Determining the Characteristic Impedance of a Transmission Line When Modeling in the Frequency Domain

Matthew D. Rankin
IBM, Austin, TX 78758

Abstract— **When performing simulations in the frequency domain using methods such as the Partial Element Equivalent Circuit (PEEC) or the Method of Moments (MoM), the characteristic impedance of a transmission line is often needed to be known. As an alternative to calculating the characteristic impedance using an equation, the procedure described herein provides a simple and efficient method to determine the characteristic impedance using a signal test simulation.**

Index Terms— **Characteristic impedance, PEEC, MoM, simulation.**

I. Introduction

IN many simulations it is desirable to have the transmission lines terminated with a matched load. In order to determine the proper value of the load resistance to provide a matched load, the characteristic impedance of the transmission line must be known. For the stripline and microstrip cases, the characteristic impedance could be calculated using equations, such as those given in [1], [2], or [3]. However, the simulated configuration may be such that these equations are not applicable. Therefore, instead of calculating the characteristic impedance based on the physical dimensions of the transmission line using an equation, the method described herein requires a test simulation to be performed from which the characteristic impedance is then determined.

II. Procedure

To calculate the characteristic impedance of a transmission line simulated in the frequency domain use the following procedure:

- Create the transmission line model using a source voltage $V_S = 1 \angle 0°$ V with no source resistance $R_S = 0$ Ω and an arbitrary load resistance Z_L. A good choice for the load resistance is $Z_L = 100$ Ω.

- Simulate over a frequency range which encompasses the frequency at which the transmission line appears to be a quarter wavelength long. To calculate the quarter wavelength frequency use

$$f\Big|_{l=\frac{\lambda}{4}} = \frac{v_0}{4l\sqrt{\epsilon_r}} \tag{1}$$

where l is the length of the transmission line. For cases with an inhomogeneous medium, such as microstrip, use the highest relative permittivity in the entire domain in (1). This will result in a frequency which is higher than the actual quarter wavelength frequency, but the desired frequency will still be encompassed in the simulation.

- Measure the absolute value of the imaginary component of the load voltage Z_L at the frequency where the imaginary component is at its minimum. The minimum value of the imaginary component will coincide with the frequency at which the transmission line appears to be a quarter wavelength, and it will coincide with the frequency at which the real component of the load voltage will be zero, $\mathrm{Re}\{V_L\} = 0$.

$$V_x = \left| \mathrm{Im}\left\{ V_L\Big|_{l=\frac{\lambda}{4}} \right\} \right|$$

- To solve for the characteristic impedance, divide the load impedance by the absolute value of the imaginary component of the load voltage at a quarter wavelength.

$$Z_0 = \frac{Z_L}{V_x} \tag{2}$$

III. Verification

The procedure was verified using a PEEC method solver to simulate in the frequency domain. The model used for verification is shown in Figure 1. The trace is 6 inches long, 5 mils wide, and 5 mils above the ground plane with $\rho = 1.74 \times 10^{-6}$ Ω-cm and an effective thickness of 1 mil. The ground plane is 8 inches long, 3 inches wide, and 1 mil thick with $\rho = 1.74 \times 10^{-6}$ Ω-cm.
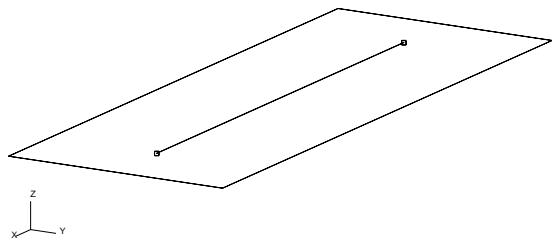

Fig. 1. Example Model Configuration

The simulations were performed from 100 MHz to 2.0 GHz using 50 MHz steps, gridded with a minimum of 20

cells/wavelength. The arbitrary load impedance was chosen as $Z_L = 100$ Ω. The result of the frequency domain simulation is shown in Figure 2.
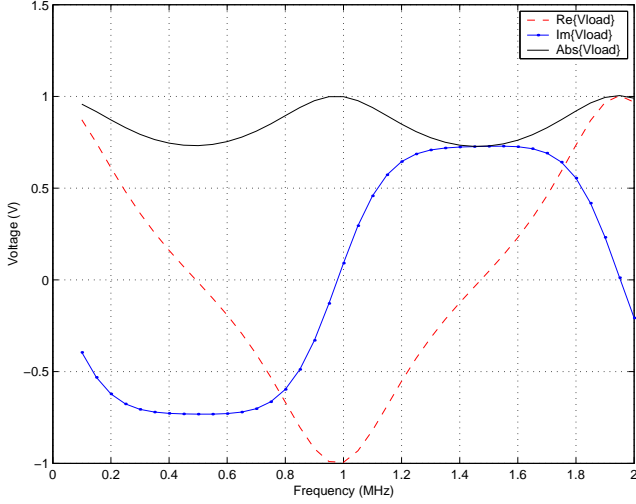

Fig. 2. Load Voltage V_L vs. Frequency with $Z_L = 100$ Ω

The minimum value of the imaginary component of the load voltage $\text{Im}\{V_L\} = -0.7316$ V. Since the absolute value of the load voltage is less than the source voltage $V_S = 1$ V, the load impedance in the first simulation was lower than the characteristic impedance; therefore $Z_0 > 100$ Ω. Using (2), the characteristic impedance was calculated to be $Z_0 = 136.7$ Ω. The results of using this value for the load impedance are shown in Figure 3.
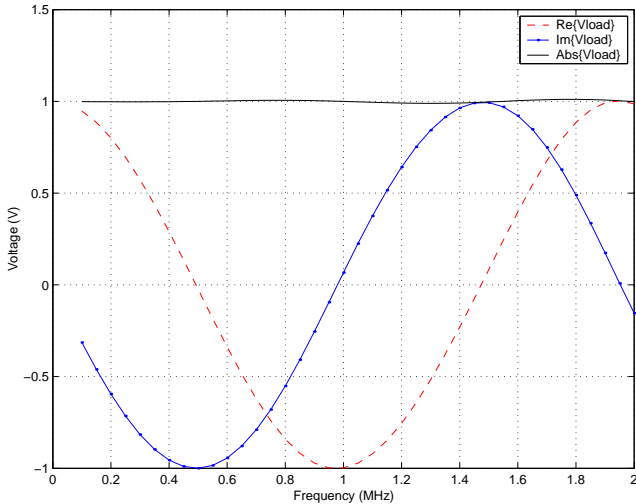

Fig. 3. Load Voltage V_L vs. Frequency with $Z_L = 136.7$ Ω

Across the simulated frequency range, 100 MHz to 2.0 GHz, the three largest variations in the load magnitude and their respective relative errors are shown in Table I.

IV. Derivation

To derive (2), the transmission line is assumed to be lossless. Even though the line is assumed to be lossless, the procedure can still be valuable in finding the closest

TABLE I
MAXIMUM VARIATIONS IN THE LOAD MAGNITUDE AND RELATIVE ERROR

Freq (GHz)	Voltage (V)	Rel Error
0.8	1.0056	0.56 %
1.3	0.98913	-1.09 %
1.8	1.0112	1.12 %

matched load over the desired frequency range for simulations containing a lossy transmission line, as shown by the previous example which contained conductor losses.

Since the transmission line is assumed lossless, the characteristic impedance must be purely real. The voltage as a function of position on the lossless transmission line is given as [2]

$$\hat{V}(z) = \hat{V}_o^+ \left[e^{-j\beta z} + \hat{\Gamma}_L e^{j\beta z} \right], \tag{3}$$

where $z = 0$ is defined to be at the load and the caret symbol denotes a complex quantity. Using (3), the voltage at the source and the load for a quarter wavelength transmission line is

$$\hat{V}\left(z = -\tfrac{\lambda}{4}\right) = \hat{V}_S = j\hat{V}_o^+ \left[1 - \hat{\Gamma}_L\right], \tag{4}$$

$$\hat{V}(z = 0) = \hat{V}_L = \hat{V}_o^+ \left[1 + \hat{\Gamma}_L\right], \tag{5}$$

respectively. Dividing (4) by (5) results in

$$\frac{\hat{V}_S}{\hat{V}_L} = j\frac{1 - \hat{\Gamma}_L}{1 + \hat{\Gamma}_L}. \tag{6}$$

Substituting

$$\hat{\Gamma}_L = \frac{\hat{Z}_L - Z_0}{\hat{Z}_L + Z_0}$$

for the load reflection coefficient into (6) yields

$$\frac{\hat{V}_S}{\hat{V}_L} = j\frac{Z_0}{\hat{Z}_L}.$$

Solving for Z_0 yields

$$Z_0 = -j\hat{Z}_L \frac{\hat{V}_S}{\hat{V}_L}, \tag{7}$$

which is the solution for the characteristic impedance of a lossless, quarter wavelength transmission line where the source voltage, load voltage, and load impedance can be complex quantities. Using a real load impedance and a source voltage of $\hat{V}_S = 1 \angle 0°$ V in (7) yields

$$Z_0 = -j\frac{Z_L}{\hat{V}_L}. \tag{8}$$

14

Since Z_0 and Z_L are purely real,

$$\hat{V}_L = -jV_x, \qquad (9)$$

where V_x is also purely real. Substituting (9) into (8) provides the final form as shown in (2). Equation (9) shows that the load voltage is purely imaginary and negative at the quarter wavelength frequency, which makes the quarter wavelength frequency easy to identify from the frequency domain plot.

V. Conclusions

The procedure described herein provides a quick method to determine the characteristic impedance of a transmission line when simulating in the frequency domain. Even though the assumption that the transmission line is lossless has been made, the procedure still yields an acceptable value for the characteristic impedance. Two of the main benefits of this method are that it is applicable to a wide range of physical geometries and that after performing two simulations the effectiveness of the transmission line match is known across the desired frequency range.

Acknowledgments

References

[1] C. R. Paul, *Introduction to Electromagnetic Compatibility*. New York: Wiley, 1992.
[2] D. M. Pozar, *Microwave Engineering*, 2nd ed. New York: Wiley, 1998.
[3] C. A. Balanis, *Advanced Engineering Electromagnetics*. New York: Wiley, 1989.