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## Grid-Based SCT Approach for the Global Electromagnetic Simulation and Design of Finite-Size and Thick Dichroïc Plate

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*Abstract* — In the work presented in this paper, a grid-enabled environment for the scale changing technique (SCT) simulations is proposed. This approach allows the fast design of a finite size and thick dichroïc metallic mirror based on global electromagnetic simulation. The computed transmission and reflection coefficients, as well as radiation pattern, satisfying to some predefined technical requirements in X and Ka frequency bands are presented. Simulation results are shown in order to demonstrate the validity of the approach.

*Index Terms* – Dichroïc mirror, distributed computing, electromagnetic analysis, frequency selective surfaces, grid computing, SCT.

#### I. INTRODUCTION

The trend in radio frequency (RF) and microwave design is towards the accurate prediction of system-level performance and behavior. Engineers simulate larger and more sophisticated design problems in support of that goal. When it involves both large structures (in terms of wavelength) and fine details, the system or structure is said to be complex. The higher the number of scale levels in the system the higher is the complexity. Well-known examples of complex structures are provided by multi-band frequencyselective surfaces, finite-size arrays of nonidentical cells and fractal planar objects. Electromagnetic modeling of such structures has been recently discussed [1-2]. In order to simulate such complex multi-scale structures, an efficient electromagnetic approach, named thus scale changing technique (SCT) [3], is used here. The SCT method allows the global simulation of a finite-size thick dichroïc mirror as shown in [4], and detailed here in this article. The scope is to define the most appropriate design of a uniform mirror satisfying to predefined technical requirements of transmission and reflection coefficients in X and Ka frequency bands. Consequently, parametric simulations must be performed.

The parametric distributed study application is now a good candidate to execute in a grid computing (GC) environment [5]. GC has emerged as an important new field, distinguished from conventional distributed computing by its focus on large-scale resource sharing, innovative applications, and, in some cases, high-performance orientation [5-6]. GC provides a safe and pervasive access to dynamically distributed computing resources. providing greater availability, reliability, and cost efficiencies that may exist with dedicated servers. In addition to the central processing unit (CPU) and storage resources, it can provide access to increased quantities of other (shared) resources and to special equipment, software, licenses, and other services.

The SCT has proven itself as a powerful tool for electromagnetic simulation of frequency selective surfaces. Various dimensions of the dichroïc mirror have been simultaneously simulated on GC nodes in order to minimize the simulation time. Overall computation time for deriving transmission, reflection coefficients, and the radiation pattern of arrays was significantly reduced compared to sequential computation while keeping the solver accuracy by deploying these simulations in distributed environments as GC.

This article is composed of two main parts. First, the frequency selective surfaces and modeling issues are discussed in section II, followed by the scale changing technique (SCT) modeling description in section III.

In the second part, the SCT algorithm is discussed and distributed computing in grid environments is presented in two different approaches. Simulation results, compared to other simulation tools, are shown here.

Finally, in section V, an overview of the key advantages of this approach coupled with SCT in the global electromagnetic analysis and design of the frequency selective surfaces is reported.

#### II. THE FREQUENCY SELECTIVE SURFACES

The frequency selective surfaces (FSSs) are a key component in the design of multi-frequency systems. They are used as frequency filters, and find applications, e.g. radomes and Cassegrain antenna reflectors. By nature, the FSS play the role of filters, allowing the transmission of certain frequencies and the reflection of others. They generally consist of a periodic array of conducting elements printed on a dielectric substrate, or of an array of apertures in a metallic plate. The nature and the geometry of the element (patch or aperture) are very important in the determination of the frequency response of the structure, like its bandwidth and transfer function. FSS performances depend strongly on the angle of incidence and polarization of the incident electromagnetic waves [7-8]. In our case study, we consider only a perforated metallic plate or roasts metal illuminated by a plane wave (with arbitrary angles of incidence).

Traditionally, FSS performances are assessed by making the assumption of infinite size and periodical FSS by using Floquet modes. Consequently, the computing time is being reduced almost to that of the basic cell.

To take into account the finite size of the array, several methods have been developed [9-10]. Note also that most of the methods treat only

FSS with small thickness. However, the simulation of the FSS by the traditional numerical methods based on spatial meshing (finite element method, finite difference method, and method of the moments) or spectral discretization (modal methods) often leads to badly ill-matrix, numerical convergence problems and/or excessive computation times. To avoid all these problems, we adapt to the identified problem an original technique, called the scale changing technique, based on various scales modeling of complicated structures and whose principle consist in the idea that it is desirable to dissociate (separate) the complex problem into several simpler problems (Cf. Section III).

For numerous applications, the FSS is generally in the near field region, so that the operation of the cell strongly depends on its position in the array. The possibility offered by SCT to optimize a non-periodic grid, with each element selected according to its position in the array, should make it possible to improve the response of the FSS usually used, in particular to reduce their impact on the radiation diagram of the source in transmission (deformation of the mean lobe, high cross-polarization...).

Since the SCT allows global electromagnetic simulation of FSS, it will be used here to define the best dimensions design of dichroïc mirror satisfying to predefined reflection and transmission conditions in X and Ka frequency bands without using Floquet modes. It allows taking into account the finite size of the array as well as its thickness. The grid computing nodes will be used to run this parametric study.

#### III. THE SCALE CHANGING TECHNIQUE

The SCT is an efficient monolithic (unique) formulation for the electromagnetic modeling of complex (multi-scale) structures i.e., structures that exhibit multiple metallic patterns whose sizes cover a large range of scales [3].

SCT consists of interconnecting scalechanging networks (SCN), each network models the electromagnetic coupling between adjacent scale levels. The cascade of the scale changing networks allows the global electromagnetic simulation of multi-scale structures, from the smallest to the highest scale.



Fig. 1. Topology of a FSS consisting of uniform rectangular waveguides perforating a thick metallic plate.

The global electromagnetic simulation of multi-scale structures via the cascade of the scale changing networks has been applied with success to the design and electromagnetic simulation of specific planar structures such as reconfigurable phase-shifters [11, 12], fractal selective surfaces [13], discrete self-similar (pre-fractal) scatterers [14, 15], and patch antennas [16, 17]. This scale-changing technique is a very fast technique and this makes it a very powerful investigation, design and optimization tool for engineers who design complex circuits [18 - 20].



Fig. 2. An example of partitioning of finite size and thick dichroïc metallic mirror.

The proposed topology consists of a metallic grid of thickness h = 5 mm, with a = b = 10 mm (Cf. Figure 1). The dimensions of the apertures are uniform and the cells have the same thickness. To

demonstrate the potential of the method, we consider the transmission and reflection coefficients of a finite size (256 cells) uniform FSS illuminated by a normal incident plane wave, as well as the radiation pattern.

## A. Partitioning of the finite size and thick dichroïc metallic mirror

The starting point of the proposed approach consists of the coarse partitioning of the dichroïc mirror into large-scale (called scale level smax) subdomains of arbitrary shape; in each sub-domain a second partitioning is then performed by introducing smaller sub-domains at scale level s<sub>max</sub>-1; again, in each sub-domain introduced at scale level s<sub>max</sub>-1 a third partitioning is performed by introducing smaller sub-domains at scale level smax-2; and so on. Such hierarchical domaindecomposition, which allows us to focus rapidly on increasing detail in the dichroïc mirror plane, is stopped when the finest partitioning (scale level s=0) is reached. An illustration of the partitioning of a metallic grid of 265 cells is sketched in Figure 2. We can easily see that a high scale ratio exists between the highest and the smallest dimensions. Four different scale levels exist when cells are grouped by four. Boundary conditions (perfect electric or perfect magnetic boundary conditions) are artificially introduced at the contour of all the sub-domains generated by the partitioning process, taking into account the physics of the problem.



Fig. 3. A scale changing network (SCN) [1].

The next step consists of computing the electromagnetic coupling between two successive scale levels via the scale changing network (SCN) (Cf. Figure 3). The cascade of SCNs allows crossing the scale from the lowest scale (s=0) to the highest one (s= $s_{max}$ ).

The scale changing technique can be applied several times to different scale levels. The transition from one level to another level is modeled by multiport associated with active modes of the two scales.

#### **B.** Surface impedance matrix

Consider the pattern given in Figure 4(a), where the thickness is taken into account. It presents a passive sub-domain Ds containing many fine geometrical details and composed of: (1) the perfect conductor domain  $D_M$  and (2) the aperture domain  $D_A$ , with  $D_S=D_M$  U DA (Figure 4(b)).



Fig. 4. (a) Initial lossless metallic pattern with a passive sub-domain DS presenting many fine geometrical details; (b) artificial rectangular waveguide of cross section DS in which the set of propagating and evanescent modes constitutes the basis for the electromagnetic field.

The electromagnetic field in Ds can be expanded on a local modal electromagnetic field in basis. Such basis can be viewed as the set of propagating and evanescent modes in the artificial rectangular waveguide of cross section. The time average electromagnetic energy stored by higherorder evanescent modes is localized in the vicinity of discontinuities or geometrical details of the planar structure. Consequently the contribution of such modes in the expansion of the electromagnetic field is expected to be insensitive to the choice of the surface, as far as the boundary conditions enclosing this surface are applied sufficiently far from discontinuities. We consider here that such modes are shunted by their (pure imaginary) impedance and are called *passive* by analogy with variational techniques in waveguide discontinuities. The electromagnetic coupling between the lower-order -or active- modes is characterized by the surface impedance matrix Zs (or admittance matrix Ys) and consequently, the original sub-domain is replaced by surface impedance (or admittance) boundary conditions.

#### **C. S-parameters calculation**

The thick metallic grid presents a symmetry plane cutting the thickness of the plate in two equal parts (Figure 5(a)).

The grid impedance matrix  $[Z_{total}]$  can then be derived from the combination of the two impedance matrices  $[Z_{even}]$  and  $[Z_{odd}]$  of halfstructures obtained by inserting respectively a magnetic wall and electric wall in the symmetry plane, as follows [5]:

$$[Z_{total}] = \frac{1}{2} \begin{pmatrix} [Z_{even}] + [Z_{odd}] & [Z_{even}] - [Z_{odd}] \\ [Z_{even}] - [Z_{odd}] & [Z_{even}] + [Z_{odd}] \end{pmatrix}.$$
(1)

The scattering matrix  $[S_{total}]$  is then deduced from the following relationship:

$$[S_{total}] = ([Z_{total}] - [Z_0])([Z_{total}] + [Z_0])^{-1}$$
(2)

where  $[Z_0]$  designates the diagonal matrix of active mode impedances. The scale changing technique is applied for the derivation of matrices  $[Z_{even}]$  and  $[Z_{odd}]$  associated with half-structures.



Fig. 5. (a) Side-view of half of the unit-cell for magnetic and electric wall configurations. (b) Surface impedance multiport for modeling the unit-cell. N1 denotes the number of active modes at the smallest scale.

#### **D.** Radiation pattern calculation

We consider the scattering of the wave plane incident on the FSS, formed by a thick metal grid. The electromagnetic problem consists in calculating the field scattered by a perfect thick metallic surface when illuminated by a normal plane wave incidence:

$$\vec{E}^{inc} = E^{inc} \exp\left(jk_0 z\right) \vec{y}.$$
(3)

We consider here the field backscattered by the half-structure obtained by inserting a magnetic wall and electric wall in the symmetry plane z = 0.



Fig. 6. Scattering problem in free space in the conventional coordinates system [14].

The induced current on the surface is defined while having the total electric field  $E^{total}$  equal zero. The integral equation in the electric field is obtained with the equivalence principle application.

With the SCT, we substitute the actual current *J* by an equivalent current *Jeq* defined by the lower-order modes (active modes) of the orthogonal modal-basis of domain S (FSS metallic grid).

The S domain is then characterized by a surface impedance matrix  $[Z_s]$  (which fixes the boundary conditions of the problem) such as:

$$[E^{total}] = [Z_s][J_{eq}] \tag{4}$$

$$\vec{J}_{eq} = \sum_{i=1}^{N \times N} I_{eq_{-}i} \, \vec{g}_{eq_{-}i}, \tag{5}$$

where  $\vec{g}_{eq_i}$  is an entire trial function of the modal basis.

To determine the solution of the boundary value problem in free space, we are led to calculate the electric field radiated by the equivalent current. This last item being expressed on a modal basis, it is better to solve the problem in the spectral domain. Under these conditions, the equation of the boundary value problem in free space becomes:

$$\vec{E}^{inc} + \hat{G}\vec{J}_{eq} = z_s\vec{J}_{eq},\tag{6}$$

where  $\hat{G}$  is the dyadic Green's function associated with free space in the spectral domain.

The application of the method of Galerkin makes it possible to establish the following matrix equation:

$$[V^{inc}] - [Z][I_{eq}] = [z_s]$$
<sup>(7)</sup>

$$[I_{eq}] = ([Z] + [z_s])^{-1} [V^{inc}].$$
(8)

The terms are obtained by the following scalar product:

$$[V_{inc}]_{i} = \langle \vec{g}_{e_{j}}, E^{inc} \rangle.$$
<sup>(9)</sup>

The goal is to calculate the  $J_{eq}$  which will give the field diffracted by the grid. This current can be found from equation (8) if Z and Zs are known.

Z is calculated from free space Green's functions; Zs is calculated by applying the SCT and using the induction theorem and the images equivalent principle to make use of the symmetry of the problem along the z-axis [21].

The SCT is applied by introducing an artificial rectangular waveguide enclosed with periodic boundary conditions. Zs represents the surface impedance matrix of the entire structure after the connection of all bifurcation multiports. The computation is performed for even configuration. Since the structure is symmetric along z-axis, we wish to take this symmetry to consider only half space in the description of the problem.

#### IV. THE GRID-PARAMETRIC SCT SIMULATIONS

#### A. Structure of the SCT simulation

As mentioned before, the SCT is a hierarchical domain-decomposition. Consider the metallic grid sketched in Figure 2, with 256 cells. Four different scale levels exist when cells are grouped by four.

First, the dimensions of the studied structure, frequency band, as well as the number of modes (calculated in convergence study) are defined. After an initialization phase, the different SCNs are computed independently.

The algorithm calculates the different correspondent modules,  $M, 1, M, 2, \dots, M256$ , as well as the four modules representing the scale changing, M257, M258, M259, and M260. Once these modules are computed, the global electromagnetic simulation of multi-scale

structures is done via the cascade represented by  $C, 1 \ C2, \dots \ C64, \ C65, \dots \ C80, \ C81, \dots \ C84$ , and  $C85. \ C1, \ C2, \dots \ C64$  are executed at the first cascade level,  $C65 \ \dots \ C80$  at the second level,  $C81 \ \dots \ C84$  at the third level and C85 at the final one. Note that the number of cascades and SCNs depends on the partitioning of the problem and chosen sub-domains defined by the user. The *SCT* program serial execution:

#### **B.** Grid execution

Two scenarios of grid computing can be applied here with SCT simulations. The first one consists of executing in parallel the independent modules, at every level. This is the parallel application. The second one is based on exploring several different structures simultaneously, by executing the same algorithm but with different parameters. That is the parametric application.

#### 1) Parallel application

The idea here consists of creating for every computing stage of SCT a service. In fact, the whole simulation requires only three types of services, since M1, M2, ... M256 are based on the same computing function. M257, M258, M259, and M260 are based on another computing function. The cascade is the same at all levels. To run a SCT simulation, the services are called with the corresponding parameters. Each service produces its results which can be used by other services. To ensure the good execution of the SCT simulation, an adapted scheduler, knowing the

level scales, organize the relation between the services.



Fig. 7. SCT flowchart.



Fig. 8. Distributed parametric computing.

The services are created with the DIET tool [22], an application service provider (ASP) able to ensure the scalability of the solution on several thousand grid computing nodes if necessary. The deployment of the application on the grid is realized by TUNe [23], an efficient tool able to repair some parts of the deployment and execution (which is a critical point of the Grid).

#### 2) Parametric application

The parametric execution (also known as embarrassingly parallel), involves execution of the same code with different input parameters, which do not request data transfer during executions. Some typical examples include frequency sweeps for antenna characterization, or design of optimal antenna geometries.

In this case, a multiple frequency band analysis (the pass band and the stop band) is required. Normally, this case is not very suitable for frequency domain software since the pass band and the stop band are 2 to 3 octaves apart.

In distributed parametric SCT simulations, a wide range of design parameters are evaluated in a single analysis run with the goal of exploring the entire design space and selecting the optimized design without need for the normal iterative process (Figure 8). In our case, the idea is to modify the aperture length, and verify the corresponding transmission coefficient.



Fig. 9. Design dimension sweep of the aperture length L of 16 x 16-element uniform thick dichroïc plate.

#### C. Results

Figure 9 displays the simulated transmission coefficients of a uniform metallic grid (a = b = 10 mm) (Figure 1) with a thickness h=5mm in the case of a normal plane wave incidence for different aperture length L. In this figure only a few curves are presented.

Since the transmission we are looking for is in the range of frequency going from 20 GHz to 30 GHz, an aperture of 8.2 mm will give such performance. Increasing the aperture length L of the cells, which is related to the cutoff frequencies, increased the bandwidth.

Figure 10 displays the simulated reflection and transmission coefficients of a uniform metallic grid (Figure 1) with a thickness h = 5mm and an aperture of L = 8.2 mm in the case of a normal

plane wave incidence. Ansoft HFSS [24] (version 11.2) was used for FEM implementation with 0.02 stopping criterion for the adaptive convergence solution. An excellent agreement between the HFSS- and SCT-results can be observed.





Figure 11 shows the radiation patterns in the case of a  $16 \times 16$ -element uniform thick dichroïc plate shown in Figure 1 simulated at a single point of frequency in the case of a normal plane wave incidence.

Figure 12 represents the simulation time evolution on the same computer for these two techniques calculating the transmission and reflection coefficient when the number N of cells increases.

For a given thick dichroïc plate and simulation technique, the computation time in this figure is normalized to the time required for calculating these coefficients in the case of a 4–cell array. For the scale changing technique computation, time increases very slowly as the number N of cells increases. Meanwhile, in the case of the finite element method, they increase dramatically, because of the mesh refinement needed to insure convergence.

As expected, for the parametric application, the overall execution time of the simulations depends on the number of nodes that are used, or more precisely, on the number of simulations that are executed on each node. A speed-up of  $\sim$ 2(compared to the time needed for the computation on one computer only) is reached by using two computers. Increasing the number of computing nodes to solve the problem increases the speed-up. Speedup up to 10 or more could be easily reached by distributing the simulations on less than one hundred computers.



Fig. 11. Co-polarization in (a) E-plane, and (b) H-plane @ 15 GHz.

Users must choose in advance the number of grid nodes reserved in order to accommodate heavier or lighter electromagnetic simulation requests. The best performance is obtained while distributing the simulations in a way to execute one simulation per computing node. These results point out the benefit from using a large number of computational nodes for running SCT simulations.



Fig. 12. Evolution of computing time compared to the standard time for the calculation of two cells of uniform arrays on the same computer.

#### V. CONCLUSION

Two efficient solutions contributing to the electromagnetic modeling and design of a thick dichroïc plate have been presented in this paper.

First, the scale changing technique modeling method, a well-adapted method for the problems of complex multi-scale structures and which demonstrated very good computing performance even in sequential on one computer, compared to existing commercial codes.

Second, the use of grid computing to solve electromagnetic problems was presented in an example of distributed parametric simulation of thick dichroïc plates. A wide range of design parameters are evaluated in a single analysis run with the goal of satisfying to predefined technical requirements of transmission and reflection coefficients in X and Ka frequency bands. Regardless of the number of possible parametric configurations antenna elements may have, their full-wave analysis can be carried out at the computational costs of a single one, by distributing them over a corresponding number of nodes.

Consequently, the approach consisting of combining the advantages of the scale changing technique and grid computing is so promising.

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#### Preconditioned MDA-SVD-MLFMA for Analysis of Multi-Scale Problems

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Abstract- Multilevel fast multipole algorithm (MLFMA) has been widely used to solve electromagnetic scattering problems from the electrically large size objects. However, it consumes very large memory to store near the interaction matrix for the object with fine structures because the "low frequency breakdown" phenomenon would happen when the finest level box's size is below 0.2 wavelengths. The matrix decomposition algorithm - singular value decomposition (MDA-SVD) is one remedy to alleviate this pressure because it has no limit of the box's size. However, the matrix assembly time of MDA-SVD is much longer than that of the MLFMA. In this paper, a hybrid method called MDA-SVD-MLFMA is proposed to analyze multi-scale problems, which uses the main framework of MLFMA but adopts the MDA-SVD to deal with the near interaction of MLFMA. This method takes advantage of the virtues of both MLFMA and MDA-SVD and is more efficient than either conventional MLFMA or conventional MDA-SVD. An efficient preconditioning technique is combined into the inner-outer flexible generalized minimal residual (FGMRES) solver to speed up the convergence rate. Numerical results are presented to demonstrate the accuracy and efficiency of the proposed method.

Index Terms- Flexible generalized minimal

residual (FGMRES), matrix decomposition algorithm - singular value decomposition (MDA-SVD), multilevel fast multipole algorithm (MLFMA).

#### I. INTRODUCTION

In electromagnetic wave scattering calculations, a classical problem is to compute the equivalent surface currents induced by a given incident plane wave. Such calculations, relying on the Maxwell equations, are required in the simulation of many industrial processes ranging from antenna design, electromagnetic compatibility, computation of back-scattered fields, and so on. All these simulations require fast and efficient numerical methods to compute an approximate solution of Maxwell's equations. The method of moments (MoM) [1-2] is one of the most widely used techniques for electromagnetic problems. It is basically impractical to analyze electrically large problems using MoM because its memory requirement and computational complexity both are  $O(N^2)$ , where N refers to the number of unknowns. Recently, a wide range of fast methods have been developed for accelerating the iterative solution of the electromagnetic integral equations discretized by MoM. One of the most popular techniques is MLFMA [3-6], which has O(NlogN) complexity for a given accuracy.

The increased power and availability of

computational resources and acceleration schemes have enabled the solution of problems with a very large number of unknowns, varying from a few thousands to a few millions [3-6]. Another class of problems arises when analyzing structures which require a high local density of unknowns to accurately capture geometric features. This class of problems is referred to as multi-scale problems exhibit multiple scales in length. For example, small length scale discretizations are required to capture sharp or fine geometric features that are embedded within large and smooth geometries discretized at a coarser length scale. Generally, the characteristic of a multi-scale problem is the concentration of large number of unknowns in electrically small domains. However, when the finest level box's size is below 0.2  $\lambda$  ( $\lambda$ indicates the incident wavelength), MLFMA will suffer from the "low frequency breakdown" phenomenon [4].

MDA-SVD is another popular technique used to analyze the scattering/radiation [7-8], which has no limit of the box's size. However, the matrix assembly time of MDA-SVD is much larger than that of MLFMA. In this paper, a hybrid method called MDA-SVD-MLFMA is proposed, which uses the main framework of MLFMA but adopts the MDA-SVD to deal with the near interaction of MLFMA. This method takes advantage of both MLFMA and MDA-SVD and is more efficient than either conventional MLFMA or conventional MDA-SVD for analyzing the multi-scale problems.

It is well known that the matrix condition number of the electric field integral equation (EFIE) for an electrically large problem is large. Furthermore, for multi-scale problems, the matrix condition number is even larger due to the mixed discretization. Therefore, the system has poor convergence history and requires urgently a good solver or preconditioner. In this paper, an efficient preconditioning technique is combined into the inner-outer flexible generalized minimal residual (FGMRES) solver to improve the property of EFIE [9-12].

The remainder of this paper is organized as follows. Section II demonstrates the formulation of EFIE and the theory of MDA-SVD briefly. Section Ш describes the theory and implementation of MDA-SVD-MLFMA in more details and gives a brief introduction to the inner-outer flexible generalized minimal residual (FGMRES) method. Numerical experiments are presented to demonstrate the efficiency of this proposed method in Section IV. Conclusions are provided in Section V comments.

#### II. THE THEORY OF MDA-SVD

#### A. The formulation of EFIE

In this paper, the electric field integral equation (EFIE) is used to analyze electromagnetic scattering problems. The EFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [2]. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows

$$\sum_{n=1}^{N} Z_{mn} I_n = V_m, \qquad m = 1, 2, ..., N$$
(1)

where

$$Z_{mn} = \frac{jk}{4\pi} (\iint_{s} \Lambda_{m}(\mathbf{r}) \cdot \iint_{s} G(\mathbf{r}, \mathbf{r}') \Lambda_{n}(\mathbf{r}') dS' dS - \frac{1}{k^{2}} \iint_{s} \nabla \cdot \Lambda_{m}(\mathbf{r}) \cdot \iint_{s} G(\mathbf{r}, \mathbf{r}') \nabla \cdot \Lambda_{n}(\mathbf{r}') dS' dS)$$
(2)

and

$$V_m = \iint_{S} \Lambda_m(\mathbf{r}) \cdot \left(\frac{1}{\eta} \mathbf{E}^i(\mathbf{r})\right) dS,$$
$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

Here,  $G(\mathbf{r}, \mathbf{r}')$  refers to the Green's function in free space and  $\{I_n\}$  is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. Also, as usual,  $\mathbf{r}$  and  $\mathbf{r}'$  denote the observation and source point locations.  $E^i(\mathbf{r})$  is the incident excitation plane wave, and  $\eta$  and k denote the free space impedance and wave number, respectively. N is the number of unknowns used to discretize the object.

Once the matrix equation (1) is solved, the expansion coefficients  $\{I_n\}$  can be used to calculate the scattered field and RCS. In the following, we use Z to denote the coefficient matrix in equation (1),  $I = \{I_n\}$  and  $V = \{V_n\}$  for simplicity. Then, the EFIE matrix equation (1) can be symbolically rewritten as

$$ZI = V \tag{3}$$

To solve the above matrix equation by an iterative method, the matrix-vector products are needed at each iteration. Traditionally, a matrix-vector production requires the operation  $\cos t O(N^2)$ .

#### **B.** The theory of MDA-SVD

The multilevel matrix decomposition algorithm originally proposed (MLMDA) was for two-dimensional geometries in [13], which utilizes the idea of equivalent point sources. The extension of this algorithm is presented in [14-19] for analyzing arbitrary three-dimensional geometries. However, it is efficient only for planar or piecewise planar objects and is inefficient for analyzing the general electrically large scatterer. MDA-SVD presented in [7-8] shows a better efficiency by recompressing the matrix of MDA using the SVD technique.

Consider, there exists two subdomains, the first one is an observation box *i* that contains  $m_1$  basis functions; whereas, the second one is a source box *j* that contains  $m_2$  test functions. When the two boxes are sufficiently separated, the impedance matrix associated with them can be expressed using low rank representations [20]. In MDA implementation, the impedance matrix which is gotten through the EFIE of two well-separated regions can be expressed as three small matrices

$$[Z_{mn}] = [U_{mr}][\omega_{rr}]^{-1}[V_{rn}]$$
(4)

where  $[Z_{m n}]$  is the interaction matrix between observation and source subdomains, *r* denotes the number of equivalent RWG sources, which is much smaller than *n* and *m*. Therefore, the matrix-vector product operation of the three matrices is much smaller than the operation of the direct multiplication [16].

Since the matrices  $[U_{mr}]$  and  $[V_{rn}]$  generated by MDA are usually not orthogonal, they may contain redundancies, which can be removed by the following algebraic recompression technique. This method may be regarded as the singular value decomposition optimized for rank-*k* matrices. Utilize QR and SVD to reorthonormalize  $[U_{mr}]$  and  $[V_{rn}]$  and the equation (4) can be obtained as

$$[Z_{mn}] = [U][\omega][V]^T$$
<sup>(5)</sup>

where [U] and [V] are both orthogonal. These techniques can reduce the required amount of storage of MDA, while the asymptotic complexity of the approximation remains the same.

MDA-SVD is one of the most popular methods for analyzing three-dimensional electromagnetic problems, but the far-field matrix assembly time of MDA-SVD is much longer than that of MLFMA. In order to reduce the matrix assembly time of MDA-SVD, a new hybrid method is proposed in the following.

#### **III. FORMULATION**

According to [3-6], MLFMA has been widely used to solve the scattering from the electrically-large size objects. When it is applied into analyzing the scattering from the multi-scale objects where dense discretization is necessary to capture geometric features accurately, the memory usage of MLFMA is very large. MDA-SVD is another popular technique for solving the three dimensional problems in [7-8], but the far-field matrix assembly time of MDA-SVD is much longer than that of MLFMA. In this section, a hybrid method called MDA-SVD-MLFMA is proposed.

Take three dimensional problems into account, both MLFMA and MDA-SVD are based on the data structure of the octree. In Fig.1, the box enclosing the object is subdivided into smaller boxes at multiple levels, in the form of an octal tree. The largest boxes not touching each other are at level 2, while the smallest boxes are at level *L*. The subdivision process runs recursively until the finest level *L*.



Fig. 1. The sketch of the octree structure.

It is well known that when the box size is less than 0.2  $\lambda$ , MLFMA will suffer from the "low frequency breakdown" phenomenon. The relative error of MLFMA and MDA-SVD corresponding to the size of the finest level box is analyzed. The formulation of the relative error is described by

Relative error = 
$$||T - M|| / ||M||_{1}$$

where M denotes the bistatic scattering from PEC sphere computed by Mie series while T is the

bistatic scattering computed by MLFMA or MDA-SVD. The incident direction is  $\theta_i = 0^\circ, \phi_i = 0^\circ$  and the scattered angles vary from

 $0^{\circ}$  to  $180^{\circ}$  in azimuth direction when pitch

angle is fixed at 0°. The number of unknowns is 15918 and the MDA-SVD truncating tolerance is  $10^{-3}$  relative to the largest singular value. Figure 2 shows the relative error of MLFMA will increase greatly when the size of the finest level box is less than 0.2  $\lambda$ . It can be seen that MDA-SVD has an acceptable precision even when the finest level box size is below 0.2  $\lambda$ .



Fig. 2. Relative error of MLFMA and MDA-SVD for a sphere corresponding to the size of finest level box for bistatic scattering.

When the multi-scale problems are analyzed, MDA-SVD is adopted for the level with the box size smaller than 0.2  $\lambda$  while MLFMA is adopted for other cases. The details of the hybrid method are shown in algorithm I.

#### Algorithm I: MDA-SVD-MLFMA

 Grouping on the target to achieve multilevel structures with the largest level *L*. When the number of basis functions of the box is less than or equal to the number of equivalent RWG sources of the box, the finest level L is gained. At each level, the sizes of the boxes are same.

- 2) For each level, determine which algorithm is applied according to the box size. MDA-SVD is adopted for the level with the box size smaller than 0.2  $\lambda$  while MLFMA is adopted for other cases.  $L_{\text{MLFMA}}$  is the level beginning of the MLFMA.
- 3) Calculation of the impedance matrix
  - a) MoM is used to calculate the near interaction impedance matrix.
  - b) From  $l = L : L_{MLFMA}+1$ MDA-SVD is applied to calculate the impedance matrix End
  - c) From  $l = L_{\text{MLFMA}}$ : 2

MLFMA is used to calculate the impedance matrix

End

- 4) Iterative solution of the matrix equation
  - d) The direct matrix-vector production is used to the near interaction impedance matrix.
  - e) From  $l = L : L_{\text{MLFMA}} + 1$

MDA-SVD is applied to speed up matrix-vector production End

f) From  $l = L_{\text{MLFMA}}$ : 2

MLFMA is used to speed up matrix-vector production

End

This new method takes advantages of the virtues of both MLFMA and MDA-SVD, which uses MLFMA to reduce the matrix assembly time of MDA-SVD and utilizes MDA-SVD to alleviate the near field burden of MLFMA. The efficiency of the method is demonstrated by the numerical

results.

In this paper, the FGMRES is used as the iterative solver for the EFIE to further accelerate the convergence [9-12]. Consider the iterative solution of equations of the form Ax=b. The GMRES algorithm with right preconditioning solves the modified system  $AM^{-1}(Mx)=b$ , where the preconditioner *M* is constant. However, in FGMRES, the preconditioner is allowed to vary from one step to another in the outer iteration. We have GMRES for the inner iterations whose preconditioner is chosen as the near interaction of MDA-SVD-MLFMA.

#### **IV. NUMERICAL RESULTS**

To validate and demonstrate the accuracy and efficiency of the proposed MDA-SVD-MLFMA, some numerical results are presented in this section. All the computations are carried out on a personal computer with 1.86 GHz CPU and 1.96GB RAM in single precision and the MDA-SVD truncating tolerance is 10<sup>-3</sup> relative to the largest singular value. The restart number of GMRES is set to be 30 and the stop precision for restarted GMRES is denoted to be 10<sup>-3</sup>. Both the inner and outer restart numbers of FGMRES are 30. The stop precision for the inner and outer iteration in the FGMRES algorithm is 10<sup>-2</sup> and 10<sup>-3</sup>, respectively. The normalized RCS is defined as

$$\frac{RCS(\theta,\phi)}{\lambda^2} = \frac{1}{\pi} \lim_{r \to \infty} \left\{ (kr)^2 \frac{\left| E^s(r,\theta,\phi) \right|^2}{\left| E^i(r,\theta,\phi) \right|^2} \right\} \quad , \qquad (6)$$

for any direction  $(\theta, \phi)$ , where  $E^s$  and  $E^i$  represent the scattered and incident electric fields.

#### A. The plane-cylinder geometry

The multi-scale examples are analyzed in the following. The first multi-scale example is plane-cylinder geometry. The edge length of the square plane is 4 m, the radius of the small column is 0.1 m, and the height of small column is

2 m. The rotation axis is *z*-axis. Here, the small column is densely discretized in comparison to the plane part of the structure. The incident and observed angles are ( $\theta_i = 0^\circ, \phi_i = 0^\circ$ ) and

 $(0^{\circ} \le \phi_s \le 180^{\circ}, \theta_s = 90^{\circ})$ , respectively. The size

of the lowest-level box of the MDA-SVD-MLFMA is 0.16  $\lambda$ , while the size of the lowest-level box of the MLFMA is  $0.33 \lambda$ . Figure 3(a) shows that the result of MDA-SVD-MLFMA agrees very well with the FEKO [21].

Table 1 summarizes the matrix assembly time of and the memory storages the MDA-SVD-MLFMA, MDA-SVD, and MLFMA. "MVP time" in the table indicates the time of one matrix-vector production. It can be observed that assembly time of the matrix the MDA-SVD-MLFMA is half less than that of MDA-SVD and is, also, less than that of MLFMA. The total consumption of memory MDA-SVD-MLFMA is half less than that of MLFMA and is less than that of MDA-SVD. The MVP time of MDA-SVD-MLFMA is, also, much less than that of either MLFMA or MDA-SVD.

Figure 3(b) gives the convergence history curves of MDA-SVD-MLFMA solved with GMRES and FGMRES. In this numerical experiment, GMRES requires 6190 s with 5896 iterative steps, while FGMRES requires only 667 s with 46 outer iterative steps. The solving time of GMRES is 9 times longer than that of FGMRES in this example.



Fig. 3. (a) Bistatic scattering cross section of plane-cylinder geometry. (b) Convergence histories of MDA-SVD-MLFMA solved with GMRES and FGMRES.

Table 1: The total memory, the matrix assembly time, and one matrix-vector multiplication time of MLFMA, MDA-SVD, and MDA-SVD-MLFMA of plane-cylinder geometry

| Frequency<br>(MHz) | Unknowns | Algorithms    | Matrix assembly<br>time (s) | Memory<br>(MB) | MVP time (s) |
|--------------------|----------|---------------|-----------------------------|----------------|--------------|
| 200                | 28756    | MDA-SVD-MLFMA | 400                         | 458            | 1.05         |
|                    |          | MDA-SVD       | 1101                        | 666            | 1.53         |
|                    |          | MLFMA         | 415                         | 968            | 1.70         |

#### **B.** The missile geometry

The second multi-scale example is missile geometry. The height of the cylinder is 4.7 m, and the radius of the cylinder is 0.5 m. The rotation axis of missile geometry is *z*-axis. The incident and scattered angles are ( $\theta_i = 0^\circ, \phi_i = 0^\circ$ ) and

 $(0^{\circ} \le \phi_{\circ} \le 180^{\circ}, \theta_{\circ} = 90^{\circ})$ , respectively. The size

of the lowest-level box of the MDA-SVD-MLFMA is 0.14  $\lambda$ , while the size of the lowest-level box of MLFMA is 0.29  $\lambda$ . Table 2 shows the matrix assembly time, the memory storages, and MVP time of MDA-SVD-MLFMA, MDA-SVD, and MLFMA. Again, the matrix assembly time of MDA-SVD-MLFMA is half less than that of MDA-SVD and is, also, less than that MLFMA. The memory of usage of MDA-SVD-MLFMA is half less than that of MLFMA and is less than that of MDA-SVD. The MVP time of MDA-SVD-MLFMA is also much less than that of MLFMA and MDA-SVD. The bistatic RCS by use of MDA-SVD-MLFMA is shown in Fig. 4(a), and is agreed well with FEKO. The convergence curves are plotted for MDA-SVD-MLFMA solved with GMRES and FGMRES in Fig. 4(b). GMRES requires 7964 s with 4958 iterative steps, while FGMRES requires only 1095 s with 65 outer iterative steps. The solving time of GMRES is 7 times longer than that of FGMRES, in this example.



Fig. 4. (a) Bistatic scattering cross section of missile geometry. (b) Convergence histories of MDA-SVD-MLFMA solved with GMRES and FGMRES.

Table 2: The total memory, the matrix assembly time, and one matrix-vector multiplication time of MLFMA, MDA-SVD, and MDA-SVD-MLFMA of missile geometry

| Frequency<br>(MHz) | Unknowns | Algorithms    | Matrix assembly<br>time (s) | Memory<br>(MB) | MVP time (s) |
|--------------------|----------|---------------|-----------------------------|----------------|--------------|
| 300                | 38145    | MDA-SVD-MLFMA | 600                         | 638            | 1.60         |
|                    |          | MDA-SVD       | 1757                        | 949            | 2.01         |
|                    |          | MLFMA         | 643                         | 1272           | 2.22         |

#### C. The VIAS geometry

The third multi-scale example is the VIAS geometry [22]. The geometry fits within a cuboid with aspect radio 6:5:0.5 and the maximum dimension is  $6\lambda$  at 300 MHz. The incident and scattered angles are ( $\theta_i = 0^\circ, \phi_i = 0^\circ$ ) and

 $(0^{\circ} \le \phi_s \le 180^{\circ}, \theta_s = 90^{\circ})$ , respectively. The size

of the lowest-level box of the MDA-SVD-MLFMA is 0.18  $\lambda$ , while the size of the lowest-level box of MLFMA is 0.37  $\lambda$  . The matrix assembly time, the memory storages and MVP time of MDA-SVD-MLFMA, MDA-SVD, and MLFMA are shown in Tab. 3. The matrix assembly time of MDA-SVD-MLFMA is much less than that of MDA-SVD and is, also, less than that of MLFMA, while the memory usage of MDA-SVD-MLFMA is much less than that of MLFMA and is less than that of MDA-SVD. The bistatic RCS by use of MDA-SVD-MLFMA is shown in Fig. 5(a), and is agreed well with that of MLFMA and MDA-SVD. The convergence curves are plotted for MDA-SVD-MLFMA solved with GMRES and FGMRES in Fig. 5(b). GMRES requires 2403 s with 1190 iterative steps, while FGMRES requires only 411 s with 84 outer iterative steps. The solving time of GMRES is 5 times longer than that of FGMRES in this example.



(b)

Fig. 5. (a) Bistatic scattering cross section of VIAS geometry. (b) Convergence histories of MDA-SVD-MLFMA solved with GMRES and FGMRES.

| Frequency | Unknowns    | Algorithms    | Matrix assembly | Memory |              |
|-----------|-------------|---------------|-----------------|--------|--------------|
| (MHz)     | Ulikilowiis | Aigoritinis   | time (s)        | (MB)   | MVP time (s) |
| 300       | 61099       | MDA-SVD-MLFMA | 811             | 661    | 2.02         |
|           |             | MDA-SVD       | 2631            | 922    | 2.43         |
|           |             | MLFMA         | 827             | 1284   | 2.90         |

Table 3: The total memory, the matrix assembly time, and one matrix-vector multiplication time of MLFMA, MDA-SVD, and MDA-SVD-MLFMA of VIAS geometry

In the memory of summary, MDA-SVD-MLFMA is much less than that of MLFMA and the matrix assembly time of MDA-SVD-MLFMA is much less than that of MDA-SVD. The matrix-vector multiplication of MDA-SVD-MLFMA is more efficient than that of either MLFMA or MDA-SVD. It can be applied to the monostatic RCS calculation of the complex object in future. The MDA-SVD-MLFMA is much more efficient than either MLFMA or MDA-SVD for the multi-scale problems. It is observed that the convergence rate of GMRES is remarkably accelerated by the application of FGMRES algorithm.

#### V. CONCLUSIONS

In this paper, a new efficient hybrid method named MDA-SVD-MLFMA is proposed. The MDA-SVD-MLFMA takes advantage of the virtues of both MLFMA and MDA-SVD, which uses MLFMA to reduce the matrix assembly time of MDA-SVD and utilizes MDA-SVD to alleviate the near field burden of MLFMA. The numerical results demonstrate that the memory of MDA-SVD-MLFMA is much less than that of MLFMA and the matrix assembly time of MDA-SVD-MLFMA is much less than that of MDA-SVD. It is observed that the convergence rate of GMRES is remarkably accelerated by the application of **FGMRES** algorithm. MDA-SVD-MLFMA combined with FGMRES is very efficient for analyzing the multi-scale problems.

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### Parallel SAI Preconditioned Adaptive Integral Method For Analysis of Large Planar Microstrip Antennas

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- An efficient parallel Abstract sparse approximate inverse (PSAI) preconditioning of the adaptive integral method (AIM) is proposed to analyze the large-scale planar microstrip antennas. The PSAI preconditioner is based on the parallelized Frobenius-norm minimization, and is used to speed up the convergence rate of the loose generalized minimal residual method (LGMRES) iterative solver. The parallel AIM is used to accelerate the required matrix vector product operations. Numerical results demonstrate that the PSAI preconditioner is effective with the AIM and can increase the parallel efficiency significantly when analyzing the large planar microstrip antennas.

*Index Terms* – Adaptive integral method, microstrip antennas, parallel sparse approximate inverse.

#### I. INTRODUCTION

In electromagnetic (EM) calculations, the calculation of the currents generated on the surface of an object when illuminated by a given incident plane wave or fed by a microstrip line is essential for the simulation of microstrip structures. For the microstrip structures, the finite element method (FEM), and the finite-difference time-domain (FDTD) method often have a large number of unknowns due to the volumetric discretization. The method of moments (MoM) is a preferred method to solve this problem, since by using the integral equation (IE) and the layered media Green's functions, it only discretizes the metallic surface, which leads to a relative small number of unknowns. The implementation of the MoM requires  $O(N^3)$  operations and  $O(N^2)$  memory storage [1], where N is the number of unknowns.

The size of the MoM matrix increases so rapidly with the increase of the number of unknowns that the computation will be intractable for the computational capacity. The difficulty can be overcome by use of Krylov iterative methods, and the required matrix-vector product operations can be accelerated by AIM [2]. The application of AIM reduces the memory requirement to O(N) and the computational complexity to O(NlogN).

With the increase of the dimensions of the object to be solved, the impedance matrix associated with the linear systems becomes larger. The computation will be time consuming for a processor. Fortunately, single the parallel technique is applied in many fast EM methods to circumvent the above difficulty. The parallel multilevel fast multipole method (MLFMA) is proposed in [3, 4] for the solution of scattering from large-scale objects, the mpi-based parallel precorrected fast Fourier transform (FFT) algorithm is proposed in [5] for analyzing scattering from arbitrary shaped three-dimensional objects, a methodology for designing a high performance parallel 3-D finite element method (FEM) is proposed in [6]. All of these techniques provide an efficient parallel scheme to deal with the large-scale problems.

Although the computational complexity and memory requirement is decreased in the fast EM methods and the computational capacity is increased by use of the parallel technique, the number of iterations needed to achieve the desired precision remains the same as the original MoM. It is natural to use preconditioning techniques [7-9] to improve the convergence of the linear systems. There are many preconditioning techniques. Simple diagonal or diagonal blocks of the impedance matrix can be parallelized easily, while they are effective only when the matrix has some degree of diagonal dominance [10]. Block diagonal preconditioner is generally more robust, which requires matrix permutations or renumbering of the grid points to cluster the large entries close to the diagonal. Incomplete LU (ILU) has been used for solving nonsysmmetric dense systems [11] and the threshold-based incomplete LU (ILUT) has been applied in the AIM [12], while the factorization is often ill conditioned that leads to the triangular solvers unstable. Shifted symmetric successive over-relaxation (SSOR) [13] and spectral two-step preconditioning techniques [14] are efficient while they are difficult to be parallelized. The SAI preconditioner based on Frobenius-norm minimization is chosen in this paper because it allows the decoupling of the constrained minimization problem into independent linear least-squares problems for each rows of the preconditioner. This is convenient to be used by parallelization. Recently, PSAI is proposed to be combined with MLFMA for solving scattering problems with a large number of unknowns [15, 16]. For layered media spatial domain Green's function, using MLFMA is much more involved than in the surface scattering problems where the free space Green's would be employed [17]. Therefore, in this paper a synthesis of the AIM and PSAI preconditioning technique is proposed for analyzing large-scale planar microstrip antennas. To the best of our knowledge, it is the first time that the parallel SAI preconditioning technique was applied into the AIM in our work. As a result, the parallel efficiency is greatly enhanced.

The paper is organized as follows. Section 2 describes the essential algorithms for the analysis of the planar microstrip antennas. The workflow of the parallel AIM is described in section 2.1; the construction of the PSAI preconditioner in the parallel AIM is described in section 2.2. Numerical results in section 3 demonstrate the efficiency of the proposed method. Finally, a brief conclusion is given in Section 4.

#### **II. THEORY**

#### A. The parallel AIM algorithm

For the sake of brevity, the summary, and sequence of the operations in the parallel AIM will be only described, the basic principles of the AIM can be found in [2]. It is known that the computation of the AIM technique mainly contains four parts: evaluate the near field impedance matrix, evaluate the expansion coefficients (i.e. projecting the RWG basis functions onto the rectangular matrix-vector grids). evaluate the product operations of the near field sparse matrix, and evaluate matrix-vector product operations of the far field. The AIM is able to be parallelized, since the above four operations are independent and there is independence in each operations. In this paper, the computation task is decomposed by distributing an equal number of unknowns to each processor for balancing the four operations simultaneously. The scheme is described below:

Step 1. The elements of the near field matrix are evaluated by MPIE and stored in a compressed sparse column format. The basis functions are distributed equally to every processor, and the relative elements of the near field impedance matrix are computed and stored, respectively. There is no inner-processor communications in this step.

Step 2. The basis functions are mapped onto a regular grid in order to apply the FFT to a Toeplitz matrix for speedup the matrix-vector product operations. The solution can be highly parallelized, since every basis function is independent during computation. As step1, the basis functions are distributed equally to every processor. Only part of the expansion coefficients just need to be computed for each processor.

Step 3. The direct matrix-vector product operations of the near field sparse matrix are parallelized as step 1 and step 2 by distributing the basis functions. However, frequent inner-processor communications are required during the iterative solution of the linear system, which leads to frequent inner-processor communications. The inner-processor communications will take the most of the computational time if the number of unknowns is small, and leads to a low parallel efficiency.

Step 4. The matrix-vector product operation of the Toeplitz matrix of the far field is accelerated by FFT. In the code, the two dimensional Toeplitz matrix is first extended to a one dimensional circulant matrix, and the one dimensional FFT [18] is implemented in the circulant matrix, thus the matrix-vector product operation of the far field can be parallelized by the parallel one dimensional FFT. It should be noted that the inner-processor communication time and the synchronization time should, also, be considered as step 3.

From the above steps, it is found that there is a balance between the computational time and innerprocessor communication time. For objects with a small number of unknowns, the inner-processor communication takes the most of the total solution time which leads to a low efficiency of the parallel algorithm. For objects with a large number of unknowns, the computation takes the most of the total solution time which leads to a high efficiency of the parallel algorithm. Since a fast convergence solution of the liner systems plays an important role for the efficiency of the parallel scheme, the PSAI preconditioning technique will be proposed in detail in the next part.

#### **B.** The PSAI preconditioning technique

First, we consider the SAI preconditioner combined with the AIM. In the context of AIM, the  $N \times N$  dense impedance matrix is decomposed as  $Z=Z_s + Z_{AIM}$  [2], where  $Z_s = Z_{near} - Z_{near}$  AIM.  $Z_{near}$  is the MoM interaction between elements,  $\mathbf{Z}_{near AIM}$  is the inaccurate contribution from grid and  $\mathbf{Z}_{AIM}$  is the matrix related to interaction from far field. Since  $\mathbf{Z}_s$  is already stored in the memory,  $\mathbf{Z}_s$  is chosen instead of  $\mathbf{Z}_{\text{near}}$  for the construction of the SAI preconditioner and the approximation is of the form  $\mathbf{M} \approx \mathbf{Z}_{s}^{-1}$ . It is found that the iterative steps of the LGMRES with SAI preconditioner constructed by  $\mathbf{Z}_{s}$  are similar with constructed by  $\mathbf{Z}_{near}$ , and it is shown in section 3. The SAI preconditioner in this paper is based on a Frobenius-norm minimization. The approximate inverse of  $\mathbf{Z}_s$  is computed by minimizing

$$\left\|\mathbf{I} - \mathbf{M}\mathbf{Z}_{s}\right\|_{F}.$$
 (1)

The Frobenius norm is usually chosen because it allows the decoupling of the constrained minimization problem into n independent linear least-squares problems for each row of M:

$$\left\|\mathbf{I} - \mathbf{M}\mathbf{Z}_{s}\right\|_{F}^{2} = \left\|\mathbf{I} - \mathbf{M}\mathbf{Z}_{s}^{T}\right\|_{F}^{2} = \sum_{j=1}^{n} \left\|e_{j} - \mathbf{Z}_{s}m_{j}\right\|_{2}^{2}, \quad (2)$$

where  $e_j$  is the *j* th unit vector and  $m_j$  is the column vector representing the *j* th row of **M**.

The main issue for the computation of the SAI preconditioner is the selection of the nonzero pattern of **M** that is the set of indices:

$$S = \{(i, j) \in [1, N^2] \text{ s.t. } m_{ii} \neq 0\}.$$
 (3)

If the sparsity of  $\mathbf{M}$  is known, the none zero structure for the *j* th column of  $\mathbf{M}$  can be automatically determined and defined as

$$J = \{i \in [1, N] \text{ s.t.} (i, j) \in S\}.$$
 (4)

The solution of (2) involves only the columns of  $\mathbf{Z}_s$  indexed by J, which can be denoted by  $\mathbf{Z}_s(:,J)$ . Since  $\mathbf{Z}_s$  is sparse, many rows in  $\mathbf{Z}_s(:,J)$ are usually null, not affecting the solution of the least-square problems. Thus, if I is the set of indices corresponding to the nonzero rows in  $\mathbf{Z}_s(:,J)$ , and if we defined  $\tilde{\mathbf{Z}} = \mathbf{Z}_s(I,J)$  by  $\tilde{m}_j = m_j(J)$  and  $\tilde{e}_j = e_j(J)$ , the "reduced" least-square problems to solve are

$$\min \left\| \tilde{\boldsymbol{e}}_{j} - \tilde{\boldsymbol{Z}}_{s} \tilde{\boldsymbol{m}}_{j} \right\|_{2}^{2}, \qquad j = 1, \dots N.$$
(5)

In general, the size of problems (5) is much smaller than problems (2). The above procedure is shown clearly in Figure 1.

As shown in Figure 1(a), "X" stands for none zero impedance matrix elements, and  $m_4$  is chosen to be computed as an example. For row 4, since the none zero columns are 2, 4, 6, 8, the rows of 2, 4, 6, 8 in Figure 1(a) are chosen. Then, the submatrix in Figure 1(b) is obtained for constructing the preconditioner. Excluding the columns with null elements in the above submatrix, after which the final least matrix in Figure 1(c) will be obtained. It can be seen that the size of the "reduced" matrix in Figure 1(c) is much smaller than the original matrix in Figure 1(a). Although the null columns are excluded, the size of the "reduced" matrix in (5) is still very large for the targets with a large number of unknowns. The difficulty can be circumvented with a prior sparsity pattern selection strategy. In the code, a constant number  $K_{\text{max}}$  is set to select the most informative elements in each row of Zs for the construction of the preconditioner by checking the value of the elements [19, 20].

The core of PSAI preconditioner is to store the minimum near field impedance matrix in each processor. The PSAI preconditioner is constructed by three steps. First, the elements of the sparse near field impedance matrix are distributed evenly on the processors. Second, the required elements which are distributed on the other processors are appended on the present processor by the all-to-all communication of the rows and columns list and the value of the elements, and the process is shown in Figure 2 to Figure 4. Third, the least-squares minimizations are solved independently on each processor as the construction of the SAI preconditioner described above.

|   | 1 | 2 | 3 | 4 | 5   | 6 | 7 | 8 |
|---|---|---|---|---|-----|---|---|---|
| 1 | Х | Х |   |   | Х   |   |   |   |
| 2 | Х | Х |   | Х |     |   |   | Х |
| 3 |   |   | Х |   |     |   |   |   |
| 4 |   | Х |   | Х |     | Х |   | Х |
| 5 | Х |   |   |   |     |   |   |   |
| 6 |   |   |   | Х |     |   |   | Х |
| 7 |   |   |   |   |     |   |   |   |
| 8 |   | Х |   | Х |     | Х |   | Х |
|   |   |   |   |   | (a) |   |   | - |

|   | 1 | 2 | 3 | 4   | 5 | 6 | 7 | 8 |
|---|---|---|---|-----|---|---|---|---|
| 2 | Х | Х |   | Х   |   |   |   | Х |
| 4 |   | Х |   | Х   |   | Х |   | Х |
| 6 |   |   |   | Х   |   |   |   | Х |
| 8 |   | Х |   | Х   |   | Х |   | Х |
|   |   |   |   | (b) |   |   |   |   |

|   | 1   | 2 | 4 | 6 | 8 |  |  |  |  |  |
|---|-----|---|---|---|---|--|--|--|--|--|
| 2 | Х   | Х | Х |   | Х |  |  |  |  |  |
| 4 |     | Х | Х | Х | Х |  |  |  |  |  |
| 6 |     |   | Х |   | Х |  |  |  |  |  |
| 8 |     | Х | Х | Х | Х |  |  |  |  |  |
|   | (c) |   |   |   |   |  |  |  |  |  |

Fig. 1. The configuration of the SAI, (a) the original impedance matrix, (b) the selected rows of the matrix for construction of the SAI preconditioner, (c) the reduced impedance matrix for construction of the SAI preconditioner.

The above algorithm is shown in Figure 5 clearly. Considering the impedance matrix of the near field, the elements of the matrix  $Z_s$  in rows 1-4 and 5-8 are distributed in two processors respectively.  $m_4$  is to be solved as discussed above. The elements of rows 2, 4, 6, 8 of the impedance matrix should be selected, while rows 6 and 8 are in the other processor; thus, the necessary inner-processor communications are implemented to append rows 6 and 8 to the first processor as shown in Figure 2(c). After the procedure of the all-to-all communication, all the processors store the

minimum elements of the matrix  $\mathbf{Z}_{s}$ . It should be noted that the inner-processor communication in the construction of PSAI preconditioner is small since there is a trade-off for controlling the selected number of elements in each row of Zs. The flow chart of the PSAI preconditioned AIM is shown in Figure 6 clearly. As shown in Figure 6, there is inner-processor communication in the process to evaluate the near field, project the unknowns onto the regular grid. And there is inner-processor communication in the process of construction of the SAI preconditioner and the matrix vector multiplication.



Fig. 2. The process that finds and exchanges the rows list of the elements should be appended of each processor, where rowSendList(P), rowRecvList(P) is the row list to be sent and received of processor *p*.

| For $i \in rowSendList$ , do                |  |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|--|
| Append column indices of row $i$ to         |  |  |  |  |  |  |  |  |
| sendColIndices                              |  |  |  |  |  |  |  |  |
| Endfor                                      |  |  |  |  |  |  |  |  |
| Send sendColIndices , receive               |  |  |  |  |  |  |  |  |
| recColIndices at the same time ! All-to-All |  |  |  |  |  |  |  |  |
| communication                               |  |  |  |  |  |  |  |  |

Fig. 3. The process that finds and exchanges the columns list of the elements should be appended of each processor, where *sendColIndices*, *recColIndices* is the column list to be sent and received.

| For $i \in rowSendList$ , do              |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|
| Append the matrix element $z_{ij}$ of row |  |  |  |  |  |  |  |
| i to sendColValues                        |  |  |  |  |  |  |  |
| Endfor                                    |  |  |  |  |  |  |  |
| Send sendColValues , receive              |  |  |  |  |  |  |  |
| recvColValues at the same time !          |  |  |  |  |  |  |  |
| All-to-All communication                  |  |  |  |  |  |  |  |

Fig. 4. The process that finds and exchanges the values of the elements to be sent and received by processor *p*, where *sendcolIndices*, *reccolIndices* is the value of the elements to be sent and received.

|   | 1 | 2 | 3 | 4   | 5 | 6 | 7 | 8 |
|---|---|---|---|-----|---|---|---|---|
| 1 | Х | Х |   |     | Х |   |   |   |
| 2 | Х | Х |   | Х   |   |   |   | Х |
| 3 |   |   | Х |     |   |   |   |   |
| 4 |   | Х |   | Х   |   | Х |   | Х |
| 5 | Х |   |   |     |   |   |   |   |
| 6 |   |   |   | Х   |   |   |   | Х |
| 7 |   |   |   |     |   |   |   |   |
| 8 |   | Х |   | Х   |   | Х |   | Х |
|   |   |   |   | (a) |   |   |   |   |

|   | 1   | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |
|---|-----|---|---|---|---|---|---|---|--|
| 1 | Х   | Х |   |   | Х |   |   |   |  |
| 2 | Х   | Х |   | Х |   |   |   | Х |  |
| 3 |     |   | Х |   |   |   |   |   |  |
| 4 |     | Х |   | Х |   | Х |   | Х |  |
|   | (b) |   |   |   |   |   |   |   |  |

|     | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |  |
|-----|---|---|---|---|---|---|---|---|--|--|
| 1   | Х | Х |   |   | Х |   |   |   |  |  |
| 2   | Х | Х |   | Х |   |   |   | Х |  |  |
| 3   |   |   | Х |   |   |   |   |   |  |  |
| 4   |   | Х |   | Х |   | Х |   | Х |  |  |
| 6   |   |   |   | Х |   |   |   | Х |  |  |
| 8   |   | Х |   | Х |   | Х |   | Х |  |  |
| (c) |   |   |   |   |   |   |   |   |  |  |

Fig. 5. The configuration of the parallel SAI, (a) the original impedance matrix, (b) the rows of the impedance matrix stored in one processor, (c) the appended minimum impedance matrix been stored in one processor for construction of the PSAI preconditioner.



Fig. 6. The flow chart for the PSAI preconditioned AIM.

#### **III. RESULTS AND DISCUSSIONS**

In this section, some microstrip antennas are analyzed by the PSAI preconditioned AIM. The resultant linear systems are solved by the LGMRES solver [21] and its tolerance is 10<sup>-4</sup>. The results presented here are all computed on 2-node clusters connected with an infiniband network. Each node includes a quad-core Intel processor and 8 GB of RAM.

First, an 8×8 microstrip corporate-fed planar antenna is considered, the parameters are depicted in Figure 6. It is discretized with 61, 345 RWG unknowns. As shown in Figure 7, the reflection coefficients versus frequency simulated by the proposed method and the Ansoft Designer® are plotted. It can be seen the results agree well which demonstrate the accuracy of the proposed method. The total simulation time of one frequency point for the PSAI preconditioned AIM is 2, 512 s, and the time for the Ansoft Designer<sup>®</sup> is 12, 560s. We, also, compare the H-plane far field pattern of the 8  $\times 8$  microstrip antenna at the frequency of 9.42 GHz with the CGFFT approach [22] in Figure 8 to verify the proposed method, where reasonable agreements are observed. The time for constructing the PSAI preconditioner is 613 s and the solution time is 1, 576 s. Figure 9 shows the residual norm histories for the  $8 \times 8$  microstrip corporate-fed planar antenna at 9.42 GHz simulated by AIM with and without the PSAI preconditioner. The label "PSAI-0" represents the PSAI preconditioner constructed by Znear, and "PSAI-1" represents the PSAI preconditioner constructed by  $Z_s$ . It is found that the LGMRES PSAI-0 and PSAI-1 have the similar iterative steps which can greatly improve the convergence by a factor of 3.8. In order to describe the proposed method clearly, the label "PSAI" shown below denotes the PSAI preconditioner constructed by  $Z_s$ . The advantage of the PSAI preconditioner can be found significantly when solving large dense linear systems with multiple right-hand sides arising in monostatic RCS, since the PSAI preconditioner.



Fig. 7. The geometry of the 8 × 8 microstrip corporate-fed planar antenna,  $L_1$ =12.32mm,  $L_2$ =18.48mm, W=10.08mm, L=11.79mm,  $d_1$ =1.3mm,  $d_2$ =3.93mm,the thickness of substrate h=1.59mm,  $\varepsilon_r$  = 2.2.



Fig. 8. The reflection coefficients versus frequency for the  $8 \times 8$  corporate-fed planar antenna.



Fig. 9. The H-plane far field pattern of the  $8 \times 8$  microstrip corporate-fed planar antenna compared with [22].



LGMRES iterative steps

Fig. 10. Residual norm histories for the  $8 \times 8$  microstrip corporate-fed planar antenna at 9.42 GHz simulated by LGMRES with and without the PSAI preconditioner.

To examine the parallel efficiency of the proposed method, the monostatic RCS of a series of microstrip antennas [23] are simulated. The layout of the microstrip antennas are shown in Figure 10, the configuration of the unit of the arrays is L = 3.66cm, W = 2.60cm, a = b = 5.517cm, the dielectric constant and the thickness of the substrates is  $\varepsilon_r = 2.17$ , d = 0.158cm. The microstrip antennas with  $7 \times 7$ ,  $20 \times 20$  arrays are simulated. The number of unknowns is 8, 428 and 84, 000, respectively. Table 1 and Table 2 list the CPU time of the above antennas simulated by AIM with and without the PSAI preconditioner at

the frequency of 3.7 GHz. The notations used in the tables are denoted below.

• *near* denotes the time used for filling the near field impedance matrix.

• *cof* denotes the time used for computing the expansion coefficients.

• *set* denotes the time used for constructing the PSAI preconditioner.

• *steps* denotes the average number of iterative steps of the LGMRES for  $\varphi = 0^{\circ}$  and  $\theta$  varying from 0° to 85°.

• *sol* denotes the time used for solving the liner systems.

• *tol* denotes the total CPU time for simulation.

• *mem* denotes the memory usage for one processor.

• *ef* denotes the efficiency of the parallelization which is defined as  $\frac{T_{Total}}{nT_{nTotal}}$ , where  $T_{Total}$  is the total CPU time computed by one processor,  $T_{nTotal}$  is the total CPU time computed by *n* processors.

As shown in Table 1, the time for filling near field matrix and solving the expansion coefficients in columns 3, 4 is almost a linear reduction with the increase of the number of processors. In column 5, little solving time of the linear system is saved when the number of unknowns is small (i.e. the  $7 \times 7$  arrays), since the inner-processor communication takes the most of the CPU time. Much solving time of the linear systems is saved when the number of unknowns is large (i.e. the 20  $\times$  20 arrays), since the computation takes the most of the CPU time. Similarly, as shown in Table 2, the time for filling the near field matrix, solving the expansion coefficients and construction of the preconditioner in columns 3, 4, and 5 is also linear reduction with the increase of the number of processors.



Fig. 11. The layout of the microstrip antenna.

Little solving time of the linear system in columns 6 is saved, when the number of unknowns is small (i.e. the  $7 \times 7$ ), and much solving time of the linear systems is saved when the number of unknowns is large (i.e. the  $20 \times 20$  arrays). Comparing the memory storage, the total solving time, and the parallel efficiency of Table 1 and Table 2, it can be found that by using the PSAI preconditioner only increases small memory usage while the total solving time and the parallel efficiency are improved significantly due to the decrease of the number of iterative steps of the LGMRES.

Finally, we verify the proposed method by a  $30 \times 30$  microstrip antenna with 231, 300 RWG basis functions. The monostatic RCS computed by the AIM with and without PSAI preconditioner is plotted in Figure 11, where reasonable agreements are observed. The time for constructing the PSAI preconditioner and per iteration is 1, 304, and 5 seconds, respectively. Figure 12 shows the iterative steps of the LGMRES with and without a PSAI preconditioner for the 30 × 30 microstrip arrays. Where  $\varphi = 0^{\circ}$  and  $\theta$  is varying from  $0^{\circ}$  to 85°. It can be found that the PSAI preconditioning technique can greatly improve the convergence by at least a factor of 4.1 compared with no preconditioned LGMRES.

Table 1: The CPU time of the 7×7 and 20×20 arrays simulated by the AIM without PSAI preconditioner

|              | processors | near(sec) | cof(sec) | steps | sol(sec)    | tol(sec)    | mem(Mb) | ef  |
|--------------|------------|-----------|----------|-------|-------------|-------------|---------|-----|
|              | 1          | 312       | 144      |       | 8, 319      | 8, 790      | 43      | -   |
| $7 \times 7$ | 4          | 78        | 36       | 243   | 8,094       | 8, 215      | 30      | 26% |
|              | 8          | 39        | 18       |       | 7,710       | 7, 770      | 25      | 14% |
| 20 ×         | 1          | 12, 724   | 5, 842   | - 257 | 102,<br>017 | 120,<br>684 | 410     | -   |
| 20           | 4          | 3,862     | 1, 367   |       | 47, 355     | 52, 685     | 171     | 57% |
|              | 8          | 1,977     | 723      |       | 43, 771     | 46, 572     | 149     | 32% |
| Processors | near (sec)                         | cof(sec)  | set(se<br>c)   | steps   | sol(sec)  | tol(sec)   | mem(Mb)   | ef  |
|------------|------------------------------------|---|--|---|---|--|---|---|
| 1          | 312                                | 144   | 886  |   | 891   | 2, 241   | 69  | -   |
| 4          | 78                                 | 36  | 222  | 13  | 655   | 996  | 42  | 56%   |
| 8          | 39                                 | 18  | 111  |   | 608   | 782  | 30  | 36%   |
| 1          | 12, 724                            | 5, 842  | 10,<br>251   | 19  | 7, 413  | 36, 588  | 510   | -   |
| 4          | 3, 862                             | 1, 367  | 2,<br>447  |   | 6, 314  | 11, 737  | 210   | 78%   |
| 8          | 1, 977                             | 723   | 1,<br>310  | _   | 4, 830  | 8, 959   | 158   | 51%   |
|            | Processors  1  4  8  1  4  8  8  8 | Processors         near (sec)           1         312           4         78           8         39           1         12, 724           4         3, 862           8         1, 977 | Processors         near (sec)         cof(sec)           1         312         144           4         78         36           8         39         18           1         12, 724         5, 842           4         3, 862         1, 367           8         1, 977         723 | Processorsnear (sec) $cof(sec)$ $set(se)$ 13121448864783622283918111112, 7245, 84210,<br>25143, 8621, 3672,<br>44781, 9777231,<br>310 | Processors         near (sec) $cof(sec)$ $set(se)$ $steps$ 1         312         144         886           4         78         36         222           1         39         18         111           1         12, 724         5, 842         251           4         3, 862         1, 367 $\frac{2}{447}$ 8         1, 977         723         1, 310 | Processorsnear (sec) $cof(sec)$ $set(se)$ $steps$ $sol(sec)$ 13121448868914783622213583918111655112,7245,8422517,41343,8621,367 $\frac{2}{447}$ 196,31481,977723 $\frac{1}{310}$ 4,830 | Processorsnear (sec) $cof(sec)$ $set(se)$ $steps$ $sol(sec)$ $tol(sec)$ 13121448868912, 241478362221365599683918111655996112, 7245, 8422517, 41336, 58843, 8621, 367 $447^2$ 196, 31411, 73781, 977723 $310^1$ 4, 8308, 959 | Processorsnear (sec) $cof(sec)$ $set(se)$ $steps$ $sol(sec)$ $tol(sec)$ $mem(Mb)$ 13121448868912,241694783622213655996428391811160878230112,7245,84210,<br>2517,41336,58851043,8621,367 $\frac{2}{447}$ 196,31411,73721081,977723 $\frac{1}{310}$ 4,8308,959158 |

Table 2: The CPU time of the 7×7 and 20×20 arrays simulated by the AIM with PSAI preconditioner

#### **IV. CONCLUSION**

In this paper, the PSAI preconditioned AIM method is proposed for analyzing the large scale microstrip antennas. The parallel AIM is used to accelerate the matrix vector multiplication. The PSAI is used to improve the iterative convergence of the LGMRES. The PSAI is based on the parallelized Frobenius-norm minimization, and the construction time of the preconditioner is further saved by selecting the most informative elements of the sparse near field impedance matrix. Numerical results prove that by using the PSAI preconditioner. the parallel efficiency is significantly improved.



Fig. 12. The monostatic RCS of the  $30 \times 30$  microstrip arrays simulated by the AIM with and without PSAI preconditioner,  $\varphi = 0^{\circ}$  and  $\theta$  is varying from  $0^{\circ}$  to  $85^{\circ}$ , and the frequency of the incident plane wave is 300MHz.



Fig. 13. The iterative steps of the LGMRES for the 30 × 30 microstrip arrays simulated by the AIM with and without PSAI preconditioner,  $\varphi = 0^{\circ}$  and  $\theta$  is varying from 0° to 85°, and the frequency of the incident plane wave is 300MHz. And the tolerance of LGMRES is 10<sup>-4</sup>.

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# Well-Conditioned CFIE for Scattering from Dielectric Coated Conducting Bodies above a Half-Space

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Abstract — In this paper, a well-conditioned coupled combined-field integral equation called the electric and magnetic current combined-field integral equation (JCFIE-JMCFIE) is proposed for the analysis of electromagnetic scattering from coated targets above a lossy half-space. The halfspace multilevel fast multipole algorithm (MLFMA) is used to reduce computational complexity. The inner-outer flexible generalized minimal residual method (FGMRES) was used as the iterative solver to further speed up the convergence rate. Numerical results were presented to demonstrate the accuracy and efficiency of the proposed method.

*Index Terms* – Electric and magnetic current combined-field integral equation (JMCFIE), flexible generalized minimal residual method (FGMRES), half-space, multilevel fast multipole algorithm (MLFMA).

## I. INTRODUCTION

There is significant interest in scattering from conducting bodies coated with dielectric materials situated in the presence of a lossy half-space [1-5]. It has applications in many domains such as communications, target identification, and so on. One of the principal tools for the analysis of such scattering is the method of moments (MoM) [6]. The electromagnetic integral equation is first discretized into a matrix equation using the Galerkin-based MoM with subdomain basis functions such as Rao-Wilton-Glisson (RWG) functions [7] for triangular patches. When iterative solvers are used to solve the MoM matrix equation; the fast multipole algorithm (FMA) or multilevel fast multipole algorithm (MLFMA) [8-11] can be used to accelerate the calculation of matrix-vector products. The half-space MLFMA differs from the free-space MLFMA. In half-space MLFMA, the near interaction terms are evaluated efficiently via the discrete complex-image method (DCIM) [12, 13]. The far interaction terms are evaluated efficiently by employing the asymptotic form of the dyadic Green's function. Each component of the approximate Green's function is expressed in terms of the direct-radiation term plus radiation from an image source in real space [14]. The former accounts for the radiation of currents into the medium in which it resides, while the latter accounts for interactions with the half-space interface. The half-space MLFMA remains the same computational complexity of O(NlogN) both in RAM and computational requirement (per iteration) as free-space MLFMA.

Although the calculation complexity and memory will be decreased in MLFMA, the number of iterations needed to achieve desired precision cannot be reduced. Actually the number of iterations largely depends on the spectral properties of the integral operator or the distribution of the impedance matrix's eigenvalues. To effectively reduce the number of iterations, there are mainly two methods. One is to construct a new integral equation leading to a well-conditioned matrix equation. The other is to use fast iteration techniques and efficient preconditioning techniques to reduce the condition number of the operator equations. Researchers have investigated various integral equations for 3D coated conducting objects [15-17]. Employing the surface equivalence principle, the problem is formulated in terms of a set of coupled integral equations involving equivalent electric and magnetic surface currents which represent boundary fields. The most familiar formulation for this problem is the Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) integral equation combined with electric field integral equation (EFIE) [18]. The PMCHWT formulation belongs to the integral equations of the first kind and it is found to be free of interior resonances and yields accurate and stable solutions; however, its iteration convergence rate is found to be slow [17]. To solve this problem, a new electric and magnetic combined-field integral equation current (JMCFIE) is developed leading to a well tested system with the RWG basis functions and Galerkin's method for scattering problems in freespace [16, 19-20]. Özgür Ergül and Levent Gürel have, also, applied the JMCFIE formulation with MLFMA for fast analysis of scattering from dielectric objects [19]. Researchers have, also, developed efficient iteration techniques and robust preconditioning techniques for the Krylov subspace iterative methods; among which, the GMRES (generalized minimal residual) method proposed in [21] is the most popular and efficient method for the iterative solution of sparse linear systems with an unsymmetric nonsingular matrix. For the GMRES algorithm, this can be easily accomplished with the help of a rather simple modification of the standard algorithm, referred to as the inner-outer flexible generalized minimum residual method (FGMRES) [21-24]. An important property of FGMRES is that it satisfies the residual norm minimization property over the preconditioned Krylov subspace just as in the standard GMRES algorithm.

The objective of this paper is to achieve a fast and accurate solution to the electromagnetic wave scattering from an arbitrary shaped coated conducting target situated in the presence of a lossy half-space. In this paper, we extended the JMCFIE method combined with MLFMA to efficiently analyze electromagnetic scattering from coated targets above a lossy half-space. This paper is outlined as follows. Section 2 gives an introduction of well-conditioned coupled surface integral equation. Numerical examples are given to demonstrate the accuracy and efficiency of the proposed method in radar cross section (RCS) calculations in Section 3. Section 4 gives some conclusions.

#### **II. FORMULATION AND THEORY**

As shown in Figure 1, the configuration of an

arbitrarily shaped conducting body coated with dielectric materials in half-space. The dielectric parameters of space are  $(\varepsilon_1, \mu_1, \sigma_1)$  and  $(\varepsilon_{half}, \mu_{half}, \sigma_{half})$ . As shown in Figure 1, the dielectric surface is  $S_d$ , and the metallic surface is  $S_c$ . The composite structure is illuminated by an incident plane wave  $(E^{inc}, H^{inc})$ . By invoking the equivalence principle [18], two equivalent problems are formulated, each valid for regions external and internal to the dielectric material as shown in Figure 1.



Fig. 1. Configuration of an arbitrarily shaped conducting body coated with dielectric materials (a) original problem (b) outer problem (c) inner problem.

In the equivalence problem for the external region, dielectric surface  $S_d$  is replaced by a fictitious mathematical surface. The entire space is filled with external medium  $(\varepsilon_1, \mu_1, \sigma_1)$ . The field inside the surface  $S_d$  is set to zero. Equivalent electric current  $\mathbf{J}_d$  and equivalent magnetic current  $\mathbf{M}_d$  are presented on  $S_d$ . The so-called T equations T-EFIE<sub>1</sub> and T-MFIE<sub>1</sub> for the exterior equivalent problem can be derived by taking the tangential boundary continuity conditions on the object's surface [18]:

$$\left(\mathbf{E}_{1}^{inc} + \mathbf{E}_{1}^{ref}\right)\Big|_{tan} = \left[-\mathbf{E}_{1}^{s}\right]\Big|_{tan}$$
(1)

$$\left(\mathbf{H}_{1}^{inc} + \mathbf{H}_{1}^{ref}\right)\Big|_{tan} = \left[-\mathbf{H}_{1}^{s}\right]\Big|_{tan}$$
(2)

$$\mathbf{E}_{1}^{s} = -(j\omega\mu_{1}\int_{T'}\overline{\overline{\mathbf{K}}}^{A}\cdot\mathbf{J}_{d}dS' - \frac{\nabla}{j\omega\varepsilon_{1}}\int_{T'}K^{\phi e}\nabla_{s}^{'}\cdot\mathbf{J}_{d}dS' - \int_{T'}\overline{\overline{\mathbf{G}}}^{EM}\cdot\mathbf{M}_{d}dS' + \frac{1}{2}\mathbf{n}_{1}\times\mathbf{M}_{d})$$
(3)

$$\mathbf{H}_{1}^{s} = -(j\omega\varepsilon_{1}\int_{T'}\overline{\mathbf{\bar{K}}}^{F} \cdot \mathbf{M}_{d}dS' - \frac{\nabla}{j\omega\mu_{1}}\int_{T'}K^{\phi m}\nabla_{s}' \cdot \mathbf{M}_{d}dS' - \int_{T'}\overline{\mathbf{\bar{G}}}^{HJ} \cdot \mathbf{J}_{d}dS' + \frac{1}{2}\mathbf{n}_{1} \times \mathbf{J}_{d}),$$
(4)

where the half-space Green's functions  $\overline{\mathbf{\bar{K}}}^{\scriptscriptstyle A}$   $\overline{\mathbf{\bar{K}}}^{\scriptscriptstyle F}$  are for the vector potentials and  $K^{\phi e}$   $K^{\phi m}$  are for the scalar potentials, meanwhile,  $\overline{\overline{\mathbf{G}}}^{EM}$  is dyadic Green's function for electric field due to magnetic currents and  $\overline{\mathbf{G}}^{\scriptscriptstyle HJ}$  is for magnetic field due to electric current. For detailed spectral domain expressions of these half-space Green's functions please refer to [25]. In general, the spatial domain Green's functions are expressed in terms of Sommerfeld integrals. Due to the highly oscillatory nature of the integrand, numerical integration is very time consuming. In this paper, a two-level generalized pencil of function (GPOF) method is utilized to realize DCIM [13]. In the two-level GPOF, dense sampling is adopted to get full information in the quick variant area where  $k_{o}$  is relative small, while a sparse sampling in the slow variant area. In this way, high precision can be realized with fewer samples [13]. Then the spatial domain Green's functions can be obtained in closed forms from their spectral-domain counterparts via the Sommerfeld identity. By operating with  $\hat{n}_1 \times$  to (1) and (2) on the object's surface the so-called N equations, N-EFIE<sub>1</sub> and N-MFIE<sub>1</sub>, for the exterior equivalent problem can be derived

$$\mathbf{n}_{1} \times \left( \mathbf{E}_{1}^{inc} + \mathbf{E}_{1}^{ref} \right) = \mathbf{n}_{1} \times \left( -\mathbf{E}_{1}^{s} \right)$$
(5)

$$\mathbf{n}_{1} \times \left( \mathbf{H}_{1}^{inc} + \mathbf{H}_{1}^{ref} \right) = \mathbf{n}_{1} \times \left( -\mathbf{H}_{1}^{s} \right).$$
(6)

In the equivalence problem for the interior region in Figure 1(c), the medium outside is given the same material parameters  $(\varepsilon_2, \mu_2, \sigma_2)$  as the coated dielectric medium. The conducting surfaces located inside the dielectric body are also replaced by fictitious mathematical surfaces. The equivalent electric current  $-\mathbf{J}_d$  and equivalent magnetic current  $-\mathbf{M}_d$  are presented on  $S_d$ . The equivalent current  $\mathbf{J}_c$  is introduced on the conducting surface  $S_c$ . Based on the boundary conditions, T-EFIE<sub>2</sub> and T-MFIE<sub>2</sub> are presented:

On dielectric surface  $S_d$ :

$$\mathbf{E}_{2}^{inc}\Big|_{tan} = \left(-\mathbf{E}_{2}^{s}\right)\Big|_{tan} \tag{7}$$

$$\left. \mathbf{H}_{2}^{inc} \right|_{tan} = \left( -\mathbf{H}_{2}^{s} \right) \right|_{tan}.$$
(8)

On metallic surface  $S_c$ :

$$\mathbf{E}_{2}^{inc}\Big|_{tap} = \left(-\mathbf{E}_{2}^{s}\right)\Big|_{tap} \tag{9}$$

$$\mathbf{E}_{2}^{s} = - \begin{pmatrix} j\omega\mu_{2} \int_{T^{*}} g_{2} \cdot (-\mathbf{J}_{d}) dS' - \frac{\nabla}{j\omega\varepsilon_{2}} \int_{T^{*}} g_{2} \nabla_{s}^{'} \cdot (-\mathbf{J}_{d}) dS' + \int_{T^{*}} \nabla g_{2} \times (-\mathbf{M}_{d}) dS' \\ + \frac{1}{2} \mathbf{n}_{2} \times (-\mathbf{M}_{d}) + j\omega\mu_{2} \int_{T^{*}} g_{2} \cdot (\mathbf{J}_{c}) dS' - \frac{\nabla}{j\omega\varepsilon_{2}} \int_{T^{*}} g_{2} \nabla_{s}^{'} \cdot (\mathbf{J}_{c}) dS' \\ \end{pmatrix}$$

$$(10)$$

$$\mathbf{H}_{2}^{s} = - \begin{pmatrix} j\omega\varepsilon_{2} \int_{T^{*}} g_{2} \cdot (-\mathbf{M}_{d}) dS' - \frac{\nabla}{j\omega\mu_{2}} \int_{T^{*}} g_{2} \nabla_{s}^{'} \cdot (-\mathbf{M}_{d}) dS' \\ - \int_{T^{*}} \nabla g_{2} \times (-\mathbf{J}_{d}) dS' - \frac{1}{2} \mathbf{n}_{2} \times (-\mathbf{J}_{d}) - \int_{T^{*}} \nabla g_{2} \times (\mathbf{J}_{c}) dS' - \frac{1}{2} \mathbf{n}_{2} \times (\mathbf{J}_{c}) \end{pmatrix}$$

$$(11)$$

where  $g_2$  is the Green's function of infinity dielectric free-space. By operating with  $\hat{n}_2 \times \text{to}$  (7), (8), and (9), N-EFIE<sub>2</sub> and N-MFIE<sub>2</sub> for the exterior equivalent problem can be derived: On dielectric surface  $S_d$ :

 $\mathbf{n}_2 \times \mathbf{E}_2^{inc} = \mathbf{n}_2 \times \left(-\mathbf{E}_2^s\right) \tag{12}$ 

$$\mathbf{n}_2 \times \mathbf{H}_2^{inc} = \mathbf{n}_2 \times \left(-\mathbf{H}_2^s\right). \tag{13}$$

On metallic surface  $S_c$ :

$$\mathbf{n}_2 \times \mathbf{E}_2^{inc} = \mathbf{n}_2 \times \left(-\mathbf{E}_2^s\right). \tag{14}$$

Note that  $\hat{n}_1 = -\hat{n}_2 = \hat{n}$ . The negative sign is due to the direction of the unit normal vector pointing into the external medium.

# A. EFIE-PMCHWT2 formulation for scattering problems in half-space

Using formulation (1)-(2), (7)-(9), the traditional PMCHWT combined with EFIE can be formed as follows:

$$\begin{cases} T-EFIE_2 \\ T-EFIE_1 - T-EFIE_2 \\ T-MFIE_1 - T-MFIE_2 \end{cases}$$
 (15)

After it is expanded and tested with the RWG basis functions  $f_n$ , a matrix equation will be:

$$\begin{bmatrix} \begin{bmatrix} Z_{nm}^{J_c J_c} \\ Z_{nm}^{J_d} \end{bmatrix} \begin{bmatrix} Z_{nm}^{J_d J_d} \end{bmatrix} \begin{bmatrix} T_{nm}^{J_c M_d} \\ T_{nm}^{J_d M_d} \end{bmatrix} \begin{bmatrix} I_{cn} \\ I_{dn} \\ M_{dn} \end{bmatrix} = \begin{bmatrix} V_{cm} \\ V_{dm} \\ H_{dm} \end{bmatrix}.$$
(16)

In the equation,  $V_{cm}$ ,  $V_{dm}$ , and  $H_{dm}$  is the incident field and reflect field summation.  $\mathbf{J}_d = \sum_{n=1}^{N_d} I_{dn} \mathbf{f}_n$   $\mathbf{J}_{c} = \sum_{n=1}^{N_{c}} I_{cn} \mathbf{f}_{n}$ ,  $\mathbf{M}_{d} = \eta_{0} \sum_{n=1}^{N_{c}} M_{dn} \mathbf{f}_{n}$ ,  $I_{dn}$ ,  $I_{cn}$ ,  $M_{dn}$  are the current coefficients needed to solve. To improve the condition number of the matrix, a coefficient adjustment is made to form the PMCHWT2 formulation:

$$\begin{bmatrix} \begin{bmatrix} Z_{mn}^{J_c J_c} \\ Z_{mn}^{J_c J_c} \end{bmatrix} & \begin{bmatrix} Z_{mn}^{J_c J_d} \end{bmatrix} & \eta_1 \begin{bmatrix} T_{mm}^{J_c M_d} \\ T_{mn}^{J_c J_c} \end{bmatrix} \begin{bmatrix} Z_{mn}^{J_d J_d} \end{bmatrix} & \eta_1 \begin{bmatrix} T_{mm}^{J_d M_d} \\ I_{dn} \\ M_{dn} \\ \eta_1 \begin{bmatrix} T_{mn}^{M_d J_c} \end{bmatrix} & \eta_1 \begin{bmatrix} T_{mn}^{M_d J_d} \end{bmatrix} & \eta_1^2 \begin{bmatrix} Y_{mn}^{M_d M_d} \end{bmatrix} \end{bmatrix} \begin{bmatrix} I_{cn} \\ I_{dn} \\ M_{dn} \\ \eta_1 H_{dm} \end{bmatrix},$$
(17)
where the  $n_{cn} = \sqrt{\mu_c / \varepsilon_{cn}}$ 

where the  $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ .

# **B. JCFIE-JMCFIE formulation for scattering** problems in half-space

In this paper, the novel integral equation JMCFIE [16] combined with JCFIE is extended to the half-space situation to realize accurate and fast solution of scattering from coated conducting targets. The JMCFIE formulation combined with JCFIE for half-space problems can be obtained by combining the T and N equations as the following form:

$$\begin{vmatrix} -\frac{1}{\eta_2} \text{ T-EFIE}_2 - \text{N-MFIE}_2 \\ \frac{1}{\eta_1} \text{ T-EFIE}_1 - \frac{1}{\eta_2} \text{ T-EFIE}_2 + \text{N-MFIE}_1 - \text{N-MFIE}_2. (18) \\ \eta_1 \text{ T-MFIE}_1 - \eta_2 \text{ T-MFIE}_2 - \text{N-EFIE}_1 + \text{N-EFIE}_2 \end{vmatrix}$$

After expanding the unknowns  $J_c$ ,  $J_d$ , and  $M_d$  in (18) with the RWG basis functions and using the Galerkin's method, a well-conditioned matrix equation will be:

$$\begin{bmatrix} \begin{bmatrix} Z_{nm}^{J_{c}J_{c}} \\ mn \end{bmatrix} \begin{bmatrix} Z_{nm}^{J_{c}J_{d}} \\ mn \end{bmatrix} \begin{bmatrix} T_{nm}^{J_{c}M_{d}} \\ mn \end{bmatrix} \begin{bmatrix} T_{nm} \\ I_{dn} \\ M_{dn} \end{bmatrix} = \begin{bmatrix} F_{ecm} \\ F_{ecm} \\ F_{edm} \end{bmatrix}.$$
(19)  
$$Z_{nm}^{J_{d}J_{c}} \end{bmatrix} \begin{bmatrix} T_{nm}^{M_{d}J_{d}} \\ mn \end{bmatrix} \begin{bmatrix} Y_{nm}^{M_{d}M_{d}} \\ M_{dn} \end{bmatrix} = \begin{bmatrix} F_{ecm} \\ F_{edm} \\ F_{hdm} \end{bmatrix}.$$
(19)  
$$Z_{nm}^{J_{d}J_{c}} = \frac{1}{\eta_{i}} j \omega \mu_{i} \langle \int_{T_{r}} \bar{\mathbf{K}}^{A} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{\eta_{i} j \omega \varepsilon_{i}} \langle \int_{T_{r}} K^{A} \nabla_{s} \cdot \mathbf{f}_{n} dS', \nabla \cdot \mathbf{f}_{m} \rangle$$
$$- \langle \mathbf{n} \times \int_{T_{r}} \bar{\mathbf{G}}^{HJ} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{2} \langle \mathbf{f}_{n} \cdot \mathbf{f}_{m} \rangle$$
$$+ \frac{1}{\eta_{2}} j \omega \mu_{2} \langle \int_{T_{r}} g_{2} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{\eta_{2} j \omega \varepsilon_{2}} \langle \int_{T_{r}} g_{2} \nabla_{s} \cdot \mathbf{f}_{n} dS', \nabla \cdot \mathbf{f}_{m} \rangle$$
$$+ \langle \mathbf{n} \times \int_{T_{r}} \nabla g_{2} \times \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{2} \langle \mathbf{f}_{n} \cdot \mathbf{f}_{m} \rangle$$

$$Z_{mn}^{J_{d}J_{c}} = -\frac{1}{\eta_{2}} j\omega\mu_{2} \left\langle \int_{T'} g_{2} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle - \frac{1}{\eta_{2} j\omega\varepsilon_{2}} \left\langle \int_{T'} g_{2} \nabla_{s} \cdot \mathbf{f}_{n} dS', \nabla \cdot \mathbf{f}_{m} \right\rangle - \left\langle \mathbf{n} \times \int_{T'} \nabla g_{2} \times \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle - \frac{1}{2} \left\langle \mathbf{f}_{n} \cdot \mathbf{f}_{m} \right\rangle$$

$$(21)$$

$$T_{nm}^{J_{\mu}M_{d}} = j\omega\varepsilon_{1} \langle \mathbf{n} \times \int_{T} \mathbf{\bar{K}}^{F} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle - \frac{1}{j\omega\mu_{1}} \langle \mathbf{n} \times \nabla \int_{T} K^{dm} \nabla_{s} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle$$

$$- \frac{1}{\eta_{1}} \langle \int_{T} \mathbf{\bar{G}}^{EM} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{2\eta_{1}} \langle \mathbf{n} \times \mathbf{f}_{n}, \mathbf{f}_{m} \rangle$$

$$- j\omega\varepsilon_{2} \langle \mathbf{n} \times \int_{T^{*}} g_{2} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{j\omega\mu_{2}} \langle \mathbf{n} \times \nabla \int_{T} g_{2} \nabla_{s} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle$$

$$+ \frac{1}{\eta_{2}} \langle \int_{T^{*}} \nabla g_{2} \times \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle - \frac{1}{2\eta_{2}} \langle \mathbf{n} \times \mathbf{f}_{n}, \mathbf{f}_{m} \rangle$$

$$Z_{mm}^{J,J_{d}} = \frac{1}{\eta_{2}} j\omega\mu_{2} \langle \int_{T^{*}} g_{2} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \rangle + \frac{1}{\eta_{2} i\omega\varepsilon_{2}} \langle \int_{T^{*}} g_{2} \nabla_{s}^{*} \cdot \mathbf{f}_{n} dS', \nabla \cdot \mathbf{f}_{m} \rangle$$

$$+ \langle \mathbf{n} \times \int_{T} \nabla g_2 \times \mathbf{f}_{\mathbf{n}} dS', \mathbf{f}_{\mathbf{m}} \rangle + \frac{1}{2} \langle \mathbf{f}_{\mathbf{n}} \cdot \mathbf{f}_{\mathbf{m}} \rangle$$

$$(23)$$

$$Z_{mn}^{J_{cJ_{c}}} = -\frac{1}{\eta_{c}} j \alpha \mu_{2} \left\langle \int_{T} g_{2} \cdot \mathbf{f_{n}} dS', \mathbf{f_{m}} \right\rangle - \frac{1}{\eta_{c} j \alpha \varepsilon_{2}} \left\langle \int_{T} g_{2} \nabla_{s} \cdot \mathbf{f_{n}} dS', \nabla \cdot \mathbf{f_{m}} \right\rangle$$
$$- \left\langle n \times \int_{T} \nabla g_{2} \times \mathbf{f_{n}} dS', \mathbf{f_{m}} \right\rangle - \frac{1}{2} \left\langle \mathbf{f_{n}} \cdot \mathbf{f_{m}} \right\rangle$$
(24)

$$T_{nn}^{J_{c}M_{d}} = -j\omega\varepsilon_{2} \left\langle \boldsymbol{n} \times \int_{T'} g_{2} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle + \frac{1}{j\omega\mu_{2}} \left\langle \boldsymbol{n} \times \nabla \int_{T'} g_{2} \nabla_{s} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle$$
$$+ \frac{1}{\eta_{2}} \left\langle \int_{T'} \nabla g_{2} \times \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle - \frac{1}{2\eta_{2}} \left\langle \mathbf{n} \times \mathbf{f}_{n}, \mathbf{f}_{m} \right\rangle$$
(25)

$$T_{mn}^{M_{a}J_{d}} = -j\omega\mu_{l}\left\langle \boldsymbol{n} \times \int_{T'} \overline{\mathbf{K}}^{A} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle + \frac{1}{j\omega\varepsilon_{l}} \left\langle \boldsymbol{n} \times \nabla \int_{T'} K^{\phi_{\mathbf{c}}} \nabla_{s}^{'} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle$$
$$-\eta_{l} \left\langle \int_{T'} \overline{\mathbf{G}}^{HJ} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle - \frac{\eta_{l}}{2} \left\langle \mathbf{n} \times \mathbf{f}_{n}, \mathbf{f}_{m} \right\rangle$$
$$+ j\omega\mu_{2} \left\langle \boldsymbol{n} \times \int_{T'} g_{2} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle - \frac{1}{j\omega\varepsilon_{2}} \left\langle \boldsymbol{n} \times \nabla \int_{T'} g_{2} \nabla_{s}^{'} \cdot \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle$$
$$-\eta_{2} \left\langle \int_{T'} \nabla g_{2} \times \mathbf{f}_{n} dS', \mathbf{f}_{m} \right\rangle + \frac{\eta_{2}}{2} \left\langle \mathbf{n} \times \mathbf{f}_{n}, \mathbf{f}_{m} \right\rangle$$
(26)

$$T_{nm}^{M_{d}J_{c}} = -j \alpha \mu_{2} \langle \boldsymbol{n} \times \int_{T} g_{2} \cdot \mathbf{f_{n}} dS', \mathbf{f_{m}} \rangle + \frac{1}{j \alpha \varepsilon_{2}} \langle \boldsymbol{n} \times \nabla \int_{T} g_{2} \nabla_{s} \cdot \mathbf{f_{n}} dS', \mathbf{f_{m}} \rangle + \eta_{2} \langle \int_{T} \nabla g_{2} \times \mathbf{f_{n}} dS', \mathbf{f_{m}} \rangle - \frac{\eta_{2}}{2} \langle \mathbf{n} \times \mathbf{f_{n}}, \mathbf{f_{m}} \rangle$$

$$(27)$$

$$Y_{m_{d}M_{d}}^{M_{d}} = \eta, j \omega \varepsilon_{s} \langle \left[ \overline{\mathbf{k}}^{F} \cdot \mathbf{f_{n}} dS', \mathbf{f_{m}} \right] + \frac{\eta_{1}}{2} \langle \left[ K^{\phi m} \nabla \right] \cdot \mathbf{f_{n}} dS', \nabla \cdot \mathbf{f_{m}} \rangle$$

$$mn \qquad (15) \text{ for } \mathbf{1} \langle \mathbf{J}_{T}, \mathbf{n} \rangle = \mathbf{n} \langle \mathbf{J}_{T}, \mathbf{g} \rangle$$

$$F_{ecm} = \left\langle \left( -\frac{1}{\eta_2} \mathbf{E}_2^{inc} + \hat{\mathbf{n}} \times \mathbf{H}_2^{inc} \right), \mathbf{f}_m \right\rangle$$
(29)

$$F_{edm} = \left\langle \left( \frac{1}{\eta_1} \mathbf{E}_1^{inc} - \frac{1}{\eta_2} \mathbf{E}_2^{inc} + \hat{\mathbf{n}} \times \mathbf{H}_1^{inc} + \hat{\mathbf{n}} \times \mathbf{H}_2^{inc} \right), \mathbf{f}_m \right\rangle$$
(30)

$$F_{hdm} = \left\langle \left( \eta_1 \mathbf{H}_1^{inc} - \eta_2 \mathbf{H}_2^{inc} - \hat{\mathbf{n}} \times \mathbf{E}_1^{inc} - \hat{\mathbf{n}} \times \mathbf{E}_2^{inc} \right), \mathbf{f}_m \right\rangle.$$
(31)

It's obvious the JCFIE-JMCFIE that formulation contains well-tested identity operators. The impedance matrix of JCFIE-JMCFIE is wellconditioned since the well-tested identity operators lead to diagonally-dominant matrices. This is an essential requirement for a formulation with a high convergence rate. This explains why the developed JCFIE-JMCFIE formulation leads to a better conditioned matrix equation than the traditional EFIE-PMCHWT formulations and hence gives more rapidly converging iterative solutions. Finally, the linear system of equations in (19) can be solved by the FGMRES method [21-24] using half-space MLFMA to accelerate the matrix-vector products [8].

#### **III. NUMERICAL EXAMPLES**

In this section, we show some numerical results for the electromagnetic characteristics of a conducting body coated with dielectric materials in half-space that illustrate the accuracy and effectiveness of the proposed JCFIE-JMCFIE formulation. The JCFIE-JMCFIE linear systems based on the RWG basis functions are solved with MLFMA accelerated Krylov iterative methods. All numerical experiments are performed on a Pentium 4 with 2.9 GHz CPU and 2GB RAM in single precision. In this paper, the inner-outer FGMRES [21-24] is used as the iterative solver for the **JCFIE-JMCFIE** and **EFIE-PMCHWT2** formulation to accelerate the convergence rate compared with GMRES. In the flowing examples, the inner and outer restart numbers of FGMRES are both 10. The stop precision of restarted GMRES is denoted to be 1.E-4. In the FGMRES algorithm, the stop precision for the inner and outer iteration is 1.E-2, 1.E-4, respectively. Additional details and comments on the implementation are given as follows:

• Zero vector is taken as an initial approximate solution for all examples and all systems in each example.

At first, the developed formulations are verified by the conducting body coated with dielectric materials in free-space. The first example is a metallic sphere covered with dielectric material in free-space as shown in Fig. 2(a). The metallic sphere's radius is  $a_1 = 0.3423\lambda_0 (\lambda_0)$  is the wavelength in free-space). The thickness of the coated dielectric is  $a_2 = 0.1017\lambda_0$  and the relative permittivity is with  $\varepsilon_r = 4$ . The incident angles of plane wave are  $\theta_i = 180^\circ$  and  $\phi_i = 180^\circ$  at a frequency of f = 300 MHz. Fig. 2(b) gives the  $\theta\theta$ co-polarized bi-static RCS for the above freespace coated metallic sphere solved by the JCFIE-JMCFIE, EFIE-PMCHWT2, and Mie. respectively. The scattering angle is  $0^{\circ} \le \theta_{a} \le 180^{\circ}, \phi_{a} = 0^{\circ}$ . MLFMA with 2 levels is used to accelerate the matrix-vector products. The coated metallic sphere is discretized with 3240 triangular patches leading to 7720 unknowns. It can be found that the results using the JCFIE-JMCFIE, EFIE-PMCHWT2 method are in good agreement with the Mie series solution. Figure 3 shows the convergence rates of the JCFIE-JMCFIE and EFIE-PMCHWT2 formulations solved by GMRES method for the above example. From Fig. 3, it can be found that the EFIE-PMCHWT method does not reach convergence after 700 iteration steps. However, the JCFIE-JMCFIE can reach convergence in 100 iteration steps. It can be concluded that the condition number of the proposed JCFIE-JMCFIE is much better than the EFIE-PMCHWT2.



Fig. 2. (a) Geometry of a metallic sphere covered with dielectric material in free-space. (b) The co-

polarized bi-static RCS ( $\theta\theta$ ) of a metallic sphere covered with dielectric material for  $0^{\circ} \le \theta_s \le 180^{\circ}, \phi_s = 0^{\circ}$  at 300MHz.



Fig. 3. The convergence history of JCFIE-JMCFIE, EFIE-PMCHWT2 solved with FGMRES for a metallic sphere covered with dielectric material in free-space at 300MHz.

Next, we extended the JCFIE-JMCFIE method to analyze the half-space problems to verify the accuracy and efficiency of the JCFIE-JMCFIE. As shown in Fig. 4(a), we consider a metallic sphere covered with spherical dielectric material situated 0.3577m above the lossy half space characterized by  $\varepsilon_{half} = 5.0 - j0.2$  and  $\mu_{half} = 1.0$ . The metallic sphere's radius is  $a_1 = 0.3423\lambda_0$ . The thickness of the coated dielectric material is  $a_2 = 0.1017\lambda_0$  and the relative permittivity is with  $\varepsilon_r = 4$ . The incident angles of plane wave are  $\theta_i = 180^\circ$  and  $\phi_i = 180^\circ$  at f = 300 MHz. The metallic sphere coated with spherical dielectric is, also, discretized with 3240 triangular patches leading to 7720 unknowns. Figure 4(b) gives the bi-static RCS for the above half-space metallic sphere coated with spherical dielectric solved by the JCFIE-JMCFIE and EFIE-PMCHWT2, respectively. The scattering angle is  $0^{\circ} \le \theta_s \le 90^{\circ}, \phi_s = 0^{\circ}$ . The two levels MLFMA are used to accelerate the matrixvector products. The simulated results obtained from the method of moment for bodies of revolution (BORMoM) and FEKO software are given to compare to them from JCFIE-JMCFIE and EFIE-PMCHWT2. In the BORMoM computation, the coated metallic sphere is discretized along the generating arc leading to 114 unknowns. The order of Fourier mode is 10 and

the computation time is 2308.3 seconds in BORMOM. From Fig. 4(b), the results of JCFIE-JMCFIE and EFIE-PMCHWT2 agree well with the BORMOM and FEKO results. Furthermore, Fig. 5 gives the convergence rates of the JCFIE-JMCFIE and EFIE-PMCHWT2 formulations solved by the FGMRES method for the above half-space example. It is obvious that the application of the novel integral equation JCFIE-JMCFIE greatly accelerates the convergence rate compared with the EFIE-PMCHWT2 formulation.



Fig. 4. (a) Geometry of a metallic sphere covered with spherical dielectric material situated above the lossy half space. (b) The co-polarized bi-static RCS ( $\theta\theta$ ) of a metallic sphere covered with spherical dielectric material situated above the lossy half space for  $0^{\circ} \le \theta_s \le 90^{\circ}, \phi_s = 0^{\circ}$  at 300MHz.



Fig. 5. The convergence history of JCFIE-JMCFIE, EFIE-PMCHWT2 solved with FGMRES for a metallic sphere covered with spherical dielectric material situated above the lossy half space at 300MHz.



Fig. 6. (a) Geometry of a metallic sphere covered with cubic dielectric material situated above the lossy half space. (b) The co-polarized bi-static RCS ( $\theta\theta$ ) of a metallic sphere covered with cubic dielectric material situated above the lossy half space for  $0^{\circ} \le \theta_s \le 90^{\circ}$ ,  $\phi_s = 0^{\circ}$  at 300MHz.

The another example shown in Fig. 6(a) is a metallic sphere covered with cubic dielectric situated 0.4 m above the lossy half space characterized by  $\varepsilon_{half} = 5.0 - j0.2$  and  $\mu_{half} = 1.0$ . The metallic sphere's radius is 0.3m. The coated cubic dielectric is lossy. The conductivity and relative permittivity of dielectric are  $\sigma = 0.001$  s/m and  $\varepsilon_r = 4$ , respectively. The incident angles of plane wave are  $\theta_i = 180^\circ$  and  $\phi_i = 180^\circ$  at a frequency of f = 300 MHz. The metallic sphere coated with spherical dielectric is also discretized with triangular patches leading to 10896 unknowns. Figure 6(b) gives the bi-static RCS for the above half-space metallic sphere coated with cubic dielectric solved by the JCFIE-JMCFIE, EFIE-PMCHWT2 finite element-boundary integral method (FE-BI) and FEKO software, respectively. The scattering angle is  $0^{\circ} \le \theta_{s} \le 90^{\circ}, \phi_{s} = 0^{\circ}$ . MLFMA with 2 levels is used to accelerate the matrix-vector products. In the FE-BI computation, the number of unknowns is 23651 and the computation time is 432.2 seconds. From Figure 6(b), it can be found that the results of JCFIE-JMCFIE are in good agreement with them obtained from EFIE-PMCHWT2, FE-BI and FEKO. Figure 7, also, shows the convergence rates of the JCFIE-JMCFIE and EFIE-PMCHWT2 formulations solved by FGMRES method for the last half-space example. It can be found that the proposed JCFIE-JMCFIE method only needs 139 iteration steps to reach

convergence while the EFIE-PMCHWT2 method can not converge after 700 iteration steps.



Fig. 7. The convergence history of JCFIE-JMCFIE, EFIE-PMCHWT2 solved with FGMRES for a metallic sphere covered with cubic dielectric material situated above the lossy half space at 300MHz.

In order to further investigate the performance of the proposed JCFIE-JMCFIE method, the above half-space metallic sphere coated with spherical dielectric material with the larger dielectric constants is considered. Figure 8 gives the copolarized bi-static RCS of a metallic sphere covered with spherical dielectric material with  $\varepsilon_r = 10$  and  $\varepsilon_r = 12$  situated above the lossy half space solved by the JCFIE-JMCFIE. The simulated results obtained from BORMoM are given to compare to them from JCFIE-JMCFIE. In the BORMoM computation, the coated metallic spheres with  $\varepsilon_r = 10$  and  $\varepsilon_r = 12$  are both discretized along the generating arc leading to 114 unknowns. The order of Fourier mode is 20 and the computation time of BORMoM is 5128.3 seconds and 6340.1 seconds for  $\varepsilon_r = 10$  and  $\varepsilon_r = 12$ cases, respectively. The scattering angle is  $0^{\circ} \le \theta_{s} \le 90^{\circ}, \phi_{s} = 0^{\circ}$ . The two levels MLFMA is used to accelerate the matrix-vector products. Figure 9 and Fig. 10 give the convergence rates of the **JCFIE-JMCFIE** and **EFIE-PMCHWT2** formulations solved by FGMRES method for the above half-space example with  $\varepsilon_r = 10$  and  $\varepsilon_r = 12$ , respectively. It can be found that the JCFIE-JMCFIE method only needs 713 iteration steps to reach the convergence for the  $\varepsilon_r = 10$  example, while the EFIE-PMCHWT2 method can only converge to the error of 0.1 after 2000 iteration.

For the  $\varepsilon_r = 12$  example, the JCFIE-JMCFIE method can reach convergence after 5581 iteration steps, while the EFIE-PMCHWT2 method only converge to the error of 0.1 after 10000 iteration steps. This demonstrates the efficiency of the proposed JCFIE-JMCFIE method for the half-space metallic sphere coated with larger dielectric constants dielectric material.



Fig. 8. The co-polarized bi-static RCS ( $\theta\theta$ ) of a metallic sphere covered with spherical dielectric material with  $\varepsilon_r = 10$  and  $\varepsilon_r = 12$  situated above the lossy half space for  $0^\circ \le \theta_s \le 90^\circ, \phi_s = 0^\circ$  at 300MHz.



Fig. 9. The convergence history of JCFIE-JMCFIE, EFIE-PMCHWT2 solved with FGMRES for a metallic sphere covered with spherical dielectric material with  $\varepsilon_r = 10$  situated above the lossy half space at 300MHz.



Fig. 10. The convergence history of JCFIE-JMCFIE, EFIE-PMCHWT2 solved with FGMRES for a metallic sphere covered with spherical dielectric material with  $\varepsilon_r = 12$  situated above the lossy half space at 300MHz.

Figure 11 gives the convergence rates of the **JCFIE-JMCFIE** and **EFIE-PMCHWT2** formulations solved by the FGMRES method for the above half-space metallic sphere covered with spherical dielectric material with  $\varepsilon_r = 4$  in Figure 4(a). The metallic sphere coated with spherical dielectric is also discretized with triangular patches leading to 37302 unknowns. Due to the limit of computer memory (2GB RAM), larger problems (>40000) can not be computed. Furthermore, compared with the free-space MLFMA, aggregation and disaggregation for the image groups as well as calculations of the image translation operator between and observation group centers are additionally required during implementing half-space MLFMA in JCFIE-JMCFIE. As a result, the storage and CPU of half-space MLFMA are approximately twice that of a free-space MLFMA, due to the extra set of image clusters. From Figure 11, it can be found that the proposed JCFIE-JMCFIE method only needs 167 iteration steps to reach convergence while the EFIE-PMCHWT2 method can only converge to the error of 0.6 after 700 iteration steps.



Fig. 11. The convergence history of JCFIE-JMCFIE, EFIE-PMCHWT2 solved with FGMRES for a metallic sphere covered with spherical dielectric material with situated above the lossy half space at 300MHz. The number of unknown is 37302.

Tables 1-3 list the iteration steps and computation time of the JCFIE-JMCFIE and EFIE+PMCHWT2 solved by FGMRES for above three examples where \* refers to no convergence after maximum 20000 iterations. When compared in terms of solution time, the JCFIE-JMCFIE have a gain over the EFIE+PMCHWT2 by a factor of more than 160 on the first free-space example, 32.2 on the second half-space metallic sphere example coated with spherical dielectric, 61.9 on the third half-space metallic sphere example coated with cubic dielectric. It can be found that the JCFIE-JMCFIE reduces the computation time significantly while it maintains high accuracy when compared with the traditional EFIE+PMCHWT2

#### **IV. CONCLUSION**

In this paper, a well-conditioned coupled surface integral equation called JCFIE-JMCFIE is proposed for the analysis of electromagnetic scattering from coated targets above a lossy halfspace. The coupled formulation can lead to a well tested system with the RWG basis functions and Galerkin's method. To efficiently analyze electrically large scattering problems in half-space, the half-space MLFMA is implemented to reduce the computational complexity and memory requirement. The inner-outer FGMRES is used as the iterative solver to further enhance the convergence rate. Numerical results are demonstrated to validate the proposed method and

show the accuracy and high efficiency compared with the traditional EFIE-PMCHWT2 method. Further investigations deserve to be undertaken to study the parallelization of the proposed JCFIE-JMCFIE with MLFMA.

Table 1: Comparison of the iteration steps and computation time of JCFIE-JMCFIE and EFIE-PMCHWT2 for the metallic sphere coated with spherical dielectric on free space

|              | Unknowns | Iteration steps | Computation time(s) |
|--------------|----------|-----------------|---------------------|
| EFIE-PMCHW12 | 7720     | *               | >20000              |
| JCFIE-JMCFIE | 7720     | 100             | 121.8               |

Table 2: Comparison of the iteration steps and computation time of JCFIE-JMCFIE and EFIE-PMCHWT2 for the metallic sphere coated with spherical dielectric situated 0.3577m above the lossy half space

|              | Unknowns | Iteration steps | Computation time(s) |
|--------------|----------|-----------------|---------------------|
| EFIE-PMCHWT2 | 7720     | 9082            | 7054.16             |
| JCFIE-JMCFIE | 7720     | 173             | 219.28              |

Table 3: Comparison of the iteration steps and computation time of JCFIE-JMCFIE and EFIE-PMCHWT2 for the metallic sphere coated with cubic dielectric situated 0.4m above the lossy half space

|              | Unknowns | Iteration steps | Computation time(s) |
|--------------|----------|-----------------|---------------------|
| EFIE-PMCHWT2 | 10896    | 16024           | 13559.17            |
| JCFIE-JMCFIE | 10896    | 139             | 219.28              |

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# Robust Beamforming Technique with Sidelobe Suppression Using Sparse Constraint on Beampattern

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Abstract — In this paper, we propose two beamforming algorithms which impose sparse constraint on beampattern to suppress the sidelobe level and enhance robustness of the beamformer against steering vector error. The sparse constraint is given by an  $l_p$ -norm, where  $p \le 2$  holds. It is shown that the proposed algorithms can be solved using iterative algorithm. Computer simulations show that the proposed algorithms not only yield lower sidelobe level than that of the conventional beamformer, but also show better robustness against steering vector error than the conventional beamformers.

*Index Terms* – Beamforming, robust, sidelobe, sparse, steering vector error.

### I. INTRODUCTION

Adaptive beamforming is an important research topic in array signal processing. It is to enhance the source from the desired direction, while suppressing the background noise and all the interference from other directions. Α representative beamformer is the minimum variance and distortionless response (MVDR) beamformer [1] which casts deep nulls in the directions of strong interferences and at the meantime guarantees the desired signal distortionless. One of the disadvantages of the MVDR beamformer is its high sidelobe level

which results in significant performance degradation in case of unexpected interference or increase of the noise power [2]. In practice, look direction mismatch due to imperfect array occurs. In such case, the signal of interest (SOI) will be mistaken as interference and the performance of the MVDR beamformer is known to degrade dramatically [3].

There has been much effort dedicated to design robust beamforming algorithms against imperfect array [4]. To address the uncertainty of the steering vector and the direction-of-arrival (DOA) of SOI, a set of unity-gain constraints for neighboring directions of the nominal look direction can be imposed [5]. Besides, the derivative of the array output can be imposed to zero at the desired look angle, which is called the derivative mainbeam constraints [6-8]. Some regularization methods [9, 10] are also proposed to enhance the robustness of the MVDR beamformer against steering vector error [11]. Beamformers using eigenvalue thresholding methods to achieve robustness have also been discussed [12].

In this paper, we propose two new beamforming algorithms which are robust against look direction mismatch and steering vector error. Besides, the achieved beampatterns have lower sidelobe level compared with the MVDR beamformer. Sidelobe suppression is realized via imposing a sparse constraint on beampattern with respect to potential interference directions. Due to imperfect knowledge of the array, the presumed directions of SOI maybe vary from the real ones, thereby causes cancelation of SOI. To solve this problem, mainlobe control is considered. Instead of maintaining distortionless response on one look direction, the proposed algorithm attempts to maintain distortionless response on a wide angular range so that sources impinging on the array from nearby directions of look direction can be retained. The two algorithms may be solved iteratively. The validity and advantages of the new algorithms are verified via computer simulations.

### II. ALGORITHM I: SIDELOBE SUPPRESSION WITH SPARSE CONSTRAINT ON BEAMPATTERN

The mathematical formulation of the MVDR beamformer is given by

$$\min_{\mathbf{w}} \mathbf{w}^{H} \mathbf{R} \mathbf{w}, \qquad (1a)$$
  
subject to  $\mathbf{w}^{H} \mathbf{a}(\theta_{0}) = 1, \qquad (1b)$ 

where **R** is the covariance matrix of data, and **w** denotes the beamforming vector. From the perspective of beampattern, it is observed from (1b) that there is only an explicit constraint on the look direction, i.e.  $\theta_0$ , while no constraint is imposed on the directions of interference. To repair this drawback, we propose the following optimization problem with an additional constraint on the sparsity of the beampattern with respect to potential interference directions:

$$\min_{\mathbf{w}} \mathbf{w}^{H} \mathbf{R} \mathbf{w}/2, \qquad (2a)$$
  
subject to  $\mathbf{w}^{H} \mathbf{a}(\theta_{0}) = 1, \qquad (2b)$ 

$$\|\mathbf{w}^H \mathbf{A}\|_p^p < \varepsilon, \qquad (2c)$$

where **A** is an  $L \times N$  matrix which consists of steering vectors in the angular range which contains all the possible interference directions, *L* is the number of sensors, and *N* denotes the number of spatial samplings over the angular range with respect to the interference. It is noticed that we have imposed (2c) to the original formulation of the MVDR beamformer (1).

In (2c),  $\|\mathbf{x}\|_p = (\sum_i |x_i|^p)^{1/p}$  is the  $l_p$ -norm of the vector  $\mathbf{x}$ . When p < 2, the  $l_p$ -norm can be defined as "dispersion" of the super-Gaussian distributions. When  $p \le 1$ , the  $l_p$ -norm can be interpreted as the diversity measurement [13]. The smaller the value of  $\|\mathbf{x}\|_p^p$ , the sparser the  $\mathbf{x}$  is, which means the number of trivial entries in  $\mathbf{x}$  is larger. For beamforming, the smaller the value of

 $\|\mathbf{w}^{H}\mathbf{A}\|_{p}^{p}$ , the lower the sidelobe level, since most of the entries in  $\mathbf{w}^{H}\mathbf{A}$  are forced to some trivial values. For our sparse constraint, we require the value of *p* to be smaller than 2.

To derive the solution of (2), the Lagrange multiplier technique is used:

 $J(\mathbf{w}) = \frac{\mathbf{w}^{H}\mathbf{R}\mathbf{w}}{2} + \lambda \|\mathbf{w}^{H}\mathbf{A}\|_{p}^{p} + \gamma(\mathbf{w}^{H}\mathbf{a}(\theta_{0}) - 1),(3)$ where  $\lambda$  and  $\gamma$  are Lagrange multipliers.

Define  $\overline{\mathbf{A}} = [\mathbf{A} | \alpha \mathbf{a}(\theta_0)], \mathbf{d}^T = [\mathbf{0}^T | \alpha \mathbf{1}]$ , where  $\alpha = \gamma / \lambda$ . (3) is equivalent to

$$J(\mathbf{w}) = \frac{\mathbf{w}^{H}\mathbf{R}\mathbf{w}}{2} + \lambda \| \overline{\mathbf{A}}^{H}\mathbf{w} - \mathbf{d}^{*} \|_{p}^{p}, \quad (4)$$

where \* denotes complex conjugate.

Calculating the gradient of  $J(\mathbf{w})$  with respect to  $\mathbf{w}$ , we have

 $\nabla_{\mathbf{w}} \mathbf{J}(\mathbf{w}) = \mathbf{R}\mathbf{w} + \widetilde{\lambda} \,\overline{\mathbf{A}} \boldsymbol{\Pi}(\mathbf{w}) (\,\overline{\mathbf{A}}^{H}\mathbf{w} - \mathbf{d}^{*}), \quad (5)$ where  $\widetilde{\lambda} = \lambda \mathbf{p}$ , and  $\boldsymbol{\Pi}(\mathbf{w})$  is given by  $\boldsymbol{\Pi}(\mathbf{w}) =$ diag{ $|(\,\overline{\mathbf{A}}^{H}\mathbf{w} - \mathbf{d}^{*})_{1}|^{p-2}, \dots, |(\,\overline{\mathbf{A}}^{H}\mathbf{w} - \mathbf{d}^{*})_{N}|^{p-2}$ }.

Equating (5) to zero, and using the gradient factorization approach [13], the update formula of  $\mathbf{w}$  is given by

 $\mathbf{w}(i+1) =$ 

 $\widetilde{\lambda} \left( \mathbf{R} + \widetilde{\lambda} \,\overline{\mathbf{A}} \boldsymbol{\Pi}(\mathbf{w}(i)) \,\overline{\mathbf{A}}^{H} \right)^{-1} \overline{\mathbf{A}} \boldsymbol{\Pi}(\mathbf{w}(i)) \mathbf{d}^{*}, \quad (6)$ where *i* denotes the iteration index.

### III. ALGORITHM II: SIDELOBE SUPPRESSION WITH ROBUSTNESS CONSIDERATION

Notice that the above formulation is proposed under the assumption that the real steering vector is known. When the presumed steering vector is used instead of the real steering vector, the above formulation may break down when **A** contains the real steering vector of SOI. This happens when the presumed look direction is erroneous.

To amend this drawback, we generalize the above formulation by mainlobe control. The direction of SOI is assumed to be contained within a spatial sector instead of a particular direction. Let  $\theta_s$  denote the spatial sector where the directions of SOI is assumed to be contained. The extension of the optimization problem (2) is given as follows:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}/2 , \qquad (7a)$$

subject to 
$$\mathbf{A}_{s}^{H}\mathbf{w} = 1$$
, (7b)

$$\|\mathbf{w}^H \mathbf{A}\|_{\mathcal{D}}^p < \varepsilon, \tag{7c}$$

where  $\mathbf{A}_{s} = [\mathbf{a}(\theta_{s,1}), ..., \mathbf{a}(\theta_{s,N_{s}})]$  consists of possible steering vectors of SOI within the spatial sector  $\mathbf{\theta}_{s}$ .

The solution to the above optimization can be derived in a similar way as the derivation presented in Section II. Firstly, we combine the constraints (7b) and (7c):

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}/2 \tag{8a}$$

subject to  $\| \mathbf{\Gamma}^{H} \mathbf{w} - \mathbf{d}^{*} \|_{p}^{p}$ , (8b) where  $\mathbf{\Gamma} = [\mathbf{A} | \alpha \mathbf{A}_{s}]$ . Next, we formulate the Lagrange function and equate its gradient to zero. Finally, the weight vector update equation is obtained as follows:

$$w(i + 1) =$$

 $\widetilde{\lambda} \left( \mathbf{R} + \widetilde{\lambda} \, \boldsymbol{\Gamma} \boldsymbol{\Pi} \left( \mathbf{w}(i) \right) \boldsymbol{\Gamma}^{H} \right)^{-1} \boldsymbol{\Gamma} \boldsymbol{\Pi} \left( \mathbf{w}(i) \right) \mathbf{d}^{*}, \quad (9)$ where  $\boldsymbol{\Pi}(\boldsymbol{w})$  is given by  $\boldsymbol{\Pi}(\boldsymbol{w}) = \text{diag}\{ |(\boldsymbol{\Gamma}^{H} \mathbf{w} - \mathbf{d}^{*})_{1}|^{p-2}, \dots, |(\boldsymbol{\Gamma}^{H} \mathbf{w} - \mathbf{d}^{*})_{N}|^{p-2} \}.$ 

It is worth mentioning that the matrix  $\Gamma$  can be implemented by considering only a few directions representing the directions of the SOI within the spatial sector  $\mathbf{\theta}_s$ . In fact, the number of directions within  $\mathbf{\theta}_s$  can be as small as two, i.e.  $N_s = 2$ . To do so, the first direction  $\theta_{s,1}$  is taken at the left boundary while the other  $\theta_{s,2}$  is taken at the right boundary. It is demonstrated by computer simulations that the optimization problem (7), compared to optimization problem (2), has several advantages: 1) better robustness against look direction mismatch; 2) smaller value of N to generate A with respect to interference directions; 3) better performance when the real interference directions are not precisely located at the sampling directions of A.

### IV. IMPLEMENTATION OF THE PROPOSED ALGORITHMS

From (7) and (10), it is observed that there are three parameters: p,  $\lambda$ , and  $\gamma$ , impacting the performance of the proposed beamformers. The parameter p determines the sparsity of the beampattern. The smaller the value of p, the sparser the derived beampattern (or equivalently, the lower the sidelobe level). The parameter  $\lambda$ determines the effectiveness of the sparse constraint. Using a large  $\lambda$  emphasizes the impact of the sparse constraint and will result in a trivial solution  $\mathbf{w}$  ( $\mathbf{w} = \mathbf{0}$  gives the smallest value of  $\|\mathbf{w}^H \mathbf{A}\|_p^p$ ), since the constraint on distortionless response in (2b) and (7b) will be neglected in this situation. Similarly, using a large  $\gamma$  emphasizes the impact of the distortionless response constraint and will attenuate the effect of the sparse

constraint. Therefore, these parameters should be chosen properly in order to have good compromise between suppression of interferences and maintenance of the desired source.

In order to appropriately choose  $\lambda$ , we compute the gradient of (3) with respect to  $\lambda$ , and equating it to zero yields

$$\| \overline{\mathbf{A}}^{H} \mathbf{w} - \mathbf{d}^{*} \|_{p}^{p} = 0.$$
 (10)

Left multiplying (5) with  $\mathbf{w}^H$  yields

 $\mathbf{w}^{H} \mathbf{R} \mathbf{w} + \tilde{\lambda} \mathbf{\bar{A}} \mathbf{\Pi}(\mathbf{w}) (\mathbf{\bar{A}}^{H} \mathbf{w} - \mathbf{d}^{*}) = 0.$  (11) With the definition of  $\mathbf{\bar{A}}$  and  $\mathbf{d}$ , we may equivalently express (10) as

$$\|\mathbf{A}^{H}\mathbf{w}\|_{p}^{p} + \alpha^{p} |\mathbf{w}^{H}\mathbf{a}(\theta_{0}) - 1|^{p} = 0.$$
(12)  
Also, (11) is equivalent to

 $\mathbf{w}^{H} \mathbf{R} \mathbf{w} + \| \mathbf{A}^{H} \mathbf{w} \|_{p}^{p} + \alpha^{p} \mathbf{w}^{H} \mathbf{a}(\theta_{0})$ 

 $|\mathbf{w}^H \mathbf{a}(\theta_0) - 1|^{p-2} (\mathbf{w}^H \mathbf{a}(\theta_0) - 1) = 0, (13)$ which can be written as

 $\mathbf{w}^{H} \mathbf{R} \mathbf{w} + \| \mathbf{A}^{H} \mathbf{w} \|_{p}^{p} + \alpha^{p} | \mathbf{w}^{H} \mathbf{a}(\theta_{0}) - 1 |^{p} +$ 

$$\alpha^{p} | \mathbf{w}^{H} \mathbf{a}(\theta_{0}) - 1 |^{p-2} (\mathbf{w}^{H} \mathbf{a}(\theta_{0}) - 1) = 0.$$
 (14)  
With (12), (14) can be simplified as

 $\mathbf{w}^{H} \mathbf{R} \mathbf{w} + \alpha^{p} | \mathbf{w}^{H} \mathbf{a}(\mathbf{\theta}_{0}) - 1 |^{p-2}$ 

$$(\mathbf{w}^{H}\mathbf{a}(\theta_{0}) - 1) = 0.$$
 (15)

Solving (15) gives the solution of  $\lambda$  as

$$\lambda = \gamma \left( \frac{\mathbf{w}^H \, \mathbf{R} \mathbf{w}}{|\mathbf{w}^H \mathbf{a}(\theta_0) - 1|^{p-2} (\mathbf{w}^H \mathbf{a}(\theta_0) - 1)} \right)^{1-p}.$$
(16)

When implementing the algorithm, we may alternatively update  $\mathbf{w}(i)$  and  $\lambda(i)$  or  $\gamma(i)$  using (6) and (16), respectively. For the second algorithm, the relationship between  $\lambda$  and  $\gamma$  is the same as (16) by replacing  $\overline{\mathbf{A}}$  with  $\mathbf{\Gamma}$ , and setting the denominator as  $\|\mathbf{A}_{S}^{H}\mathbf{w} - \mathbf{d}\|^{p-1}$ .

Because we have deduced the relationship between  $\lambda$  and  $\gamma$ , we may properly determine one of them and compute the other one using (16). Based on the existing literature, when p is fixed, some principles of choosing a proper  $\lambda$ , such as the L-curve [13], have been introduced. Therefore, when implementing the algorithm, we may firstly choose a proper value of p. Then, we use existing principles to determine  $\lambda$ . When  $\lambda$  is derived, we use (16) to compute  $\gamma$ . It should be emphasized that how to precisely choose  $\lambda$  is still an open issue. Current algorithms do not guarantee the optimal results. Fortunately, from our computer simulations, we see that the proposed algorithms are not sensitive to the choice of  $\lambda$  and  $\gamma$ . Therefore, they can be chosen empirically. When doing so, we should keep in mind that if a large  $\lambda$  or a small p is used, the value of  $\gamma$  should not be too small so that a trivial **w** can be avoided.

We summarize the proposed algorithms in Table 1.

| Table 1: Summary of the proposed algorithms                                |
|--|
| Step 1: Parameter setting:   |
| 1) Set up p, $\lambda$ and $\gamma$ ( $\lambda$ and $\gamma$ can be chosen |
| empirically);  |
| 2) Generate $\overline{\mathbf{A}}$ according to the presumed look         |
| direction and sidelobe sector;   |
| Step 2: Initialize: $i=0$ , $w(0)$ ;                                       |
| Step 3: Iterations:  |
| 1) Compute $\Pi(\mathbf{w}(i))$ ;  |
| 2) Update <b>w</b> (i) using (6) or (9);                                   |
| 3) If $\lambda$ is not chosen empirically, using the L-                    |
| curve principle to determine $\lambda$ ;                                   |
| 4) Compute γ using (16);   |
| 5) <i>i=i</i> +1;  |
| Step 4: Terminate the algorithm if the stopping                            |
| criterion is satisfied, otherwise go to Step 3.                            |

The stopping criterion can be chosen as  $i>N_{iter}$ , where  $N_{iter}$  is a preset number. The other usually adopted stopping rule is to evaluate the value of  $\|\mathbf{w}(i+1) - \mathbf{w}(i)\|_2^2 / \|\mathbf{w}(i)\|_2^2$ . If it is smaller than a preset threshold, terminate the algorithm.

#### **V. COMPUTER SIMULATIONS**

In this section, computer simulations are conducted to verify validity and advantages of the proposed algorithms. In all the simulations, the beampattern is computed by  $20 \log_{10} |\mathbf{w}^H \mathbf{a}(\theta)|$  for  $\theta \in [-90^o, 90^o]$ .

# A. Implementation of algorithm I to array synthesis

It is observed that the proposed algorithms can also be applied to the array synthesis algorithm [14] with  $\mathbf{R}=\mathbf{I}$  in (2). The sparse constraint (2c) plays a role to suppress the assigned sidelobe sector. To verify this, we conduct the following simulations. A uniform linear array (ULA) with 32 half-wavelength spaced sensors is assumed.

Firstly, we assume sidelobe suppression sector as  $[-90^{\circ}, -1^{\circ}] \cup [1^{\circ}, 90^{\circ}]$ . The array beampattern obtained using the proposed algorithm is shown in Fig. 1.

One can see from Fig. 1 that the proposed algorithm reduces the sidelobe level several

decibels from the original beampattern. Also, it is noticed that the smaller the value of p, the greater the sidelobe suppression. This coincides with our previous analysis that p controls the sparsity of the beampattern.



Fig. 1. Array beampatterns for a 32-sensor ULA with sidelobe suppression sector  $[-90^{\circ}, -1^{\circ}] \cup [1^{\circ}, 90^{\circ}]$ .



Fig. 2. Array beampatterns for a 32-sensor ULA with sidelobe suppression sector  $[-90^{\circ}, -5^{\circ}] \cup [5^{\circ}, 90^{\circ}]$ .

In certain applications, one may like to maintain the mainlobe of the array response at a certain width to ensure that the signal coming from the desired look direction will be received by the array with sufficient array gain. In Fig. 2, the proposed algorithm is used in the design of array pattern under this situation. The sidelobe suppression sector is chosen to be  $[-90^{\circ}, -5^{\circ}] \cup [5^{\circ}, 90^{\circ}]$ .

It is seen that compared with the original beampattern, the beamwidth is increased slightly, while the sidelobe level is improved by the order of tens of decibels. The sidelobes beside the mainlobe is significantly improved by more than 5 dB.

# **B.** Evaluation of algorithm I on narrowband beamforming

In this section, a ULA with eight halfwavelength inter-element spaced sensors is used. The DOA of the desired signal is supposed to be  $0^{\circ}$ , and the DOAs of three interferences are set to --30°, 30° and 70°. The signal-to-noise ratio (SNR) is set to 10 dB, and the signal-to-interference ratio (SIR) is -20 dB. 100 snapshots are used to compute the covariance matrix R. The regularization parameter  $\lambda$  is set to 0.2 in all the simulations. The matrix A consists of array steering vectors in the angular range from  $[-90^{\circ}, -1^{\circ}] \cup [1^{\circ}, 90^{\circ}]$ with 1° sampling interval. In this case, all the potential interferences are contained in A and the sidelobe level of the whole angular range is under control.

In the first simulation, we assume mismatch does not occur. The proposed method is compared with the MVDR beamformer. With different p and  $\alpha$ , the derived beampatterns are shown in Fig. 3.



Fig. 3. Array beampatterns for SNR=10 dB in the no mismatch case (Algorithm I).

It is observed that all of the beamformers cast deep nulls in directions of interference and generate distortionless response at the look direction. However, the sidelobe level of the MVDR beamformer is much higher than that of the proposed algorithm. For the proposed beamformer, the sidelobe level decreases as p decreases. This is because the smaller the p, the smaller the values of the beampattern are. This simulation result coincides with our previous analysis.

Figure 4 shows the plots of output SINR versus SNR with 1000 independent trials. The output signal to interference plus noise ratio (SINR) is calculated via

$$SINR_{output} = \frac{\sigma_s^2 \mathbf{w}^H \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) \mathbf{w}}{\mathbf{w}^H \left( \sum_{j=1}^J \sigma_j^2 \mathbf{a}(\theta_j) \mathbf{a}^H(\theta_j) + \sigma_n^2 \mathbf{I} \right) \mathbf{w}'}$$
(17)

where J denotes the number of interferences.



Fig. 4. Plots of output SINR versus SNR in the no mismatch case (Algorithm I).

It is noted that the proposed beamformer can achieve an output SINR approximately 6 dB larger than that of MVDR beamformer for all of the SNRs. For different p and  $\alpha$ , the proposed algorithm shows similar performance results.

In the second simulation, we assume there is a  $3^{\circ}$  mismatch in the look direction. The desired signal impinges the array from broadside, while the look direction of the beamformer is assumed to be  $3^{\circ}$ . Figure 5 shows the obtained beampatterns of different algorithms.

It is observed from Fig. 5 that the MVDR beamformer introduced a deep null at  $0^{\circ}$ , which means the desired source is suppressed due to imprecise knowledge of the look direction. In contrast, the proposed beamformer only slightly

shifts its maximum point of the beampattern, and is still capable of generating a distortionless response at the direction of SOI. At interference directions, all the algorithms are able to cast deep nulls.

Figure 6 plots the output SINR versus SNR with 1000 independent trials. From the figure, we see that the performance of MVDR beamformer and the proposed one differ a lot. It is observed that the proposed algorithm gives an output SINR about 10 dB, while the MVDR beamformer fails to work in all the cases. Similar to Figure 4, for different p and  $\alpha$ , the proposed algorithm has very similar performance.



Fig. 5. Array beampatterns for SNR=10 dB with 3° mismatch (Algorithm I).



Fig. 6. Plots of output SINR versus SNR with 3° mismatch (Algorithm I).

# C. Evaluation of algorithm II on narrowband beamforming

In this section, we evaluate the performance of the proposed Algorithm II when directions of interferences and the SOI are not precisely known. An ULA with ten half-wavelength inter-element spaced sensors is used. The actual source DOA is supposed to be 0°, and the DOAs of four interference signals are set to  $-30^{\circ}$ ,  $-10^{\circ}$ ,  $20^{\circ}$  and  $45^{\circ}$ . The SIR is assumed to be -20 dB, and the SNR is assumed to be 10 dB. The matrix **A** consists of array steering vectors in the DOA range  $[-90^{\circ}, -4^{\circ}) \cup (4^{\circ}, 90^{\circ}]$  with  $10^{\circ}$  sampling interval, while the matrix **A**<sub>s</sub> consists of only two array steering vectors that is defined at  $-3^{\circ}$  and  $3^{\circ}$ .



Fig. 7. Array beampatterns for SNR=10 dB with 3° mismatch (Algorithm II).



Fig. 8. Plots of output SINR versus SNR with 3° mismatch (Algorithm II).

Note that unlike the simulation settings in the previous section, we greatly relax our interferences constraints by using a  $10^{\circ}$  sampling interval. Also, the interference from  $45^{\circ}$  is not located on the spatial sampling grid.

In the first simulation, we assume a  $3^{\circ}$  look mismatch, i.e., the SOI impinges the array from broadside direction, while we assume it comes from  $3^{\circ}$ . Figure 7 shows the derived beampatterns using different algorithms.

It is observed from Fig. 7 that the MVDR castsa deep null at  $0^{\circ}$ , while the proposed algorithm still performs satisfactorily in this case. For interference directions, the algorithm casts deep nulls.

Figure 8 plots the output SINR versus SNR for 1000 independent trials. We see that the proposed algorithm outperforms the MVDR beamformer. The MVDR beamformer almost fails to work in all the cases. The proposed algorithms with different values of p again achieve similar performance to each other.



Fig. 9. Array beampatterns for SNR=10 dB with 3° mismatch and sensor position error (Algorithm II).

In the second simulation, besides the look direction mismatch, we further assume sensor position error which is a Gaussian variable with zero mean and standard deviation 0.1 times the sensor spacing appears in the steering vector. Figure 9 depicts the obtained beampatterns using different algorithms. Figure 10 shows the output SINR versus SNR for 1000 independent trials.

From Fig. 9, we see that the proposed algorithm is capable of maintaining distortionless

response at the direction of the SOI, while the MVDR beamformer suppresses the SOI about 25 dB in order to yield a distortionless response at the look direction. Figure 10 clearly demonstrates that the proposed algorithm is robust against look direction error and sensor position error, while the MVDR beamformer is very sensitive to these errors.



Fig. 10. Plots of output SINR versus SNR with 3° mismatch and sensor position error (Algorithm II).

#### **VI. CONCLUSIONS**

In this paper, we propose two beamforming algorithms which use sparse constraint to suppress a sidelobe level of beampattern. Taking imperfect array into account, we also add robust constraint on the new beamformers. All these algorithms are implement. Computer simulations easy to demonstrate that these algorithms work satisfactorily in the presence of steering vector error. Furthermore, the proposed algorithms are not very sensitive to the values of parameters used in the algorithms.

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## A Frequency-Dependent Hybrid Implicit-Explicit FDTD Scheme for Dispersive Materials

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Abstractfrequency-dependent А hybrid finite-difference time-domain implicit-explicit (HIE-FDTD) method for dispersive materials is presented. This method has higher computation efficiency than the conventional FDTD method, because the time step in this method is only determined by two space discretizations. The accuracy of this method is demonstrated by computing the incident at a planar air-water interface over a wide frequency band including frequency-dependent effects of the the permittivity of water.

*Index Terms-* Dispersive materials, HIE-FDTD method, weakly conditional stability.

#### I. INTRODUCTION

The finite-difference time-domain (FDTD) method [1] has been proven to be an effective scheme that provides accurate predictions of field behaviors for varieties of electromagnetic interaction problems. However, as it is based on an explicit finite-difference algorithm, the Courant–Friedrich–Levy (CFL) condition [2] must be satisfied when this method is used. Therefore, a maximum time-step size is limited by the minimum cell size in a computational domain, which makes this method inefficient for the problems where fine scale structures are involved.

To overcome the CFL constraint on the time step size of the FDTD method, some unconditionally stable methods [3-7] and weakly conditionally stable [8-17] schemes have been studied, among which, the hybrid implicit-explicit finite-difference time-domain (HIE-FDTD) method, has been applied extensively [13-17]. In the HIE-FDTD method, the time step size is only determined by two space discretizations, which is useful for problems with very fine structures in one direction. The accuracy and computational efficiency of the HIE-FDTD method have been well validated in [13-15] and [17].

In this paper, the HIE-FDTD method will be extended to frequency-dependent materials. The formulations of HIE-FDTD for a frequency-dependent complex permittivity are presented and an example of calculation of a wave incident at a planar air-water interface over a wide frequency band is showed. The extension of the HIE-FDTD method to frequency-dependent permeability is similar.

#### **II. FORMULATION**

For this paper, we will assume that our materials are linear and isotropic, and only the permittivity is frequency-dependent. Extension to nonlinear or anisotropic dispersive materials should be possible.

The displacement vector D is related to the electric field E in the time domain by the following equation:

$$D(t) = \varepsilon_{\infty}\varepsilon_{0}E(t) + \varepsilon_{0}\int_{0}^{t}E(t-\tau)\chi(\tau)d\tau.$$
(1)

 $\varepsilon_0$  is permittivity of free space,  $\chi(\tau)$  is the electric susceptibility, and  $\varepsilon_{\infty}$  is the infinite frequency relative permittivity.

Using Yee's notation, we let  $t = n\Delta t$  in (1), and each vector component of D and E can be written as:

$$D(t) \approx D(n\Delta t) = D^{n} =$$

$$\varepsilon_{\infty}\varepsilon_{0}E^{n} + \varepsilon_{0}\int_{0}^{n\Delta t}E(n\Delta t - \tau)\chi(\tau)d\tau.$$
(2)

All field components are assumed to be constant over each time interval  $\Delta t$ . Therefore, we have, assuming D(t) and E(t) are zero for t < 0:

$$D^{n} = \varepsilon_{\infty}\varepsilon_{0}E^{n} + \varepsilon_{0}\sum_{m=0}^{n-1}E^{n-m}\int_{m\Delta t}^{(m+1)\Delta t}\chi(\tau)d\tau.$$

$$D^{n+1} = \varepsilon_{\infty} \varepsilon_0 E^{n+1} + \varepsilon_0 \sum_{m=0}^n E^{n+1-m} \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau) d\tau.$$

(4)

(3)

When (3) is substituted in (4), we find:

$$D^{n+1} - D^{n} = \varepsilon_{\infty} \varepsilon_{0} \left[ E^{n+1} - E^{n} \right]$$
  
+  $\varepsilon_{0} E^{n+1} \int_{0}^{\Delta t} \chi(\tau) d\tau$   
+  $\varepsilon_{0} \sum_{m=0}^{n-1} E^{n-m} \left\{ \int_{(m+1)\Delta t}^{(m+2)\Delta t} \chi(\tau) d\tau - \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau) d\tau \right\}.$ 
(5)

For simplicity, we let:

$$\chi_m = \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau) d\tau.$$
 (6)

$$\Delta \chi_m = \chi_m - \chi_{m+1}. \tag{7}$$

Then, we have:

$$E^{n+1} = \frac{\varepsilon_{\infty}}{\left(\varepsilon_{\infty} + \chi_{0}\right)} E^{n}$$

$$+ \frac{1}{\left(\varepsilon_{\infty} + \chi_{0}\right)} \sum_{m=0}^{n-1} E^{n-m} \Delta \chi_{m} \qquad (8)$$

$$+ \frac{D^{n+1} - D^{n}}{\left(\varepsilon_{\infty}\varepsilon_{0} + \varepsilon_{0}\chi_{0}\right)}.$$

The three HIE-FDTD scalar equations that relate the components of the electric field E to the components of the magnetic field H can be readily obtained from (8) by using the discretized Maxwell-Ampere equation to replace  $D^{n+1} - D^n$  in (8) with the curl of the H field,

$$E_{y}^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k\right) = \frac{\varepsilon_{\infty}}{(\varepsilon_{\infty}+\chi_{0})}E_{y}^{n-\frac{1}{2}}\left(i,j+\frac{1}{2},k\right) + \frac{1}{(\varepsilon_{\infty}+\chi_{0})}\sum_{m=0}^{n-\frac{3}{2}}E_{y}^{n-\frac{1}{2}-m}\left(i,j+\frac{1}{2},k\right)\Delta\chi_{m} + \frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0}+\varepsilon_{0}\chi_{0})\Delta z}\begin{bmatrix}H_{x}^{n}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)\\-H_{x}^{n}\left(i,j+\frac{1}{2},k-\frac{1}{2}\right)\end{bmatrix} - \frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0}+\varepsilon_{0}\chi_{0})\Delta x}\begin{bmatrix}H_{z}^{n}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)\\-H_{z}^{n}\left(i-\frac{1}{2},j+\frac{1}{2},k\right)\end{bmatrix}.$$
(9)

$$E_{x}^{n+1}\left(i+\frac{1}{2},j,k\right) = \frac{\varepsilon_{\infty}}{(\varepsilon_{\infty}+\chi_{0})} E_{x}^{n}\left(i+\frac{1}{2},j,k\right) + \frac{1}{(\varepsilon_{\infty}+\chi_{0})} \sum_{m=0}^{n-1} E_{x}^{n-m}\left(i+\frac{1}{2},j,k\right) \Delta \chi_{m}$$

$$-\frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0}+\varepsilon_{0}\chi_{0})} \frac{1}{\Delta z} \begin{bmatrix} H_{y}^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right) \\ -H_{y}^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right) \end{bmatrix} + \frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0}+\varepsilon_{0}\chi_{0})} \frac{1}{2\Delta y} \begin{bmatrix} H_{z}^{n+1}\left(i+\frac{1}{2},j+\frac{1}{2},k\right) \\ -H_{z}^{n+1}\left(i+\frac{1}{2},j-\frac{1}{2},k\right) \end{bmatrix} + \frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0}+\varepsilon_{0}\chi_{0})} \frac{1}{2\Delta y} \begin{bmatrix} H_{z}^{n}\left(i+\frac{1}{2},j+\frac{1}{2},k\right) \\ -H_{z}^{n+1}\left(i+\frac{1}{2},j-\frac{1}{2},k\right) \end{bmatrix}$$

$$+ \frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0}+\varepsilon_{0}\chi_{0})} \frac{1}{2\Delta y} \begin{bmatrix} H_{z}^{n}\left(i+\frac{1}{2},j+\frac{1}{2},k\right) \\ -H_{z}^{n}\left(i+\frac{1}{2},j-\frac{1}{2},k\right) \end{bmatrix}$$

$$(10)$$

$$E_{z}^{n+1}\left(i, j, k+\frac{1}{2}\right) = \frac{\varepsilon_{\infty}}{(\varepsilon_{\infty} + \chi_{0})} E_{z}^{n}\left(i, j, k+\frac{1}{2}\right) + \frac{1}{(\varepsilon_{\infty} + \chi_{0})} \sum_{m=0}^{n-1} E_{z}^{n-m}\left(i, j, k+\frac{1}{2}\right) \Delta \chi_{m} + \frac{\Delta t}{(\varepsilon_{\infty}\varepsilon_{0} + \varepsilon_{0}\chi_{0})\Delta x} \begin{bmatrix} H_{y}^{n+\frac{1}{2}}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right) \\ -H_{y}^{n+\frac{1}{2}}\left(i-\frac{1}{2}, j, k+\frac{1}{2}\right) \end{bmatrix} - \frac{\Delta t}{2(\varepsilon_{\infty}\varepsilon_{0} + \varepsilon_{0}\chi_{0})\Delta y} \begin{bmatrix} H_{x}^{n+1}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) \\ -H_{x}^{n+1}\left(i, j-\frac{1}{2}, k+\frac{1}{2}\right) \end{bmatrix} - \frac{\Delta t}{2(\varepsilon_{\infty}\varepsilon_{0} + \varepsilon_{0}\chi_{0})\Delta y} \begin{bmatrix} H_{x}^{n}\left(i, j+\frac{1}{2}, k+\frac{1}{2}\right) \\ -H_{x}^{n}\left(i, j-\frac{1}{2}, k+\frac{1}{2}\right) \end{bmatrix}.$$
(11)

It can be seen from these equations that the eqs. (10) and (11) can't be used for direct numerical calculation, because they all include the unknown components defined at the same time, thus, modified equations are derived from the original equations.

Updating of  $E_x^{n+1}$  component, as shown in eq. (10), needs the unknown  $H_z^{n+1}$  component at the same time. In the nonmagnetic media, the updating for H component is unchanged. The equation of the  $H_z^{n+1}$  component in the HIE-FDTD method is as follows:

$$H_{z}^{n+1}\left(i+\frac{1}{2},j+\frac{1}{2},k\right) = H_{z}^{n}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)$$
$$+\frac{\Delta t}{2\mu\Delta y}\left[E_{x}^{n+1}\left(i+\frac{1}{2},j+1,k\right) - E_{x}^{n+1}\left(i+\frac{1}{2},j,k\right)\right]$$
$$+E_{x}^{n}\left(i+\frac{1}{2},j+1,k\right) - E_{x}^{n}\left(i+\frac{1}{2},j,k\right)\right]$$
$$-\frac{\Delta t}{\mu\Delta x}\left[E_{y}^{n+\frac{1}{2}}\left(i+1,j+\frac{1}{2},k\right) - E_{y}^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k\right)\right].$$
(12)

Thus, the  $E_x^{n+1}$  component has to be updated implicitly. Substituting eq. (12) into eq. (10), the equation for  $E_x^{n+1}$  field is given as,

$$\begin{split} \left[1 + \frac{\Delta^{2}}{2\varepsilon\mu\Delta y^{2}}\eta\right] E_{x}^{n+1}\left(i + \frac{1}{2}, j, k\right) \\ - \frac{\Delta^{2}}{4\varepsilon\mu\Delta y^{2}}\eta\left[E_{x}^{n+1}\left(i + \frac{1}{2}, j + 1, k\right) + E_{x}^{n+1}\left(i + \frac{1}{2}, j - 1, k\right)\right] \\ = \eta\varepsilon_{\infty}E_{x}^{n}\left(i + \frac{1}{2}, j, k\right) + \eta\sum_{m=0}^{n-1}E_{x}^{n-m}\left(i + \frac{1}{2}, j, k\right)\Delta\chi_{m} \\ + \eta\frac{\Delta t}{\varepsilon\Delta y}\left[H_{z}^{n}\left(i + \frac{1}{2}, j + \frac{1}{2}, k\right) - H_{z}^{n}\left(i + \frac{1}{2}, j - \frac{1}{2}, k\right)\right] \\ - \eta\frac{\Delta t^{2}}{2\mu\varepsilon\Delta x\Delta y}\left[E_{y}^{n+\frac{1}{2}}\left(i + 1, j + \frac{1}{2}, k\right) - E_{y}^{n+\frac{1}{2}}\left(i, j + \frac{1}{2}, k\right)\right] \\ + \eta\frac{\Delta t^{2}}{2\mu\varepsilon\Delta x\Delta y}\left[E_{y}^{n+\frac{1}{2}}\left(i + 1, j - \frac{1}{2}, k\right) - E_{y}^{n+\frac{1}{2}}\left(i, j - \frac{1}{2}, k\right)\right] \\ + \eta\frac{\Delta t^{2}}{4\varepsilon\mu\Delta y^{2}}\left[E_{x}^{n}\left(i + \frac{1}{2}, j - 1, k\right) - E_{x}^{n}\left(i + \frac{1}{2}, j, k\right)\right] \\ + \eta\frac{\Delta t^{2}}{4\varepsilon\mu\Delta y^{2}}\left[E_{x}^{n}\left(i + \frac{1}{2}, j - 1, k\right) - E_{x}^{n}\left(i + \frac{1}{2}, j, k\right)\right] \\ - \eta\frac{\Delta t}{\varepsilon\Delta z}\left[H_{y}^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j, k + \frac{1}{2}\right) - H_{y}^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j, k - \frac{1}{2}\right)\right]. \end{split}$$
(13)

here,  $\eta = 1/(\varepsilon_{\infty} + \chi_0)$ . The updating equation of  $E_z^{n+1}$  is similarly derived and calculated in the same way as eq.(13).

Note that if we assume the relative permittivity is independent of frequency, namely,  $\chi(\tau) = 0$ ,  $\varepsilon_{\infty} = \varepsilon_r$ ,  $\chi_m = 0$  for all *m*, then, the above equations reduce to the standard HIE-FDTD formulations [13].

The above formulation is valid for electrically dispersive media. The extension to magnetically dispersive media would be similar.

In [15], the weakly conditional stability of the HIE-FDTD method is well validated. The time step of the HIE-FDTD method is only determined by two spatial increments  $\Delta x$  and  $\Delta z$ , namely:

$$\Delta t \le 1 / \left( c \sqrt{\left( 1 / \Delta x \right)^2 + \left( 1 / \Delta z \right)^2} \right).$$
(14)

 $c = 1/\sqrt{\epsilon\mu}$  is the speed of light in the medium. This is especially useful when the simulated structure has fine-scale dimensions in one direction, which will be validated in next section.

#### **III. SIMULATION RESULTS**

In order to demonstrate the validity and accuracy of the above formulation, a small current source above an incident at a planar air-water interface is presented here. The geometric configuration of the numerical simulation is shown in Fig.1. The dimensions of the perfect-electric-conductor box are  $15 \text{cm} \times 15 \text{cm} \times 3 \text{cm}$ . The box is filled with water up to height 7.5cm. A small current source applied along y direction is placed at the upper part of the box. The time dependence of the excitation function is:

$$g(t) = \exp[-4\pi(t-t_0)^2/t_1^2], \qquad (15)$$

where  $t_0$  and  $t_1$  are constants, and both equal to  $0.6 \times 10^{-9}$ . The observation point B is set in the water, and 2 cm far from the source point A.



Fig. 1. Geometric configuration of the numerical simulation.

Applying the FDTD method to compute the time domain electric field component  $E_y$  at the observation point B, the cell size is chosen as  $5\Delta y = \Delta x = \Delta z = 0.5$  cm, so that the computational domain is  $30 \times 30 \times 30$  cells. To satisfy the stability condition of the FDTD

algorithm, the time-step size for conventional FDTD [18, 19] is  $\Delta t \leq 3.20$  ps. Here, the conventional FDTD is the FDTD method using recursive convolutional (RC) method to implement the dispersive media. For the HIE-FDTD scheme, the maximum time increment is only related to the space increments  $\Delta x$  and  $\Delta z$ , that is,  $\Delta t \leq 11.78$  ps. The HIE-FDTD method is using the recursive convolutional method with Debye model to implement the dispersive properties of the media.

For water, the complex permittivity  $\varepsilon^*(\omega)$  can be described as

$$\varepsilon^{*}(\omega) = \varepsilon_{0} \Big[ \varepsilon_{\infty} + (\varepsilon_{s} - \varepsilon_{\infty}) / (1 + j\omega\tau_{0}) \Big], \quad (16)$$

where  $\varepsilon_s$  is the "static" permittivity, and  $\tau_0$  is the "relaxation time" constant. The water parameters used here are  $\varepsilon_s = 81$ ,  $\varepsilon_{\infty} = 1.8$ , and  $\tau_0 = 9.4 \times 10^{-12}$ .

The summation (convolution) term of eq.(8) can be updated recursively by utilizing the equation (20) in [18], because the susceptibility function is an exponential. So, it doesn't require storing a large number of past time values  $E^n$ , and the computational time is saved. Only one additional number needs to be stored for each electric field component at each spatial index.

First, we validate the numerical stability condition (14). Figure 2 shows the electric field component  $E_y$  at observation point B calculated by using the HIE-FDTD methods with the time-step size  $\Delta t = 11.78$ ps for a long time history. No instability problem is observed even for 5,000 steps, which validates the weakly conditional stability of the HIE-FDTD method numerically.

To demonstrate the high computational efficiency of HIE-FDTD method, we perform the numerical simulations for an 8 ns time history by using the conventional FDTD, and HIE-FDTD methods, and compare the CPU times taken by using these two methods. In the conventional FDTD method, the time-step size keeps a constant of 3.20 ps; while in the HIE-FDTD method, the time step size is 11.78ps.

Figure 3 shows the electric field component  $E_y$  at observation point B calculated by using the conventional FDTD and HIE-FDTD methods. It can be seen from this figure that the result calculated by the HIE-FDTD method agrees with

the result calculated by the conventional FDTD method. The HIE-FDTD method has almost the same accuracy as that of the conventional FDTD method. To validate this further, the divergence between these two methods are presented, as



Fig. 2. The electric field component  $E_y$  at observation point B calculated by using the HIE-FDTD method with the time-step size  $\Delta t = 11.78$  ps for a long time story.



Fig. 3. The electric field component  $E_y$  at observation point B calculated by using the conventional FDTD ( $\Delta t = 3.20$ ps), and HIE-FDTD methods ( $\Delta t = 11.78$ ps).

shown in Fig.4. It can be seen from this figure that the error curve is limited and it doesn't increase as the addition of the computational time. It should be noted that the simulation takes 364 s for the conventional FDTD method and 121 s for the HIE-FDTD method. The time cost for the HIE-FDTD simulation is 1/3 times as that for the conventional FDTD simulation. So, we can conclude that the HIE-FDTD has higher efficiency than the conventional FDTD method, due to larger time step size used.



Fig. 4. The divergence between conventional FDTD ( $\Delta t = 3.20$ ps), and HIE-FDTD methods ( $\Delta t = 11.78$ ps).

#### **IV. CONCLUSION**

A frequency-dependent HIE-FDTD method for dispersive materials is presented. It is found that the technique is weakly conditionally stable and supports time steps greater than the CFL limit. A numerical example demonstrates that computation efficiency of the HIE-FDTD method is higher than the conventional FDTD method, and the accuracy of the HIE-FDTD is almost the same as that of the conventional FDTD method.

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## Adaptive Mixed-Form Fast Multipole Method for the Analysis of Electromagnetic Scattering

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Abstract- To analyze an electrically large object with local fine structures, the conventional mixed form fast multipole algorithm requires that the boxes of the finest level are all with the same size, which belongs to the low-frequency region. This scheme is deficient since the mesh size is limited by the box size of the finest level, which is related to the finest parts of the object. In this paper, an efficient adaptive grouping scheme is introduced into a mixed-form fast multipole algorithm. In an adaptive mixed-form fast multipole algorithm, the number of unknowns in each non-empty box of the finest level is almost the same which results in the box of the finest level may be in a different frequency regime with a different size. Hence multipole expansions are employed, if the boxes located in the low frequency regime while the plane wave expansions are employed if the boxes located in the mid-frequency regime. Numerical results are given to show that the proposed approach is efficient to analyze the objects with many fine structures.

*Index Terms*- Adaptive grouping scheme, electromagnetic scattering, mixed-form fast multipole algorithm, multilevel fast multipole algorithm (MLFMA).

## I. INTRODUCTION

The integral equation formulation is one of the most commonly used methods for solving the Maxwell's equations in electromagnetic scattering problems. Due to the fast increasing capability of computers, computational electromagnetics has developed to be more and more powerful. As a result, a number of algorithms have been presented for the fast computation of the large linear matrix equation systems resulting from the discretization of integral equations for electromagnetic scattering, such as conjugate gradient fast Fourier transform (CG-FFT) [1], adaptive integral method (AIM) [2], multilevel fast multipole algorithm (MLFMA) [3], etc.

Although many methods can simulate EM field accurately of electrically large objects, it remains to be difficult to simulate the EM field accurately where the size of the subscatterers is a small fraction of a wavelength. Many traditional methods suffer from a so-called "low frequency breakdown", such as method of moments (MoM), finite element method (FEM) and so on [3]. In order to capture the fine details of the structure accurately, e.g., in circuit components such as inductors and capacitors, this problem becomes more and more important and poses a pressing to simulate EM phenomena in circuits and antennas. Therefore, it becomes a popular topic for researchers in this field to overcome the low-frequency breakdown problem. Some fast methods have been proposed to cover the regime from low frequency to mid frequency, such as loop and tree basis decomposition introduced for EFIE [15, 16], the plane wave methods based on the generalized Gaussian quadrature rules proposed by Greengard [4], and the low-frequency fast inhomogeneous plane wave algorithm (LF-FIPWA) for wide-band fast computations [5, 6]. However, much memory is required for LF-FIPWA, since evanescent waves are highly direction dependent.

The fast multipole algorithm is numerically unstable due to the oscillatory characteristic of the spherical Hankel function for small arguments. As an attempt for a possible remedy, the mixed-form fast multipole algorithm (MF-FMA) [3, 7] is proposed to cover a wide band from the low-frequency to mid-frequency. However, it is noticed that the MF-FMA need to construct multipole expansions for each nonempty boxes at the low frequency regime [3, 7]. As a result, the finest level boxes' size will all be less than  $0.2\lambda$ (" $\lambda$ " stands for wavelength). This results in a source distribution of high-density for the whole object. However, AMF-FMA just applies MF-FMA for some part of the object which belongs to the low frequency regime and the rest of the object which belongs to the mid frequency regime is still analyzed by MLFMA. Hence for an object containing fine structure, AMF-FMA takes advantage of adaptive grouping scheme [8, 13] to set the finest level box size smaller than  $0.2\lambda$  for fine structure while for the rest of the object, the finest level box size is larger than  $0.2\lambda$ . This frees MLFMA from low-frequency breakdown. As a result, the characteristics of both long-wavelength and short-wavelength are considered simultaneously for the object. In this way, the proposed algorithm is memory efficient since the adaptive grouping scheme is used.

This paper is organized as follows. Section II gives a brief introduction to the processing of near interaction of MF-FMA and the EFIE formulation in electromagnetic wave scattering. Section III describes the theory and implementation of the adaptive MF-FMA in more details. Numerical experiments with a few electromagnetic wave scattering problems are presented to demonstrate the efficiency of the AMF-FMA in Section IV. Section V gives some conclusions and comments.

#### II. EFIE FORMULATION

The EFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [13]. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows:

$$\sum_{n=1}^{N} Z_{mn} a_n = V_m, \qquad m = 1, 2, ..., N, \qquad (1)$$

where

$$Z_{mn} = \frac{jk\eta}{4\pi} \iint_{s} \mathbf{f}_{m}(\mathbf{r}) \cdot \iint_{s} G(\mathbf{r}, \mathbf{r}') \mathbf{f}_{n}(\mathbf{r}') dS' dS$$
$$-\frac{j\eta}{4\pi k} \iint_{s} \nabla \cdot \mathbf{f}_{m}(\mathbf{r}) \cdot \iint_{s} G(\mathbf{r}, \mathbf{r}') \nabla \cdot \mathbf{f}_{n}(\mathbf{r}') dS' dS,$$

and

$$V_m = \int_s \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{E}^i(\mathbf{r}) ds, \quad G(\mathbf{r},\mathbf{r'}) = \frac{e^{-jk|\mathbf{r}-\mathbf{r''}|}}{4\pi |\mathbf{r}-\mathbf{r'}|}.$$

Here,  $G(\mathbf{r}, \mathbf{r}')$  refers to the Green's function in free space and  $\{a_n\}$  is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. Also, as usual,  $\mathbf{r}$  and  $\mathbf{r}'$  denote the observation and source point locations.  $\mathbf{E}^i(\mathbf{r})$  is the incident excitation plane wave, and  $\eta$  and k denote the



Fig. 1. The general translation used for LF-FMA.

free space impendence and wave number, respectively. Once the matrix equation (1) is solved by numerical matrix equation solvers, the expansion coefficients  $\{a_n\}$  can be used to calculate the scattered field and RCS. In the following, we use **A** to denote the coefficient

matrix in equation (1),  $\mathbf{x} = \{a_n\}$ , and  $\mathbf{b} = \{V_m\}$  for simplicity. Then, the EFIE matrix equation (1) can be symbolically rewritten as:

$$\mathbf{A}\mathbf{x} = \mathbf{b}.$$
 (2)

To solve the above matrix equation by an iterative method, the matrix-vector products are

needed at each iteration. Physically, а matrix-vector product corresponds to one cycle of interactions between the basis functions. The basic idea of the mixed-form FMA (MF-FMA) is to combine the LF-FMA with the MLFMA into one uniform expression and convert the interaction of element-to-element to the interaction of group-to-group. Here, a group includes the elements residing in a spatial box. The mathematical foundation of the mixed-form FMA is, also, the addition theorem for the scalar Green's function in free space. Using the MF-FMA, the matrix-vector product Ax can be written as:

$$\mathbf{A}\mathbf{x} = \mathbf{A}_{N}\mathbf{x} + \mathbf{A}_{F}\mathbf{x}.$$

Here,  $\mathbf{A}_N$  is the near part of  $\mathbf{A}$  and  $\mathbf{A}_F$  is the far part of  $\mathbf{A}$ .

In the MF-FMA, the calculation of matrix elements in  $\mathbf{A}_N$  remains the same as in the MoM procedure. However, those elements in  $\mathbf{A}_F$  are not explicitly computed and stored. Hence, they are not numerically available in the MF-FMA.

### III. ADAPTIVE MIXED-FORM FAST MULTIPOLE ALGORITHM

It is well known that the diagonalized translation matrix of the 3-D MLFMA can greatly decrease the memory requirements and expedite the matrix-vector multiplication. However, it can not avoid low-frequency breakdown [3, 4]. In order to overcome the low-frequency breakdown, the low-frequency fast multipole algorithm (LF-FMA) is developed [12] and it bridges the gap from static to electrodynamic.

#### A. Low-frequency fast multipole algorithm

As shown in Fig. 1,  $\mathbf{r}_{ji} = \mathbf{r}_{jJ} + \mathbf{r}_{JI} + \mathbf{r}_{Ii}$  and the addition theorem [3, 9, 10] is utilized for LF-FMA. The general translation equation is obtained as follows:

$$\alpha_{LL'}\left(\mathbf{r}_{ji}\right) = \sum_{L_1} \sum_{L_2} \beta_{LL_1}\left(\mathbf{r}_{jJ}\right) \alpha_{L_1L_2}\left(\mathbf{r}_{JI}\right) \beta_{L_2L'}\left(\mathbf{r}_{Ii}\right)$$
(3)

The above equation is a recursive equation and

 $L = (l, m), 0 \le l \le P, -l \le m \le l$ , P is truncation number of multipoles. Here,  $L_1$ ,  $L_2$  and L'have similar definitions as L. L and L' are used as the subscripts for the  $\alpha$  operator and the  $\beta$  operator.  $\alpha_{LL}$  and  $\beta_{LL}$  are defined as

$$\alpha_{L',L}(\mathbf{r}) = \sum_{L''} 4\pi (-j)^{(l'+l''-l)} \Psi_{L'}(k,\mathbf{r}) A_{L,L',L''} \quad (4)$$
$$\beta_{L',L}(\mathbf{r}) = \sum_{L''} 4\pi (-j)^{(l'+l''-l)} \Re_g \Psi_{L'}(k,\mathbf{r}) A_{L,L',L''}, \quad (5)$$

where

$$\Psi_{L}(k,\mathbf{r}) = h_{l}^{(2)}(kr)Y_{l}^{m}(\theta,\phi)$$
  

$$\Re_{g}\Psi_{L}(k,\mathbf{r}) = j_{l}(kr)Y_{l}^{m}(\theta,\phi).$$
(6)

 $A_{L,L',L''}$  is the Gaunt coefficient,  $h_l^{(2)}(x)$  is the spherical Hankel function of the second kind,  $Y_l^m(\theta,\phi)$  is the spherical harmonic function [10, 11].

 $G(\mathbf{r}_{j}, \mathbf{r}_{i})$  is free space green's function and corresponds to  $\alpha_{00}(\mathbf{r}_{ji})$ 

$$G(\mathbf{r}_{j},\mathbf{r}_{i}) = -jk\alpha_{00}(\mathbf{r}_{ji}).$$
(7)

# **B.** Adaptive mixed-form fast multipole algorithm

LF-MLFMA is developed for very low-frequency problems based the on non-diagonalized form of the fast multipole translator and the multipoles are efficient in grouping and translating waves among much smaller objects or boxes [3, 7]. However, LF-MLFMA will lose its accuracy and efficiency when the box size is above  $0.2\lambda$ . Multipoles and plane waves are combined into one octree browsing process to form the MF-FMA for the efficient analysis in both low frequency and mid-frequency bands. When the frequency is low, or the MLFMA box sizes are small compared to wavelength, the non-diagonalized form of the fast multipole translator is used. When the box sizes are comparable to wavelength, the diagonalized form of the fast multipole (plane wave) translator is used for translation. In this manner, the multilevel algorithm can efficiently work for box sizes to be a small fraction of a wavelength and the whole simulation objects can be electrically large enough when compared to wavelength.

The fast multipole translator  $\alpha_{LL}$  is always in the form of dense matrix and can be diagonalized into the plane wave integrations (8) and (9).

Inserting (9) into (3), the Mixed-form translation containing two-level multipoles translation and two-level diagonal translation in

the form of matrix is obtained in (10) below, where  $S_i$  is a sample number of propagating waves over a unit sphere at a level for diagonal translations, [I] is the interpolation matrix and  $[I]^T$ denotes the transpose of matrix [I].  $J_i$  and  $I_i$ denote the different boxes in "i" level. "Diag" is short for the diagonal translation matrix.  $[D]_{S\times L}$  is transformer from multipoles to plane waves, and  $[D]_{S\times L}^{\dagger}$  is transformer from plane waves to multipoles [7].

$$\alpha_{LL'}\left(\mathbf{r}_{ji}\right) = \int d\Omega_{k}\left(-j\right)^{l} Y_{L}^{*}\left(\Omega_{k}\right) e^{-jk\cdot\mathbf{r}_{jJ}} \tilde{T}_{JI}\left(\Omega_{k},\mathbf{r}_{JI}\right)\left(-j\right)^{-l'} Y_{L'}\left(\Omega_{k}\right) e^{-jk\cdot\mathbf{r}_{Ii}}$$
(8)

$$\tilde{T}_{JI}\left(\Omega_{k},\Omega_{k}'\right) = \sum_{l=0}^{L_{max}} \left(-j\right)^{l} \left(2l+1\right) h_{l''}^{(2)}\left(kr_{JI}\right) P_{l}\left(\hat{k}\cdot\hat{r}_{JI}\right)$$
(9)

$$\begin{bmatrix} \alpha_{LL'} \left( \mathbf{r}_{ji} \right) \end{bmatrix}_{L \times L'} = \begin{bmatrix} \beta_{LL_{1}} \left( \mathbf{r}_{jJ_{1}} \right) \end{bmatrix}_{L \times L_{1}} \cdot \begin{bmatrix} \beta_{L_{1}L_{2}} \left( \mathbf{r}_{J_{1}J_{2}} \right) \end{bmatrix}_{L_{1} \times L_{2}} \\ \cdot \begin{bmatrix} \mathbf{D} \end{bmatrix}_{S_{3} \times L_{2}}^{\dagger} \cdot diag \begin{bmatrix} e^{-jk \cdot \mathbf{r}_{J_{2}J_{3}}} \end{bmatrix}_{S_{3} \times S_{3}} \cdot \begin{bmatrix} \mathbf{I} \end{bmatrix}_{S_{4} \times S_{3}}^{T} \cdot diag \begin{bmatrix} e^{-jk \cdot \mathbf{r}_{J_{3}J_{4}}} \end{bmatrix}_{S_{4} \times S_{4}} \\ \cdot diag \begin{bmatrix} \tilde{T} \left( \Omega_{s4}, \mathbf{r}_{J_{4}I_{4}} \right) \omega_{S_{4}} \end{bmatrix}_{S_{4} \times S_{4}} \\ \cdot diag \begin{bmatrix} e^{-jk \cdot \mathbf{r}_{I_{4}I_{3}}} \end{bmatrix}_{S_{4} \times S_{4}} \cdot \begin{bmatrix} \mathbf{I} \end{bmatrix}_{S_{4} \times S_{3}} \cdot diag \begin{bmatrix} e^{-jk \cdot \mathbf{r}_{I_{3}I_{2}}} \end{bmatrix}_{S_{3} \times S_{3}} \cdot \begin{bmatrix} \mathbf{D} \end{bmatrix}_{S_{3} \times L_{2}} \\ \cdot \begin{bmatrix} \beta_{L_{2}L_{1}} \left( \mathbf{r}_{I_{2}I_{1}} \right) \end{bmatrix}_{L_{2} \times L_{1}} \cdot \begin{bmatrix} \beta_{L_{1}L'} \left( \mathbf{r}_{I_{1}i} \right) \end{bmatrix}_{L_{1} \times L'} \end{bmatrix}$$

$$(10)$$

$$\frac{e^{-jkr_{ji}}}{r_{ji}} = -jk\alpha_{00}\left(\mathbf{r}_{ji}\right)$$

$$= -jk\left[\beta_{0L_{1}}\left(\mathbf{r}_{jJ_{1}}\right)\right]_{I\times L_{1}} \cdot \left[\beta_{L_{1}L_{2}}\left(\mathbf{r}_{J_{1}J_{2}}\right)\right]_{L_{1}\times L_{2}}$$

$$\cdot \left[\mathbf{D}\right]_{S_{3}\times L_{2}}^{\dagger} \cdot diag\left[e^{-jk\cdot\mathbf{r}_{J_{2}J_{3}}}\right]_{S_{3}\times S_{3}} \cdot \left[\mathbf{I}\right]_{S_{4}\times S_{3}}^{T} \cdot diag\left[e^{-jk\cdot\mathbf{r}_{J_{3}J_{4}}}\right]_{S_{4}\times S_{4}}$$

$$\cdot diag\left[\tilde{T}\left(\Omega_{S_{4}},\mathbf{r}_{J_{4}I_{4}}\right)\omega_{S_{4}}\right]_{S_{4}\times S_{4}}$$

$$\cdot diag\left[e^{-jk\cdot\mathbf{r}_{I_{4}I_{3}}}\right]_{S_{4}\times S_{4}} \cdot \left[\mathbf{I}\right]_{S_{4}\times S_{3}} \cdot diag\left[e^{-jk\cdot\mathbf{r}_{I_{3}I_{2}}}\right]_{S_{3}\times S_{3}} \cdot \left[\mathbf{D}\right]_{S_{3}\times L_{2}}$$

$$\cdot \left[\beta_{L_{2}L_{1}}\left(\mathbf{r}_{I_{2}I_{1}}\right)\right]_{L_{2}\times L_{1}} \cdot \left[\beta_{L_{1}0}\left(\mathbf{r}_{I_{1}i}\right)\right]_{L_{1}\times I}$$
(11)

$$\frac{e^{-jk\mathbf{r}_{ji}}}{\mathbf{r}_{ji}} = diag \left[ e^{-jk\cdot\mathbf{r}_{jJ_3}} \right]_{S_3 \times S_3} \cdot \left[ \mathbf{I} \right]_{S_4 \times S_3}^{\mathrm{T}} \cdot diag \left[ e^{-jk\cdot\mathbf{r}_{J_3J_4}} \right]_{S_4 \times S_4} \\
\cdot diag \left[ \tilde{T} \left( \Omega_{S_4}, \mathbf{r}_{J_4I_4} \right) \omega_{S_4} \right]_{S_4 \times S_4} \\
\cdot diag \left[ e^{-jk\cdot\mathbf{r}_{I_4J_3}} \right]_{S_4 \times S_4} \cdot \left[ \mathbf{I} \right]_{S_4 \times S_3} \cdot diag \left[ e^{-jk\cdot\mathbf{r}_{I_2j}} \right]_{S_3 \times S_3}.$$
(12)

The free space Green's function is expanded by mixed-form translation through equation (10) as (11) above, and the free space Green's function is expanded by plane wave translation for MLFMA as (10) above.

It can be found from the above equations that both the LF-MLFMA and the MF-FMA uses the same grouping structure as MLFMA. Therefore, the simulated object is first enclosed by the smallest possible box, called root boxes. Then the root box is divided into eight equal boxes and each of the child boxes is recursively subdivided into eight smaller boxes until we reach the finest level. For MF-FMA, when the multipole expansion is applied for each nonempty box in the finest level, it is inevitable to produce a large number of unknowns because of the source distribution for the whole object is high-density at the low-frequency regime and the number of the finest level box is consequentially larger than



Fig. 2. Point positions.

MLFMA. Therefore, the MF-FMA is not necessary for the whole object but only used for the fine structures connected with the electrically large object.

In the conventional grouping scheme described above, the parent boxes are divided regardless of the number of sources inside them [9]. However, for some fraction of the object, the source distribution is high-density and it is in fact a low-frequency problem. As a result, the uniform partitioning often leads to the whole object divided with the same resolution and the simulation result is not accurate. When MF-FMA is used for the object, the box size of finest level can be smaller than  $0.2\lambda$ , but source distribution is high-density for the whole object. To improve the efficiency of the method, the MF-FMA is applied for fine structures of the object which is considered at the low-frequency regime and the conventional MLFMA is used for the rest of the object which is considered in the mid-frequency regime. As MF-FMA and MLFMA is used at the same time, the size of leafy groups is not always the same. Therefore, a new grouping scheme has to be developed.

Different from the method described in [12], if a box doesn't contain sources in the low-frequency regime, it will be considered as a leaf box and the subdivision is terminated for that box, otherwise it will continue to be divided until satisfying the low-frequency grouping requirement. The following text gives a tree structure where the leaf boxes can be in any of the refinement levels. The pseudocode of the adaptive tree construction is given as follows:

#### **Algorithm I: Adaptive tree structure**

Step 1: Enclose the sources in the smallest possible box (computational domain)

Step 2: Decompose the computational domain to eight equal child boxes

Step 3: For each child box k, do k = 1 to 8

Step 4: If it contains no source, then eliminate it,

Step 5: If it contains sources that belong to the

mid-frequency regime and the box size is  $0.2\lambda$ , **then** 

Step 6: The box is considered a mid-frequency leaf box and is not further divided

Step 7: end if
Step 8: else
Step 9: Divide it into its 8 child boxes
Step 10: end if
Step 11: end do

Step 12: Repeat until the boxes at the finest level agree with the low-frequency requirement.



Fig. 3. Adaptive grouping.

In the adaptive scheme, the interaction sublists for the boxes are formed similar to the traditional fast multipole algorithm. As the leaf boxes can be in any of the refinement levels, the construction of the near list is more involved. To define the near list, we consider a box i at level 1. The descents of box i are boxes in levels  $l_d$ ,  $l_d < l$ , that are contained in box i. These include the children, grandchildren etc. of box i. The ancestors of box i are boxes in levels  $l_a$ ,  $l_a > l$ , that contain box i.

For each leaf box b, we define the following sets of boxes:

1. The leaf boxes which are descents of

box b's near neighbors, which is denoted by the set  $N_d$ .

- 2. The leaf boxes which are near neighbors of box b's ancestors, which is denoted by the set  $N_a$ .
- 3. The leaf boxes which are near neighbors of box b, denoted by the set  $N_n$ . The near list of a leaf box b consists of boxes  $b_n$ , where  $b_n$  belongs to  $\{N_n, N_a, N_d\}$ .

As is shown in Fig. 3, the adaptive partitioning of the computational domain for a nonuniform distribution of sources is shown in two dimensions. Consider a leaf box b, the boxes in the near list are lightly shaded, and the boxes in deep color are b's far groups.

Usually, the mesh density for the small cylinder is higher than the cubic for the object shown in Fig. 5. As a result, if the box size is set  $0.2\lambda$  for the finest level, it will contain many elements in a box which results in large memory consumption for the near interaction matrix. To alleviate the press of near interaction, according to the adaptive grouping scheme in [8, 12] and making some modification as the above description, MF-FMA is applied for in the purple small cylinder and MLFMA is applied for in the cubic at the bottom. Thus, the finest level box size for the MF-FMA can be set smaller than  $0.2\lambda$  while the finest level box size is at least  $0.2\lambda$  for MLFMA.



Fig. 4. Bistatic RCS of a PEC sphere working at 0.18 GHz simulated by Mie series and adaptive mixed-form FMA.

#### **IV. NUMERICAL RESULTS**

In this section, several numerical examples are presented to demonstrate the efficiency of adaptive MF-FMA (AMF-FMA) for fast analysis of electromagnetic scattering. All experiments are performed on a Core-2 6300 with 1.86 GHz CPU and 1.96GB RAM in single precision. The iteration process is terminated when the normalized backward error is reduced by 10<sup>-3</sup> for all examples.

In the implementation of the MF-FMA, the adaptive grouping method is used to reduce the memory consumption and captures the fine details of the structure. The incident wave is considered to be a plane wave at  $\theta = 0.0$ ,  $\varphi = 0.0$ . All geometries are modeled by plane triangles panels. As shown in Fig. 4, the numerical results of bistatic RCS for vertical polarization are given to

illustrate the performance of our AMF-FMA. It

consists of a sphere with radius 1m at 180MHz, and the average element size is 0.048 for the top half sphere with 3042 unknowns while the average element size is 0.06 for the bottom half sphere with 843 unknowns. The MF-FMA and MLFMA are applied to the top half sphere and the bottom, respectively. The numerical result of AMF-FMA is compared with the Mie series and we can see the result agrees with the Mie series vigorously.

In Fig. 6, the geometry is a cube connected with a small cylinder with 11862 unknowns. The size of the cube is  $1m \times 1m \times 1m$  with a small cylinder whose radius is 0.03m and the height is 1m. The average element size is 0.03 for the small cylinder with 826 unknowns and the cubic is 0.043 at 300MHz. The numerical results of the


Fig. 5. A cubic with a small cylinder.



Fig. 6. Monostatic RCS of the cube-cylinder simulated by MoM and adaptive mixed-form FMA.



Fig.7. Bistatic RCS of the cube-cylinder working at 60MHz simulated by MoM and adaptive mixed-form FMA.

monostatic RCS for vertical polarization are given to illustrate the performance of our AMF-FMA at 60 MHz, 120 MHz, 180 MHz, 240 MHz, and 300MHz. The finest level box size corresponding to the frequencies is  $0.05\lambda$ ,  $0.05\lambda$ ,  $0.15\lambda$ ,  $0.1\lambda$ , and  $0.125\lambda$ .

Two levels LF-FMA is applied for the whole object at 60 MHz. Mixed-form translation containing two levels multipoles translation and one level diagonal translation is utilized for the small cylinder and the cubic is analyzed by one level MLFMA at 120 MHz. For 180 MHz and 240 MHz, mixed-form translation containing one level multipoles translation and one level diagonal translation is used for the small cylinder and the cubic is analyzed by one level MLFMA. At 300MHz, mixed-form translation containing one



Fig. 8. A cuboid with many fine structures.

level multipoles translation and two levels diagonal translation is used for the small cylinder while the cubic is analyzed by two levels MLFMA.

At 60 MHz, the electrical size of the cube-cylinder is  $0.4\lambda$  and the average element size is 0.006 for the small cylinder and 0.0086 for the cubic. AMF-FMA degenerates to two levels LF-FMA with the finest level box size is 0.05 $\lambda$ . It is shown in Fig. 7 that the result has a good agreement with MoM.

When the proportion of fine structures in an object goes up as shown in Fig. 8, the total memory consumption for **MLFMA** and AMF-FMA is described in Fig. 9. The cuboid of the object in Fig. 8 is of the size 3m×3m×0.5m and the small cylinders are all in the same size with radius 0.05m and the height 2m. The average element size for the cuboid is around 0.093 and set the average element size for all cylinders varies from 0.033 to 0.01, corresponding with the unknowns from 8490 to 49011. The box size at the finest level for MLFMA is set  $0.25\lambda$ . The finest level box sizes of AMF-FMA are  $0.05\lambda$  for fine structures and  $0.2\lambda$  for the rest of the object at 200 MHz. Using AMF-FMA for the object, MF-FMA is applied for the cylinders and MLFMA is for the cuboid. With the total number of unknowns increasing, it can be seen from Fig. 9(a) that the memory consumption of storage near part matrix





Fig. 9. (a) The memory consumption of the object shown in Fig. 8 between AMF-FMA and MLFMA, (b) CPU times for one matrix-vector operation versus the number of unknowns for the object shown in Fig. 8, and (c) bistatic RCS of the object shown in Fig. 8 simulated by MLFMA and AMF-FMA at 200MHz.

elements of  $\mathbf{A}_N$  is large for MLFMA when leaf box size is 0.25 $\lambda$ . In other words, the more unknowns for the fine structures, the less memory requirement for AMF-FMA compared with MLFMA. In Fig. 9(b), it can be seen that the CPU time of AMF-FMA for one matrix-vector operation is more than MLFMA, but the CPU time consumption of AMF-FMA is close to MLFMA with the increasing of the unknowns for the fine structures. In Fig. 9(c), the bistatic RCS of the object in Fig. 8 for vertical polarization is given at 200 MHz with the unknowns 8490. It can be seen from the figure that the result of AMF-FMA agrees with MLFMA vigorously.

| Source point | Field point   | Green's function           | Multipoles expansion       |
|--------------|---------------|----------------------------|----------------------------|
| (0, 0, 0.)   | (1.0, 0, 0.)  | (0.9980267,-6.2790520E-02) | (0.9917871,-6.2790506E-02) |
| (0, 0, 0.)   | (1.0,0.5,0.)  | (0.8922212,-6.2780187E-02) | (0.8950982,-6.2780164E-02) |
| (0, 0, 0.)   | (1.0,0.5,0.5) | (0.8140802,-6.2769860E-02) | (0.8088611,-6.2769853E-02) |
| (0, 0, 0.)   | (1.0,0.,0.5)  | (0.8922212,-6.2780187E-02) | (0.8950983,-6.2780179E-02) |

Table 1: Test the LF-FMA translator

Table 2: Test the MF-FMA translator

| Source point | Field point      | Green's function        | Multipole expansion     |
|--------------|------------------|-------------------------|-------------------------|
| (0.25,0,0)   | (1,0.1,0)        | (-0.4293182,-1.249964)  | (-0.4293646,-1.249869)  |
| (0.25,0,0)   | (1,0.24,0.24)    | (-0.5804493,-1.067079)  | (-0.5811364,-1.066991)  |
| (0.25,0,0)   | (1.24,0.24,0.24) | (-0.8333122,-0.4675281) | (-0.8338917,-0.4674568) |
| (0.25,0,0)   | (1.24,0,0.25)    | (-0.8216858,-0.5328912) | (-0.8221112,-0.5328190) |

In Tables 1 and 2, we test the accuracy of equation (6) and equation (10). In Table 1, we choose the leafy box size is 0.5m and the source point is located at (0, 0, 0.). We take field point at 1, 2, 3, 4, as shown in Fig. 2, the frequency is 0.003GHz. In Table 2, we choose the leafy box is 0.25m and the frequency is 0.12GHz. The source point is fixed at (0.25, 0, 0.) and the field points are selected at random. From the two tables, we can conclude that the error of the results is no more than 1%.

# V. CONCLUSIONS

In this paper, we have described an adaptive grouping scheme combined with MF-FMA (AMF-FMA) for solving electromagnetic wave scattering problems. One advantage of our presented AMF-FMA is that it can cover a wide band from low frequency to mid-frequency without low-frequency breakdown. Another improvement is that for the object with fine structures, combined with adaptive grouping method, MF-FMA and MLFMA can be used flexibly for different parts of the object simultaneously. MF-FMA is applied for capturing the fine details of the structure accurately, and MLFMA is implemented for the rest of the object. As a result, the memory consumption is greatly reduced compared with MLFMA. Numerical results are performed and compared to verify the AMF-FMA is flexible and efficient.

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# **Improved Microstrip Folded Tri-Section Stepped Impedance Resonator Bandpass Filter using Defected Ground Structure**

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Abstract – In this paper, a study of a microwave microstrip bandpass filter (BPF), based on folded tri-section stepped impedance resonator (FTSIR) and defected ground structure (DGS) is presented. Based on the resonance characteristics of a stepped impedance resonator (SIR), the first spurious resonant frequency can be tuned over a wide range by adjusting its structure parameters. The SIR bandpass filter is designed to have a quasi-elliptic function response and a wide upper stopband. Furthermore, five ground slots (DGS) are used in order to omit the first spurious resonant frequency. Compared to similar microstrip filter without defected ground. the simulated performances of this novel structure indicate some functional advantages. To validate the design and analysis, a prototype of the bandpass filter with DGS was fabricated and measured. It is shown that the measured and simulated performances are in a good agreement.

*Index Terms* – Defected ground structure (DGS), folded tri-section stepped impedance resonator (FTSIR), microstrip bandpass filter.

# **I. INTRODUCTION**

By using a SIR the first and second spurious response can be pushed far beyond  $2f_0$  and  $3f_0$ , respectively (where  $f_0$  is the fundamental resonant frequency); this is achieved by properly choosing the impedance ratio. In addition, bandpass filters based on stepped impedance resonators usually have high selectivity and low internal losses in pass band width. For this reason, these filters are extensively used in a most of the communication applications nowadays. Stepped impedance resonators are use in circuits like duplexers and

mixers because they have smaller dimensions, relative to conventional resonators, and high quality factor.

There are many types of transfer functions for filters (Butterworth, Chebyshev, Elliptic, Quasielliptic, etc.). Among these different types, filters with quasi-elliptic transfer function have higher selectiveness, because of having the same ripples in pass band and high slope nearby transmission zeros. In addition, the ability to create transfer zeros in nearby of pass band causes to improve the slope of reject band. So, quasi-elliptic filters are extensively used in modern applications.

Poles degree of the filter and consequently the quantity of resonators must be increased in order to obtain higher selectiveness, but internal losses increase because resonators are not ideal. Here, in the design process of SIR filters, we have used quasi-elliptic four-pole cross-coupled resonators (reciprocal coupling between nonadjacent resonators) in order to obtain high selectiveness and decrease the internal loss. Whereas the first spurious resonant frequency is near to pass band in this structure, we have to consider an approach to eliminate it. The selected approach to eliminate the first spurious resonant frequency is etching defects in the ground plane of the structure.

In [1], a FTSIR bandpass filter is designed, but here we will use a different structure and designing method; also, we will discuss the DGS design process in detail.

# II. QUASI-ELLIPTIC BANDPASS FILTER BASED ON SIR

We used folded tri-section stepped impedance resonators to design a four-pole quasi-elliptic bandpass filter. The basic structure of tri-section SIR is shown in Fig. 1(a). The folded SIR is also shown in Fig. 1(c). These resonators not only have smaller dimensions but also have wider rejection bandwidth comparing to other popular resonators.



Fig. 1. (a) Basic structure of tri-section SIR resonator. (b) An equivalent circuit of proposed SIR resonator. (c) Proposed folded SIR resonator.

At first, let us consider a capacitively loaded lossless transmission line resonator of Fig. 1(b).  $C_L, Z_a, \beta_a$ , and d represent the loaded capacitance, the characteristic impedance, the propagation constant, and the length of the unloaded line, respectively. Thus, the electric length is  $\theta_a = \beta_a d$ . The related equations of folded SIR are derived in [1] and [2], such that:

$$\begin{cases} \theta_{a0} = 2 \tan^{-1} \left( \frac{1}{\pi f_0 Z_a C_L} \right) \\ \theta_{a1} = 2\pi - 2 \tan^{-1} (\pi f_1 Z_a C_L) \end{cases}$$
(1)

The fundamental resonant frequency and the first spurious resonant frequency are  $f_0$  and  $f_1$ , respectively. If  $C_L$  equals to zero in the above equation, then  $\theta_{a0}$  and  $\theta_{a1}$  equals to  $\pi$  and  $2\pi$ , respectively. If  $C_L \neq 0$ , the resonant frequencies shift down as the loaded capacitance increases.

Here, we have assumed  $Z_a = 50 \Omega$ , d = 18.15 mm,  $W_a = 0.71 \text{ mm}$ ,  $V_{Pa} = 1.149 \times 10^8 \text{ m/s}$ and  $V_{Pa}$  is associated phase velocity. We considered a Rogers RT/duroid 3010 substrate to design our filter; the substrate has a relative dielectric constant of 10.2, a thickness of 0.635 mm and a loss tangent of 0.0035.

The resonator of Fig. 1(c), which is composed of a microstrip line with both ends loaded with folded open-stubs, is used. The folded arms of open-stubs are applied for increasing the loading capacitance to ground for the purpose of cross coupling.



Fig. 2. The fundamental, first spurious resonant frequencies and their ratio versus the loading capacitances.

According to (1), Fig. 2 plots the calculated resonant frequencies, as well as their ratio for different capacitance loading with the above mentioned values. As can be seen when the loading capacitance is increased, in addition to the decrease of both resonant frequencies, the ratio of the first spurious resonant frequency to the fundamental one is increased.

Shown in Fig. 2 are the fundamental and first spurious resonant frequencies as well as their ratio against the loading capacitances, obtained using a full-wave advanced design system (ADS) simulator. According to this Fig, the ratio of the first spurious resonant frequency to the main resonant frequency has direct relation to the loading capacitance. Increasing of the loading capacitance will increase the ratio of the first spurious resonant frequency to the main resonant frequency but the main and the first spurious resonant frequencies will decrease. It is possible to increase the distance between the first spurious and the main resonant frequencies by increasing the loading capacitance. This improves the filter characteristics in rejection band.

Figure 3 shows the fundamental and spurious resonant frequencies together with their ratio versus the length of folded open-stubs. The length of folded open-stubs (L) is obtained from [2]:

$$\begin{cases} L = L_1 & for \quad L \le 6.5 \ mm \\ L = 6.5 + L_2 & for \quad L > 6.5 \ mm \end{cases}$$
(2)



Fig. 3. The fundamental, spurious resonant frequencies and their ratio versus the length of the folded open-stub.

The increase profile of the ratio of the first resonant frequency to the main resonant frequency with the increase of L is similar to the increasing profile of this ratio with  $C_L$ . This matter was predictable because both graphs are discussing about an unloaded microstrip line width 18.15 mm length and 0.71 width which exists on a substrate with a relative dielectric constant of 10.2 and a thickness of 0.635 mm, only the open stub of the resonator of Fig. 3 is approximated by a capacitance (Fig. 2). It is important to say that the open-stub should have a wider line or lower characteristic impedance. In this case, according to Fig. 1(c), we have  $W_1 = 2 mm$  and  $W_2 = 3.66 mm$  for the folded open-stub.

To achieve high selectiveness, the order of filter must increase for the Butterworth and Chebyschev filters. Elliptic filter has ripple in the both pass band and rejection band due to having limited transmission zeroes; as a result, it is highly selective and its slope is much more than the above two filters. However, the realization of this type of filter with the printed circuits structure is complicated. The four-pole cross-coupled SIR filter is designed using the proposed basic resonator structure [3].

Figure 4 shows the arrangement of resonators in the quasi-elliptic four-pole SIR filter. Referring to Fig. 4, the coupling between resonators 1 and 4 is of electric type, between 2 and 3 is of magnetic type and between 1 and 2 is of mixed type as well as 3 and 4. More details about electric, magnetic, and mixed coupling between the resonators are discussed in [2].



Fig. 4. Schematic of a quasi-elliptic four-pole SIR filter.

In order to design a quasi-elliptic four-pole SIR filter, it is essential to characterize the coupling between adjacent resonators. Fig. 5 shows a typical graph of magnetic coupling between two SIRS's for a desired distance between them, in a certain frequency range. This graph is derived via simulation.



Fig. 5. Typical frequency responses simulated for extracting the coupling coefficient.

The coupling  $K_{ij}$  of any pair of adjacent SIR's is determined by [3].

$$K_{ij} = \frac{f_{P2}^2 - f_{P1}^2}{f_{P2}^2 + f_{P1}^2}.$$
(3)

For the example in Fig. 5, the resonant frequencies  $f_{P_1}$  and  $f_{P_2}$  are obviously measurable based on amplitude frequency responses. In this sample, the values of resonant frequencies are  $f_{P_1} = 2.115 \ GH_z$  and  $f_{P_2} = 2.995 \ GH_z$ , so according to equation (2), we have:



Fig. 6. Coupling coefficients against space between adjacent resonators (a) magnetic coupling, (b) electric coupling and (c) mixed coupling.

$$K_{23} = 0.334$$

The graphs of coupling coefficient against space between adjacent resonators are derived according to equation (3) and Fig. 5 for three types of couplings which are used in the design of our filter. These graphs are shown in Fig. 6.

The equations for calculating coupling coefficient of different types of coupling may be obtained as described in [7]:

$$M_{14} = \frac{FBW \cdot J_1}{g_1}$$

$$M_{23} = \frac{FBW \cdot J_2}{g_2}$$

$$M_{12} = M_{34} = \frac{FBW}{\sqrt{g_1 \ g_2}}.$$
(4)

The existing factors in (4) are specified for each certain filter characteristics in some tables which are presented in [4]. So the coupling coefficients of our desired filter may be obtained.

By using Fig. 6 spaces between each adjacent resonator may be obtained according to the above specified coupling coefficients. Our desired bandpass filter may be designed by this method.

Based on the substrate we selected, the design parameters of the resonator in Fig. 1(c) and the distance between the resonators of Fig. 4 are summarized in Table 1, with all dimensions shown in millimeters.

The results of the preliminary designed fourpole SIR filter are seen in Fig. 7. One can observe bandpass filter behaviour with central frequency of 0.965 GHz, and bandwidth of 85 MHz. The

insertion loss is, also, 1.55 dB and the return loss is better than 23 dB.

Table 1: The values of design parameters of the desired filter

| $L_1$ | 7.06 | $W_2$ | 3.66  | $d_{14}$               | 1.45 |
|-------|------|-------|-------|------------------------|------|
| $L_2$ | 6.58 | $W_a$ | 0.71  | <i>d</i> <sub>23</sub> | 2.11 |
| $W_1$ | 2.04 | d     | 18.15 | $d_{12} = d_{34}$      | 0.11 |



Fig. 7. Simulation of preliminary designed fourpole SIR filter.

Figure 8 shows the simulation results of the rejection band for the four-pole SIR filter. Referring to Fig. 8, the first spurious resonant frequency occurred at 3.2 GHz.

As can be seen, the rejection band width of the designed filter is almost short. Insertion loss at first spurious resonant frequency 3.2 GHz reaches to 5 dB. In order to improve the rejection band

width, we have to eliminate the first spurious frequency. To achieve this, we utilize the defected ground structures (DGS).



Fig. 8. Simulation of rejection band of the SIR filter.

### **III. DEFECTED GROUND STRUCTURE**

Microstrips structures, with special shapes etched in their ground planes are one of the approaches that researchers use to improve the characteristics of filters. This generic structure is called defected ground structure (DGS). Some advantages of using DGS in filters are simple fabrication, the ability to transfer of high power, low insertion loss and minimization of the filter size. Among various suggested shapes of DGS structures is the rectangular structure due to its simplicity.

An LC equivalent circuit can represent the unit DGS circuit. The equivalent circuit parameters are affected by the physical dimensions of the DGS. The equivalent circuit and parameters of the DGS should be extracted in order to design a circuit with DGS section. One can derive the equivalent circuit of the DGS section using the ADS simulator. The equivalent inductive reactance can be easily calculated by using the prototype element value of the one-pole response.



Fig. 9. Equivalent circuit of the DGS circuit.

The parallel capacitance value for the given DGS section can be extracted from the attenuation pole location frequency. The slot in ground plane excited by the microstrip line behaves as parallel resonance. Fig. 9 shows the equivalent circuit of the DGS circuit, where the dotted box shows the DGS section [5].

In Fig. 9, the parallel capacitance value  $C_p$  and the parallel inductance value  $L_p$  are as follows:

$$C_{P} = \frac{\omega_{C}}{Z_{0} g_{1}(\omega_{0}^{2} - \omega_{C}^{2})} = \frac{5 f_{C}}{\pi (f_{0}^{2} - f_{C}^{2})} \qquad pf, \quad (5a)$$

$$L_{P} = \frac{250}{C_{P} (\pi f_{0})^{2}} \qquad nH.$$
(5b)

The equivalent reactance value for DGS unit can be expressed as below:

$$jX_{LC} = \frac{j\omega L_P \times \frac{1}{j\omega C_P}}{j\omega L_P + \frac{1}{j\omega C_P}} = \frac{j\omega L_P}{1 - \omega^2 L_P C_P}$$
(6*a*)

$$\omega_0^2 = \frac{1}{L_p C_p} \tag{6b}$$

$$X_{LC} = \frac{1}{\omega_0 C_p \left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)}.$$
 (6c)

DGS structures etched under microstrip line increase the inductance and capacitance of line in the rejection band and this, consequently, decreases the propagation velocity. In this situation, the slots in the ground plane cause a portion of density of electrical flow to couple to the side line which in normal design it couples to the ground plane. This will increase the capacitance coupling in coupled lines. Also, because of the type of etched slot on the ground plane, phase velocity of the even and odd modes will be equal. By equalizing the even and odd mode phase velocities it is possible to suppress the first harmonic.

# A. Rectangular slots in the direction of coupling line

According to Fig. 10, in this situation of arrangement of DGS unit in the ground plane of coupling lines, slots do not cause any changes in the field lines of the odd mode, but for the even mode, part of electrical field will propagate in the

space under the slot because of the distance between two DGS structures  $(S_M)$ , Fig. 10 (b). This will decrease the effective relative dielectric coefficient in even mode  $(\varepsilon_{eff}^e)$  [6].



Fig. 10. (a) Electrical filed lines in odd and even modes, in structure of coupling lines without DGS.(b) Electrical filed lines in odd and even modes, in structure of coupling lines with DGS [6].

Etching a slot in a direction of the coupled lines decreases  $\varepsilon_{eff}^{e}$  and makes it closer in values to  $\varepsilon_{eff}^{o}$ (the effective relative dielectric coefficient in odd mode). As Fig. 10 shows,  $\varepsilon_{eff}^{o}$  does not change. So, the difference between phase velocity at even mode and odd mode ( $\varepsilon_{eff}^{even} > \varepsilon_{eff}^{odd}$ ), which exists in normal design, will decrease and the phase velocity of the two modes will become approximately equal. Rectangular slots can be located in different positions in direction of coupling lines which will be discussed. Width, length and location of slots are the main design parameters which increase the coupling and equalize the phase velocity in odd and even modes.

To determine each one of these parameters, two other parameters should be considered as a constant and the influence of changes in the first one on the coupling will be studied. According to Fig. 11 among situations 1, 2, and 3, no. 1 is the most effective one on the coupling. Regarding the width of the slots, three different conditions will be discussed which have better performance than others [5]; first equal to the width of coupling line (W), second equal to the distance between coupling line (S), and third equal to the average of the last two ones (W+S/2). Among these three cases, the second one has the most degree of influence on the coupling. Regarding the effect of length of the slots on the coupling, there are three different options; first equal to half of the length of the coupling line, second equal to the length of the coupling line, and third equal to the total length of the ground plane. The last two options have more influence on coupling compared to the first one. It is notable that between these two, the length equal to the length of the coupling line is more common.

Situation no. 4 in Fig. 11 is a more effective structure to equalize the phase velocity of even and odd phases as well as having great influence on increasing the coupling like the previous study in [5].



Fig. 11. Different situations of DGS location in ground plane of two coupling lines.

# **B.** Rectangular slots perpendicular to the direction of coupling lines

In this situation, as shown in Fig 10(b), the electrical field lines are similar to the case without DGS; so  $\varepsilon_{eff}^{o}$  will not change. In even mode, the signal path increases for a special physical length due to the existence of the DGS in the ground plane. This increase in signal path is equal to the decrease of phase velocity in even mode. In other words,  $\varepsilon_{eff}^{e}$  and  $\varepsilon_{eff}^{o}$  increase equally. This increase of signal path is similar to the increase of return currents path in ground plane for a simple line with DGS.

In fact, the increase of  $\varepsilon_{eff}^{e}$  causes more difference between phase velocities in two modes. On the other hand, this increase of  $\varepsilon_{eff}^{e}$  and no change of  $\varepsilon_{eff}^{o}$  causes the coupling between the resonator lines to increase, compared to the case without DGS.

# IV. IMPROVED MICROSTRIP SIR BANDPASS FILTER USING DEFECTED GROUND STRUCTURES

One method to improve the filter characteristics is to use microstrip structures with defected ground plane. DGS cells are used as complementary of the main filter and also independently in executing the filter, due to having natural resonant characteristics [5]. DGS structures generally affect the filter rejection band. The effects of these structures on improving the filter characteristic in pass band are also discussed. Easily fabrication, low insertion loss, and size reduction of the filters are some advantages of using DGS structure in design of filters [7].

In order to prevent radiation from DGS, we have to know distribution of surface currents on the ground plane of the filter which is shown in Fig. 12.



Fig. 12. The distribution of surface currents on the ground plane of the filter.



Fig. 13. Schematic of designed four-pole SIR filter with DGS.

For realization of desired DGS structure, considering discussed issues in [7], we utilized a simple linear slot in the ground plane of the preliminary designed filter. Two simple linear slots are etched in the ground plane, one between resonators No. 1 and 2 and the other between

resonators No. 3 and 4, in order to improve the mixed coupling between them. The other slots are applied to improve the coupling between the input and output of the filter.

Dimensions of the used DGS slots in Fig. 13 are identified in Table 2, with all dimensions shown in millimeters.

Table 2: Dimension of the DGS slots of Fig. 13

|     |            | U         |
|-----|------------|-----------|
| DGS | DGS Length | DGS Width |
| a   | 2.8        | 1.24      |
| b   | 4.19       | 0.78      |
| с   | 2.63       | 2.11      |

Figure 14 shows the simulation results of the passband for the four-pole SIR filter with DGS.



Fig. 14. Simulation results of the pass band for the improved four-pole SIR filter with DGS.



Fig. 15. Simulation results of the SIR filter with DGS in the range 0.5-5 GHz.

As observed in Fig. 14, insertion loss is 1.6 dB and return loss is better than 24 dB. The bandwidth of the filter reached to 85 MHz at 0.965

GHz central frequency which is in a good agreement with the case without DGS.

We can see 16 dB improvements in the insertion loss of the first spurious resonant frequency of the filter according to Fig. 15. In fact, insertion loss at 3.2 GHz reaches 21 dB. In addition, comparing the frequency response of the preliminary SIR filter (without DGS) to the SIR filter with DGS in Fig. 8, we observe elimination of the spurious frequency response in frequency 3.2 GHz. Frequency responses of the filter with and without DGS are compared in Table 3.

Table 3: Comparing the frequency responses of the filter with and without DGS

| S parameter  | Without<br>DGS | With<br>DGS |
|--|----------------|-------------|
| Insertion loss at the fundamental frequency (dB)       | 1.55           | 1.6         |
| Return loss at the<br>fundamental frequency (dB)       | 23             | 24          |
| Insertion loss at the first<br>spurious frequency (dB) | 5              | 21          |

# V. MEASURED RESULTS AND COMPARISON WITH SIMULATED

A sample of four-pole SIR bandpass filter with DGS was fabricated and measured. Photographs of the fabricated filter with DGS are shown in Fig. 16. The prototype circuit size of the filter is around 44.6 mm  $\times$  28.4 mm.



Fig. 16. Photograph of the fabricated four-pole SIR bandpass filter with DGS (a) top and (b) bottom.

Measurement was carried out using an Agilent HP 8720B Network Analyzer. The results of S-

parameter in pass band for the fabricated sample compared to the simulated results are shown in Fig. 17.

As it is observed in Fig. 17, insertion loss of the fabricated model is 2.6 dB at the center frequency, which is about 1 dB higher than the simulation results. In addition, return loss of the fabricated model is 20 dB, which is close to simulation results, as seen in Fig. 17.



Fig. 17. Measured and simulated results of Sparameter response in pass band.

The measured sample presented a good response in rejection band, as illustrated in Fig. 18. By comparing Fig. 18 to the simulation result (Fig. 15), we observe a good accordance between them.



Fig. 18. Measured results in rejection band width.

#### **IV. CONCLUSION**

We have designed and fabricated a microstrip quasi-elliptic four-pole cross-coupled FTSIR filter for one of the GSM frequency bands, which may be used in further communication applications. The folded tri-section SIR not only makes the resonator more compact, but also enables the flexibility of introducing cross coupling in a filter configuration. A comprehensive treatment of a capacitively loaded transmission line resonator is described, which leads us to apply a microstrip folded tri-section stepped impedance resonator. By etching five simple linear slots in the ground plane, we have obtained a great improvement in elimination of the first spurious resonant frequency. The measured results in the pass band and rejection band are in a good agreement with the simulated ones.

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# Impact of the Mobile Phone Dimensions on the Hearing Aids Compatibility

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Abstract- In this work, we have investigated the influence of the mobile phone physical dimensions on the hearing aids compatibility (HAC). In our study, we have considered an inverted F antenna (IFA) and a planar inverted F antenna (PIFA) both fitting into a typical candy bar mobile phone. We have used a generic cubical head model to investigate the user impact on the near fields (NF). The field values are obtained by using the finite difference time domain (FDTD) method. We have observed significant difference in the peak field values between free space and with the head included although only free space values are specified in the relevant standard. Important outcome for the physical dimensions of the mobile phone is that the increase of the length of the handset significantly decreases the peak H value.

#### **I. INTRODUCTION**

Several studies have shown that the electromagnetic interference in the hearing aids (HA) due to the GSM digital mobile phones has an annoying effect on the user and a negative influence on the intelligibility of the speech [1-3]. In [4], the presence of discrete peaks in the output spectrum corresponding to the time division multiple access frame rate and its harmonics was shown. In the same study, the interference has been characterized as a buzzing sound. These results were confirmed by a more detailed investigation [5]. A comprehensive study of the NF for typical antennas in mobile handsets does not exist in the open literature. Moreover, the great variety

of the HA available on the market complicates the HAC of the mobile phones. This article investigates the influence of the mobile handsets dimensions on the HAC. The IFAs and PIFAs have been investigated for the low band (850 MHz) and high band (1900 MHz) frequency ranges. The conducted analysis is based on the available HAC standard which has been defined for free space [6]. Further, a comparison between free space and with the impact of the head has been carried out to see how well the standard reflects the real case.

# **II. PREDICTION METHOD**

An implementation of a generic mobile handset in the FDTD for computing the electromagnetic field distribution has been presented in [7]. In the same study, the "thin wire" method is used to model the antenna and avoid the "staircasing" effect in FDTD. A parallel FDTD code developed at the Antennas, Propagation and Radio-Networking (APNET) group at Aalborg University has been used for the investigation. The code has been used in a previous study [8]. A cell size of 1 mm which ensures at least twenty samples per wavelength was used. For the termination of the simulation space the perfectly matched layer absorbing boundaries [9] were used.

# III. ANTENNA DESIGN AND HEAD MODEL

Figure 1 shows the IFA and PIFA for the low band.



Fig. 1. (a) IFA for the low band and (b) PIFA for the low band.

The initial dimension of the mobile handset has been chosen to be  $40 \ge 100 \ge 10$  mm (width  $\ge$  length  $\ge$  thickness). The ground plane and the metallic antenna elements have been modelled as a perfect electric conductor. The heights of the antennas have been chosen in such a way that the proper bandwidth has been established for low and high band cases. The width has been varied to 50, 60, 70 mm; the length to 120, 140, 160, 180, 200 mm; and the thickness to 8, 9, 11, and 12 mm. The IFAs has been placed on the top, on the side or at the bottom of the box phone. The PIFAs has been located on the back top or back bottom of the phone.



Fig. 2. Generic cubical head model (dimensions in mm).

For simplicity, the head and the ears have been modelled as homogeneous tissue bricks (Fig. 2). The dimensions of the ears have been chosen according to [10]. The thickness of the cubical ear is 5 mm. The electrical properties of the head material have been taken from [11]. We have assumed that the ear consists of fat tissue [12]. In our analysis, we have assumed horizontal orientation of the mobile phone with respect to the head and no gap between the phone and the ear. The mobile phone is positioned in such a way that the speaker, the center of the ear, and the center of the calculation plane (defined in the next part) lie on a line perpendicular to the phone. We have assumed that the speaker is positioned 15 mm from the top edge of the phone.

#### **IV. HAC STANDARD**

According to the last version of the available standard, the common operation of a mobile phone and a hearing aid is classified as normal if the HAC category is at least 5. The latter is defined as a sum of the category of the mobile phone (dependent on the measured NF) and the category of the HA (dependent on the HA immunity). Owing to the fact that for the present HA, the achievement of HA category equal to 2 is very easy; the aim for the mobile phone category of 3 and above. The standard specifies measurement of the NF in a plane 50 by 50 mm at distance 15 mm from the mobile phone (Fig. 3).



Fig. 3. Set up for the NF measurement according to the available HAC standard.

The phone speaker and the center of the measurement plane lie on a line perpendicular to the phone. After the data are obtained, for both E and H fields, the measurement plane is divided into nine equal sub-grids. For each E and H sub-grid, the maximum value is estimated. In our analysis, the maximum value of the entire measurement plane will have an index m. Then the exclusion procedure has to be applied following the rules:

- The center sub-grid cannot be excluded.
- Three E and three H sub-grids have to be eliminated as at least 4 out of the 6 left sub-grids have to be common for both E and H field.
- For each E and H field, the excluded sub-grids have to be contiguous.

The mobile phone category is defined by the maximum E and H values after the exclusion (these values are called characteristic and in this article are denoted with an index c).

Table 1: Phone emission limits and categories (for frequencies below 960 MHz the values are 10 dB higher)

| Category   | Telephone RF          |              |  |  |  |  |
|------------|-----------------------|--------------|--|--|--|--|
|            | parameters (above 960 |              |  |  |  |  |
|            | MHz)                  |              |  |  |  |  |
| Near Field | E field               | H field      |  |  |  |  |
|            | [dB V/m]              | [dB A/m]     |  |  |  |  |
| M1         | 48.5 to               | -1.9 to 3.1  |  |  |  |  |
|            | 53.5                  |              |  |  |  |  |
| M2         | 43.5 to               | -6.9 to -1.9 |  |  |  |  |
|            | 48.5                  |              |  |  |  |  |
| M3         | 38.5 to               | -11.9 to -   |  |  |  |  |
|            | 43.5                  | 6.9          |  |  |  |  |
| M4         | < 38.5                | < -11.9      |  |  |  |  |

Table 2: System performance classification

ł

| System classification | HAC category |
|-----------------------|--------------|
| Useable               | 4            |
| Normal use            | 5            |
| Excellent performance | 6            |

## V. RESULTS AND DISCUSSION

The HAC standard has been defined for the GSM working frequency bands at 850 MHz (low band) and 1900 MHz (high band). These frequencies are used in the USA, Canada, and other countries in the Americas. Of particular interest is to compare the NF produced at these GSM frequency bands with the ones used in the GSM system in Europe. The comparison has been made with respect to the peak electric and magnetic values as we have taken into an account the planar values as defined in the standard and explained in the previous section. To obtain an antenna resonating at lower or higher frequency, we have changed only the length of the radiating element and all other dimensions have been kept constant. Moreover, to be able to fulfill the bandwidth requirements for all four investigated frequency bands, the distance from the wire to the ground-plane for the IFAs has been chosen to be 9 mm. In the case of PIFAs, the minimum investigated height has been 8 mm. The latter was enough to cover the necessary bandwidth. The analysis has been made between all possible antenna configurations obtained via combination of the different physical dimensions of the antennas width, length, and thickness.

| Comparison between the NF<br>for close frequencies |          |       | Difference in the peak NF<br>between 850 and 900 MHz,<br>[dB] |                |                     |                     |  |
|--|----------|-------|---|----------------|---------------------|---------------------|--|
| Type   | Position | Head? | Me  | ean            | Devi                | ation               |  |
| 51   |          |       | $\Delta E_m$  | $\Delta H_{m}$ | $\Delta E_{\sigma}$ | $\Delta H_{\sigma}$ |  |
| IFA  | Тор      | No    | 0.3   | 0.8            | 0.6                 | 0.6                 |  |
| IFA  | Тор      | Yes   | 0.3   | 0.5            | 0.2                 | 0.4                 |  |
| IFA  | Side     | No    | 0.5   | 1              | 0.1                 | 0.1                 |  |
| IFA  | Side     | Yes   | 0.05  | 0.2            | 0.2                 | 0.2                 |  |
| IFA  | Bottom   | No    | 0.2   | 0.7            | 0.2                 | 0.2                 |  |
| IFA  | Bottom   | Yes   | 0.1   | 0.1            | 0.4                 | 0.2                 |  |
| PIFA   | Тор      | No    | 0.2   | 0.3            | 0.5                 | 0.4                 |  |
| PIFA   | Тор      | Yes   | 0.3   | 0.3            | 0.8                 | 1.1                 |  |
| PIFA   | Bottom   | No    | 0.3   | 0.8            | 0.2                 | 0.2                 |  |
| PIFA   | Bottom   | Yes   | 0.3   | 0.3            | 0.1                 | 0.1                 |  |

Table 3: Similarity between the NF at close frequencies in free space

| Table 4: Similarity between the NF at close | е |
|---|---|
| frequencies in free space                   |   |

| Compa | arison betwee | erence in | e in the peak NF           |              |                     |                     |  |
|-------|---------------|-----------|----------------------------|--------------|---------------------|---------------------|--|
| for   | close freque  | ncies     | between 1800 and 1900 MHz, |              |                     |                     |  |
|       |               |           | [dB]                       |              |                     |                     |  |
| -     |               |           | Me                         | ean          | Devi                | ation               |  |
| Туре  | Position      | Head?     | 15                         |              |                     |                     |  |
|       |               |           | $\Delta E_m$               | $\Delta H_m$ | $\Delta E_{\sigma}$ | $\Delta H_{\sigma}$ |  |
| IFA   | Тор           | No        | 0.4                        | 0.2          | 0.6                 | 0.4                 |  |
|       |               |           |                            |              |                     |                     |  |
| IFA   | Тор           | Yes       | 0.3                        | 0.2          | 0.4                 | 0.3                 |  |
| IFA   | Side          | No        | 0.2                        | 0.1          | 0.3                 | 0.2                 |  |
| IFA   | Side          | Yes       | 0.4                        | 0.3          | 0.3                 | 0.2                 |  |
| IFA   | Bottom        | No        | 0.6                        | 0.7          | 0.1                 | 0.3                 |  |
| IFA   | Bottom        | Yes       | 0.3                        | 0.1          | 0.5                 | 0.3                 |  |
| PIFA  | Тор           | No        | 0.3                        | 0.1          | 0.6                 | 0.3                 |  |
| PIFA  | Тор           | Yes       | 0.2                        | 0.1          | 0.3                 | 0.2                 |  |
| PIFA  | Bottom        | No        | 0.4                        | 0.3          | 0.1                 | 0.2                 |  |
| PIFA  | Bottom        | Yes       | 0.2                        | 0.4          | 0.3                 | 0.3                 |  |

As it can be seen from the table, the difference in the peak values is negligible. This is an important conclusion because it gives us the possibility to investigate the NF at the frequency bands defined in the HAC standard and then translate the obtained results for the neighbour frequency bands.

In the analysis of the NF between free space and with the head model, in Table 2 and 3 are shown only the cases for the initial physical dimension of the antennas. Further, if changing any of the dimensions causes a significant difference in the NF for either free space or with the head present, it will be outlined and explained in the text.

For both frequency bands of interest and in the case of top positioned IFAs and PIFAs, the presence of the head model decreases the peak E value compared to the free space cases. For

|               | Table 5. Feak electric and magnetic fvr at low band |                               |            |                |                         |                |            |           |  |
|---------------|---|-------------------------------|------------|----------------|-------------------------|----------------|------------|-----------|--|
| Antenna type, | Electric field [dB V/m]                             |                               |            |                | Magnetic field [dB A/m] |                |            |           |  |
| position      |   | E <sub>m</sub> E <sub>c</sub> |            | H <sub>m</sub> |                         | H <sub>c</sub> |            |           |  |
|               | Free space  | With head                     | Free space | With head      | Free space              | With head      | Free space | With head |  |
| IFA, top      | 60.3  | 43.3                          | 58.3       | 43.3           | 17.4                    | 7.7            | 1.8        | 7.6       |  |
| IFA, bottom   | 58.4  | 55.5                          | 57.5       | 55.3           | 11.4                    | 8.5            | 3.5        | 8.3       |  |
| PIFA, top     | 48.6  | 42                            | 47.9       | 41.8           | 4.4                     | 7.8            | 3.6        | 7.1       |  |
| PIFA, bottom  | 55  | 42.2                          | 54.8       | 42             | 8.6                     | 6.8            | 0.6        | 6.8       |  |

Table 5: Peak electric and magnetic NF at low band

| Table 6: Peak electric and magnetic NF at high band |            |            |               |           |            |                         |            |                |  |
|---|------------|------------|---------------|-----------|------------|-------------------------|------------|----------------|--|
| Antenna type,                                       |            | Electric f | field [dB V/m | ]         |            | Magnetic field [dB A/m] |            |                |  |
| position  |            | Em         |               | Ec        |            | H <sub>m</sub>          |            | H <sub>c</sub> |  |
|   | Free space | With head  | Free space    | With head | Free space | With head               | Free space | With head      |  |
| IFA, top  | 52.8       | 42.5       | 52.5          | 42.5      | 3          | 7.6                     | 3          | 7              |  |
| IFA, bottom   | 50.1       | 55.5       | 46.8          | 53.8      | -1.4       | 3.6                     | -3.2       | 2.8            |  |
| PIFA, top   | 53         | 39.8       | 52.5          | 38.8      | 1.6        | 5.2                     | 0.8        | 3              |  |
| PIFA, bottom  | 48.8       | 35.0       | 48.5          | 34.8      | 7.8        | 0.4                     | 5.1        | 0.1            |  |

both bands and top positioned radiating elements, except the case of top positioned IFA, when comparing the free space with a user, the opposite trend has been observed for the magnetic peak value meaning that the presence of the user increases the peak H value. For that case of top positioned IFA, the interesting fact is the large margin between the peak value H<sub>m</sub> and the characteristic value H<sub>c</sub> in free space. The peak E and H values are decreased when the head is present compared to free space for bottom low band IFAs and bottom positioned PIFAs. The latter is not the case for bottom located IFA working at high band as both peak electric and magnetic field values increase in the case of a user compared to free space.

In the cases of low and high band bottom positioned IFAs and PIFAs and high band top located IFA for both free space and with an user, increasing the width of the phone from 40 to 70 mm causes reduction of between 3 and 4 dB V/m (A/m) in the peak electric (magnetic) value. For all of the other investigated cases, the width of the handset does not have a significant impact on the peak field values.

The length of the mobile phone appears to be the most important physical dimension. As a general conclusion, increasing the length of the mobile phone from 100 to 200 mm reduces the peak H values. We have not observed a strict tendency for the influence of the length of the mobile phone on the peak E value. In free space for all different antenna configurations and types, due to the increased length of the mobile phone the reduction of the peak magnetic values is within 4 to 5 dB dB A/m. The decrease in the peak values is stronger with the head present compared to the free space. A simple example has been shown in the next figure.

Figure 4 shows the impact of the length of the mobile phone on the maximum E and H values before and after the exclusion rule was applied for top positioned low band IFA (a) and PIFA (b) with the head presented. The left axis is the E scale, the right axis is the H scale, and the top axis is the mobile phone category specified by the characteristic values  $E_c$  (H<sub>c</sub>). Each group of four values is specified for a concrete length of the mobile phone as in a consecutive way are shown the electric and magnetic values before and after the exclusion procedure has been applied. For both types of antennas, increasing the length from 100 to 200 mm leads to significant reduction by more than 10 dB A/m of the peak H value, which also will increase the mobile phone category to the value of 3 which will ensure normal common work of the mobile phone with a typical hearing aid. Of all the investigated physical dimensions, the thickness of the mobile phone has the least influence on the NF



Fig. 4. Bar graphs of maximum E and H values for 850 MHz top positioned antenna with the head present: (a) IFA and (b) PIFA.

values as we have not observed difference in the NF larger than 0.5 dB V/m (dB A/m) when changing the thickness from 8 to 12 mm.

## **VI. CONCLUSION**

In this paper, we have shown that there is a difference in the NF values between free space and with a head. The length of the mobile phone appears to be an important parameter as with its increase, the peak H value decreases. The width and the thickness of the mobile phone do not have significant impact on the NF. The influence of the hand on the NF may, also, need to be explored.

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# Parametric Analysis of Multi Source Feeding Flare Rolling and Corrugating Effects for H-Plane Horn Radiator

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Abstract – In this paper, radiation characteristics of H-plane sectoral horn antenna are treated systematically by investigation of the multi source feeding, flare wall rolling and corrugating effects. The analytical regularization method (ARM) is used to solve the problem of E-polarized wave diffraction in a fast and accurate way. The numerical procedure is initially verified by the analytical solutions, and the calculated field patterns are presented for the selected horn antenna configurations. Proper choices of the geometrical parameters, such as source position, back wall shape, multi source feed, metal thickness, and ridged and corrugated flares are proposed for the designers to reach high directivity, narrow beam, and suppressed side and back lobe levels.

*Index Terms* — Analytical regularization method, corrugated horn, flare rolling, horn antenna, multi source feeding.

# I. INTRODUCTION

Horn structures are one of the most popular antenna technologies commonly used in wideband RF communication systems, high power microwave devices, electromagnetic compatibility tests, microwave and millimeter wave radars, and parabolic reflector feeder designs [1-6]. The radiation characteristics of horn antennas have been investigated starting from the analysis of principal configurations, and numerous practical types of the horn antennas have been developed in recent years [7-11]. The initial research methods were analytical techniques, which are primarily based on the mode-matching and aperture integration principals [3, 12]. However, the arbitrary shaped horn geometries used in practice were analyzed only by experiments or optical approximations [7]. Thus, direct numerical techniques such as, finite difference time domain (FDTD), finite element method (FEM), method of moments (MoM), and hybrid algorithms are considered more versatile and usable in practical models [13-15]. Nevertheless, they often require a long computing time especially for large size horn designs and wide band analysis [4,14]. Moreover, MoM and FEM can cause unstable numerical process for some structures due to reducing the boundary value problem (BVP) to the algebraic equation set of the first kind that usually has a singular kernel [16]. Hence, the analyticalnumerical approaches that transform the illconditioned integral equation of first kind into a well-conditioned one of second kind are preferred to solve the matrix equation numerically by truncation method with fast convergence to obtain fast and reliable solutions [9,16]. On this scope, the analytical regularization method (ARM) is proposed for calculation of the horn radiation [17]. The primary analysis of the H-plane horn antenna has been demonstrated by Turk with investigation of the effects of essential parameters such as, different waveguide lengths, widths, flare angles and aperture sizes to characterize the radiation pattern [18].

This study deals with the parametric analysis of radiation from two-dimensional (2D) H-plane horn structure for different geometrical configurations of line source feeder positioning, back wall shaping, multi source locating, and flare rolling and corrugating. The principal aim is to characterize the radiation pattern to achieve low side and back lobe levels. For this purpose, the BVP is essentially formulated with respect to zcomponent of the electric field satisfying Helmholtz's equation with Dirichlet boundary condition, and the ARM procedure is applied to obtain fast, accurate, and reliable results [5,18]. The numerical results are presented in figures, including performance comparisons with 3D numerical techniques.

#### **II. ARM FORMULATION**

The H-plane horn antenna is primarily considered as a perfectly conducting cylindrical which is smooth, longitudinally obstacle, homogeneous, and infinitely long in z-direction. The XOY cross section of the structure shown in Fig. 1 is denoted by the closed contour S. Let us assume the scalar wave does not vary along z-axis. In this case, the scalar diffraction problem of such an object corresponds to the 2D Dirichlet boundary condition for incidence of E-polarized wave. Hence, the incident and scattered scalar wave functions  $(u^{i}(p) \text{ and } u^{s}(p))$  should satisfy the Helmholtz equation and the Dirichlet boundary condition given in Eq. (1) and Eq. (2) [16].

$$(\Delta + k^2)u^s(p) = 0, \quad p \in \mathbb{R}^2 \setminus \mathbb{S}, \tag{1}$$

$$u^{s(+)}(p) = u^{s(-)}(p) = -u^{i}(p), \quad p \in S,$$
 (2)

where, *S* is the smooth contour of the structure domain in 2D space  $R^2$  that belongs to the smoothness class  $C^{2,\alpha}$  [16].  $u^{s(+)}(p)$  and  $u^{s(-)}(p)$ are limiting values of  $u^s(p)$  in the inner and the outer sides of *S*, respectively. The solution of the BVP is written in Eq. (3) using the Green's formula and the related boundary condition defined in Eq. (2).

$$-\frac{i}{4} \int_{S} \left[ H_{0}^{(1)}(k \mid q - p \mid) Z(p) \right] dl_{p} = -u^{i}(q), \quad q, p \in S, \quad (3)$$

where,

$$Z(p) = \frac{\partial u^{s(-)}(p)}{\partial n} - \frac{\partial u^{s(+)}(p)}{\partial n}, \quad p \in S.$$
 (4)

*n* is the unit outward with respect to *S* normal of the point *p*. The unknown function Z(p) is constructed by solving Eq. (3) and using parameterization of the *S* contour specified by the function  $\eta(\theta) = (x(\theta), y(\theta))$  that smoothly parameterizes the contour *S* by the points of  $\theta \in [-\pi, \pi]$ . The functions  $x(\theta)$ ,  $y(\theta)$  are Cartesianlike coordinates of the point  $\eta(\theta) \in S$ , which are supposed to be infinitely smooth functions after their  $2\pi$ -periodical continuation on  $(-\infty,\infty)$ . The parametric representation of the integral equation of the first kind in Eq. (3) can be equivalently rewritten as,

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ \ln \left| 2\sin \frac{\theta - \tau}{2} \right| + K(\theta, \tau) \right\} Z_D(\tau) d\tau = g(\theta), \quad \theta \in [-\pi, \pi],$$
 (5)

by means of the following  $\eta(\theta)$  transformations:

$$Z_D(\theta) = \left( [x'(\theta)]^2 + [y'(\theta)]^2 \right)^{1/2} Z(\eta(\theta)), \ \theta \in [-\pi, \pi], (6)$$

$$|q-p| = \left\{ [x(\theta) - x(\tau)]^2 + [y(\theta) - y(\tau)]^2 \right\}^{1/2}, \ \theta \in [-\pi, \pi],$$
(7)

$$g(\theta) = -u^{i}(\eta(\theta)), \qquad \theta \in [-\pi, \pi].$$
(8)

In Eq. (5), the unknown function is  $Z_D(\tau)$ , the given function is  $g(\theta)$ , and  $K(\theta, \tau)$  function is the rather smooth section of the Green's function in comparison with the logarithmic part that contains the main singularity of this equation [17]. The functions are represented by their Fourier series expansions with  $k_{s,m}$ ,  $z_m$ ,  $g_m$  coefficients, as defined in set of Eqs. (9)-(12).

$$K(\theta,\tau) = \sum_{s=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} k_{sm} e^{i(s\theta+m\tau)}, \quad \theta \in [-\pi,\pi], \quad (9)$$

$$\ln|2\sin\frac{\theta-\tau}{2}| = -\frac{1}{2}\sum_{n=-\infty}^{\infty} |n|^{-1} e^{in(\theta-\tau)}, \quad \theta \in [-\pi,\pi], \quad (10)$$

$$Z_D(\tau) = \sum_{n=-\infty}^{\infty} z_n e^{in\tau}, \quad \tau \in [-\pi, \pi],$$
(11)

$$g(\theta) = \sum_{n=-\infty}^{\infty} g_n e^{in\theta}, \quad \theta \in [-\pi, \pi].$$
(12)

Finally, one can obtain an infinite system of the linear algebraic equations of the second kind given in Eq. (13).

$$\hat{z}_s + \sum_{m=-\infty}^{\infty} \hat{k}_{s,m} \hat{z}_m = \hat{g}_s, \quad s = \pm 1, \pm 2, \dots$$
 (13)

where,

$$\hat{k}_{s,m} = -2\tau_s \tau_m \Big[ k_{s,-m} + (1/2)\delta_{s,0}\delta_{m,0} \Big], 
\hat{z}_n = \tau_n^{-1} z_n, \qquad \hat{g}_s = -2\tau_s g_s.$$
(14)

Here,  $\delta_{s,n}$  is the Kronecker delta function. As a result, Eq. (13) leads to the generic form of equation of the second kind that guarantees the convergence and stability of this numerical process. Therefore, the scattered field  $u^s(q)$  for  $q \in \mathbb{R}^2$  are calculated by the integral equation representation of the Eq. (3) with any required

accuracy by the truncation method [17].

# III. PARAMETRIC DESIGN OF H-PLANE HORN ANTENNA

The described ARM procedure is derived for the selected horn antenna types, and the geometrical cross-section of the horn is modeled by ARM as a closed contour *L* that goes from point *A* to point *T* and back to *A* corresponding to  $\theta \in [-\pi, \pi]$ , as illustrated in Fig. 1. The relation between *l* and  $\theta$  is formulated in Eq. (15).

$$\left. \begin{array}{c} l = (\theta + \pi)L/2\pi \\ l \in [0, L] \to (\theta, \tau) \in [-\pi, \pi] \end{array} \right\}.$$

$$(15)$$

Then, the effects on radiation characteristics of the treated design parameters such as, multi source positioning, feeder back wall shaping, and flare wall rolling and corrugating are determined.



Fig. 1. XOY-plane geometry of H-plane horn.

The horn structure consists of totally 18 contour parts, which are defined in Table 1 and 2. The parameterization of the contour line is implemented separately from point *A* to *T*, and back to *A* by means of the variable  $l \in [0, L]$  as follows:

$$x = 0, y = l - L_{AB} + a, l \in [AB)$$
 (16)

$$x = l - L_{AB}, \quad y = a \quad , \ l \in [BC) \tag{17}$$

$$x = (l - L_{BC}) \cos \alpha_1 + b$$
  

$$y = (l - L_{BC}) \sin \alpha_1 + a$$
,  $l \in [CD)$ 
(18)

$$x = \left[\frac{d}{2}\sin\alpha_{1} + b + c\cos\alpha_{1}\right] + R\frac{d}{2}\cos\left[\left(\frac{\pi}{2} + \alpha_{1}\right) - 2\left(\frac{l - L_{CD}}{Rd}\right)\right] + X_{0}$$

$$y = \left[-\frac{d}{2}\cos\alpha_{1} + a + c\sin\alpha_{1}\right] + R\frac{d}{2}\sin\left[\left(\frac{\pi}{2} + \alpha_{1}\right) - 2\left(\frac{l - L_{CD}}{Rd}\right)\right] + Y_{0}$$

$$X_{0} = -(R - 1)\frac{d}{2}\sin\alpha_{1}, \quad Y_{0} = (R - 1)\frac{d}{2}\cos\alpha_{1}$$
(19)

$$x = b + d \sin \alpha_1 + [c - l + L_{DE}] \cos \alpha_1$$
  

$$y = a - d \cos \alpha_1 + [c - l + L_{DE}] \sin \alpha_1$$
,  $l \in [EF)$  (20)

$$x = d/\sin\alpha_1 + b - (l - L_{EF})\cos\varphi_1 \\ y = a - (l - L_{EF})\sin\varphi_1$$
,  $l \in [FG)$  (21)

$$x = -l + L_{FG} + b - db_1, \quad y = a - d \quad , l \in [GH)$$
 (22)

$$x = (-l + L_{GH}) \cos(\psi_1) + d + dx_1, y = a - d - (l - L_{GH}) \sin(\psi_1),$$
 (23)

$$x = d, \quad y = -l + L_{HJ} + a - d - dy_1, l \in [JK]$$
 (24)

$$x = d, \quad y = -l + L_{JK}, \ l \in [KL)$$
 (25)

$$x = d + (l - L_{KL}) \cos(\psi_2) y = -a + d + dy_2 - (l - L_{KL}) \sin(\psi_2)$$
,  $l \in [LM)$  (26)

$$x = l - L_{LM} + d + dx_1, \quad y = -a + d , \ l \in [MN]$$
(27)

$$x = b - db_2 + (l - L_{MN}) \cos \varphi_2 y = -a + d - (l - L_{MN}) \sin \varphi_2 , \ l \in [NO)$$
(28)

$$x = d/\tan \alpha_{2} + b + (l - L_{NO}) \cos \alpha_{2} + 0.5d \sin \alpha_{2} \\ y = -a - (l - L_{NO}) \sin \alpha_{2}$$
 (29)

$$x = \frac{d}{2}\sin\alpha_{2} + b + c\cos\alpha_{2} + R\frac{d}{2}\cos\left[\frac{\pi}{2} - \alpha_{2} - 2\left(\frac{l - L_{oP}}{Rd}\right)\right] + X_{0}$$

$$y = -\left(-\frac{d}{2}\cos\alpha_{2} + a + c\sin\alpha_{2}\right) + R\frac{d}{2}\sin\left[\frac{\pi}{2} - \alpha_{2} - 2\left(\frac{l - L_{oP}}{Rd}\right)\right] + Y_{0}$$

$$X_{0} = -(R - 1)\frac{d}{2}\sin\alpha_{2}, \quad Y_{0} = -(R - 1)\frac{d}{2}\cos\alpha_{2}$$
(30)

$$x = -(l - L_{PR})\cos\alpha_2 + b + c\cos\alpha_2$$
  

$$y = (l - L_{PR} - c)\sin\alpha_2 - a$$
,  $l \in [RS]$  (31)

$$x = -l + L_{RS} + b$$
  $y = -a$  ,  $l \in [ST)$  (32)

$$x = 0, \quad y = l - L_{ST} - a \quad , \ l \in [TA]$$
 (33)

The horn antenna is supposed to be excited by line source(s) located in the waveguide region at specified pin inset apart from the back wall. Thus, the incident wave at any (x,y) coordinate of the horn structure  $(u^i(x, y))$  is calculated by the line source radiation formula.

Table 1: Parametric definitions of horn contour

| No | Region | Definition   |
|----|--------|--|
| 1  | AB     | $-\pi \le 	heta < -\pi + 2L_{\scriptscriptstyle AB} \pi/L$   |
| 2  | BC     | $-\pi + 2L_{AB} \pi/L \le \theta < -\pi + 2L_{BC} \pi/L$   |
| 3  | CD     | $-\pi + 2L_{\scriptscriptstyle BC} \ \pi/L \le \theta < -\pi + 2L_{\scriptscriptstyle CD} \ \pi/L$ |
| 4  | DE     | $-\pi + 2L_{CD} \pi/L \le \theta < -\pi + 2L_{DE} \pi/L$   |
| 5  | EF     | $-\pi + 2L_{DE} \pi/L \le \theta < -\pi + 2L_{EF} \pi/L$   |
| 6  | FG     | $-\pi + 2L_{\rm EF} \ \pi/L \le \theta < -\pi + 2L_{\rm FG} \ \pi/L$                               |
| 7  | GH     | $-\pi + 2L_{FG} \pi/L \le \theta < -\pi + 2L_{GH} \pi/L$   |
| 8  | HJ     | $-\pi + 2L_{_{GH}} \pi/L \le \theta < -\pi + 2L_{_{HJ}} \pi/L$                                     |
| 9  | JK     | $-\pi + 2L_{_{HJ}} \pi/L \le \theta < -\pi + 2L_{_{JK}} \pi/L$                                     |
| 10 | KL     | $-\pi + 2L_{_{J\!K}} \pi/L \le \theta < -\pi + 2L_{_{K\!L}} \pi/L$                                 |
| 11 | LM     | $-\pi + 2L_{\rm KL} \ \pi/L \le \theta < -\pi + 2L_{\rm LM} \ \pi/L$                               |
| 12 | MN     | $-\pi + 2L_{\rm LM} \ \pi/L \le \theta < -\pi + 2L_{\rm MN} \ \pi/L$                               |
| 13 | NO     | $-\pi + 2L_{_{MN}}\pi/L \le \theta < -\pi + 2L_{_{NO}}\pi/L$                                       |
| 14 | OP     | $-\pi + 2L_{_{NO}}\pi/L \le \theta < -\pi + 2L_{_{OP}}\pi/L$                                       |
| 15 | PR     | $-\pi + 2L_{_{OP}}\pi/L \le \theta < -\pi + 2L_{_{PR}}\pi/L$                                       |
| 16 | RS     | $-\pi + 2L_{_{PR}}\pi/L \le \theta < -\pi + 2L_{_{RS}}\pi/L$                                       |
| 17 | ST     | $-\pi + 2L_{RS} \pi/L \le \theta < -\pi + 2L_{ST} \pi/L$   |
| 18 | TA     | $-\pi + 2L_{ST} \pi/L \le \theta < \pi$  |

#### **IV. NUMERICAL RESULTS**

The parametric analysis of the horn antenna is focused on the investigation of the effects of feeder positioning, back wall shaping, multi source locating, and rolling and corrugating of the flare wall to characterize the radiation pattern. First aim of this work is to minimize the side and back lobes. Before we demonstrate the numerical results, it is important to explain the reliability of the generic ARM algorithm, which has already been verified with analytical results by achieving very good agreements for the cases of plane wave scattering from a circular cylinder and radiation from open ended waveguide [5,18]. The ARM results of one horn type were also compared with analytical solutions additionally, and good agreements were obtained as shown in Fig. 2.

| T 11 A    | <b>a</b>    | 1 .1     | 0.1    |                       | •         |
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|           |             |          |        |                       |           |

| No   | Region | Segment Length                               |  |
|--|--------|--|--|
| 1  | AB     | $L_{AB} = a$                                 |  |
| 2  | BC     | $L_{BC} = L_{AB} + b$                        |  |
| 3  | CD     | $L_{CD} = L_{BC} + c$                        |  |
| 4  | DE     | $L_{DE} = L_{CD} + \pi Rd / 2$               |  |
| 5  | EF     | $L_{EF} = L_{DE} + c - d / \tan \alpha_1$    |  |
| 6  | FG     | $L_{FG} = L_{EF} + d / \sin \varphi_1$       |  |
| 7  | GH     | $L_{GH} = L_{FG} + b - db_1 - d - dx$        |  |
| 8  | HJ     | $L_{HJ} = L_{GH} + \sqrt{(dx_1^2 + dy_1^2)}$ |  |
| 9  | JK     | $L_{JK} = L_{HJ} + a - d - dy_1$             |  |
| 10   | KL     | $L_{KL} = L_{JK} + a - d - dy_2$             |  |
| 11   | LM     | $L_{LM} = L_{KL} + \sqrt{(dx_2^2 + dy_2^2)}$ |  |
| 12   | MN     | $L_{MN} = L_{LM} + b - db_2 - dx_2 - d$      |  |
| 13   | NO     | $L_{NO} = L_{MN} + d / \sin \varphi_2$       |  |
| 14   | OP     | $L_{OP} = L_{NO} + c - d / \tan \alpha_2$    |  |
| 15   | PR     | $L_{PR} = L_{OP} + \pi Rd / 2$               |  |
| 16   | RS     | $L_{RS} = L_{PR} + c$                        |  |
| 17   | ST     | $L_{ST} = L_{RS} + b$                        |  |
| 18   | TA     | $L = L_{ST} + a$                             |  |
| $\overline{db_{1,2}} = d / \tan \varphi_{1,2} - d \tan \alpha_{1,2}$ |        |  |  |

The analysis results for different back wall corner angle, ridged flare, wall thickness, corrugated teeth length, line source positioning at horizontal and vertical axis, and multi source feeding configurations are presented in Fig. 3, Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8, and Fig. 9, respectively. The major effects are briefly commented as that; the back wall corner can suppress the first side lobes up to 5 dB (see Fig. 3), the flare rolling can decrease both side and back lobes between 5-10 dB (see Fig.4), the thick metal walls can yield 15 dB suppressed back lobes (see Fig. 5), and the corrugating teeth can reach narrowed main beam width with about 5-10 dB improved side and back lobe levels (see Fig. 6). Figsures 7-9 indicate the back lobe suppression and side lobe asymmetry performances of pin inset and multi-pin localizations along x and y axis.

Figure 10 proposes a flare ridged horn design combined with triple source feeding to obtain 20 dB more suppressed side lobe levels. The multi source fed corrugated horn design shown in Fig. 11 is another antenna proposal that can successfully achieve -40 dB back lobe and -15 dB side lobe levels.



Fig. 2. Half-power beam width plots of ARM calculations and analytical results [3]. (Type 1:  $a = 0.6\lambda$ ,  $b = 4\lambda$ ,  $d = 0.1\lambda$ , R = 1,  $\alpha_1 = \alpha_2 = \varphi_1 = \varphi_2 = \alpha$ ,  $\Psi_1 = \Psi_2 = 0^{\circ}$ )



Fig. 3. Normalized radiation patterns of the horn for different back wall corner angles.

(Type 2:  $a=1.1\lambda$ ,  $b=2\lambda$ ,  $c=4\lambda$ ,  $d=0.1\lambda$ , R=1,  $a_1=a_2=45^\circ$ ,  $\phi_1=\phi_2=45^\circ$ ,  $dx=0.25\lambda$ )



Fig. 4. Normalized radiation pattern of flare rolled type horn.

(Type 2:  $a = 1.1\lambda$ ,  $b = 2\lambda$ ,  $c = 4\lambda$ ,  $d = 0.1\lambda$ , R = 1,  $a_1 = a_2 = 45^\circ$ ,  $\varphi_1 = \varphi_2 = 45^\circ$ ,  $\Psi_1 = \Psi_2 = 0^\circ$ )



Fig. 5. Normalized radiation patterns of the horn for different wall thicknesses. (Type 2:  $a=1.1\lambda$ ,  $b=2\lambda$ ,  $c=4\lambda$ , R=1,  $\alpha_1=\alpha_2=45^\circ$ ,

 $\varphi_1 = \varphi_2 = 45^\circ, \ \Psi_1 = \Psi_2 = 0^\circ)$ 



Fig. 6. Normalized radiation patterns of the corrugated horn for different lengths of the teeth. (Type 2:  $a=1.1\lambda$ ,  $b=2\lambda$ ,  $c=4\lambda$ ,  $d=0.1\lambda$ , R=1,  $a_1=a_2=45^\circ$ ,  $\Psi_1=\Psi_2=0^\circ$ )



Fig. 7. Normalized radiation patterns of the horn for different pin insets (along *x*-axis). (Type 3:  $a=0.481\lambda$ ,  $b=2.6\lambda$ ,  $c=8.664\lambda$ ,  $d=0.1\lambda$ ,  $\alpha_1=\alpha_2=11.18^\circ$ ,  $\varphi_1=\varphi_2=11.18^\circ$ ,  $\Psi_1=\Psi_2=0^\circ$ )



Fig. 8. Normalized radiation patterns of the horn for different pin insets (along *y*-axis).

(Type 3:  $a = 0.481\lambda$ ,  $b = 2.6\lambda$ ,  $c = 8.664\lambda$ ,  $d = 0.1\lambda$ ,  $\alpha_1 = \alpha_2 = 11.18^\circ$ ,  $\varphi_1 = \varphi_2 = 11.18^\circ$ ,  $\Psi_1 = \Psi_2 = 0^\circ$ )



Fig. 9. Normalized radiation patterns of the horn for different triple line source feeding configurations.

(Type 2:  $a=1.1\lambda$ ,  $b=2\lambda$ ,  $c=4\lambda$ ,  $d=0.1\lambda$ , R=1,  $a_{l}=a_{2}=45^{\circ}$ ,  $\varphi_{l}=\varphi_{2}=45^{\circ}$ ,  $\Psi_{l}=\Psi_{2}=75^{\circ}$ )



Fig. 10. Normalized radiation pattern of the flare ridged horn version with triple line source feeding.



Fig. 11. Normalized radiation pattern of the corrugated horn version with triple line source feeding.



Fig. 12. Comparisons of radiation patterns calculated by 2D ARM and 3D MoM simulator. (a) Type 1:  $a=0.6\lambda$ ,  $b=4\lambda$ ,  $c=4\lambda$ ,  $d=0.1\lambda$ , R=1,  $\alpha_1=\alpha_2=45^{\circ}$ 





Fig. 13. Average of relative errors in numerical calculations (demonstrated for two horn types).

Finally, the 2D ARM solutions of two different horn types are compared with 3D MoM simulator results. The coherency observed in Fig. 12 implies that ARM is a useful technique due to fast convergence and short calculation times (see Table 3 and Fig. 13), especially for rough model design and wide band analysis of the horn antennas.

| Table | 3:  | Computation    | times   | of   | ARM  | and | 3D |
|-------|-----|----------------|---------|------|------|-----|----|
| MoM   | sim | ulator (2.53 G | Hz, 4 C | BB I | RAM) |     |    |

| Tuno | Truncation | Computation Time |            |  |
|------|------------|------------------|------------|--|
| гуре | (N)        | 2D ARM           | 3D MoM     |  |
| 1    | 40         | 16.92 sec.       | 13 min. 48 |  |
|      | 60         | 51.18 sec.       | sec.       |  |
| 2    | 40         | 16.92 sec.       | 12 min. 45 |  |
|      | 60         | 51.18 sec.       | sec.       |  |
| 3    | 40         | 16.92 sec.       | 16 min. 23 |  |
|      | 60         | 51.18 sec.       | sec.       |  |

# **V. CONCLUSION**

This paper has presented the parametric analysis of the radiation characteristics of the Hplane horn antenna essentially for the geometric versions of multi source feeding, flare rolling, and corrugating. The analytical regularization method is used to perform 2-D analysis of the E-polarized electromagnetic wave scattering problem. The analysis results of different back wall corner angle, ridged flare, wall thickness, corrugated tooth length, vertical and horizontal line source positioning, multi source feeding configurations, and combined proposals are demonstrated on the radiation patterns. The main goal of this work is to provide systematic information for the horn designers to attain desired antenna patterns, such as high gain, narrow beam, and suppressed side and back lobes, in a fast and accurate way.

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