ANALYSIS OF TRANSIENT SCATTERING FROM CONDUCTORS USING LAGUERRE POLYNOMIALS AS TEMPORAL BASIS FUNCTIONS

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ABSTRACT

In this paper, a new method is presented for analyzing the transient electromagnetic response from a three-dimensional (3-D) perfectly electric conducting body using the time-domain electric field integral equation (TD-EFIE). Instead of the conventional marching-on in time (MOT) technique, the solution method in this paper is based on the Galerkin's method that involves separate spatial and temporal testing procedure. Triangular patch basis functions are used for spatial expansion and testing functions for arbitrarily shaped 3-D structures. The time-domain unknown coefficient is approximated as an orthonormal basis function set that is derived from the Laguerre functions. These basis functions are also used as the temporal testing. With the representation of the derivative of the time-domain coefficient in an analytic form, the time derivative of the vector potential in the TD-EFIE can be handled analytically. We also propose an alternative formulation to solve the differential form of the TD-EFIE. Two methods presented in this paper result in very accurate and stable transient responses from conducting objects. Detailed mathematical steps are included and representative numerical results are presented and compared.

I. INTRODUCTION

For a time-domain integral equation formulation, the MOT method is usually employed [1]. A serious drawback of this algorithm is the occurrence of late-time instabilities in the form of high frequency oscillation. Several MOT formulations have been presented for the solution of the TD-EFIE to calculate the electromagnetic scattering from arbitrarily shaped three-dimensional structures using triangular patch modeling technique. An explicit solution has been presented by differentiating the TD-EFIE and using second order finite difference [2]. But the results become unstable for late times. Its late time oscillations could be eliminated by approximating the average value of the current [3]. In addition, to overcome this, a backward finite difference approximation for the magnetic vector potential term has been presented for the explicit technique [4]. Recently an implicit scheme has been proposed to improve the stability problem [5]-[8], in addition matrix pencil is used in [9] to extrapolate the late time data. Even though employing an implicit technique, the accuracy and stability are dependent on the choice of the time step.

In this paper, we present a new technique to obtain accurate and stable responses of the TD-EFIE for arbitrarily shaped 3-D conducting objects using the associate Laguerre polynomials as temporal basis functions. The associate Laguerre series is defined only over the interval from zero to infinity and, hence, are considered to be more suited for the transient problem, as they naturally enforce causality [10], [11]. Using the associate Laguerre polynomials, we construct a set of orthogonal basis functions. Transient quantities that are functions of time can be spanned in terms of these orthogonal basis functions. The temporal basis functions used in this work are completely convergent to zero as time increases to infinity. Therefore, transient response spanned by these basis functions is also convergent to zero as time progresses. Using the Galerkin's method, we introduce a temporal testing procedure, which is similar to the spatial testing procedure of the method of moments (MoM). By applying the temporal testing procedure to the TD-EFIE, we can eliminate the numerical instabilities. Instead of the MOT procedure, we employ a marching-on in-degree manner as increasing the degree of temporal testing functions. Therefore, we can obtain the unknown coefficients by solving a matrix equation recursively with a finite number of basis functions. The minimum degree or number of basis functions is dependent on the time duration and the frequency bandwidth product of an incident wave. We also propose an alternative formulation to solve the differential form of TD-EFIE, which has been used in [2].

This paper is organized as follows. In the next section, we describe the general TD-EFIE and set up a matrix equation by applying MoM with spatial and temporal testing procedure. In section III, an alternative technique

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for TD-EFIE formulation is presented. In section IV, we discuss some numerical results. Finally, some conclusions based on this work are presented in section V.

II. FORMULATION

In this section we discuss the TD-EFIE and derive a matrix equation to obtain induced currents on the conducting scatterer. Let ***S*** denote the surface of a closed or open conducting body illuminated by a transient electromagnetic wave. Since the total tangential electric field is zero on the surface for all times, we have

$$\left[\mathbf{E}^{i}(\mathbf{r},t)+\mathbf{E}^{s}(\mathbf{r},t)\right]_{\tan}=0,\ r\in S, \tag{1}$$

where $\mathbf{E}^{\mathbf{i}}$ is the incident field and $\mathbf{E}^{\mathbf{S}}$ is the scattered field due to the induced current $\mathbf{J}$. The subscript 'tan' denotes the tangential component. The scattered field is

$$\mathbf{E}^{s}(\mathbf{r},t)=-\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t)-\nabla\Phi(\mathbf{r},t), \tag{2}$$

where $\mathbf{A}$ and Φ are the magnetic vector and the electric scalar potential given by, respectively,

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu}{4\pi}\int_{S}\frac{\mathbf{J}(\mathbf{r}',\tau)}{R}dS' \tag{3}$$

$$\Phi(\mathbf{r},t)=\frac{1}{4\pi\varepsilon}\int_{S}\frac{q(\mathbf{r}',\tau)}{R}dS'. \tag{4}$$

In (3) and (4), $R=\left|\mathbf{r}-\mathbf{r}'\right|$ represents the distance between the arbitrarily located observation point $\mathbf{r}$ and the source point $\mathbf{r}'$, $\tau=t-R/c$ is the retarded time, μ and ε are permeability and permittivity of the space, and c is the velocity of propagation of the electromagnetic wave in that space. The electric surface charge density q is related to the surface current density $\mathbf{J}$ by the equation of continuity

$$\nabla\cdot\mathbf{J}(\mathbf{r},t)=-\frac{\partial}{\partial t}q(\mathbf{r},t)\ . \tag{5}$$

Combining (1) and (2) gives

$$\left[\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t)+\nabla\Phi(\mathbf{r},t)\right]_{\tan}=\left[\mathbf{E}^{i}(\mathbf{r},t)\right]_{\tan},\ r\in S. \tag{6}$$

Equation (6) with (3) and (4) constitutes a TD-EFIE from which the unknown current $\mathbf{J}$ may be determined.

1. SPATIAL TESTING PROCEDURE

The surface of the structure to be analyzed is approximated by planar triangular patches. As in [12], we define the vector basis function associated with the n-th common edge as

$$\mathbf{f}_n(\mathbf{r})=\mathbf{f}_n^{+}(\mathbf{r})+\mathbf{f}_n^{-}(\mathbf{r}) \tag{7-1}$$

$$\mathbf{f}_n^{\pm}(\mathbf{r})=\begin{cases}\dfrac{l_n}{2A_n^{\pm}}\boldsymbol{\rho}_n^{\pm}, & \mathbf{r}\in T_n^{\pm}\\ 0, & \mathbf{r}\notin T_n^{\pm}\end{cases}, \tag{7-2}$$

where l_n and $A_n^{\pm}$ are the length of the edge and the area of triangle $T_n^{\pm}$. $\boldsymbol{\rho}_n^{\pm}$ is the position vector defined with respect to the free vertex of $T_n^{\pm}$. The electric current $\mathbf{J}$ on the scattering structure may be approximated in terms of the vector basis function as

$$\mathbf{J}(\mathbf{r},t)=\sum_{n=1}^{N}J_n(t)\mathbf{f}_n(\mathbf{r}), \tag{8}$$

where N represents the number of common edges, discounting the boundary edges in the triangulated model of the conducting object. When (8) is used in (6), we meet a time integral term from the relation (4) and (5). For convenience to avoid this problem and to handle the time derivative of a vector potential analytically, we introduce a new source vector $\mathbf{e}(\mathbf{r},t)$ defined by

$$\mathbf{J}(\mathbf{r},t)=\frac{\partial}{\partial t}\mathbf{e}(\mathbf{r},t), \tag{9}$$

where the relation between this source vector and charge density is given as

$$q(\mathbf{r},t)=-\nabla\cdot\mathbf{e}(\mathbf{r},t)\ . \tag{10}$$

By using (8) and (9), we may express

$$\mathbf{e}(\mathbf{r},t)=\sum_{n=1}^{N}e_n(t)\mathbf{f}_n(\mathbf{r})\ . \tag{11}$$

We now solve (6) by applying Galerkin's method in the MoM context and hence the testing functions are same as the expansion functions. By choosing the spatial expansion function $\mathbf{f}_m(\mathbf{r})$ also as the spatial testing functions, we have from (6)

$$\begin{aligned}&<\mathbf{f}_m(\mathbf{r}),\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t)>+<\mathbf{f}_m(\mathbf{r}),\nabla\Phi(\mathbf{r},t)>\\&=<\mathbf{f}_m(\mathbf{r}),\mathbf{E}^{i}(\mathbf{r},t)>,\end{aligned} \tag{12}$$

where $m=1,2,\cdots,N$. The next step in the MoM procedure is to substitute the unknown expansion functions defined in (11) into (12). In computing the inner product integrals in (12), we assume that the unknown quantity does not appreciably change within a triangle patch so that

$$\tau=t-\frac{R}{c}\rightarrow\tau_{mn}^{pq}=t-\frac{R_{mn}^{pq}}{c},\ R_{mn}^{pq}=\left|\mathbf{r}_m^{cp}-\mathbf{r}_n^{cq}\right| \tag{13}$$

where p and q are + or -. $\mathbf{r}_m^{c\pm}$ is the position vector of the center in triangle $T_n^{\pm}$. With the assumption (13) and using (3), (4), and (9)-(11), (12) can be written as

$$\sum_{n=1}^{N}\sum_{p,q}\left[\mu a_{mn}^{pq}\frac{d^2}{dt^2}e_n(\tau_{mn}^{pq})+\frac{b_{mn}^{pq}}{\varepsilon}e_n(\tau_{mn}^{pq})\right]=V_m(t),$$
$$m=1,2,\cdots,N, \tag{14}$$
where
$$a_{mn}^{pq}=\frac{1}{4\pi}\int_S \mathbf{f}_m^p(\mathbf{r})\cdot\int_S\frac{\mathbf{f}_n^q(\mathbf{r}')}{R}dS'dS \tag{15}$$
$$b_{mn}^{pq}=\frac{1}{4\pi}\int_S \nabla\cdot\mathbf{f}_m^p(\mathbf{r})\int_S\frac{\nabla'\cdot\mathbf{f}_n^q(\mathbf{r}')}{R}dS'dS \tag{16}$$
$$V_m(t)=\int_S \mathbf{f}_m(\mathbf{r})\cdot\mathbf{E}^{\mathrm{i}}(\mathbf{r},t)dS. \tag{17}$$
The integrals (15)-(17) may be evaluated by the method described in [12] and [13].

2. TEMPORAL BASIS FUNCTIONS

Consider the set of functions [14],
$$L_j(t)=\frac{e^t}{j!}\frac{d^j}{dt^j}(t^je^{-t}),\ 0\le t<\infty,\ j=0,1,2,\cdots. \tag{18}$$
These are the Laguerre functions of degree j. They are causal, i.e., exist for $t\ge 0$. They can be computed in a stable fashion recursively through
$$L_0(t)=1 \tag{19-1}$$
$$L_1(t)=1-t \tag{19-2}$$
$$L_j(t)=\frac{1}{j}\left[(2j-1-t)L_{j-1}(t)-(j-1)L_{j-2}(t)\right],\ j\ge 2. \tag{19-3}$$
The Laguerre functions are orthogonal as
$$\int_0^\infty e^{-t}L_i(t)L_j(t)dt=\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\ne j\end{cases}. \tag{20}$$
An orthonormal basis function set can be derived from the Laguerre function through the representation
$$\phi_j(t)=e^{-t/2}L_j(t). \tag{21}$$
These functions can approximate a causal response quite well. A causal electromagnetic response function $f(t)$ at a particular location in space for $t\ge 0$ can be expanded using (21) as
$$f(t)=\sum_{j=0}^{\infty}f_j\phi_j(t). \tag{22}$$
By multiplying a function $f(t)$ with (21) and integrating from zero to infinity, which we call a Laguerre transform here, we get
$$\int_0^\infty \phi_i(t)f(t)dt=f_i. \tag{23}$$
In obtaining (23), the orthogonal relation (20) was used. Also, we can obtain the result of the Laguerre transform for the derivative of the function $f(t)$ as
$$\int_0^\infty \phi_i(t)\frac{d}{dt}f(t)dt=\frac{1}{2}f_i+\sum_{k=0}^{i-1}f_k, \tag{24}$$
where $f(0)=0$ was assumed and $\phi_i(\infty)=0$ was used. Using a similar relation between (22) and (23), we can expand the derivative of the function $f(t)$ using (24) as
$$\frac{d}{dt}f(t)=\sum_{j=0}^{\infty}\left(\frac{1}{2}f_j+\sum_{k=0}^{j-1}f_k\right)\phi_j(t). \tag{25}$$
Similarly, if we assume $f'(0)=0$, the result of expanding the second derivative of the function $f(t)$ can be obtained as
$$\frac{d^2}{dt^2}f(t)=\sum_{j=0}^{\infty}\left[\left(\frac{1}{4}f_j+\sum_{k=0}^{j-1}(j-k)f_k\right)\right]\phi_j(t). \tag{26}$$

3. TEMPORAL TESTING PROCEDURE

The transient coefficient introduced in (11) can be expanded as
$$e_n(t)=\sum_{j=0}^{\infty}e_{n,j}\phi_j(st), \tag{27}$$
where s is a scaling factor. By controlling this factor s, the support provided by the expansion can be increased or decreased. Using (26), therefore, the expression of expanding the second derivative of the coefficient is given as
$$\frac{d^2}{dt^2}e_n(t)=s^2\sum_{j=0}^{\infty}\left[\frac{1}{4}e_{n,j}+\sum_{k=0}^{j-1}(j-k)e_{n,k}\right]\phi_j(st). \tag{28}$$
Substituting (27) and (28) into (14) and taking a temporal testing with $\phi_i(st)$, which is the Laguerre transform defined in (23), we have
$$\sum_{n=1}^{N}\sum_{p,q}\sum_{j=0}^{\infty}\left[\begin{matrix}\left(\frac{s^2}{4}\mu a_{mn}^{pq}+\frac{b_{mn}^{pq}}{\varepsilon}\right)e_{n,j}+\\ s^2\mu a_{mn}^{pq}\sum_{k=0}^{j-1}(j-k)e_{n,k}\end{matrix}\right]I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right)=V_{m,i}, \tag{29}$$
where
$$I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right)=\int_0^\infty \phi_i(st)\phi_j\left(st-s\frac{R_{mn}^{pq}}{c}\right)d(st); \tag{30}$$
$$V_{m,i}=\int_0^\infty \phi_i(st)V_m(t)\,d(st). \tag{31}$$
Now, we consider the integral defined in (30). For simplicity, we rewrite (30) as
$$I_{ij}(y)=\int_0^\infty \phi_i(x)\phi_j(x-y)dx. \tag{32}$$
Through the following change of variable $z=x-y$ in (32), we have

$$I_{ij}(y) = e^{-y/2}\int_{-y}^{\infty} e^{-z} L_i(z+y) L_j(z)dz\,. \tag{33}$$

Using the formula (8.971) and (8.974) in [15], we obtain

$$L_i(z+y) = \sum_{k=0}^{i} L_k(z)\left[L_{i-k}(y) - L_{i-k-1}(y)\right]. \tag{34}$$

Substituting (34) into (33), we obtain

$$I_{ij}(y) = e^{-y/2}\sum_{k=0}^{i}\left[L_{i-k}(y) - L_{i-k-1}(y)\right]\int_{-y}^{\infty} e^{-z} L_k(z) L_j(z)dz. \tag{35}$$

Because the Laguerre function is defined for $z \geq 0$, the lower limit of the integral in (35) may be changed from $-y$ to zero, and the integral can be computed easily using (20). Finally, we have

$$I_{ij}(y) = \begin{cases} e^{-y/2}\left[L_{i-j}(y) - L_{i-j-1}(y)\right], & j \leq i \\ 0, & j > i \end{cases}. \tag{36}$$

We note that $I_{ij} = 0$ when $j > i$. Therefore we can write the upper limit for the summation symbol as i instead of ∞ in (29). In this result, moving the terms including $e_{n,j}$, which is for $j < i$, to the right-hand side, we obtain

$$\begin{aligned}
&\sum_{n=1}^{N}\sum_{p,q}\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right) e_{n,i} I_{ii}\left(s\frac{R_{mn}^{pq}}{c}\right) = \\
&V_{m,i} - \sum_{n=1}^{N}\sum_{p,q}\sum_{j=0}^{i-1}\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right) e_{n,j} I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right) \\
&-\sum_{n=1}^{N}\sum_{p,q}\sum_{j=0}^{i} s^2 \mu a_{mn}^{pq}\sum_{k=0}^{j-1}(j-k)e_{n,k} I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right).
\end{aligned} \tag{37}$$

Rewriting (37) in a simpler form, we have

$$\sum_{n=1}^{N}\alpha_{mn} e_{n,i} = V_{m,i} + P_{m,i},\ m = 1, 2, \cdots, N, \tag{38}$$

where

$$\alpha_{mn} = \sum_{p,q}\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right)\exp\left(-s\frac{R_{mn}^{pq}}{2c}\right) \tag{39}$$

$$P_{m,i} = -\sum_{n=1}^{N}\sum_{p,q}\left[\begin{array}{l}\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right)\sum_{j=0}^{i-1} e_{n,j} I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right) + \\ s^2\mu a_{mn}^{pq}\sum_{j=0}^{i}\sum_{k=0}^{j-1}(j-k)e_{n,k} I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right)\end{array}\right]. \tag{40}$$

In obtaining (39), we used $I_{ii}(y) = e^{-y/2}$ from (36). Finally, we can write (38) in a matrix form as

$$\left[\alpha_{mn}\right]\left[e_{n,i}\right] = \left[\gamma_{m,i}\right],\ i = 0, 1, \cdots, \infty\,, \tag{41}$$

where $\gamma_{m,i} = V_{m,i} + P_{m,i}$. It is important to note that $[\alpha_{mn}]$ is not a function of the degree of the temporal testing function. Therefore, we can obtain the unknown coefficients by solving (41) by increasing the degree of the temporal testing functions. The coefficients of the current $J_n(t)$ oscillate for low degrees and die down for high degrees. We can solve the coefficients recursively until they are small enough. Therefore, this formulation is marching on in degree as opposed to marching –on-in time for an implicit procedure. The matrix equation is first solved for i = 0 and then continued for different values for i which corresponds to different order of the Laguerre functions.

We need the minimum degree or number of temporal basis functions, M in computing (41). This parameter is dependent on the time duration of the transient response and the bandwidth of the excitation signal. We consider a signal with a bandwidth B in frequency-domain and the duration T_f in the time-domain. When we represent this signal by a Fourier series, the range of the sampling frequency is $-B \leq k\Delta f \leq B$, where k is an integer and $\Delta f = 1/T_f$. So we get $|k| \leq B/T_f$. Hence the minimum number of temporal basis functions becomes $M = 2BT_f + 1$. We note that the upper limit of the integral in (31) can be replaced by a time duration sT_f instead of infinity.

4. *CURRENT AND FAR FIELD*

By solving the matrix equation (41) in a marching-on in degree manner, the electric transient current coefficient in (8) is expressed using the relation (9) and (11) with (25) as

$$J_n(t) = \frac{d}{dt}e_n(t) = s\sum_{j=0}^{M-1}\left(\frac{1}{2}e_{n,j} + \sum_{k=0}^{j-1} e_{n,k}\right)\phi_j(st). \tag{42}$$

Once the current coefficients have been obtained, we can compute the far field. We explain the analytic method to compute the far field directly by using the coefficient $e_n(t)$ obtained from (41). Neglecting the scalar potential term, the far field is given by

$$\mathbf{E}^s(\mathbf{r}, t) \approx -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t)\,. \tag{43}$$

Substituting (3), (9), and (11) into (43) with (7-1), we get

$$\mathbf{E}^s(\mathbf{r}, t) \approx -\frac{\mu}{4\pi}\sum_{n=1}^{N}\sum_{q}\int_S \frac{d^2}{dt^2}e_n(\tau)\frac{\mathbf{f}_n^q(\mathbf{r}')}{R}dS'. \tag{44}$$

We make the following approximation in the far field: $R \approx r - \mathbf{r}'\cdot\hat{\mathbf{r}}$ for the time retardation term $\tau = t - R/c$, $R \approx r$ for the amplitude term $1/R$, where $\hat{\mathbf{r}} = \mathbf{r}/r$ is a unit vector in the direction of the radiation. The integral in (44) is evaluated by approximating the integrand by the value at the center of the source triangle T_n^q.

Substituting (7-2) into (44) and approximating $\mathbf{r}' \approx \mathbf{r}_n^{cq}$ and $\boldsymbol{\rho}_n^q \approx \boldsymbol{\rho}_n^{cq}$, we obtain

$$\mathbf{E}^s(\mathbf{r}, t) \approx -\frac{\mu}{8\pi r}\sum_{n=1}^{N} l_n \sum_q \boldsymbol{\rho}_n^{cq} \frac{d^2}{dt^2} e_n(\tau_n^q), \quad (45)$$

where $\tau_n^q \approx t - (r - \mathbf{r}_n^{cq} \cdot \hat{\mathbf{r}})/c$ and

$$\frac{d^2}{dt^2} e_n(t) = s^2 \sum_{j=0}^{M-1}\left[\frac{1}{4} e_{n,j} + \sum_{k=0}^{j-1}(j-k)e_{n,k}\right]\phi_j(s\tau_n^q). \quad (46)$$

III. ALTERNATIVE FORMULATION

In this section, we present an alternative method of solving TD-EFIE as given in (1), which has been extensively used in the literature. The goal is to see which form provides more accurate solution as this method contains double derivatives. By differentiating (6), we get

$$\left[\frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r}, t) + \nabla \frac{\partial}{\partial t}\Phi(\mathbf{r}, t)\right]_{\tan} = \left[\frac{\partial}{\partial t}\mathbf{E}^{\mathrm{i}}(\mathbf{r}, t)\right]_{\tan}, \quad \mathbf{r} \in S.. \quad (47)$$

In a similar manner as in (12), we obtain the result of the spatial testing from (47) as

$$\begin{aligned} &< \mathbf{f}_m(\mathbf{r}), \frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r}, t) > + < \mathbf{f}_m(\mathbf{r}), \nabla \frac{\partial}{\partial t}\Phi(\mathbf{r}, t) > \\ &= < \mathbf{f}_m(\mathbf{r}), \frac{\partial}{\partial t}\mathbf{E}^{\mathrm{i}}(\mathbf{r}, t) > . \end{aligned} \quad (48)$$

Substituting (3)-(5), (7), and (8) into (48) and with the use of (13), we get

$$\sum_{n=1}^{N}\sum_{p,q}\left[\mu a_{mn}^{pq}\frac{d^2}{dt^2}J_n(\tau_{mn}^{pq}) + \frac{b_{mn}^{pq}}{\varepsilon}J_n(\tau_{mn}^{pq})\right] = V_m(t), \quad (49)$$

where a_{mn}^{pq} and b_{mn}^{pq} are same as to (15) and (16), respectively, and

$$V_m(t) = \int_S \mathbf{f}_m(\mathbf{r}) \cdot \frac{\partial}{\partial t}\mathbf{E}^i(\mathbf{r}, t)dS. \quad (50)$$

The transient current coefficient can be written as

$$J_n(t) = \sum_{j=0}^{\infty} J_{n,j}\phi_j(st), \quad (51)$$

where s is a scaling factor. Using (26), the second derivative of the current coefficient is given as

$$\frac{d^2}{dt^2}J_n(t) = s^2\sum_{j=0}^{\infty}\left[\frac{1}{4}J_{n,j} + \sum_{k=0}^{j-1}(j-k)J_{n,k}\right]\phi_j(st). \quad (52)$$

Substituting (51) and (52) into (49) with the temporal testing with $\phi_i(st)$, we get

$$\sum_{n=1}^{N}\sum_{p,q}\sum_{j=0}^{\infty}\left[\begin{aligned}&\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right)J_{n,j} \\ &+ s^2\mu a_{mn}^{pq}\sum_{k=0}^{j-1}(j-k)J_{n,k}\end{aligned}\right] I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right) = V_{m,i}, \quad (53)$$

where $V_{m,i}$ is of the same form given in (31), but $V_m(t)$ is different. Changing the upper limit of the summation symbol to i instead of ∞ in (53) and moving the terms including $J_{n,j}$, which is for $j < i$, to the right-hand side, we obtain

$$\begin{aligned} &\sum_{n=1}^{N}\sum_{p,q}\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right)J_{n,i}I_{ii}\left(s\frac{R_{mn}^{pq}}{c}\right) \\ &= V_{m,i} - \sum_{n=1}^{N}\sum_{p,q}\sum_{j=0}^{i-1}\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right)J_{n,j}I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right) \\ &\quad - \sum_{n=1}^{N}\sum_{p,q}\sum_{j=0}^{i} s^2\mu a_{mn}^{pq}\sum_{k=0}^{j-1}(j-k)J_{n,k}I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right). \end{aligned} \quad (54)$$

Rewriting (54) in a simple form, we have

$$\sum_{n=1}^{N}\alpha_{mn}J_{n,i} = V_{m,i} + P_{m,i}, \; m = 1, 2, \cdots, N, \quad (55)$$

where α_{mn} is same as (39) and

$$P_{m,i} = -\sum_{n=1}^{N}\sum_{p,q}\left[\begin{aligned}&\left(\frac{s^2}{4}\mu a_{mn}^{pq} + \frac{b_{mn}^{pq}}{\varepsilon}\right)\sum_{j=0}^{i-1}J_{n,j}I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right) \\ &+ s^2\mu a_{mn}^{pq}\sum_{j=0}^{i}\sum_{k=0}^{j-1}(j-k)J_{n,k}I_{ij}\left(s\frac{R_{mn}^{pq}}{c}\right)\end{aligned}\right]. \quad (56)$$

Lastly, we can write (55) in a matrix form as

$$[\alpha_{mn}][J_{n,i}] = [\gamma_{m,i}], \quad (57)$$

where $\gamma_{m,i} = V_{m,i} + P_{m,i}$. By solving (57) by a marching-on in degree algorithm with M temporal basis functions, we can obtain the current coefficient directly, which is given as

$$J_n(t) = \sum_{j=0}^{M-1} J_{n,j}\phi_j(st). \quad (58)$$

Substituting (3) and (8) into (43) with (7), and using (25), the far field is given as

$$\mathbf{E}^s(\mathbf{r}, t) \approx -\frac{\mu}{8\pi r}\sum_{n=1}^{N} l_n \sum_q \boldsymbol{\rho}_n^{cq}\frac{d}{dt}J_n(\tau_n^q), \quad (59)$$

where

$$\frac{d}{dt}J_n(\tau_n^q) = s\sum_{j=0}^{M-1}\left(\frac{1}{2}J_{n,j} + \sum_{k=0}^{j-1}J_{n,k}\right)\phi_j(s\tau_n^q). \quad (60)$$

The formulation provided in this section computes the coefficients of $J_n(t)$ directly. We don't need to convert $e_n(t)$ to $J_n(t)$ by (42). However, this formulation requires the derivative of the incident wave as in (50). Gaussian wave is used extensively in transient analysis, and we have the analytic form of derivative of Gaussian wave. Two formulations will obtain the same performance by using Gaussian wave, as shown in the next section. If we have an arbitrary input, the formulation introduced in section II is preferred.

It's important to notice that the minimum degree can be obtained at the optimal scaling factor s. We can estimate the range for convergence and the optimal scaling factor s by B and Tf, and solve Jn,i recursively, until Jn,i converges to zero ([16]). Stable performance can be obtained by this way.

IV. NUMERICAL EXAMPLES

In this section, we present the numerical results for three representative 3-D scatterers, viz. a sphere, a cube, and a cylinder, as shown in Fig. 1. The scatterers are illuminated by a Gaussian plane wave, in which the electric field is given by

$$\mathbf{E}^{i}(\mathbf{r},t) = \mathbf{E}_0 \frac{4}{\sqrt{\pi}T} e^{-\gamma^2}, \tag{61}$$

$$\gamma = \frac{4}{T}(ct - ct_0 - \mathbf{r}\cdot\hat{\mathbf{k}}), \tag{62}$$

where $\hat{\mathbf{k}}$ is the unit vector in the direction of wave propagation, T is the pulse width of the Gaussian impulse, and t_0 is a time delay which represents the time at which the pulse peaks at the origin. In this work, the field is incident from $\phi = 0^\circ$ and $\theta = 0^\circ$ with $\hat{\mathbf{k}} = -\hat{\mathbf{z}}$ and $\mathbf{E}_0 = \hat{\mathbf{x}}$. To avoid problems with the internal resonance of the structure, we use a pulse of width $T = 8$ lm with $ct_0 = 12$ lm, which has a frequency spectrum of 125 MHz. The unit 'lm' denotes a light meter. A light meter is the length of time taken by the electromagnetic wave to travel 1 m. We set $s = 10^9$ and $M = 80$, which is sufficient to get accurate solutions. For comparison, we present MOT solutions using the method in [8] and the results obtained by taking the IDFT solution calculated from the frequency-domain EFIE. In all figures to be shown, the legends 'form1' and 'form2' implies results computed by the formulation in section II and section III, respectively.

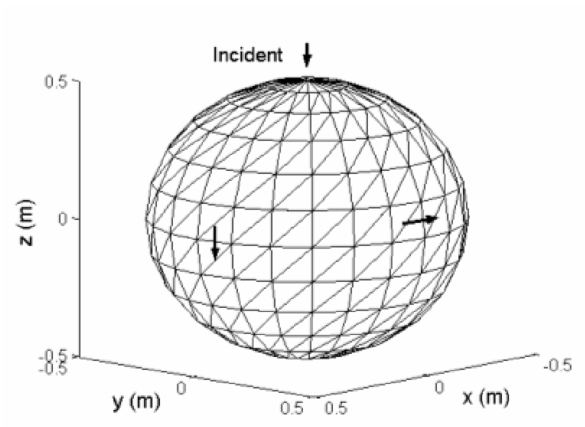
As a first example, we consider a conducting sphere of radius 0.5 m centered at the origin as shown in Fig. 1(a). The first resonant frequency of this sphere is 262 MHz. There are twelve and twenty-four divisions along the θ and ϕ directions with equal angular intervals. This results in a total of 528 patches and 792 common edges, and $R_{\min}$ =2.23 cm, where $R_{\min}$ represents the minimum distance between any two distinct patch centers. The $\theta-$ directed current at $\theta = 90^\circ$ and $\phi = 7.5^\circ$, and $\phi-$ directed current at $\theta = 7.5^\circ$ and $\phi = 90^\circ$ on the sphere are indicated by arrows in Fig. 1(a). Fig. 2 shows the transient response for the $\theta-$ directed and $\phi-$ directed current. The time step in the MOT computation is chosen such that $c\Delta t = 4R_{\min}$ in order to generate an implicit solution. It is important to note that all the four solutions show good agreements except the late-time oscillation in the MOT solution. We can see that solutions of the presented method 1 and 2 are stable and the agreement between each other is very good. Fig. 3 compares the transient response of two presented methods with the Mie series solution and the IDFT of the frequency-domain EFIE solution for the far scattered field from the sphere along the backward direction. All the four solutions agree well as is evident from the figure.

As a second example, consider a conducting cube, 1 m on a side, centered about the origin shown in Fig. 1(b). The first resonant frequency of this cube is 212 MHz. There are eight divisions along the x , y and z directions, respectively. This represents a total of 768 patches and 1,152 common edges, and $R_{\min}$ =5.57 cm. The $z-$ and $x-$directed current at the side faces are indicated by arrows in Fig. 1(b). Fig. 4 shows the transient response for the $z-$ and $x-$directed currents. The time step in the MOT computation is chosen as $c\Delta t = 2R_{\min}$ in order to generate an implicit solution. Here the agreement between the results from the IDFT and two presented methods is very good. It is important to note that the MOT solution shows some instability. Fig. 5 compares the transient response of two presented methods and the IDFT of the frequency-domain EFIE solution for the far scattered field from the cube along the backward direction. All the three solutions agree well.

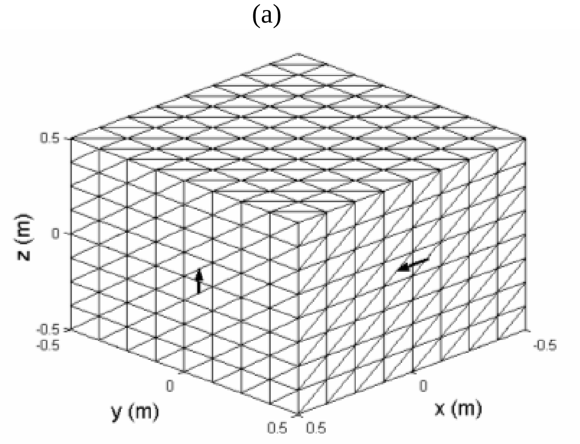
As a final example, we show the transient response from a conducting cylinder with a radius of 0.5 m and height 1 m, centered at the origin as shown in Fig. 1(c). We subdivide the cylinder into four, twenty-four, and eight divisions along r , ϕ and z directions, respectively. This represents a total of 720 patches with 1,080 common edges, and $R_{\min}$ =2.15 cm. The $z-$ and $\phi-$directed current at the side faces are indicated by arrows in Fig. 1(c). Fig. 6 shows the transient response for the $z-$ and $\phi-$directed currents. The time step in MOT computation is chosen as $c\Delta t = 4R_{\min}$ in order to generate an implicit solution. Here the agreement between the results from the IDFT and two presented methods is very good, while MOT solution shows instability. Fig. 7 compares the transient response of two presented methods and the IDFT of the frequency-domain EFIE solution for the far scattered field from the cylinder along the backward direction. All the three solutions agree well without late-time oscillation.

V. CONCLUSION

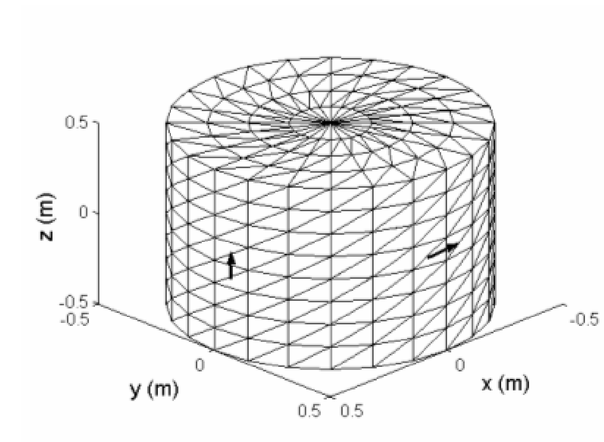
We presented two methods to solve the time-domain electric field integral equation for three-dimensional arbitrarily shaped conducting structures. To apply MoM procedure, we used triangular patch functions as spatial basis and testing functions. We introduced temporal basis function set derived from Laguerre polynomials. The advantages of proposed method is to guarantee the late time stability. The temporal derivative can be treated analytically. Transient electric current and far field obtained by the two presented methods are accurate and stable. The agreement between the solutions obtained using the two proposed methods and the IDFT of the frequency domain is excellent.



(a)

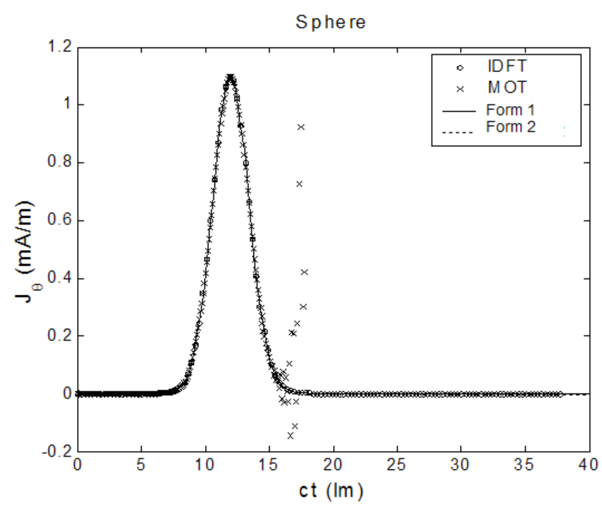


(b)

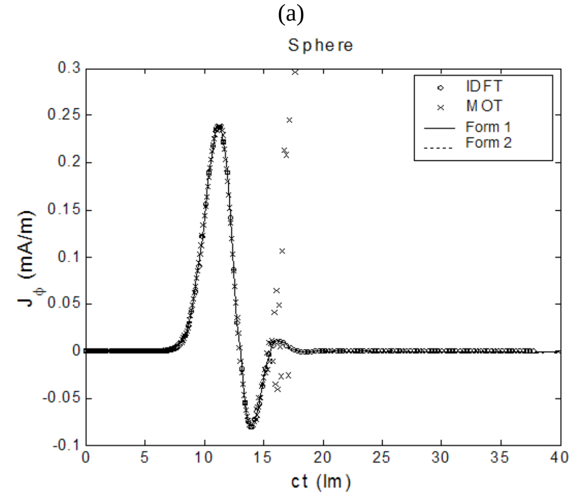


(c)

Fig. 1. Triangle patching of a conducting objects. (a) sphere. (b) cube. (c) cylinder.



(a)



(b)

Fig. 2. Transient current on the sphere. (a) θ – directed current. (b) ϕ – directed current.

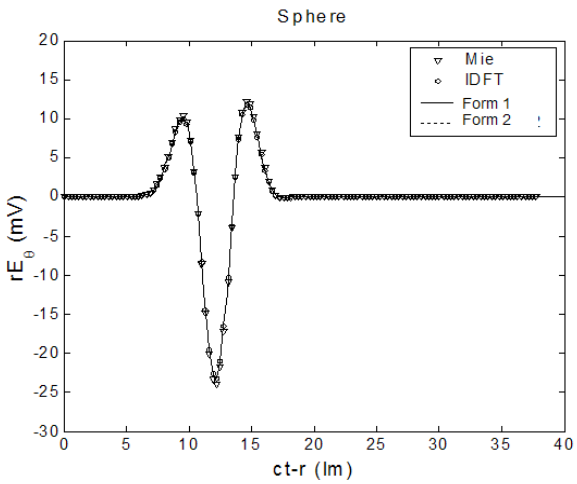


Fig. 3. Scattered far field from the sphere along backward direction.

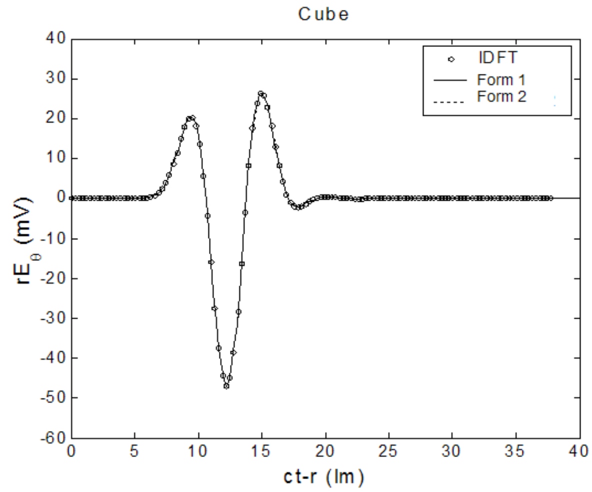
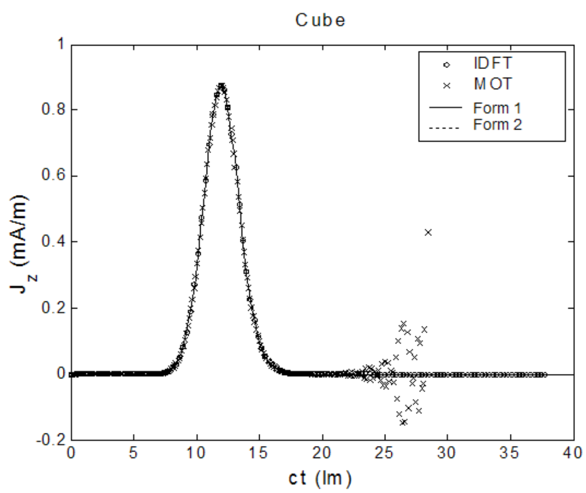
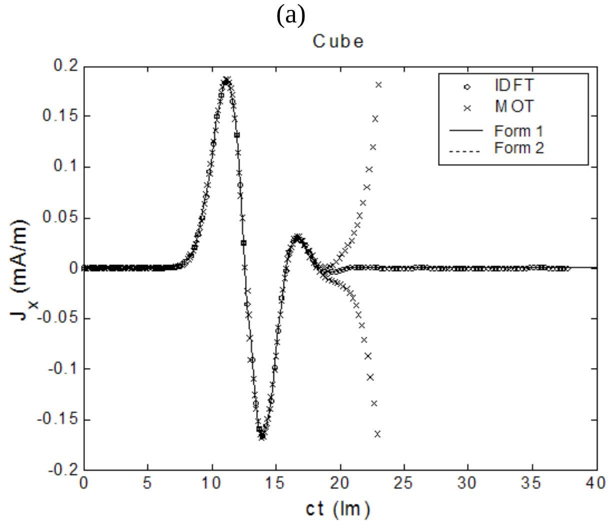


Fig. 5. Scattered far field from the cube along backward direction.

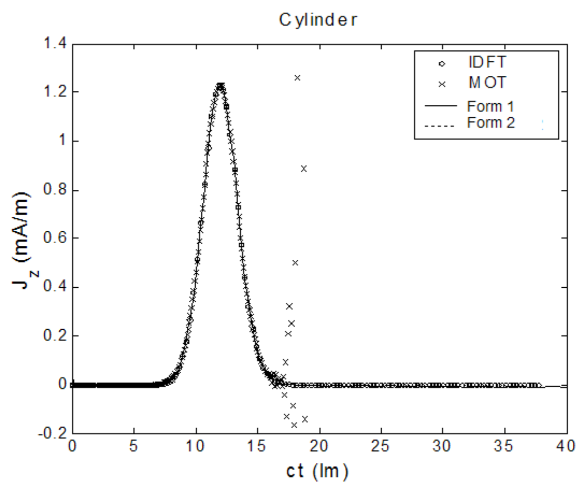


(a)

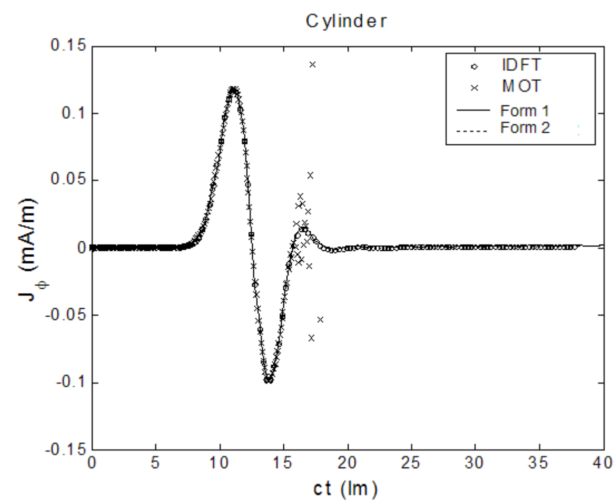


(b)

Fig. 4. Transient current on the cube. (a) $z-$directed current. (b) $x-$directed current.



(a)



(b)

Fig. 6. Transient current on the cylinder. (a) $z-$directed current. (b) $\phi-$directed current.

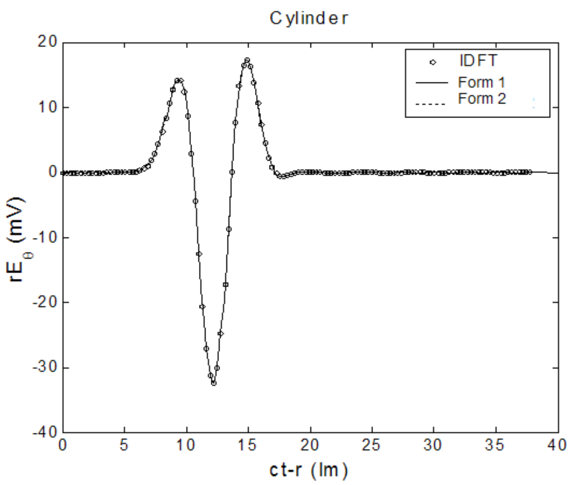


Fig. 7. Scattered far field from the cylinder along backward direction.

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