A Hybrid Finite Element-Boundary Integral-Characteristic Basis Function Method for Scattering by Multiple 3-D Cavities

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Abstract **─** An efficient hybrid finite element-boundary integral-characteristic basis function method (FE-BI-CBFM) is proposed to solve the problem of electromagnetic scattering by multiple three-dimensional (3-D) cavities embedded in a conducting plane. Specifically, the finite element method is used to obtain the solution of the vector wave equation inside each cavity and the boundary integral equation is applied on the apertures of all the cavities as a global boundary condition. The resultant coupling system of equations is solved by using an excitation independent characteristic basis function method. Some numerical results are included to illustrate the validity and capability of the proposed method.

Index Terms **─** Boundary integral equation, characteristic basis function method, finite element method, multiple cavities.

I. INTRODUCTION

Electromagnetic scattering from various cavity structures has been intensively investigated by many researchers during the past few decades. Among the many methods applied to this class of problems, the hybrid finite element-boundary integral (FE-BI) method [1–7] has been widely proved to be a general, robust, and accurate numerical method to analyze the EM scattering from open cavities. It employs the finite element method (FEM) to handle the fields in the cavity volume, while the boundary integral equation (BIE) to handle the fields on the aperture of the cavity. This method has been first applied to 2-D scattering problems [1, 2] and later extended to more challenging 3-D scattering problems [3–5]. Recently, Alavikia and Ramahi further extended it to the solution of EM scattering problems involving multiple 2-D cavities [6].

More recently, we presented a domain decomposition of the FE-BI method for solving the problem of EM scattering by multiple 3-D cavities [7]. In the implementation of the method, the vector FEM was applied inside each cavity to derive a linear system of equations associated with unknown fields. The BIE was then applied on the apertures of all the cavities to truncate the computational domain and to connect the matrix subsystem generated from each cavity. By virtue of an iterative domain decomposition method, the coupling system of equations was reduced to a small one which only includes the unknowns on the apertures. The solution to the reduced system was obtained by an iterative solver, where the multilevel fast multipole algorithm (MLFMA) was employed to speed up the matrix-vector multiplication. However, the iterative solver is inefficient when one is interested in solving the reduced system for multiple excitation vectors, as the iterations need to be started anew for each right hand side. In practice, one is often interested in analyzing the monostatic scattering characteristics of the cavities and, in such a case, the resultant FE-BI matrix equation involves a number of excitation vectors. For the purpose of efficient analysis of monostatic scattering characteristics of multiple cavities in a conducting plane, we utilize the characteristic basis function method (CBFM) [8–12] to solve the resultant FE-BI matrix equation. The use of CBFM has the advantages that it only utilizes direct solvers rather than iterative methods; hence it does not suffer from convergence problems and can solve multiple excitation problems efficiently.

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In the following, Section 2 presents the formulation of the hybrid FE-BI-CBFM. Section 3 illustrates some numerical examples and Section 4 is the conclusion.

II. FORMULATION

As illustrated in Fig. 1, let us consider the problem of EM scattering by multiple 3-D cavities embedded in a perfectly conducting plane. For the sake of convenient description, the free space region above the cavities and conducting plane is denoted as Ω_0, the region occupied by the ith cavity is denoted as $\Omega_i\left(i=1,2,\cdots,m\right)$, with m being the total number of the cavities, and the corresponding volume and area of the aperture are denoted as V_i and S_i, respectively.

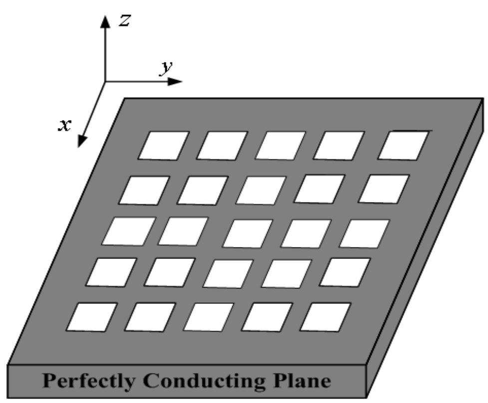


Fig. 1. Geometry of multiple 3-D cavities embedded in a perfectly conducting plane.

The field in region $\Omega_i\left(i=1,2,\cdots,m\right)$ can be formulated into an equivalent variational problem with the functional given by [13]

$$F\left(\mathbf{E}_i\right)=\frac{1}{2}\iiint_{V_i}\left[\frac{1}{\mu_{ri}}\left(\nabla\times\mathbf{E}_i\right)\cdot\left(\nabla\times\mathbf{E}_i\right)-k_0^2\varepsilon_{ri}\mathbf{E}_i\cdot\mathbf{E}_i\right]dV + jk_0Z_0\iint_{S_i}\left(\mathbf{E}_i^S\times\mathbf{H}_i^S\right)\cdot\hat{n}_i dS \tag{1}$$

where $\mathbf{E}_i^s$ and $\mathbf{H}_i^s$ denote the electric and magnetic fields on S_i, respectively, and $\hat{n}_i$ denotes the outward unit vector normal to S_i. Using FEM with edge elements, the functional can be converted into a sparse matrix equation

$$\begin{bmatrix} K_i^{II} & K_i^{IS} & 0 \\ K_i^{SI} & K_i^{SS} & B_i \end{bmatrix}\begin{Bmatrix} E_i^I \\ E_i^S \\ H_i^S \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2}$$

where $\left\{E_i^I\right\}$ is a vector containing the discrete electric fields inside V_i, $\left\{E_i^S\right\}$ and $\left\{H_i^S\right\}$ are the vectors containing the discrete electric and magnetic fields on S_i, respectively. Also, $\left[K_i^{II}\right]$, $\left[K_i^{IS}\right]$, $\left[K_i^{SI}\right]$ and $\left[K_i^{SS}\right]$ are contributed by the volume integral in (1), whereas $\left[B_i\right]$ is contributed by the surface integral.

Since Eq. (2) is independent of the excitation, we can eliminate the interior unknowns to derive a matrix equation that only includes the unknowns on S_i, as follows

$$\left[S_i\right]\left\{E_i^S\right\}+\left[B_i\right]\left\{H_i^S\right\}=\left\{0\right\} \tag{3}$$

where

$$\left[S_i\right]=\left[K_i^{SS}\right]-\left[K_i^{SI}\right]\left[K_i^{II}\right]^{-1}\left[K_i^{IS}\right] \tag{4}$$

Before further proceeding, it should be noted that the above computation in each cavity is independent. Moreover, when the cavities are uniform, this computation can be significantly reduced. Since for this case, the coefficient matrices are the same for each cavity, only one cavity needs to be dealt with.

For all the cavities, we can write the global linear system as follows

$$\begin{bmatrix} S_1 & & & \\ & S_2 & & \\ & & \ddots & \\ & & & S_m \end{bmatrix}\begin{Bmatrix} E_1^S \\ E_2^S \\ \vdots \\ E_m^S \end{Bmatrix} + \begin{bmatrix} B_1 & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_m \end{bmatrix}\begin{Bmatrix} H_1^S \\ H_2^S \\ \vdots \\ H_m^S \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{5}$$

By invoking Huygens's principle and image theory, the magnetic field in region Ω_0 can be represented as

$$\mathbf{H}_0=\mathbf{H}^{inc}+\mathbf{H}^{ref}+\sum_{i=1}^{m}\mathbf{H}_i^{sca} \tag{6}$$

where $\mathbf{H}^{inc}$ is the incident field, $\mathbf{H}^{ref}$ is the reflected field from the conducting plane and $\mathbf{H}_i^{sca}$ is the scattered field caused by the equivalent magnetic current $\mathbf{M}_i = \mathbf{E}_i^S \times \hat{n}_i$ on S_i . Also, $\mathbf{H}_i^{sca}$ is defined by [3]

$$\mathbf{H}_i^{sca}\left(\mathbf{r}\right) = -2jk_0 \frac{1}{Z_0} \iint_{S_a} \mathbf{M}_i\left(\mathbf{r}'\right) \cdot \overline{\overline{G}}_0\left(\mathbf{r},\mathbf{r}'\right) dS' \qquad (7)$$

Since the tangential magnetic field must be continuous across the apertures of all the cavities, we may enforce the boundary condition on S_i to obtain the following BIE

$$\hat{n}_i \times \mathbf{H}_i^S = 2\hat{n}_i \times \mathbf{H}^{inc} + \sum_{i=1}^{m}\left(\hat{n}_i \times \mathbf{H}_i^{sca}\right) \qquad (8)$$

Multiplying Eq. (8) by jk_0Z_0 and discretizing the resulting BIE via Galerkin's method yields

$$\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{bmatrix} \begin{Bmatrix} E_1^S \\ E_2^S \\ \vdots \\ E_m^S \end{Bmatrix} - \begin{bmatrix} B_1 & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_m \end{bmatrix} \begin{Bmatrix} H_1^S \\ H_2^S \\ \vdots \\ H_m^S \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{Bmatrix} \qquad (9)$$

where $\left[P_{ij}\right], \left[B_i\right]$ and $\left\{b_i\right\}\left(i, j = 1, 2, \cdots, m\right)$ are the resultant impedance matrices and excitation vectors due to the discretization of BIE. Combining (5) and (9), we obtain

$$\begin{bmatrix} P_{11}+S_1 & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22}+S_2 & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm}+S_m \end{bmatrix} \begin{Bmatrix} E_1^S \\ E_2^S \\ \vdots \\ E_m^S \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{Bmatrix} \qquad (10)$$

For the sake of convenient description, (10) is written in a more compact form as

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1m} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{m1} & \mathbf{Z}_{m2} & \cdots & \mathbf{Z}_{mm} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_m \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_m \end{bmatrix} \qquad (11)$$

The resultant FE-BI matrix equation can be efficiently solved by using an excitation independent CBFM presented in [10]. In accordance with the CBFM, we first characterize each aperture by using the so called primary characteristic basis functions (CBFs) that constructed by illuminating the aperture with plane waves incident from N_{PWS} angles. To be more specific, we construct the CBFs by solving the following matrix equation

$$\mathbf{Z}_{ii} \cdot \mathbf{J}_i^{\mathrm{CBFs}} = \mathbf{V}_i^{\mathrm{PWS}} \qquad (12)$$

where $\mathbf{V}_i^{\mathrm{PWS}}$ denotes the N_{PWS} plane wave excitation. Since the dimension of each block-diagonal matrix $\mathbf{Z}_{ii}$ is relatively small, the above equation can be solved by using LU decomposition. This type of factorization is highly desirable because we have to solve Eq. (12) N_{PWS} times, one for each incident plane wave, to compute the complete set of primary basis functions.

Next, we use the singular value decomposition (SVD) to express the set of solutions $\mathbf{J}_i^{\mathrm{CBFs}}$ as

$$\mathbf{J}_i^{\mathrm{CBFs}} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathrm{T}} \qquad (13)$$

and we retain the columns from the left singular value matrix $\mathbf{U}$ whose singular values are above a threshold. For simplicity, we assume that all of the apertures contain the same number K of CBFs after SVD, where K is always smaller than N_{PWS}. For the ith aperture the solution can be written as

$$\mathbf{J}_i = \boldsymbol{\alpha}_i \tilde{\mathbf{J}}_i \qquad (14)$$

where $\boldsymbol{\alpha}_i = \left(\alpha_i^1, \alpha_i^2, \cdots, \alpha_i^K\right)^{\mathrm{T}}$ are the unknown expansion coefficients to be determined by solving the reduced matrix equation, and $\tilde{\mathbf{J}}_i$ are the new CBFs after SVD. The elements of the reduced matrix take the form

$$\mathbf{A}_{ij} = \tilde{\mathbf{J}}_i^{\mathrm{T}} \mathbf{Z}_{ij} \tilde{\mathbf{J}}_j \qquad (15)$$

Thus, the reduced matrix equation can be represented as

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1m} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{m1} & \mathbf{A}_{m2} & \cdots & \mathbf{A}_{mm} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ \boldsymbol{\alpha}_m \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{J}}_1^{\mathrm{T}}\mathbf{V}_1 \\ \tilde{\mathbf{J}}_2^{\mathrm{T}}\mathbf{V}_2 \\ \vdots \\ \tilde{\mathbf{J}}_m^{\mathrm{T}}\mathbf{V}_m \end{bmatrix} \qquad (16)$$

Obviously, the dimension of the reduced matrix is much smaller than that of the original impedance matrix and hence equation (16) can be solved directly.

III. NUMERICAL RESULTS

Based on the formulation described above, we have written a computer program to demonstrate the validity and capability of the proposed method. In this program, tetrahedral elements are used to discretize the solution domain and the density of meshes is 12 parts per wavelength. In what follows, all the computations are performed on a personal computer with 3.0 GHz CPU and 2 GB memory.

To illustrate the validity of the proposed method, we first consider a 2×2 array of cavities embedded in a conducting plane. Each cavity of the array has a square $1.0\lambda\times1.0\lambda$ aperture and is 0.6λ deep, with λ being the operating wavelength. The periodicity of the array is 2.0λ in the x- and y-dimensions. For numerical solution, each cavity is subdivided into 4182 tetrahedral elements. As a result, a total of 17808 FEM unknowns and 1252 BIE unknowns are generated. Figure 2 shows the computed radar cross sections (RCS) as a function of the angle of incidence. For comparison, the result obtained using the method of moments (MOM) is given in the same figure. Good agreements are observed between them.

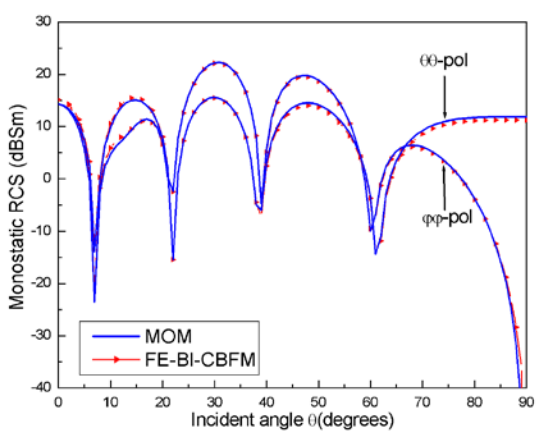
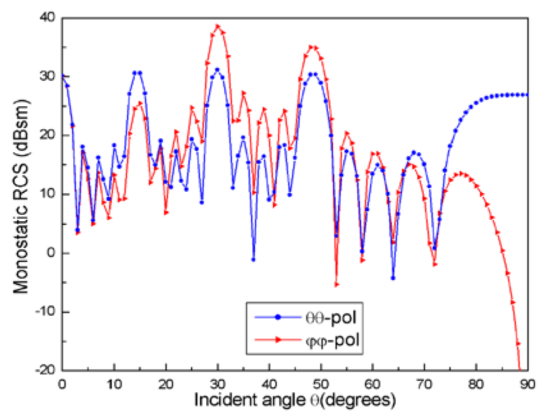
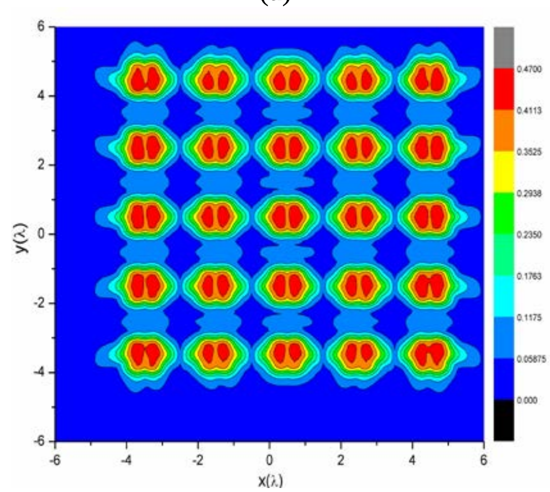


Fig. 2. Comparison of the monostatic RCS for a 2×2 array of cavities from the FE-BI-CBFM and the MOM.

Next, we examine the efficiency of the proposed method. For the problem described above, although the number of FEM unknowns is very large, only one fourth of those need to be dealt with since the cavities are uniform. Utilizing the frontal method to solve the FEM matrix equation, the memory requirement and the computational time are 5 Mb and 15 s, respectively. Furthermore, we only need to analyze a single cavity to construct the CBFs for the entire array. Using the CBFM to solve the resultant FE-BI matrix equation, the memory required is about 6 Mb and the computational time is 25 s. But when we use the iterative method in combination with MLFMA to solve the FE-BI equation, the memory requirement and the computational time are 8 Mb and 310 s, respectively. Thus the proposed FE-BI-DDM is well suitable for the analysis of monostatic scattering characteristics of multiple cavities embedded in a conducting plane.



(a)



(b)

Fig. 3. RCS and magnitude of electric field calculated for 5×5 array of cavities: (a) monostatic RCS; (b) magnitude of electric field.

Now, we consider the scattering of a plane wave from a 5×5 array of cavities embedded in a conducting plane, as depicted in Fig. 1. The size of each cavity and the periodicity are the same as those of the first example. Figure 3(a) shows the monostatic RCS of the array, while Figure 3(b) shows the magnitude of electric field calculated at a plane located at a distance of $z = 0.2\lambda$ above the apertures for the case of normal incidence.

Finally, to demonstrate the capability of the FE-BI-CBFM to handle large scale problems, we consider a 10×10 array of cavities depicted in Fig. 4 (a). The size of each cavity and the periodicity are also taken as $1.0\lambda\times1.0\lambda\times0.6\lambda$ and 2.0λ, respectively. For this example, although the number of the cavities is far more than that of the first example, the memory requirement basically remains the same. The computational time is 2200 s, which can be significantly reduced by adopting the parallel computation because the CBFM is highly parallelizable. The computed RCS is given in Fig. 4 (b) as a function of the angle of incidence.

IV. CONCLUSION

In this paper, the hybrid FE-BI method in combination with the CBFM is proposed to analyze the EM scattering from multiple 3-D cavities in a conducting plane. In the proposed method, each cavity is efficiently modeled by the edge-based FEM. The holes are coupled to each other through the BIE based on the Green's function. To reduce the computational burden, an excitation independent CBFM is used to solve the resultant FE-BI matrix equation. Since the CBFM only utilizes direct solvers rather than iterative methods, it does not suffer from convergence problems and can solve multiple excitation problems efficiently. Numerical results obtained show that the proposed method is suitable for this class of problems.

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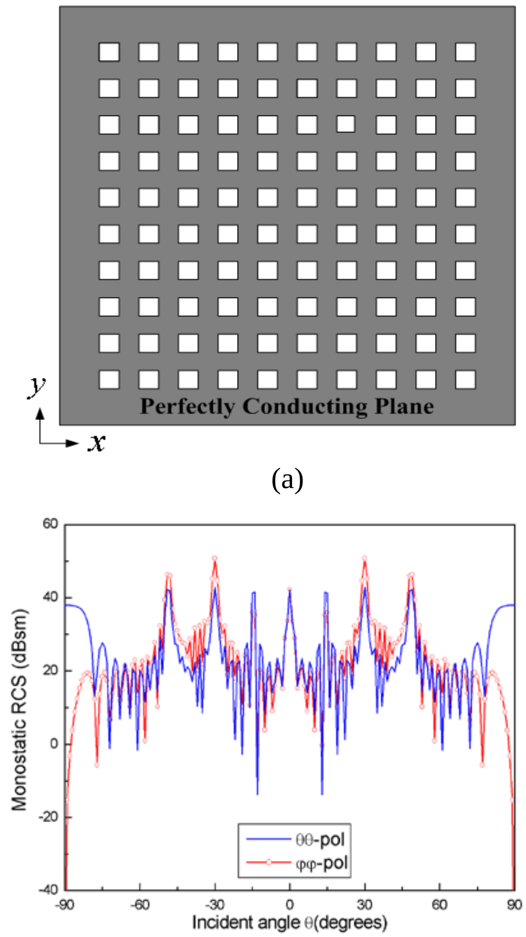

Fig. 4. A 10×10 array of rectangular cavities: (a) geometrical configuration; (b) monostatic scattering cross section.

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