

SOLUTION OF TEAM BENCHMARK PROBLEM #13 (3-D nonlinear magnetostatic model)

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Abstract

Four solutions for the TEAM magnetostatic benchmark #13 are presented. The problem was solved with the three dimensional volume integral code CORAL, formerly called GFUNET. A series of models were solved with increasing discretization in order to study the convergence and the charged CPU-time.

Problem Definition

TEAM benchmark problem #13 is a magnetostatic problem consisting of a coil and steel plates. The geometry and all the material data is specified in details by Nakata, Takahashi, Fujiwara and Olszewski /1/. The purpose is to compute the magnetic flux density \mathbf{B} along a given line outside the steel plates and in addition the average flux across certain planes inside the steel. Measured data are also provided. The geometry of the problem is shown in Fig. 1 and a schematic picture of the measurements in Fig. 2.

The Volume Integral Code CORAL

The volume integral code CORAL is based on a decomposition of the magnetic field strength \mathbf{H}

$$\mathbf{H} = \mathbf{H}_m(\mathbf{M}, \mathbf{r}') + \mathbf{H}_s(\mathbf{J}, \mathbf{r}').$$

H_m is the field due to magnetization M and H_s the field due to currents. A system of integral equations is set up using a collocation approach and the line integrals of H along the edges of tetrahedra are solved /2/. The theoretical background of the formulation is explained in detail in reference /3/.

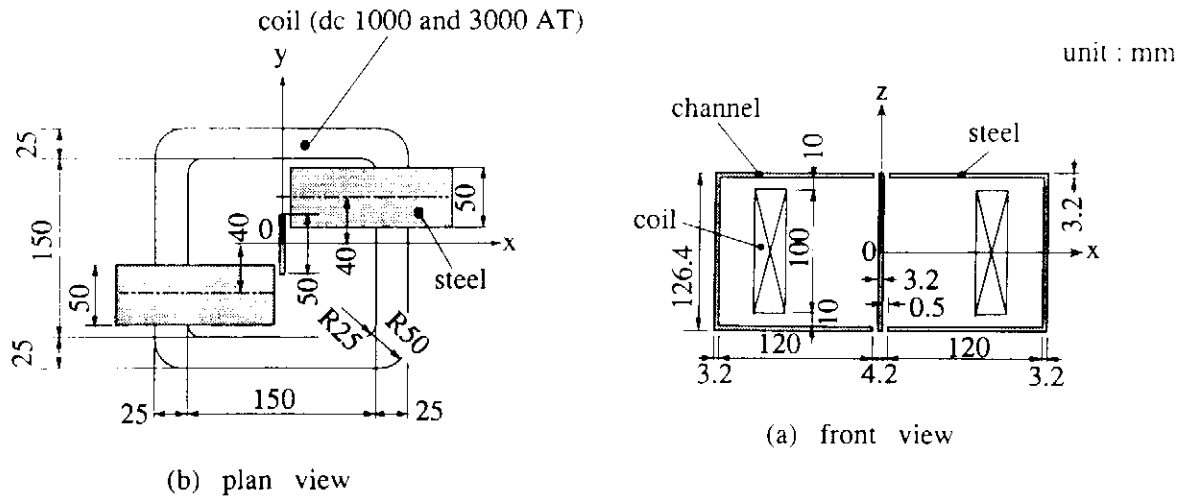


Figure 1. Geometry of the TEAM problem #13. /1/

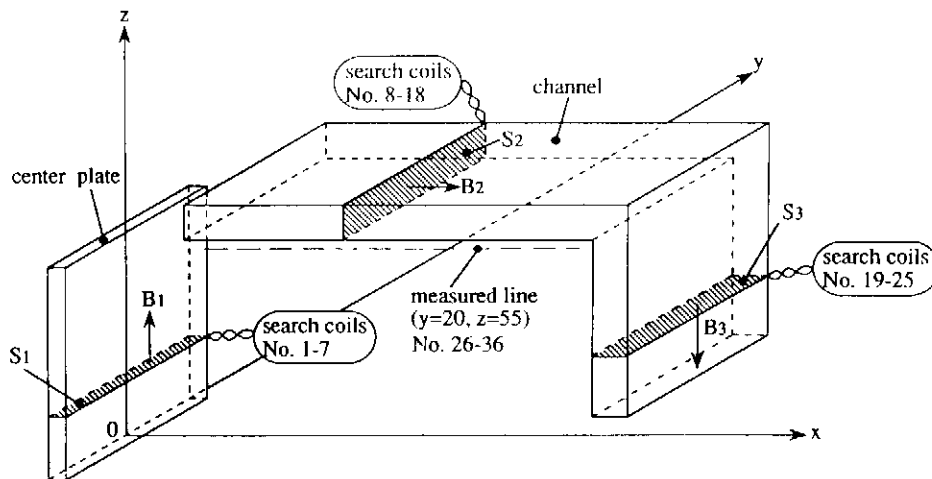


Figure 2. Schematic picture of the measurements. /1/

The main subroutines of CORAL are the integral equation matrix generation, the coil field computation routines, the solver, and the routines to update the susceptibility data and the matrix during a nonlinear iteration.

The first cycle of the nonlinear iteration, which sets up the matrix, takes always more CPU-time than the others, since vectors

$$C(r) = \int \frac{(r-r')}{\|r-r'\|^3} dv'$$

are generated only once and then stored on disc. These vectors are only geometry dependent and they are needed to compute the contribution of each tetrahedron to the scalar potential at each node. The so called paths /2/,/3/ are also created only once. They are, however, kept in the main memory all the time.

CORAL generates fairly large scratch files for the temporary data storage of the C -vectors. Vector C is integrated at each node from all the tetrahedra of the mesh. Thus the amount of disc space needed to store the C vectors is $3 \times \text{nodes} \times \text{tetrahedra} \times \text{length of the variable}$ (i.e. 4 if single precision, 8 if double precision variables are used). At the moment, the size of the scratch file is the limiting factor we have reached preventing us of running very large problems. For instance using double precision variables a problem of 4950 tetrahedra and 2200 nodes generates a scratch file of 260 MB.

The integral equation matrix is diagonal dominant and nonsymmetric. Nothing else is known. Thus the solver we have used is based on LU-decomposition and backsubstitution. The lower and upper triangular are generated "in place" using Crout's algorithm with partial pivoting. The LU-decomposition requires about $N^3/3$ operations and the backsubstitution stage $N^2/2$ executions, where N is the number of equations. Hence the solution time of large problems increases rapidly and will cause problems in addition to the disc space needed.

At the moment the finite element mesh is generated by splitting hexahedra to five tetrahedra. This means that the interior tetrahedron of each hexahedron has the volume about twice as big as the others. It is not yet clear whether the bigger elements dominate the results or not. In the near future the present mesh generator will be changed to a 3D Delaunay tetrahedral mesh generator.

Results

The results we presented in Sorrento workshop /4/ were solved with a fairly small workstation (24 MB SUN SPARCstation IPC), and hence it was dubious how the results changed if the mesh is refined. However, the integral formulation already seemed to share the same tendency as all other h -type formulations; the computed flux across the surfaces inside the steel plates is higher than the measured values.

The new results are obtained with a Kubota (Stardent) Titan 3010 computer. It has allowed tripling the number of tetrahedra and doubling the number of unknowns so far. The results seem to verify that with increasing discretization the convergence of the flux inside the steel plates is very slow; no obvious convergence was obtained. However, the magnetic field along the given line in air remained about the same all the time as shown in Fig. 2.

There has been debate on the reasons for the possible excessive values of flux in the steel plates. It is an interesting detail of the integral code that the only approximation made is the approximation of magnetic field \mathbf{H} in the space \mathbf{W}^1 (the space spanned by the "edge elements"). In fact, the \mathbf{H} is a vector field of \mathbf{W}^1 , which belongs to the class $ker(curl)$; the closed line integrals of the field vanish. Thus, if the flux is too high, the problem seems to be related directly to the type of elements used.

The data of the four discretizations are shown in Table 1. The average flux in the steel plates is shown in Fig. 3. and the field in air in Fig. 4.

Table 1.

Case	Number of tetrahedra	Number of nodes	Number of equations
1	1505	720	718
2	3080	1104	1102
3	3705	1652	1650
4	3960	1380	1378

The charged CPU-time using the old version of CORAL varied between about 2000 to 42000 seconds. With the new version the solution time of the largest problem was about 31000 CPU-seconds. The elapsed times of the main subroutines of the new version are shown in Table 2.

Table 2.

Routine	Charged CPU-time	Percentage
Coil field computation	173.61 seconds	0.553 %
Generation of paths	6696.0 seconds	21.32 %
Matrix setup	1057.7 seconds	3.368 %
Solver	233.74 seconds/cycle 69 cycles	0.744 % / cycle 51.36 %

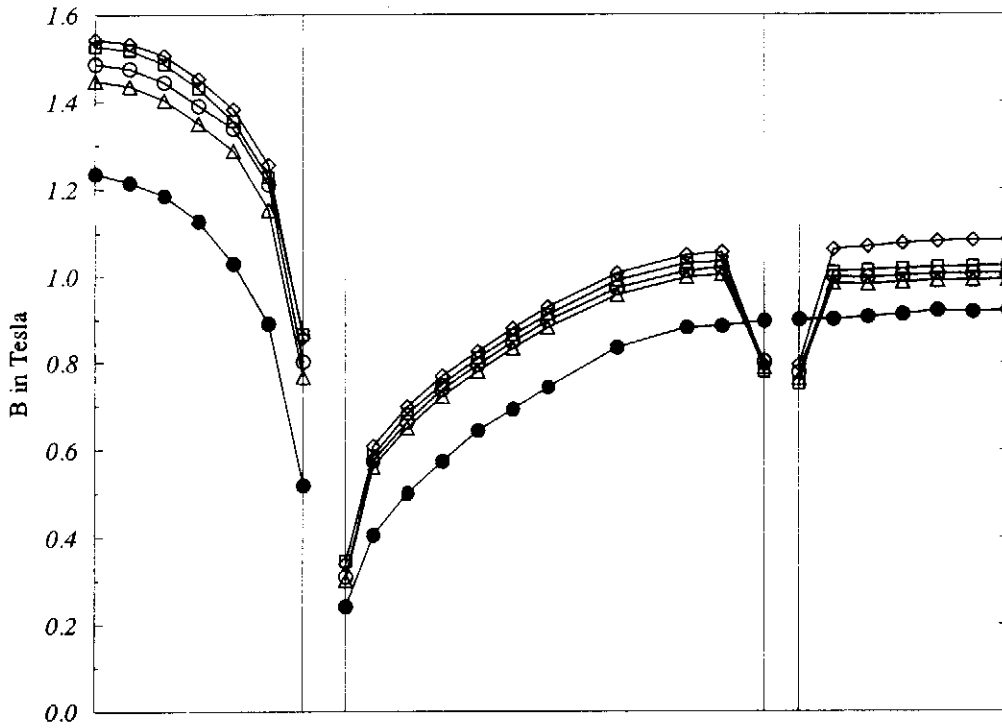


Figure 3. Average flux in the steel plates. Case 1, diamonds; case 2, squares; case 3, circles; case 4, triangles; measurements, filled circles.

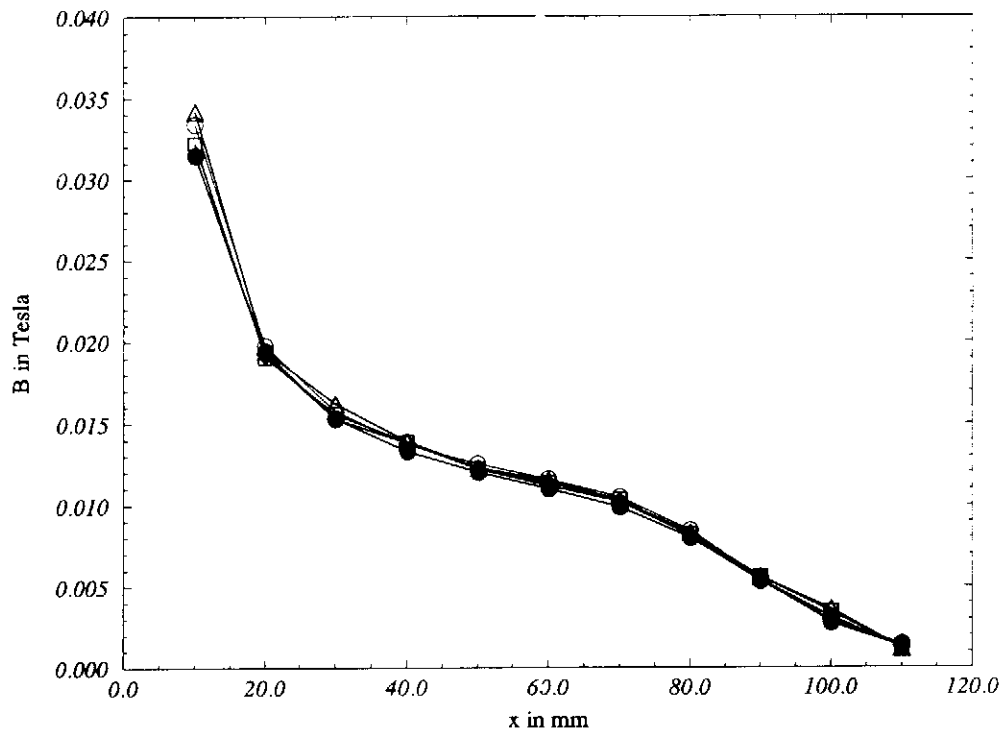


Figure 4. Magnetic field in air. Case 1, diamonds; case 2, squares; case 3, circles; case 4, triangles; measurements, filled circles.

References

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