DISPERSIVE ANALYSIS OF CIRCULAR CYLINDRICAL MICROSTRIPS AND BACKED SLOTLINES

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Abstract - In this work, the full-wave analysis of circular cylindrical microstrips and backed slotlines is performed, by using a combination of Hertz vector potentials and Galerkin method. The analysis is developed in the spectral domain.

I. INTRODUCTION

The dispersive analysis using Hertz vector potentials in the Fourier domain was first used to analyze planar structures, such as microstrip transmission lines [1] and patch antennas and resonators [1]-[4]. This work describes an extension of this technique to study non-planar structures, such as those considered in [5]-[11], in order to determine accurately their characteristics and to investigate its application in (monolithic) microwave integrated circuits (M)MIC. The analysis of circular cylindrical microwave integrated structures is usually quite complex, requiring a large amount of computer time. To overcome this problem, accurate and efficient algorithms were developed [8]. A very good agreement was observed between the results of this work and those available in the literature, in particular [2]-[4], [12].

II. THEORY

A. Circular Cylindrical Microstrip Lines (CCML)

The microstrip structure considered in this work is shown in Fig. 1, where $w = 2\alpha\, r_2$ and α is half of the strip angle. In this analysis, the following approximations are assumed: a) the dielectric substrate is isotropic, linear and homogeneous, b) the ground and conducting strip losses are neglected, c) harmonic dependence for the electric and magnetic fields is assumed, and d) the conducting strip thickness is neglected.

In this analysis, the electric and magnetic field components are expressed in terms of the electric and magnetic Hertz vector potentials, $\bar{\pi}_{ei}$ and $\bar{\pi}_{hi}$, respectively, which are defined for each dielectric region i (i=1,2 in Fig. 1) as [8],[13]

$$\bar{\pi}_{ei} = \pi_{ei}\, \hat{a}_r \quad (1)$$

$$\bar{\pi}_{hi} = \pi_{hi}\, \hat{a}_r \quad (2)$$

where $\hat{a}_r$ is the radial unit vector.

In the analytical procedure of the Hertz vector potentials technique, Maxwell's equations are used, giving

$$\bar{B}_i = j\omega\mu_0\varepsilon_i\, \nabla \times \bar{\pi}_{ei} \quad (3)$$

$$\bar{E}_i = -j\omega\mu_0 \nabla \times \bar{\pi}_{hi} \quad (4)$$

where μ_0 is the free space permeability, ε_i is the electric permittivity for dielectric region i (i = 1,2 in Fig. 1) and ω is the angular operating frequency. After some algebraic manipulation, the electric and magnetic field components are obtained. They are refered to the propagation TE and TM waves (with respect to r-direction, in Fig. 1). Then, the total electric and magnetic field expressions are obtained by superposition and given by

$$\bar{E} = -j\omega\mu_0 \nabla\times\bar{\pi}_{hi} + \omega^2\mu_0\varepsilon_i\bar{\pi}_{ei} + \nabla\nabla.\bar{\pi}_{ei} \quad (5)$$

$$\bar{H} = j\omega\varepsilon_i \nabla\times\bar{\pi}_{ei} + \omega^2\mu_0\varepsilon_i\bar{\pi}_{hi} + \nabla\nabla.\bar{\pi}_{hi} \quad (6)$$

respectively.

Furthermore, the electric and magnetic Hertz potentials should satisfy the wave equations

$$\nabla^2 \bar{\pi}_{ei} + \omega^2 \mu_0 \varepsilon_i \bar{\pi}_{ei} = 0 \quad (7)$$

$$\nabla^2 \bar{\pi}_{hi} + \omega^2 \mu_0 \varepsilon_i \bar{\pi}_{hi} = 0 \quad (8)$$

respectively.

The transformation to the spectral domain is obtained using the following definition [8]

$$\tilde{\Omega}(r, m) = \int_{-\infty}^{\infty} \Omega(r,\phi)\, \exp(-j\omega\phi)\, d\phi \quad (9)$$

$$\Omega(r,\phi) = \sum_{m=-\infty}^{m=\infty} \tilde{\Omega}(r,m)\exp(jm\phi) \quad (10)$$

where "~" means the transformed function and m is the spectral variable.

The wave equations for $\tilde{\pi}_{ei}$ and $\tilde{\pi}_{hi}$ are determined from, and are given by (7) to (10)

$$\frac{d^2}{dr^2}\tilde{\pi}_{ei,hi}(r,m) + \frac{1}{r}\frac{d}{dr}\tilde{\pi}_{ei,hi}(r,m) - \zeta_i^2\tilde{\pi}_{ei,hi}(r,m) = 0 \quad (11)$$

with

$$\zeta_i^2 = \gamma_i^2 - (m/r)^2 \quad (12)$$

$$\gamma_i^2 = k_i^2 - \beta^2 \quad (13)$$

where γ_i is the propagation constant and k_i is the wave number.

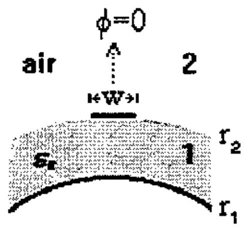

Figure 1: Cross sectional view of a circular cylindrical microstrip line.

The solutions of (11) have the general form shown below [8]

$$\tilde{\pi}_{e1}(r,m) = A(m)\,J_m(\gamma_1 r) + B(m)\,N_m(\gamma_1 r) \quad (14)$$

$$\tilde{\pi}_{h1}(r,m) = C(m)\,J_m(\gamma_1 r) + D(m)\,N_m(\gamma_1 r) \quad (15)$$

for dielectric region 1 ($r_1 < r < r_2$, in Fig. 1), and

$$\tilde{\pi}_{e2}(r,m) = E(m)\,H_m^{(2)}(r,m) \quad (16)$$

$$\tilde{\pi}_{h2}(r,m) = F(m)\,H_m^{(2)}(r,m) \quad (17)$$

for dielectric region 2 ($r > r_2$, in Fig. 1), which is air-filled.

In (14) to (17), the expressions for the unknown coefficients A(m), B(m),..., F(m) are determined from the boundary conditions; $J_m(.)$ and $N_m(.)$ are Bessel's functions of 1st and 2nd kind, respectively; and $H_m^{(2)}(.)$ is the Hankel function of 2nd kind.

In this work, the transformed field components are expressed as functions of $\tilde{\pi}_{hi}(r,m)$, for the TE modes, and $\tilde{\pi}_{ei}(r,m)$, for the TM modes. By using superposition and imposing the boundary conditions, the expressions for the total electric and magnetic field components are obtained.

At the interface dielectric-air ($r = r_2$, in Fig. 1), the transformed tangential electric field components, $\tilde{E}_\phi$ and $\tilde{E}_z$, are expressed in terms of the transformed current density components, $\tilde{J}_\phi$ and $\tilde{J}_z$, as

$$\tilde{E}_\phi(m,\beta) = \tilde{Z}_{\phi\phi}(m,\beta)\tilde{J}_\phi(m) + \tilde{Z}_{\phi z}(m,\beta)\tilde{J}_z(m) \quad (18)$$

$$\tilde{E}_z(m,\beta) = \tilde{Z}_{z\phi}(m,\beta)\tilde{J}_\phi(m) + \tilde{Z}_{zz}(m,\beta)\tilde{J}_z(m) \quad (19)$$

where, $\tilde{Z}_{\phi\phi}, \tilde{Z}_{\phi z}, \tilde{Z}_{z\phi}$ and $\tilde{Z}_{zz}$ are the transformed impedance matrix components in the spectral domain.

Once the impedance matrix $[\tilde{Z}]$ was determined, the Galerkin method [14] is used and a linear system of equations is obtained, according to

$$[K]\,[c] = 0 \quad (20)$$

where the matrix $[K]$ components are given by

$$K_{ip}^{\phi\phi} = \sum_{m=-\infty}^{m=\infty} \tilde{f}_{\phi i}(m)\tilde{Z}_{\phi\phi}(m,\beta)\tilde{f}_{\phi p}(m) \quad (21)$$

$$K_{ip}^{\phi z} = \sum_{m=-\infty}^{m=\infty} \tilde{f}_{\phi i}(m)\tilde{Z}_{\phi z}(m,\beta)\tilde{f}_{\phi p}(m) \quad (22)$$

$$K_{jp}^{z\phi} = \sum_{m=-\infty}^{m=\infty} \tilde{f}_{zj}(m)\tilde{Z}_{z\phi}(m,\beta)\tilde{f}_{\phi p}(m) \quad (23)$$

$$K_{iq}^{zz} = \sum_{m=-\infty}^{m=\infty} \tilde{f}_{zj}(m)\tilde{Z}_{zz}(m,\beta)\tilde{f}_{zq}(m) \quad (24)$$

In (21) to (24), $\tilde{f}_{\phi i}, \tilde{f}_{\phi p}, \tilde{f}_{zj}$ and $\tilde{f}_{zj}$ are basis functions, which should be properly chosen in order to reduce the computational effort. The characteristic equation for the propagation constants in the structure considered is obtained by imposing $\det[K] = 0$. Therefore, the effective permittivity is readily determined.

B. Circular Cylindrical Backed Slotline (CCBS).

The geometry of the CCBS is shown in Fig. 2, where $w = 2\alpha r_2$ and α is half of the slot angle. This analysis is performed by taking advantage of that presented for CCML structures. In the case of CCBS structures, an admittance matrix has to be derived. Nevertheless, this algebraic manipulation is avoided by setting [8]

$$[\tilde{Y}] = [\tilde{Z}]^{-1} \qquad (25)$$

where the matrix $[\tilde{Z}]$ components are those shown in (18) and (19).

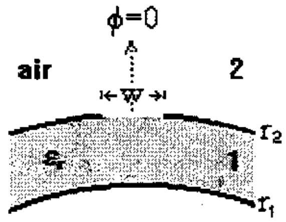

Figure 2: Cross sectional view of a circular cylindrical backed slotline (CCBS).

By using Galerkin method, the characteristic equation for the propagation constants is obtained, as well as the effective permittivity.

III. RESULTS

A new parameter was defined to show the numerical results, which is $R = r_1 / r_2$ (see Figs. 1 and 2). For small values of r_1 and r_2, R is always lower then 1, while for large values of them, R is close to 1. The dielectric thickness of region 1, H (=$r_2 - r_1$), is kept constant.

Fig. 3 shows the dispersive behavior of the normalized wavelength, λ_s / λ_0, for a circular cylindrical microstrip line (CCML) with W/H=1.0; R=0.98; ε_{r1} = 9.6 and ε_{r2}=1.0.

Results obtained for the normalized wavelength, λ_s / λ_0, and the effective permittivity, ε_{eff} , against frequency for circular cylindrical backed slotlines (CCBS) are shown in Figs. 4 and 5, respectively.

The results shown in Fig. 4, for λ_s / λ_0, were obtained for a quasi-planar CCBS, with $R = r_1 / r_2$= 0.98; ε_{r1} = 20.0 and ε_{r2} = 1.0. Results for a planar (not backed) slotline with same values for W, H, ε_{r1} and ε_{r2} obtained from [12] are presented. As expected the results for these different structures approach each other because large values for W/H (=5.568) and ε_r were considered.

Fig. 5 shows the numerical results for ε_{eff} that were obtained for a quasi-planar backed slotline, or a CCBS with a large value for R (=r_1 / r_2) , where w=40 μm, H =600 μm, ε_{r1} = 12.9, ε_{r2} = 1 and R = 0.98. The numerical

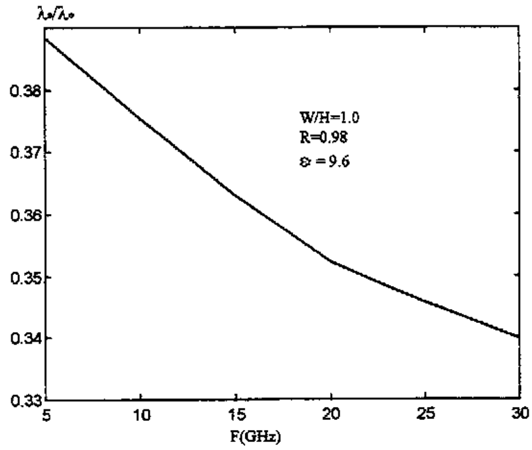

Figure 3: Dispersive behavior of the normalized wavelength, λ_s / λ_0, for a circular cylindrical microstrip line (CCML), with W/H=1.0; R=0.98; ε_{r1}= 9.6 and ε_{r2}=1.0.

results from [6], for a (planar) backed slotline with same values for w, H, ε_{r1} and ε_{r2} are presented. A close agreement is observed, as expected.

IV. CONCLUSION

The analyses of circular cylindrical microstrip lines (CCML) and circular cylindrical backed slotlines (CCBS) were performed by using a combination of Hertz vector potentials and Galerkin method, in the spectral domain. These structures are used in (M)MIC, applications, such as antennas, resonators and phase-shifters.

Numerical results were presented for the normalized wavelength and the effective permittivity versus frequency for different structural parameters.

A comparison between the results of this work and those available in the literature for the CCML showed a very good agreement. For CCBS, the results of this work were plotted with those obtained for similar structures, mainly planar structures, studied by other authors, showing agreement, as expected.

Finally, the technique used in this work is accurate efficient and can be used to analyze other non-planar structures, such as those of single and coupled transmission lines on anisotropic substrates.

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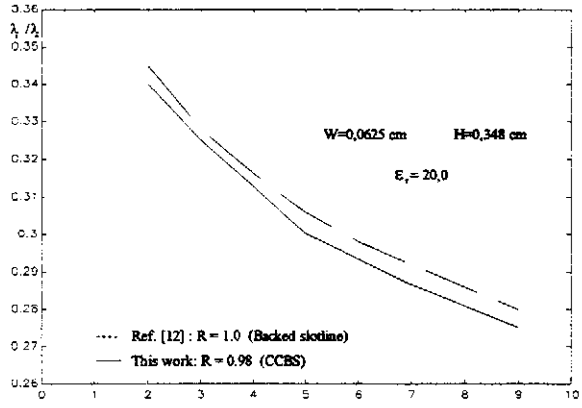

Figure 4: λ_s / λ_0 versus frequency for a CCBS and a slotline (not backed).

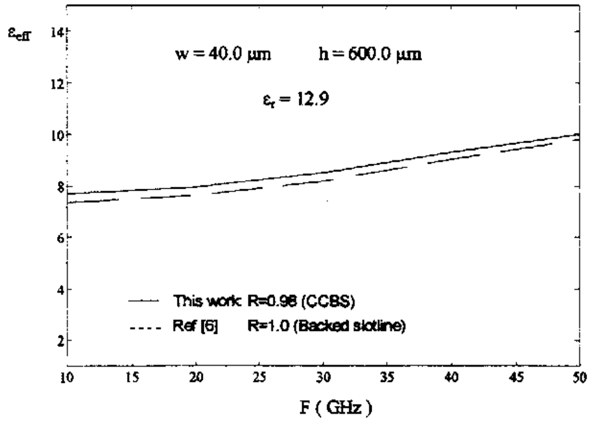

Figure 5: Effective permittivity versus frequency for a circular cylindrical backed slotline (CCBS) and a backed (planar) slotline.

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