A Mode Matching - Finite Element - Spectral Decomposition Approach for the Analysis of Large Finite Arrays of Horn Antennas

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***Abstract*─** In this paper a Mode Matching / Finite Element / Spectral Decomposition (MM/FE/SD) approach is applied to the analysis of finite but large arrays of horn antennas. The proposed methodology retains advantages from the three involved techniques: the SD reduces the finite problem to a superposition of infinite periodic ones, whereas the flexibility of the FE method allows us to model complex irregular structures providing a very high degree of generality. A key step in the analysis consists of resorting to a stepped waveguide model of the longitudinal inner profile of the elementary horn antenna. The numerical efficiency of the MM procedure ensures the continuity of transverse fields over each waveguide discontinuity and over the radiating aperture. The methodology presented here is applied to array with elements arranged in polygonal shape.

***Index Terms*─** Mode Matching (MM), Spectral Decomposition (SD), Horn antenna array.

I. INTRODUCTION

A periodic array of horn antennas consists of a large number of identical radiating elements arranged in a double periodic grid. In this paper, a hybrid numerical technique, based on the Spectral Decomposition (SD) and Mode Matching/Finite Element (MM/FE) [1]-[5] method is presented for the analysis of a periodic array of horn antennas. The SD approach is used to reduce the finite large problem to a summation of infinite periodic problems through a Fast Fourier Transform (FFT) technique or by means of a Fourier closed form solution [6]. The Floquet's expansion of the electromagnetic field enables us to reduce the analysis to a single elementary radiating element and derive the solution for the other elements of the array by a proper phase shift. The longitudinal profile of the horn antenna is discretized into a series of waveguide discontinuities including the free space transition. The MM procedure provides an accurate and effective numerical modal analysis and is employed to impose the continuity of the fields at the discontinuous interfaces. Both Floquet modes and TE/TM waveguide modes have been exploited to evaluate the Generalized Scattering Matrix (GSM) of the radiating element including the radiating aperture. A two-dimensional FE with tangential vector interpolation basis functions is employed to solve the eigenvectors and eigenvalues problem. The FE method allows us to model complex and irregular structures, providing a very high degree of generality. The hybrid MM/FE/SD approach combines the three methods in order to retain the advantages of all. Comparison with available reference and other electromagnetic solver demonstrates the effectiveness of the method. This methodology reveals to be very useful when the number of elements of the array is large and full-wave three-dimensional methods become impractical due to their high numerical effort.

II. SPECTRAL DECOMPOSITION APPROACH

The MM/FE/SD method can be summarized in some fundamental main steps:

- confine the excitation in space where it exists (source windowing);
- decompose the windowed source in spectral samples via a DFT (or FFT) algorithm or by means of a closed form solution for the Fourier transform;
- solve the doubly-periodic (infinite) problem for each spectral sample with the MM/FEM approach (infinite array configuration);

- combine the above results in order to obtain the solution of the finite problem (finite array configuration).

Let us consider an array of horn antennas radiating in a given direction, where N_x and N_y represent the number of horn antennas along the $\hat{i}_x$ and $\hat{i}_y$ directions, respectively (see Fig. 1). The grid spacings are denoted by d_x and d_y, so that the total dimensions of the array are $L_x=N_x\ d_x$ and $L_y=N_y\ d_y$. Unlike the infinite array, the truncated array is no longer periodic, and it is not possible to invoke the Floquet's theorem to analyze a single elementary radiating element. In order to circumvent this difficulty, we employ the SD approach, which enables us to synthesize the solution of a finite array problem by superposing the solutions of several corresponding infinite array configuration problems.

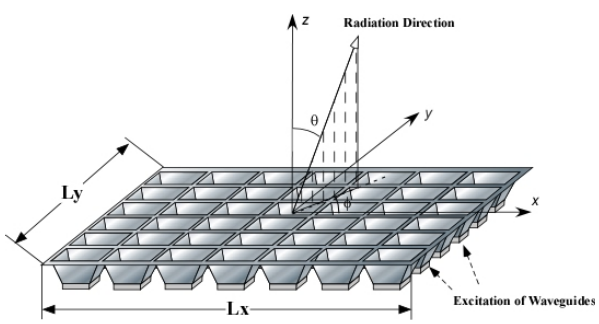


Fig. 1. Geometry and characteristic parameters of the finite array.

To this aim, the problem of a plane wave impinging on the finite array is replaced by the equivalent problem of the infinite array illuminated with an excitation confined in the working space of the array (Fig. 2). The equivalence between the above stated problem and the original one, although not strictly exact from a theoretical point of view, reveals very accurate especially when the array becomes larger and larger.

The excitation is easily represented by a two-dimensional rectangular (or polygonal) gate function $g(x,y)$ in the spatial domain. In order to apply the SD, the spectrum of the excitation $G(\beta_{kx}, \beta_{ky})$ is calculated and its numerical representation is derived via a DFT (or FFT) algorithm:

$$G(\beta_{kx},\beta_{ky})=\frac{1}{N_{Tx}N_{Ty}}\sum_{m=0}^{N_{Tx}-1}\sum_{n=0}^{N_{Ty}-1} g(x_m)g(y_n)\, e^{-j\frac{2\pi k}{N_{Tx}\Delta x}x_m}\, e^{-j\frac{2\pi k}{N_{Ty}\Delta y}y_n}\ , \tag{1}$$

where $\beta_{ku}=k/(N_{Tu}\ \Delta u)$ is the discretized spectral frequency, $\Delta u\ (u=x,\ y)$ denotes the sampling interval into the spatial domain, N_{Tu} the number of samples and $u_n=n\Delta u$ represents the discretized spatial coordinate. We also note that the methodology can be extended to finite arrays with polygonal shape. In fact, when the antennas are arranged within a planar N-sided polygonal shape, the spectrum of the excitation is available in closed-form:

$$G(\beta_{ku},\beta_{kv})=\sum_{n=1}^{N} e^{j\vec{w}\cdot\vec{\gamma}_n}\left[\frac{\hat{n}\times\hat{\alpha}_n\cdot\hat{\alpha}_{n-1}}{(\vec{w}\cdot\hat{\alpha}_n)(\vec{w}\cdot\hat{\alpha}_{n-1})}\right], \tag{2}$$

where $\vec{w}=\hat{i}_x\beta_{ku}+\hat{i}_y\beta_{kv}$, $\vec{\gamma}_n=x_n\hat{i}_x+y_n\hat{i}_y$ is the vector from origin O to corner n^{th}, $\hat{n}=\hat{i}_z$,and $\hat{\alpha}_n=\vec{\gamma}_{n+1}-\vec{\gamma}_n/|\vec{\gamma}_{n+1}-\vec{\gamma}_n|$ is the tangential vector. For the sake of clarity, in Fig. 3 we represent the N-sided planar polygon.

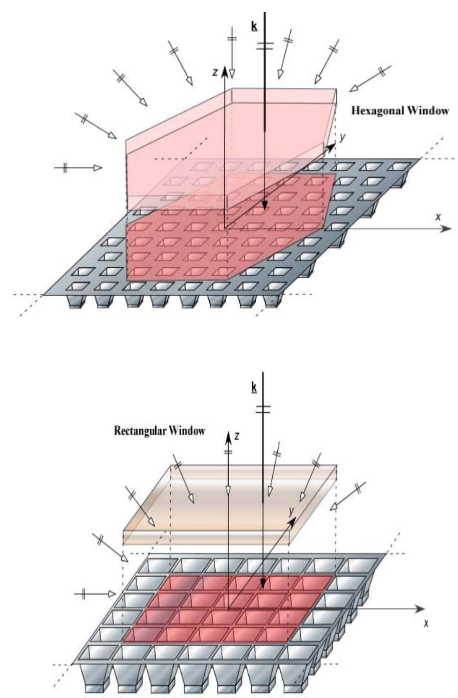


Fig. 2. Some examples of windowing functions applied to the excitation.

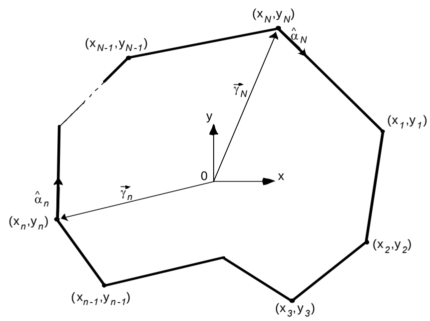


Fig. 3. *N*-sided planar polygon; $\hat{\alpha}_n$ is the tangential vector and $\hat{\gamma}_n$ describes the position of the n^{th} corner.

Each sample of the spectrum can be interpreted as an excitation of the infinite array with uniform amplitude and phase shift β_{ku}. The active reflection coefficients of the elements of the finite array can be obtained from the analysis of N_{Tu} different problems, each one corresponding to a spectral sample. The number of problems is determined by selecting the minimum number of samples required for the reconstruction of the windowing function, while avoiding the aliasing problem.

The active reflection coefficient, related to each horn antenna of the finite array, can be retrieved by applying the contribution of N_{tu} different samples as follows:

$$R(x,y) = C\sum_{i=0}^{N_{Tx}-1}\sum_{j=0}^{N_{Ty}-1} G(k_{i_x} k_{j_y})\, r(k_{i_x} k_{j_y})\, e^{-jk_{i_x}x} e^{-jk_{j_y}y}, \quad (3)$$

where C denotes a normalization factor inversely proportional to the energy of the spectrum and the exponentials account for the different positions of the single horns within the array. The reflection coefficient $r(k_{ix}, k_{iy})$, relative to the elementary radiating element of the infinite array, is obtained by applying the MM/FE procedure for each incident spectral sample and accounts for all the scattering phenomena that occur over the aperture and within the inner part of the horn.

The far field radiation pattern of the entire array is evaluated as a summation of the fields radiated by each element resorting to radiation equation. To this aim, according to [7], accounting for the single infinite problem, the radiating aperture can be replaced by an equivalent magnetic surface current density radiating in unbounded free space. The magnetic surface current density for the elementary radiating horn excited by the generic sample of the spectrum of the excitation, is obtained as a weighted superimposition of the single magnetic current distributions generated by the selected N_M TE/TM Floquet's modes over the aperture:

$$\vec{M}\left(k_{i_x}, k_{i_y}\right) = \begin{cases} -2\sum_{n=1}^{N_M} t_n\left(k_{i_x}, k_{i_y}\right)\left[\hat{i}_n \times \vec{e}_n\left(\vec{p}\right)\right] & \vec{p} \in S_{Ap} \\ 0 & \vec{p} \notin S_{Ap} \end{cases}, \quad (4)$$

where S_{Ap} is the surface of the aperture, $\vec{e}_n$ is the transverse electric field related to the Floquet's mode. The weighting coefficients t_n are evaluated from the GSM of the entire radiating element, supposing that only fundamental mode is excited in the feeding waveguide section of each array element. Referring to Eq. 3-4, the distribution of the radiating surface magnetic current density for a generic m^{th} element of the array follows:

$$\vec{M}_m\left(\vec{p}\right) = \begin{cases} \sum_{i=0}^{N_{Tx}-1}\sum_{j=0}^{N_{Ty}-1} G\left(k_{i_x}, k_{j_y}\right)\vec{M}\left(k_{i_x}, k_{j_y}\right) e^{-jk_{i_x}x_m} e^{-jk_{j_y}y_m} & \vec{p} \in S_{Ap_m} \\ 0 & \vec{p} \notin S_{Ap_m} \end{cases}, \quad (5)$$

The radiated fields are then computed according to [8].

The analysis of finite arrays by means of the Spectral Decomposition method provides accurate and reliable results even if the solution is solved as a scalar 2-D problems. However, if one is interested to the solution of the radiated fields outside the principal planes as well as to the field polarization, a vector 3D problem must be considered. To this aim, a generalization to the 3D case can be derived by considering the proper polarization of the incident field. In order to ensure the correct reconstruction of the incident electromagnetic field, each spectral sample, which defines a plane wave, can be seen as a superposition of TE and TM plane waves. Then, appropriate depolarization coefficients are introduced [9], which determine a projection of the waves along the *x*, *y* and *z* axes according to the direction of the impinging wave. The active reflection coefficients depend now on both the polarization of the samples and the Floquet's expansion.

III. MODE MATCHING / FINITE ELEMENT APPROACH FOR A SINGLE HORN

Mode Matching technique is employed to efficiently compute the scattering behavior of the elementary horn antennas. A fundamental key step is the capability of MM to reduce the original three-dimensional problem into a cascaded of two-dimensional problems with a great reduction of the numerical effort. Longitudinal continuous profile is discretized into a series of subsequent waveguide discontinuities including the free space transition. As suggested in [10], a length of the waveguide sections equals to λ/32 has demonstrated as a sufficient bound to correct interpolate the continuous inner profile of the horn. At each discontinuity of the stepped waveguide model TE/TM fields are matched to evaluate the related GSM. All the matrices are then cascaded in a conventional manner in order to obtain the GSM of the inner guided part of the antenna. To account for the radiating aperture, for each sample of the spectrum of the excitation waveguide modes are matched with Floquet modes to compute scattering parameters. The coupling of the previous GSMs furnishes the scattering behavior of the elementary antenna horn taking into account the mismatch between waveguide and free space propagation regions.

IV. NUMERICAL RESULTS

As previously mentioned, the proposed approach can efficiently take into account complex configurations and devices with large dimensions considerably reducing the computational time with respect to fully three-dimensional approach. In order to demonstrate the effectiveness of the method, a comparison is made with the results obtained through the commercial software ANSOFT HFSS[v10.1] as well as some results found in open literature. As a first example, we propose a comparison with the results obtained with the Truncated Floquet Wave Full-Wave ($T(FW)^2$) [11]. The data are relevant to a 20×20 array of open-ended rectangular waveguides with periodicities T_x=18.84 *mm* and T_y=8.7 *mm* (TE polarization case). The elements have dimension a=17.142 *mm*, b=7.62 *mm* and the analysis is performed at frequency f=10 *GHz*. In Figs. 4 and 5 the active reflection coefficient and the radiation pattern on E-plane are shown respectively. In the case of array of open-ended truncated waveguides MM procedure is performed only over the radiating aperture.

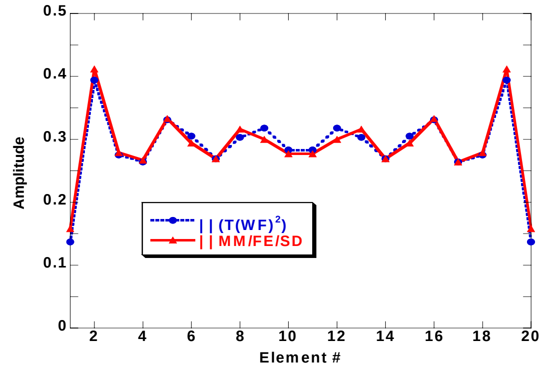


Fig. 4. Array of 20×20 open-ended rectangular waveguides compared with [11]: amplitude of active reflection coefficient relevant to one of the two central columns of the array.

Good accordance is obtained for reflection coefficient, while the far field pattern shows a mismatch especially for wide scan angles. This is probably due to a not correct reconstruction of the fringe currents by the MM/FE/SD approach. Indeed, in the presented procedure there is not any contribution or correction factor taking into account the diffraction effects produced by the edges or corners.

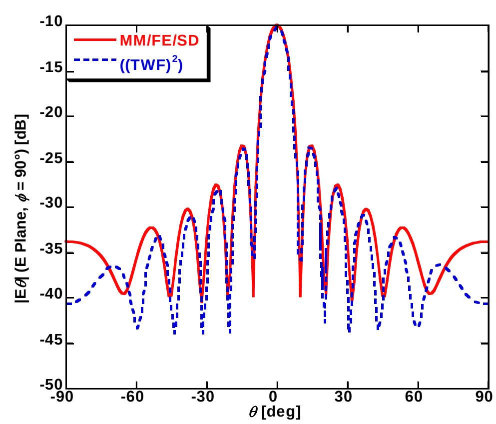
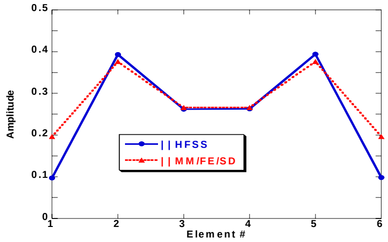
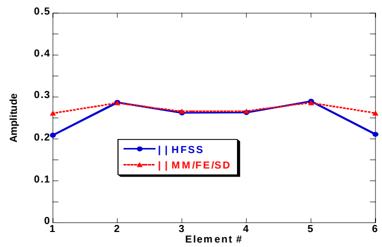


Fig. 5. Array of 20×20 open-ended rectangular waveguides compared with [11]: E-plane radiation pattern.

In Fig. 6 and Fig. 7, results are proposed for an array of previously mentioned truncated rectangular waveguides now arranged in a 6×6 spatial grid. The dimensions of the single unit element are the same as in the previous example.



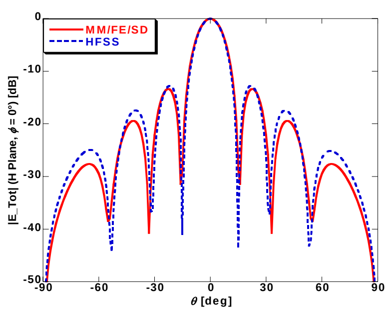
(a)



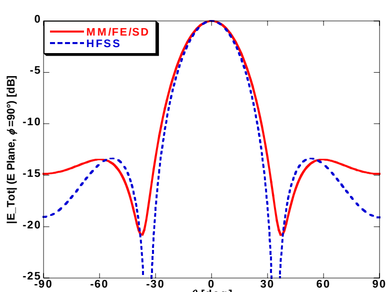
(b)

Fig. 6. Amplitude of active reflection coefficient for array of 6×6 open-ended waveguides compared with Ansoft HFSS: a) one of the two central columns, b) one of the two central rows.

Results show a good match with simulations performed by HFSS, proving the effectiveness of the proposed approach in the analysis of smaller array configurations.



(a)



(b)

Fig. 7. Normalized radiation patterns of the 6×6 open-ended rectangular waveguide array compared with Ansoft HFSS: a) H-plane pattern, b) E-plane pattern.

If horn antennas are now considered as radiating elements the overall procedure is modified only by extending the MM procedure to the scattering analysis of the inner profile of the elementary horn antenna. As a first example the analysis has been performed at a frequency f=15 GHz, and the data are relevant to a 6×6 array of pyramidal horn antennas arranged with periodicities T_x=18.84 mm and T_y=8.7 mm. The feeding waveguides have dimensions a_1=11.428 mm and b_1=5.08 mm and the radiating apertures have dimensions a_2=17.142 mm and b_2=7.62 mm, the longitudinal length l=20 mm (Fig. 8). The continuous internal shape has been arranged in a 32 cascaded waveguide discontinuities and only the fundamental exciting mode TE_{10} has been considered in the feeding waveguide section.

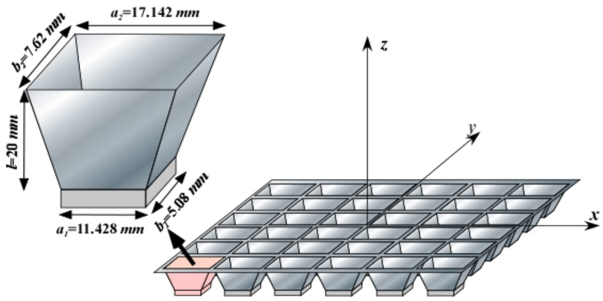


Fig. 8. Geometry of the array and dimensions of the elementary radiating element.

In Fig. 9, the magnitude of the active reflection coefficient evaluated along one of the two central columns of the array is shown. In Fig. 10, the normalized radiation patterns over principal planes are reported, all the results are compared with Ansoft HFSS, showing good agreement in the main beams region and the previous mentioned mismatch for wide scan angles.

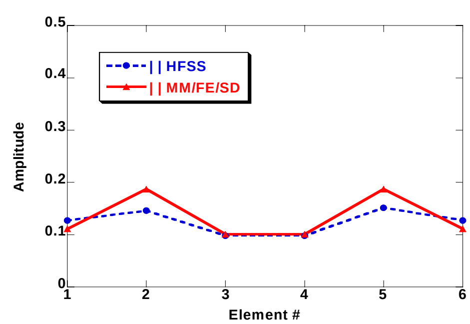
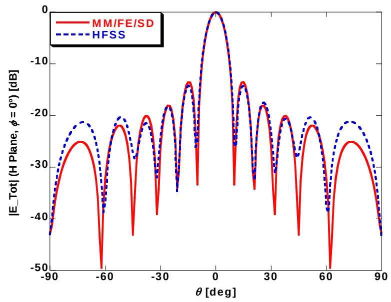
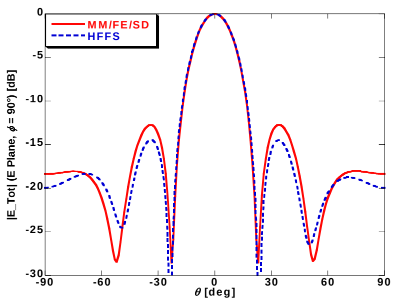


Fig. 9. Comparison with Ansoft HFSS: active reflection coefficient along one of the two central columns for the 6×6 array of pyramidal horns.



(a)



(b)

Fig. 10. Normalized radiation patterns of the 6×6 pyramidal horn array compared with Ansoft HFSS: a) H-plane pattern, b) E-plane pattern.

Another example shows the active reflection coefficient evaluated for an hexagonal array of horn antennas, operating at a frequency of 15 *GHz*. The geometry of the structure is depicted in Fig. 11. The radiating horns have the same dimensions of the previous example. The periodicity is T_x=28.57 *mm* and T_y=17.142 *mm* with a skewness angle α=50.194°. Due to the geometrical symmetry of the structure, the active reflection coefficient is shown in Fig. 11, only for the apertures belonging to the first quadrant.

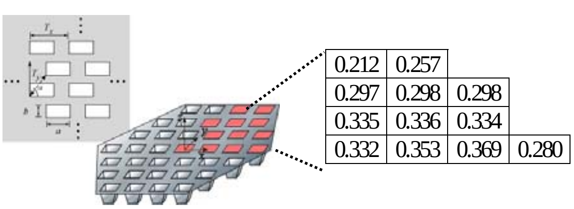


Fig. 11. Geometry of the hexagonal array and magnitude of the active reflection coefficients belonging to the first quadrant.

Finally, in Table I we show the computational time and the memory usage for the proposed MM/FE/SD technique compared against Ansoft HFSS $^{v10.1}$ for the array cases proposed in Fig. 6 and Fig. 8, respectively. Simulations have been performed on a PC with AMD ATHLON XP2400 2.4 *GHz* processor with 2 GB RAM. As apparent, the proposed MM/FE/SD technique permits a time saving of more than 50% in both cases and a considerable memory saving.

Table 1: Comparison between computational time and memory usage for the proposed MM/FE/SD and Ansoft HFSS $^{v10.1}$.

	MM/FE/SD (Time/ Memory)	**HFSS** (Time/ Memory)
Array of 6×6 truncated rectangular waveguides (Fig. 6)	75 min./11.3 MB	164 min./1.29 GB
Array of 6×6 pyramidal horns (Fig. 8)	75 min./11.3 MB	205 min./1.394 GB

V. CONCLUSION

A hybrid Mode Matching / Finite Element / Spectral Decomposition (MM/FE/SD) approach for analyzing finite but large arrays of horn antennas has been presented in this paper. The SD approach enabled us to analyze the finite problem by a superposition of equivalent infinite periodic ones. For each infinite problems Floquet's theorem allowed us to compute the GSM of the elementary radiating element, accounting the radiating aperture, by employing a MM/FE technique. The problem where horn antennas are present, has been addressed by discretizing the longitudinal continuous profile into a series of subsequent waveguide discontinuities including the free space transition. A step of λ/32 has been chosen in order to ensure the correct reconstruction of the inner profile. Several representative numerical examples have been presented to demonstrate the accuracy and efficiency of the method.

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