

## 2-D DOA Estimation with Matrix Pencil Method in the Presence of Mutual Coupling

A. Azarbar<sup>1</sup>, G. R. Dadashzadeh<sup>2</sup>, and H. R. Bakhshi<sup>2</sup>

<sup>1</sup>Department of Computer and Information Technology Engineering,  
Islamic Azad University, Parand Branch, Tehran, 37613 96361, Iran  
aliazarbar@piu.ac.ir

<sup>2</sup>Faculty of Engineering, Shahed University, Tehran 33191 18651, Iran  
gdadashzadeh@shahed.ac.ir, bakhshi@shahed.ac.ir

**Abstract**— A new 2-D Direction of Arrival (DOA) estimation algorithm in the presence of mutual coupling for the Uniform Rectangular Array (URA) based on Matrix Pencil (MP) method is presented. By setting a group of elements as auxiliary on each side of the URA, it can accurately estimate the DOAs using a single snapshot of data and the effect of mutual coupling can be eliminated by the inherent mechanism of the proposed method. Theoretical analysis and simulation results demonstrated the effectiveness of the proposed algorithm.

**Index Terms**— 2-D DOA estimation, Matrix pencil, Mutual coupling, URA..

### I. INTRODUCTION

The study of adaptive antennas in radar and wireless communications has been an attractive research topic for several decades. Furthermore, direction of arrival estimation is an important feature of adaptive antenna arrays. Multiple Signal Classification (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [1] and MP [2-5] are some popular conventional methods of DOA estimation.

In array signal processing, most adaptive algorithms assume that the array elements are isotropic sensors; thus the mutual coupling effects are ignored. However, in practical applications, each array element receives signals reradiated from other sensors within the array and the

performance of an adaptive antenna array is drastically affected by the existence of the mutual coupling effect between antenna elements [6-8]. Such an effect needs to be removed in order to achieve a high performance in an actual system [8]. Many efforts have been made to reduce or compensate for this effect on Uniform Linear Array (ULA) and Uniform Circular Array (UCA) [8-14]. But few authors have dealt with 2-D cases and considered the effect of mutual coupling or any other array errors [15-16].

Some studies have stated that using auxiliary elements can reduce the effect of mutual coupling [16-17]. It was shown in [14] and [16], that by providing a modest number of auxiliary array elements, the MUSIC algorithm can be adopted directly for DOA estimation in ULA and URA and the approach is resilient against a well-known mutual coupling model with some unknown parameters. But, these proposed algorithms suffer from two major drawbacks: first, the MUSIC algorithm is based on the covariance matrix and it requires independent identically-distributed secondary data to estimate the covariance matrix; also the estimation of the covariance matrix requires the storage and processing of the secondary data. This is computationally intensive, requiring many calculations in real time. The second is that in the proposed algorithm there may be some blind angles caused by some particular combinations of mutual coupling coefficients which should be avoided while designing the array [14].

In this paper, a simple solution is presented to settle the coupling problem of URA in 2-D DOA estimation based on the MP algorithm. This algorithm can overcome the drawbacks of statistical techniques. Because it is based on the spatial samples of the data and the analysis is done on a snapshot-by-snapshot basis, non-stationary environments can be handled easily [2]. It is proposed that the array elements on the boundary of URA should be of auxiliary elements and only use the output of the rest array to estimate the DOA of incoming signals. Through this process, the MP algorithm can be directly applied for 2-D DOA estimation.

## II. 2-D MATRIX PENCIL METHOD

The DOA estimation of several signals which simultaneously impinge on a two-dimensional planar array can also be performed using the Matrix Pencil method. Consider a URA consisting of  $M \times N$  equally spaced elements in rows and columns. The space between neighboring columns is  $d_x$  and that of neighboring rows is  $d_y$ . The array receives  $P$  narrow band signals,  $s_p(t)$ , from unknown directions,  $(\theta_p, \varphi_p)$ ,  $p=1, 2, \dots, P$ , as shown in Fig. 1.

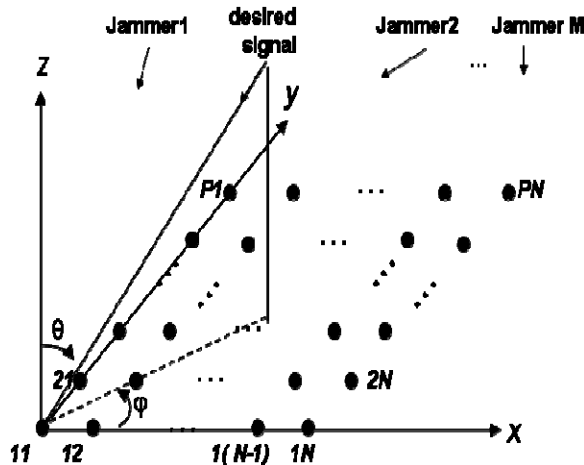


Fig. 1. URA with  $M \times N$  elements.

Hence, the voltage  $x(m, n)$  induced at the feed point of the antenna elements of the URA which can be modeled by summing the complex exponentials, i.e.,

$$y(m, n) = x(m, n) + n(m, n). \quad (1)$$

$$m = 1, \dots, M, n = 1, \dots, N.$$

where

$$x(m, n) = \sum_{p=1}^P r_p y_p^m z_p^n, \quad (2)$$

$$y_p = \exp(j \frac{2\pi}{\lambda} d_y \sin \theta_p \sin \varphi_p),$$

$$z_p = \exp(j \frac{2\pi}{\lambda} d_x \sin \theta_p \cos \varphi_p).$$

where  $r_p$  is the complex amplitude of  $p$ th signal and  $n(m, n)$  is the additive noise,

Basically, in 2-D MP method, 2-D problem is divided into two 1-D problems. Solved for each pole in each dimension and paired together to get the correct DOA angles. The formulation of the 2-D matrix pencil method was discussed in detail in [4]. The noiseless data matrix  $x(m, n)$  can be written as follows:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_M \end{bmatrix} = \begin{bmatrix} x(1,1) & x(1,2) & \dots & x(1,N) \\ x(2,1) & x(2,2) & \dots & x(2,N) \\ \vdots & \vdots & \ddots & \vdots \\ x(M,1) & x(M,2) & \dots & x(M,N) \end{bmatrix} \quad (3)$$

The data matrix  $\mathbf{X}$  can be enhanced and written in Hankel block matrix structure as follows:

$$\mathbf{D} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_3 \\ \mathbf{X}_2 & \mathbf{X}_3 & \mathbf{X}_4 \\ \vdots & \vdots & \vdots \\ \mathbf{X}_{M-2} & \mathbf{X}_{M-1} & \mathbf{X}_M \end{bmatrix} \quad (4)$$

Two matrices of  $\mathbf{D}_1$  and  $\mathbf{D}_2$  are defined in order to extract the poles associated with the one dimension.  $\mathbf{D}_1$  is obtained from  $\mathbf{X}$  by deleting the last row and  $\mathbf{D}_2$  is obtained from  $\mathbf{X}$  by deleting the first row. One can also write

$$\mathbf{D}_2 = \mathbf{Y}_1 \mathbf{R} \mathbf{Y}_0 \mathbf{Y}_2, \quad (5)$$

$$\mathbf{D}_1 = \mathbf{Y}_1 \mathbf{R} \mathbf{Y}_2. \quad (6)$$

where

$$\mathbf{Y}_2 = \begin{bmatrix} 1 & z_1 & \dots & z_1^{N-1} & y_1 & y_1 z_1 & \dots & y_1 z_1^{N-1} & \dots & y_1^2 & y_1^2 z_1 & \dots & y_1^2 z_1^{N-1} \\ 1 & z_2 & \dots & z_2^{N-1} & y_2 & y_2 z_2 & \dots & y_2 z_2^{N-1} & \dots & y_2^2 & y_2^2 z_2 & \dots & y_2^2 z_2^{N-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & z_P & \dots & z_P^{N-1} & y_P & y_P z_P & \dots & y_P z_P^{N-1} & \dots & y_P^2 & y_P^2 z_P & \dots & y_P^2 z_P^{N-1} \end{bmatrix}_{P \times (3N)} \quad (7)$$

$$\mathbf{Y}_1 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ y_1 & y_2 & \dots & y_P \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{M-2} & y_2^{M-2} & \dots & y_P^{M-2} \end{bmatrix}_{(M-1) \times P}, \quad (8)$$

$$\mathbf{Y}_0 = \text{diag}[y_1, y_2, \dots, y_P], \quad (9)$$

$$\mathbf{R} = \text{diag}[r_1, r_2, \dots, r_P]. \quad (10)$$

where  $\text{diag}[\bullet]$  represents a  $P \times P$  diagonal matrix. Now, consider the matrix pencil

$$\mathbf{D}_2 - \lambda \mathbf{D}_1 = \mathbf{Y}_1 \mathbf{R} (\mathbf{Y}_0 - \lambda \mathbf{I}) \mathbf{Y}_2. \quad (11)$$

while  $\mathbf{I}$  is the  $P \times P$  identity matrix. It was shown in [3] that this problem can be reduced to an ordinary eigenvalue problem such that  $\mathbf{Y}_0 = \text{diag}[y_1, y_2, \dots, y_P]$  is the eigenvalues of:

$$\mathbf{D}_1^+ \mathbf{D}_2 - \lambda \mathbf{I}. \quad (12)$$

where  $\mathbf{D}_1^+$  is the Moore-Penrose pseudo-inverse of  $\mathbf{D}_1$ . This, in turn, is defined as:

$$\mathbf{D}_1^+ = (\mathbf{D}_1^H \mathbf{D}_1)^{-1} \mathbf{D}_1^H \quad (13)$$

Similarly, the data  $x(m;n)$  can be written in a new matrix form as follows:

$$\mathbf{X}' = \begin{bmatrix} \mathbf{X}'_1 \\ \mathbf{X}'_2 \\ \vdots \\ \mathbf{X}'_N \end{bmatrix} = \begin{bmatrix} x(1,1) & x(2,1) & \dots & x(M,1) \\ x(1,2) & x(2,2) & \dots & x(M,2) \\ \vdots & \vdots & \ddots & \vdots \\ x(1,N) & x(2,N) & \dots & x(M,N) \end{bmatrix} \quad (14)$$

The data matrix  $\mathbf{X}'$  can be enhanced and written in Hankel block matrix structure as follow:

$$\mathbf{D}' = \begin{bmatrix} \mathbf{X}'_1 & \mathbf{X}'_2 & \mathbf{X}'_3 \\ \mathbf{X}'_2 & \mathbf{X}'_3 & \mathbf{X}'_4 \\ \vdots & \vdots & \vdots \\ \mathbf{X}'_{N-2} & \mathbf{X}'_{N-1} & \mathbf{X}'_N \end{bmatrix} \quad (15)$$

So,  $\mathbf{Z}_0 = \text{diag}[z_1, z_2, \dots, z_P]$  will be the eigenvalues of:

$$\mathbf{D}'_1^+ \mathbf{D}'_2 - \lambda \mathbf{I}. \quad (16)$$

where  $\mathbf{D}'_1$  is obtained from  $\mathbf{D}'$  by deleting the last row and  $\mathbf{D}'_2$  is obtained from  $\mathbf{D}'$  by deleting the first row.

In the presence of noise, some pre-filtering needs to be done. Noise reduction can be performed via the Singular Value Decomposition (SVD) [18].  $\mathbf{D}$  and  $\mathbf{D}'$  are decomposed using the SVD yielding:

$$\mathbf{D} = \mathbf{U}_s \boldsymbol{\Sigma}_s \mathbf{V}_s^H + \mathbf{U}_n \boldsymbol{\Sigma}_n \mathbf{V}_n^H. \quad (17)$$

where  $(\bullet)^H$  denotes conjugate transpose and the  $\mathbf{U}_s$ ,  $\boldsymbol{\Sigma}_s$  and  $\mathbf{V}_s^H$  are in the signal subspace corresponding to the  $P$  principal components whereas  $\mathbf{U}_n$ ,  $\boldsymbol{\Sigma}_n$  and  $\mathbf{V}_n^H$  are in the noise subspace.

It was shown in [3] that, for the noisy case, the eigenvalues of the following matrix was the solution for determining  $y_p$ :

$$\mathbf{U}_{s1}^+ \mathbf{U}_{s2} - \lambda \mathbf{I}. \quad (18)$$

where  $\mathbf{U}_{s1}$  is obtained from  $\mathbf{U}_s$  with its the last row of  $\mathbf{U}_s$  deleted and  $\mathbf{U}_{s2}$  is obtained by deleting the first row of  $\mathbf{U}_s$ . Second,  $\mathbf{D}'$  was decomposed using the SVD yielding:

$$\mathbf{D}' = \mathbf{U}'_s \boldsymbol{\Sigma}'_s \mathbf{V}'_s{}^H + \mathbf{U}'_n \boldsymbol{\Sigma}'_n \mathbf{V}'_n{}^H. \quad (19)$$

Similarly, the eigenvalues of the following matrix is the solution for determining  $z_p$ :

$$\mathbf{U}'_{s1}{}^+ \mathbf{U}'_{s2} - \lambda \mathbf{I}. \quad (20)$$

### III. THE PROPOSED ALGORITHM

Most DOA estimation algorithms, including MP assume an ideal, linear array of isotropic sensors. Unfortunately, such an ideal sensor is obviously not realizable. A practical antenna array is composed of the elements in some physical sizes. The elements sample and reradiate incident fields and cause mutual coupling. Mutual coupling severely degrades the accuracy of the DOA estimator [8]. Any implementation of DOA estimation requires a compensation for the mutual coupling.

In this paper, in order to nullify the effect of mutual coupling, the array sensors on the boundary of URA are set to be auxiliary sensors and only the output of the rest array are used to estimate the DOAs. Utilizing this process, the MP algorithm can be directly applied for 2-D DOA estimation.

Assuming that  $\mathbf{C}$  denotes the mutual coupling matrix of the URA, the array's output can be expressed as  $\mathbf{x}_c = \mathbf{C} \mathbf{x}_e$  where  $\mathbf{x}_e$  denotes the received signal vector in the presence of mutual coupling and is defined as  $\mathbf{x}_e = [x_c(1,1), x_c(1,2), \dots, x_c(1,N), x_c(2,1), \dots, x_c(2,N), \dots, x_c(M,1), \dots, x_c(M,N)]$

According to [19], the coupling between neighboring elements of a ULA is almost the same and the magnitude of the coupling parameters decreases very fast by increasing the sensor

spacing. Essentially, the mutual coupling coefficient between two far-apart elements can be approximated to zero. Thus, it is often sufficient to consider the ULA coupling model with only finite non-zero coefficients, and a banded symmetric toeplitz matrix can be used as a model for the mutual coupling. This model can be extended to the mutual coupling of URA. Because the mutual coupling of the URA is more complex than the ULA and UCA, in this paper we assume that one sensor is only affected by the coupling of the 8 sensors around it [16], which is shown in Fig. 2.

The mutual coupling matrix can be expressed as:

$$C = \begin{bmatrix} C_1 & C_2 & 0 & \dots & 0 & 0 & 0 \\ C_2 & C_1 & C_2 & \dots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & & \\ 0 & 0 & 0 & \dots & C_2 & C_1 & C_2 \\ 0 & 0 & 0 & \dots & 0 & C_2 & C_1 \end{bmatrix}_{MN \times MN} \quad (21)$$

where  $C_1$  and  $C_2$  are  $N \times N$  sub-matrices of  $C$  and can be given by:

$$\begin{aligned} C_1 &= \text{toeplitz}\{[1, c_x, 0, \dots, 0]\}, \\ C_2 &= \text{toeplitz}\{[c_y, c_{xy}, 0, \dots, 0]\}. \end{aligned} \quad (22)$$

where the symbol  $\text{toeplitz}\{\mathbf{v}\}$  denotes the symmetric toeplitz matrix constructed by the vector  $\mathbf{v}$ . In order to eliminate the effect of the mutual coupling, the sensors on the boundary of the URA are set to be auxiliary sensors.

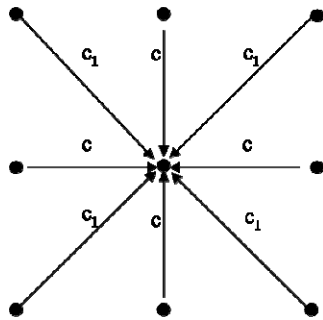


Fig. 2. Scheme of mutual coupling.

Now, a new matrix  $D_c$  can be formed, which is obtained from the output of the middle  $(M-2) \times N$  in URA:

$$D_c = \begin{bmatrix} X_{c2} \\ X_{c3} \\ \vdots \\ X_{c(M-1)} \end{bmatrix} = \begin{bmatrix} x(2,1) & x(2,2) & \dots & x(2,N) \\ x(3,1) & x(3,2) & \dots & x(3,N) \\ \vdots & \vdots & \ddots & \vdots \\ x(M-1,1) & x(M-1,2) & \dots & x(M-1,N) \end{bmatrix} \quad (23)$$

Let us define:

$$\bar{C} = \begin{bmatrix} C_2 \\ C_1 \\ C_2 \end{bmatrix}_{(3M) \times N} \quad (24)$$

After this definition, a very important relationship between  $D$  and  $D_c$  is obtained as follows:

$$D_c = D \bar{C} \quad (25)$$

So, two matrices of  $D_{c1}$  and  $D_{c2}$  are defined.  $D_{c1}$  is obtained from  $D_c$  by deleting the last row and  $D_{c2}$  is obtained from  $D_c$  by deleting the first row. Therefore:

$$\begin{aligned} D_{c1} &= D_1 \bar{C} \\ D_{c2} &= D_2 \bar{C} \end{aligned} \quad (26)$$

Using (5) and (6), the following can be obtained:

$$\begin{aligned} D_{c2} &= Y_1 R Y_0 Y_2 \bar{C} \\ D_{c1} &= Y_1 R Y_2 \bar{C} \end{aligned} \quad (27)$$

Now, the matrix pencil can be formed:

$$D_{c2} - \lambda D_{c1} = Y_1 R \{Y_0 - \lambda I\} Y_2 \bar{C} \quad (28)$$

This problem can be reduced to an ordinary eigenvalue problem and  $Y_0 = \text{diag}[y_1, y_2, \dots, y_P]$  will be the eigenvalues of:

$$D_{c1}^+ D_{c2} - \lambda I \quad (29)$$

Similar to  $D_c$ , a new matrix  $D'_c$  can be formed, which is obtained from the output of the middle  $M \times (N-2)$  in the URA:

$$D'_c = \begin{bmatrix} X'_{c2} \\ X'_{c3} \\ \vdots \\ X'_{c(N-1)} \end{bmatrix} = \begin{bmatrix} x(1,2) & x(2,2) & \dots & x(M,2) \\ x(1,3) & x(2,3) & \dots & x(M,3) \\ \vdots & \vdots & \ddots & \vdots \\ x(1,N-1) & x(2,N-1) & \dots & x(M,N-1) \end{bmatrix} \quad (30)$$

So,  $Z_0 = \text{diag}[z_1, z_2, \dots, z_P]$  will be the eigenvalues of:

$$D'_{c1}{}^+ D'_{c2} - \lambda I \quad (31)$$

where  $D'_{c1}$  is obtained from  $D'_c$  by deleting the last row and  $D'_{c2}$  is obtained from  $D'_c$  by deleting the first row.

For the noisy case, the eigen-structure of the matrices  $D_c$  and  $D'_c$  is found by considering the SVD:

$$D_c = U_c \Sigma_c V_c^H \quad (32)$$

Here,  $U_c$  and  $V_c$  are unitary matrices, composed of the eigenvectors of  $D_c D_c^H$  and,

$\mathbf{D}_c^H \mathbf{D}_c$ , respectively.  $\Sigma_c$  is the singular values of  $\mathbf{D}_c$ . For simplicity, it is assumed that the number of signals is known in this paper. After SVD of data matrix  $\mathbf{D}_c$  is computed, the matrix space is divided into two subspaces, signal subspace and noise subspace. Here, the matrices  $\mathbf{D}_{c1}$  and  $\mathbf{D}_{c2}$  are constructed from the signal subspace matrix. So, the "filtered" matrix  $\mathbf{U}_{cs}$  is constructed. It consists of the first  $P$  columns of  $\mathbf{U}_c$  and the rest of right-singular vectors, corresponding to the small singular values, are discarded. Therefore, the following can be written:

$$\begin{aligned} \mathbf{D}_{c1} &= \mathbf{U}_{cs1} \Sigma_{cs} \mathbf{V}_{cs}^H, \\ \mathbf{D}_{c2} &= \mathbf{U}_{cs2} \Sigma_{cs} \mathbf{V}_{cs}^H. \end{aligned} \quad (33)$$

where  $\mathbf{U}_{cs1}$  is obtained from  $\mathbf{U}_{cs}$  with its the last row of  $\mathbf{U}_{cs}$  deleted and  $\mathbf{U}_{cs2}$  is obtained by deleting the first row of  $\mathbf{U}_{cs}$ . Then, the eigenvalues of the following matrix is the solution for determining  $y_p$ :

$$\mathbf{U}_{cs1}^+ \mathbf{U}_{cs2} - \lambda \mathbf{I} \quad (34)$$

Similarly,  $\mathbf{D}'_c$  was decomposed using the SVD yielding:

$$\mathbf{D}'_c = \mathbf{U}'_{cs} \Sigma'_{cs} \mathbf{V}'_{cs}{}^H + \mathbf{U}'_{cn} \Sigma'_{cn} \mathbf{V}'_{cn}{}^H. \quad (35)$$

where  $\mathbf{U}'_{cs1}$  and  $\mathbf{U}'_{cs2}$  are obtained by deleting the last row and the first row of  $\mathbf{U}'_{cs}$ , respectively.

Then, the eigenvalues of the following matrix is the solution for determining  $z_p$ :

$$\mathbf{U}'_{cs1}{}^+ \mathbf{U}'_{cs2} - \lambda \mathbf{I}. \quad (36)$$

Most of adaptive algorithms which compensate mutual coupling, must be computed the inverse coupling matrix. That is computationally intensive and requires many calculations in the real time. The proposed algorithm can be directly applied for 2-D DOA estimation and don't use the inverse mutual coupling matrix. In addition, the data matrices of the proposed algorithm ( $\mathbf{D}_c$  and  $\mathbf{D}'_c$ ) are as order of  $N \times (M-2)$ . But, the data matrices of the MP algorithm ( $\mathbf{D}$  and  $\mathbf{D}'$ ) are as order of  $3N \times (M-2)$ . Hence, the proposed algorithm is faster than the MP algorithm.

#### IV. NUMERICAL SIMULATIONS

In this section,  $5 \times 5$  elements z-direction parallel identical dipoles are used, which are equally spaced in rows and columns with the distance of  $\lambda/2$ , where  $\lambda$  is the wavelength. Each dipole is  $0.5\lambda$  long and  $\lambda/200$  in radius and all the

elements are loaded with a terminal load of  $Z_L = 50 \Omega$ . The Method of Moments (MOM) is used to accurately model the interactions between antenna elements. The array receives two signals from  $(20^\circ, 15^\circ)$  and  $(35^\circ, 60^\circ)$ . The MP algorithm and the proposed algorithm use only a single snapshot. Table.1 shows the accuracy of DOA estimation using the new proposed algorithm in the presence of MC.

Table 1: Comparing Accuracy of MP and Proposed Algorithm

	The MP method in the presence of MC	The proposed algorithm in the presence of MC
Signal 1	$(19.89^\circ, 13.82^\circ)$	$(20.00^\circ, 15.00^\circ)$
Signal 2	$(32.64^\circ, 57.87^\circ)$	$(35.00^\circ, 60.00^\circ)$

In the next example, the noisy data are used. The Signal-to-Noise Ratio (SNR) was set at 20 dB. 1000 independent trials are used. The scatter plot of the estimated elevation and azimuth angles with conventional MP algorithm and the proposed algorithm in the presence of the mutual coupling are shown in Figs. 3 and 4.

As can be seen, using the proposed algorithm, the error of bias is very low and accuracy is high and very close to ideal.

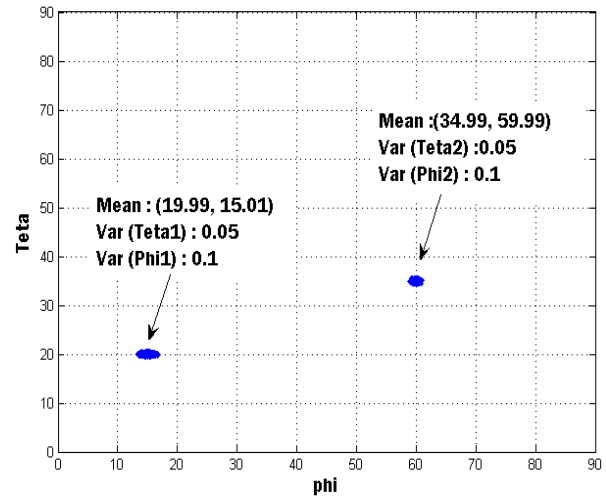


Fig. 3. The scatter plot of direction of arrival angles of 2 impinging signals in the absence of MC.

The performance of the proposed method is compared with ideal MP algorithm, under different SNR. The variances of the estimators are plotted in Fig. 5. As can be seen from Fig. 5, the

proposed algorithm in the presence of mutual coupling has a close variance to the ideal MP method.

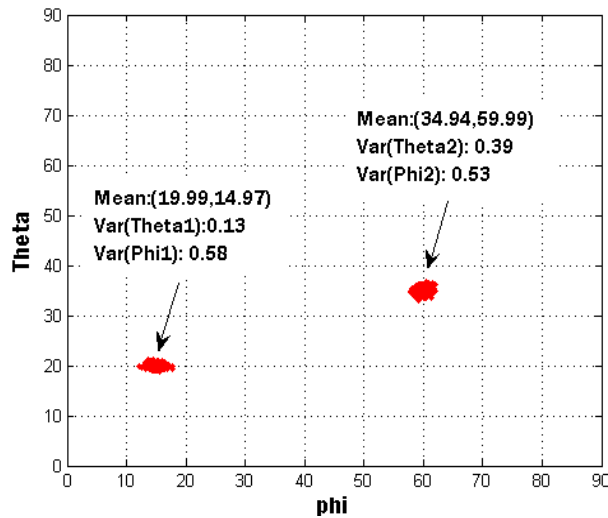


Fig. 4. The scatter plot of direction of arrival angles of 2 impinging signals with the proposed algorithm in the presence of MC.

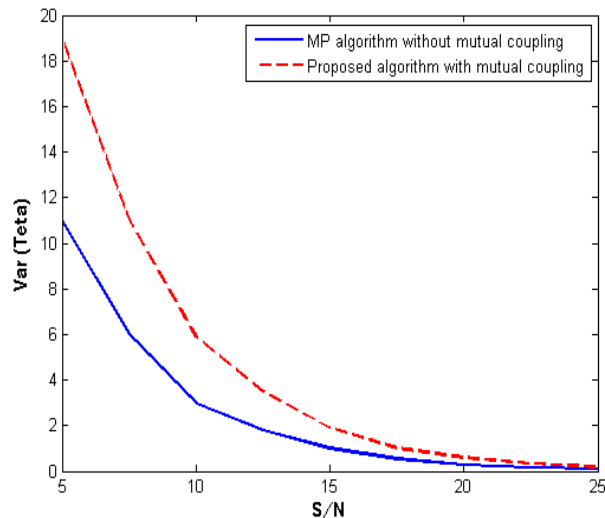


Fig. 5. Comparing of the performance of MP and the proposed algorithm for different SNR.

## V. CONCLUSION

In this paper, the problem of 2-DDOA estimation was studied for the URA in the presence of mutual coupling. By setting the sensors on the boundary of the URA as auxiliary sensors, the robustness of the proposed algorithm was proved to be against sensor coupling. Without using the mutual coupling coefficient calculation,

this method can accurately estimate the 2-D DOAs only by using one snapshot of data.

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Ali Azarbar was born in Tehran, Iran, in 1972. He received the B.Sc. and M.Sc. degree, both in communication engineering from Sharif University, Tehran, Iran in 1994 and 1997, respectively and Ph.D. degree in communication engineering from Islamic Azad University, Tehran, Iran in 2011. His research interests are in adaptive signal processing algorithms and antenna design.



**Gholamreza Dadashzadeh** was born in Urmia, IRAN, in 1964. He received the B.Sc. degree in communication engineering from Shiraz University, Shiraz, Iran in 1992 and M.Sc. and Ph.D. degree in communication engineering from Tarbiat Modarres University (TMU), Tehran, Iran, in 1996 and 2002, respectively. He is a member of IEEE, Institute of Electronics, Information and Communication Engineers (IEICE) of Japan and Iranian Association of Electrical and Electronics Engineers (IAEEE) of Iran. He honored received the first degree of national researcher in 2007 from Iran's ministry of ICT. He has published more than 70 papers in referred journals and international conferences in the area of antenna design and smart antennas.



**Hamidreza Bakhshi** was born in Tehran, Iran on April 25, 1971. He received the B.Sc. degree in electrical engineering from Tehran University, Iran in 1992, the M.Sc. and Ph.D. degree in Electrical Engineering from Tarbiat Modarres University, Iran in 1995 and 2001, respectively. Since 2001, he has been as an Assistant Professor of Electrical Engineering at Shahed University, Tehran, Iran. His research interests include wireless communications, multiuser detection, and smart antennas.