be significantly smaller than

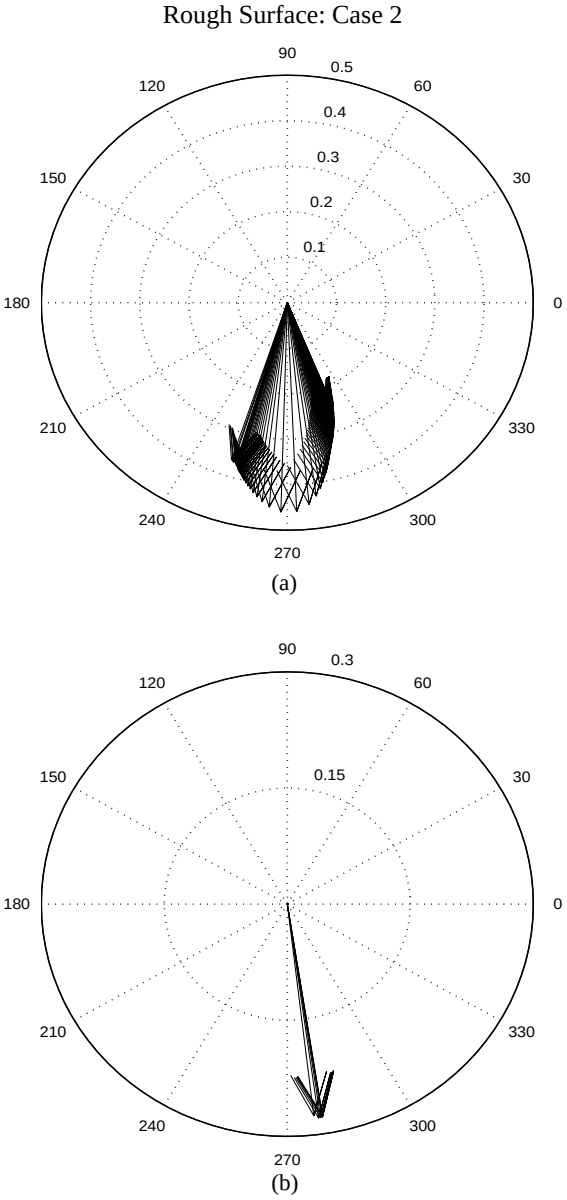


Fig. 10. Eigenvalue spectrum of G_M for the rough surface with 15 m/s wind speed. Case 2: EFIE for TM_z polarization. (a) $P = N = 128$, $D_P = 0.1\lambda$, (b) $P = 8$, $D_P = 1.6\lambda$.

the other sections which do not have the target. However, the three dimensional results that have been presented in [20] and [21] may have different numbers of modes on each section.

Figure 12 shows the eigenvalue spectrum for the MOMI (or FB method), i.e., the MSMM section size is $D_P = 0.1\lambda$. There are two large eigenvalues that cause the algorithm to diverge. However, as the section size is increased, the MSMM converges quite rapidly as seen in Fig. 13. In the case of Fig. 13(a) the ship-like geometry is part in one section, part in another, and the method still produces convergent

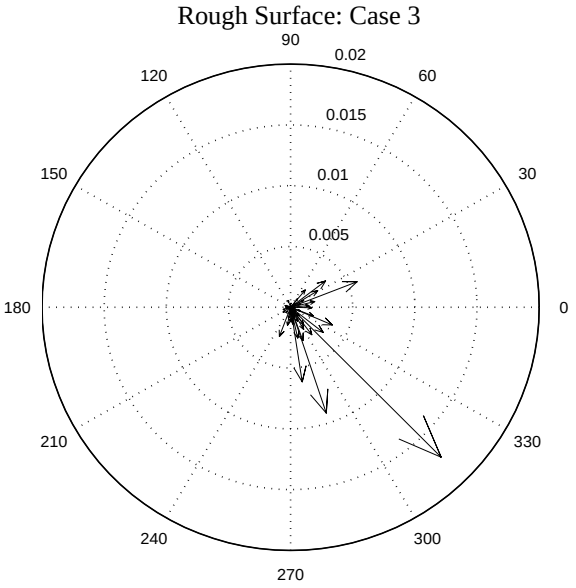


Fig. 11. Eigenvalue spectrum of G_M for the rough surface with 15 m/s wind speed. Case 3: MFIE for TE_z polarization. (a) $P = N = 128$, $D_P = 0.1\lambda$, (b) $P = 8$, $D_P = 1.6\lambda$.

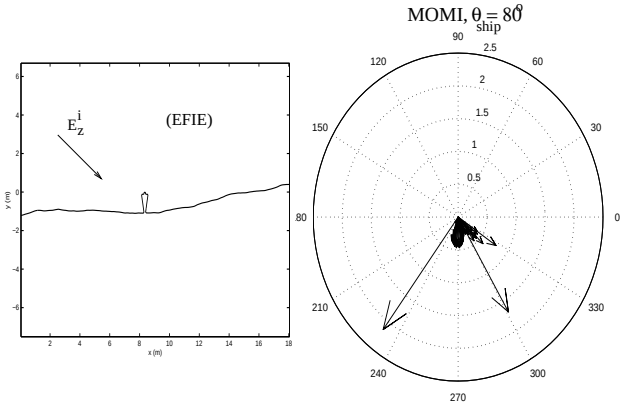


Fig. 12. MOMI: Eigenvalue spectrum of G_M for the rough surface with a 2D target. $P = N = 210$.

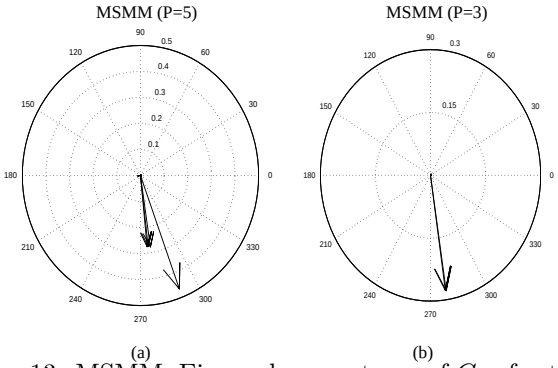


Fig. 13. MSMM: Eigenvalue spectrum of G_M for the rough surface with a 2D target. (a) $P = 5$, (b) $P = 3$.

results. However, the MOMI (or FB method) does not show convergent behavior if there is a target (or other strongly coupled region such as a breaking wave) on the surface.

V. CONCLUSIONS

The convergence properties of the Multiple Sweep Method of Moments (MSMM) has been studied both analytically and numerically for some classes of 2D scattering geometries. A connection between the MSMM and well-known classical iterative methods in mathematics is established so that the convergence properties of the MSMM can be investigated using matrix theory. The MSMM is shown to be mathematically equivalent to a block Jacobi preconditioned system of equations that results from the moment method, and solved via block symmetric successive over-relaxation (SSOR) with relaxation factor $\omega = 1$ (i.e., block Forward Backward Gauss-Seidel method). Based on this connection, the convergence difficulties (such as for the closed cylinder) can be explained by examining the eigenvalue distribution of the iteration matrix G defined for any stationary iterative method. In addition, the MSMM can be compared with other recently developed iterative methods for rough surface scattering problems such as the Method of Ordered Multiple Interactions (MOMI), or Forward-Backward (FB) Method, from both theoretical and numerical points of view. The results show that the MSMM converges for some problems for which the MOMI (and FB) fails to converge, e.g., the rough surface with a target on it, or a multi-valued surface which has large off-diagonal elements in the interaction matrix. It is also shown that the MSMM can be reduced to the MOMI if the block section size reduces to a single Method of Moments segment size, and the initial start vector is set to zero.

It has been shown in earlier works that the point SSOR (and hence MOMI and FB methods) is strongly effected by the ordering of the elements. It is because of the fact that changing the ordering of the elements results in a different iteration matrix G, that the iterative procedure may have different convergence properties. Hence, convergence may be a problem, especially for 2D surfaces (3D scattering problems) because a sequential ordering of elements is difficult to achieve. The convergence of the MSMM has no sensitivity to a change in the ordering of the elements because the ordering of groups becomes important rather than the elements. However, if the size of the blocks is reduced to the size of one element, the MSMM essentially becomes the MOMI and will have the same difficulties with convergence.

The effect of using different integral equation formulations (EFIE or MFIE) and the polarization of the incident field (TE_z or TM_z) on the convergence of the MSMM are also considered and the results are compared with the MOMI. The EFIE with TE_z polarization is found to have the worst divergence problem if a small block size is used, and the MFIE with TE_z polarization has extremely good convergence for any block size. The convergence is also affected by the geometry, and high Q structures such as the closed or partially open cylinder may not converge for any choice of block size (except, of course, when the entire geometry is in one block).

Guidelines for the implementation of the MSMM and some suggestions for the choice of an optimum P are presented in [35]. It has been found that as the MSMM section size increases (or the number of MSMM sections decreases) the rate of the convergence of the method also increases, i.e., $\rho(G_M)$ gets smaller. However, increasing the section size also increases the CPU time to factorize the $[Z_{pp}]$ block matrices. For the strip (also for the rough surface) the optimum section size is found to be $D_P \approx 1\lambda$. However, it is not easy to generalize this conclusion for an arbitrary geometry because every geometry has a different convergence behavior with respect to the section size. Furthermore, it is not easy to predict the $\rho(G_M)$ a priori for a given MSMM section size. As a rule of thumb, one can apply the following procedure. For a given a problem geometry, first include any target or high Q geometry in one MSMM section. As a result, the $[Z]$ matrix will have less significant off-diagonal blocks. Next, the smooth part of the geometry can be sectioned into $D_P = 1\lambda$ width MSMM sections. However, if the entire geometry is a high Q structure, such as a closed cylinder, then a better integral equation formulation could be used to make the MSMM converge.

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