Effective Permittivity Scheme for ADI-FDTD Method at the Interface of Dispersive Media

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Abstract **–** This paper presents an effective permittivity scheme to treat the dispersive media interfaces in ADI-FDTD method so as to avoid significant error due to improper assignment of media permittivity. In order to reduce the extra memory storage and computation operation required, a reduced-order modeling method is introduced to our scheme, which can simplify the programming work as well and therefore has a significant practical meaning. One numerical experiment will be performed to illustrate the procedure and effect of this effective permittivity scheme. The stability analysis of the updating equations will also be discussed.

Keywords**:** ADI-FDTD, biological tissues, dispersive media, and material interfaces.

I. INTRODUCTION

For accurate modeling of material interfaces in the conventional finite difference time domain (FDTD) method [1], the effective permittivity scheme has been proposed for the interfaces of non dispersive media [2, 3] and dispersive media [4]. In this paper, the effective permittivity scheme is presented for the alternating-directional-implicit (ADI) FDTD method [5] at the interface of dispersive media. During recent years a lot of research works related to the ADI-FDTD method have been carried out due to its unconditional stability, which means the time step size of this method would not be constraint by the mesh size any more. For example, the three-dimensional ADI-FDTD method as well as its stability analysis was proposed in [6]. The higher order [7] and parameter-optimized ADI-FDTD methods [8, 9] have also been developed. The discussion here is based on the ADI method for dispersive media in [10], where the treatment for the interface of dispersive media has not been mentioned. This scheme is also applicable to the cases of non dispersive-non dispersive and dispersive-non dispersive media interfaces. To the best of our knowledge, the treatment of media interfaces in ADI-FDTD method has not been discussed in the literature.

In the next section, the formulas for the effective permittivity scheme are provided. Section III performs one numerical experiment to illustrate the procedure and effect of our proposed scheme. This experiment is about the wave propagation in different biological tissues and the reflection coefficient at the interface of muscle and bone is evaluated, which are both Debye dispersive media. To reduce the extra memory storage and computation operation required, a reduced-order modeling method for discrete system will be applied to deal with the effective permittivity at the interface of two neighboring dispersive media, which can also simplify the programming work and therefore has a significant practical advantage. It can be seen that this scheme can avoid the significant error due to improper assignment of media permittivity. Thus it is meaningful to investigate the treatment of media interface in ADI-FDTD method since it will be useful for many practical problems, such as the one about bio-electromagnetics discussed here. The stability analysis of the updating equations of this scheme is discussed in Section IV, which provides the approach to investigate the stability based on the reduced-order model of the permittivity obtained.

II. FORMULATION

Following [10], we consider the case of two-dimensional (2-D) TE wave propagation in dispersive media. The permittivity $\varepsilon(\omega)$ is related to the frequency ω and the permeability μ. is assumed to be constant. Therefore we will focus the discussion on solving Maxwell's equation from Ampere's law, whose integral form is represented as,

$$\frac{\partial}{\partial t}\iint_S D\cdot dS=\oint_C H\cdot dl\,. \tag{1}$$

The electric flux density D can be related to the electric field intensity E in s domain by,

$$D(s)=\varepsilon(s)E(s)\,. \tag{2}$$

According to the second order temporal approximation of equation (1), it can be obtained that,

$$\frac{\iint_S D^{n+1}\cdot dS-\iint_S D^n\cdot dS}{\Delta t}=\frac{\oint_C H^{n+1}\cdot dl+\oint_C H^n\cdot dl}{2}+O\left(\Delta t^2\right). \quad (3)$$

Here Δt is the time step size and n is the temporal index. The D and E in discrete time domain can be related in z domain by,

$$D(z)=\varepsilon(z)E(z)\,. \quad (4)$$

Equation (4) can be derived from equation (2) by substituting,

$$s=\frac{4}{\Delta t}\frac{1-z^{-1/2}}{1+z^{-1/2}}. \quad (5)$$

Henceforth, all the poles, zeros and orders of the representation for the media permittivity in z domain are in relation to the term $z^{-1/2}$, because each field component is marching one half time step for each update sub-procedure.

Considering the field components arrangement in Fig. 1 and splitting equation (3) into two sub-procedures in discrete spatial domain, we can obtain the equations in equation (6) for the treatment of the interface of two different dispersive media in ADI-FDTD method.

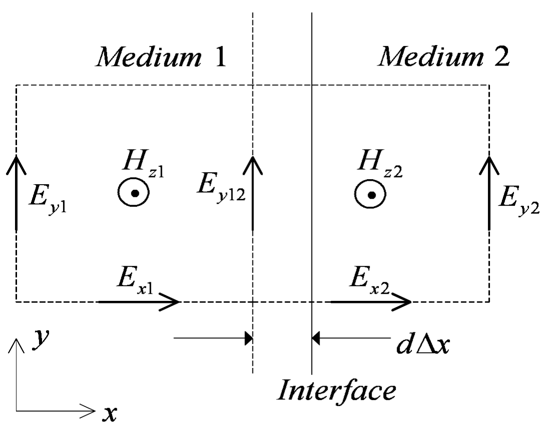

Fig. 1. TE wave field components arrangement at the interface.

$$\frac{D_{y12}{}^{n+1/2}-D_{y12}{}^{n}}{\Delta t/2}=-\frac{H_{z2}{}^{n+1/2}-H_{z1}{}^{n+1/2}}{\Delta x}, \quad (6a)$$

$$\frac{D_{y12}{}^{n+1}-D_{y12}{}^{n+1/2}}{\Delta t/2}=-\frac{H_{z2}{}^{n+1/2}-H_{z1}{}^{n+1/2}}{\Delta x} \quad (6b)$$

where Δx is the mesh size in x direction, $d\Delta x$ ($-0.5\le d\le 0.5$) is the displacement of the media interface relative to the nearest parallel FDTD mesh edge, and the subscripts *1* and *2* denote the field components in media *1* and *2*, respectively. D_{y12} can be related to the electric field component E_{y12} at the mesh edge by the effective permittivity scheme as,

$$\begin{aligned}D_{y12}(z)&=(\frac{1}{2}+d)D_1(z)+(\frac{1}{2}-d)D_2(z)\\&=(\frac{1}{2}+d)\varepsilon_1(z)E_{12}(z)+(\frac{1}{2}-d)\varepsilon_2(z)E_{12}(z).\end{aligned} \quad (7)$$

In practice, the mesh edge is usually set to overlap with the interface of different media. So here we mainly consider this case, where d = 0 and we obtain,

$$D_{y12}(z)=\frac{\varepsilon_1(z)+\varepsilon_2(z)}{2}E_{y12}(z)=\varepsilon_{12}(z)E_{y12}(z)\,. \quad (8)$$

Based on equations (6) and (8), the update equations for the interface of dispersive media can be derived as in [10]. The symbols $\varepsilon_1(z)$ and $\varepsilon_2(z)$ are the permittivity of these two neighboring media in z domain. The $\varepsilon_{12}(z)$ is the effective permittivity at the material interface and is related to the sum of $\varepsilon_1(z)$ and $\varepsilon_2(z)$. One can find that the number of the poles of $\varepsilon_{12}(z)$ is increased when the poles of $\varepsilon_1(z)$ and $\varepsilon_2(z)$ are different, which is in accordance with the statement in [4]. Since the order of the numerator and denominator of the representation for the effective permittivity will grow with the increase of the number of poles, extra memory storage and computation operation will be required. In order to reduce the extra memory storage and computation operation, here we introduce a reduced-order modeling method of discrete system [11] to deal with the effective permittivity. Through this method, the order of both the numerator and denominator of the effective permittivity in z domain can be reduced. We can select the order of the reduced-order model to be no higher than the maximum order of either neighboring media. The whole procedure of the simulation can be summarized as:

1) Determine the conditions of the simulation, such as mesh size, time step, boundary condition, and properties of media.
2) Derive the representation of each medium permittivity in z domain.
3) Derive the relation between D and E in the z domain at the interface of different media.
4) Apply the reduced-order modeling method to achieve the reduced-order model of the effective permittivity at the interface if there are different poles of the permittivity for the neighboring media.
5) Obtain the update equations in a similar way as in [10] and run simulation.

When one or both of neighboring media are replaced by non dispersive media permittivity, equation (8) will represent the effective scheme for interfaces of dispersive-non dispersive and non dispersive-non dispersive media, respectively, where the permittivity of the non dispersive media is a constant and in a simpler form compared with that of dispersive media. In the next section, numerical experiment will be performed to illustrate the procedure and effect of our scheme for the case of dispersive-dispersive media interface.

III. NUMERICAL EXPERIMENT

To show the procedure and benefit of this effective permittivity scheme for ADI-FDTD method at the interface of dispersive media, let us assume a Gaussian pulse propagating normally through the interface of the muscle (assumed to be medium *1*) and bone (assumed to be medium *2*), and study the reflection coefficient at this interface, which is parallel to *y* axis. The continuous function of the pulse is $g(t)=100\exp\left(-\left((t-320\Delta t)/64\Delta t\right)^2\right)$, where Δt = 10ps is the simulation time step. The uniform mesh size is set to be 0.5 mm and the thickness of both tissues is assumed to be 1 cm. The structure to be analyzed is truncated by 10-cell PML.

To start the illustration for the procedure of the effective permittivity scheme, the parameters of the Debye equation for the muscle and bone are obtained from [12] and [13], respectively. The Debye equation of relative permittivity can be presented in *s* domain as,

$$\varepsilon_r(s)=\varepsilon_\infty+\frac{A_1}{1+s\tau_1}+\frac{A_2}{1+s\tau_2}\,. \tag{9}$$

For the relative permittivity $\varepsilon_{msl}(s)$ of muscle, ε_∞=19, A_1=10000, τ_1=1.13×10^{-7}s, A_2=42, and τ_2=1.19×10^{-11}s. For the relative permittivity $\varepsilon_{bon}(s)$ of bone, ε_∞=3.4, A_1=309.4, τ_1=4.625×10^{-8}s, A_2=3.71, and τ_2=9.07×10^{-11}s. Their representations in *z* domain, $\varepsilon_1(z)$ and $\varepsilon_2(z)$, can be derived by the bi-linear transform in equation (5),

$$\varepsilon_1(z)=\varepsilon_{msl}(s)\Big|_{s=\frac{4}{\Delta t}\frac{1-z^{-1/2}}{1+z^{-1/2}}} =\frac{26.512901\text{-}31.324797z^{-1/2}+4.966468z^{-1}}{1\text{-}1.652734z^{-1/2}+0.652749z^{-1}}\,, \tag{10}$$

$$\varepsilon_2(z)=\varepsilon_{bon}(s)\Big|_{s=\frac{4}{\Delta t}\frac{1-z^{-1/2}}{1+z^{-1/2}}} =\frac{3.516241\text{-}6.616321z^{-1/2}+3.101916z^{-1}}{1\text{-}1.946244z^{-1/2}+0.946250z^{-1}}\,. \tag{11}$$

Here all the numbers are kept to six digits after the decimal part in order to distinguish them. It should be noted that in this section we only talk about the process of the relative permittivity for simplicity, and the final permittivity in the updating equation should be the relative permittivity obtained times the vacuum permittivity ε_0.

According to equation (9), the effective permittivity at the material interface can be achieved. One can find that the poles of the muscle permittivity in *z* domain are 0.999955 and 0.652778, and those of the bone permittivity are 0.999892 and 0.946352. Therefore the effective permittivity will possess four different poles and zeros, rather than two poles and zeros like the permittivity of each neighboring medium. So the field components *D* and *E* at two extra time steps need to be saved and processed. When the reduced-order modeling method is applied to reduce the order of the effective permittivity, a discrete model for the effective permittivity at the interface can be deduced, which possesses the same number of poles and zeros as these two media. Therefore the memory and computation operation for the extra field components of two time steps are not needed any more. Here if we assume that the number of the nodes on the interface is N_d and the memory occupied by each field component value is N_m bytes, the scheme adopting reduced-order modeling at least can save $4N_dN_m$ bytes memory and the computation operation on the extra field components is also eliminated. In addition, this reduced-order modeling process simplifies the programming work, which has a significant practical advantage.

If $z^{1/2}$ in equations (10) and (11) is replaced by $z^{1/2}$=exp($j\omega\Delta t/2$)=exp($j\theta$), where ω is the frequency we are interested in and Δt is the time step size of the FDTD scheme, the system response before the reduced-order modeling can be obtained from equation (8). Since the reduce-order model of the permittivity at the interface is supposed to possess two poles and zeros as the two neighboring media, let us substitute $z^{1/2}$=exp($j\theta$) into the reduced-order model of equation (12),

$$\varepsilon_{12}(z)=\frac{p_0+p_1z^{-1/2}+p_2z^{-1}}{q_0+q_1z^{-1/2}+q_2z^{-1}}=\frac{p_0z+p_1z^{1/2}+p_2}{q_0z+q_1z^{1/2}+q_2} \tag{12}$$

where p_0, p_1, p_2, q_0, q_1, and q_2 are the parameters to be achieved in this reduced-order model. Based on the model in equation (12) and the system response from equation (8) mentioned above, an equation similar to the equation (8) in [11] can be achieved and the reduced-order modeling can be carried out accordingly [11]. The result of the reduced-order model of the effective permittivity in *z* domain reads,

$$\varepsilon_{12}(z) = \frac{24.397910\text{-}44.359457z^{-1/2} + 19.991045z^{-1}}{1\text{-}1.877082z^{-1/2} + 0.877088z^{-1}} \cdot \quad (13)$$

With all these permittivity representations in z domain, the relation between D and E in z domain everywhere in the computational domain can be derived, and then the update equations can be obtained in a similar way as in [10].

The magnitude and phase angle (in degree) of the reflection coefficients at various frequencies are evaluated by three different schemes, which include the effective permittivity scheme, and the permittivity at the boundary simply assigned to either one of the two media. The numerical results are compared with the exact values in Figs. 2 and 3.

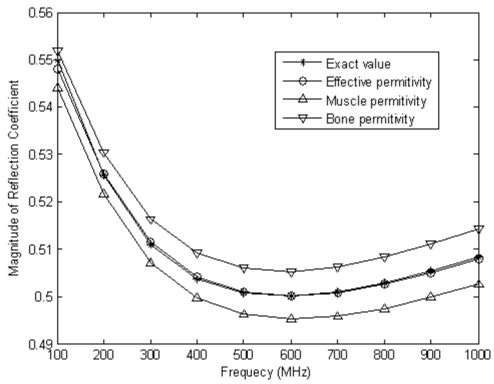

Fig. 2. Comparison of the magnitudes of reflection coefficients.

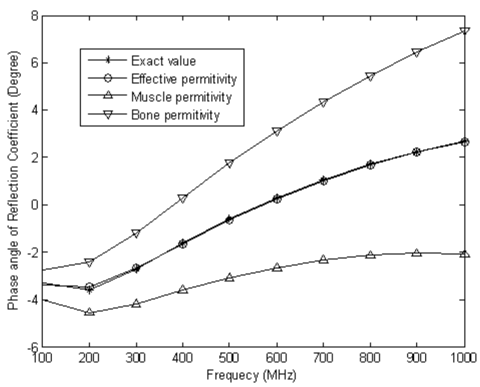

Fig. 3. Comparison of the phase angles of reflection coefficients.

From these two figures, one can find that the results of the effective permittivity scheme agree with the exact values very well, however, the results of the other two cases obviously disagree with the exact value. In order to highlight the difference between the numerical results and exact value, *ErrordB* in equation (14) defines the error of the numerical results in decibel for each sampling point,

$$ErrordB(f_m) = 20\log\left(\frac{|F(f_m) - F_0(f_m)|}{|F_0(f_m)|}\right). \quad (14)$$

$F(f_m)$ is the numerical result at the m-th frequency sampling point f_m while $F_0(f_m)$ is the corresponding exact value.

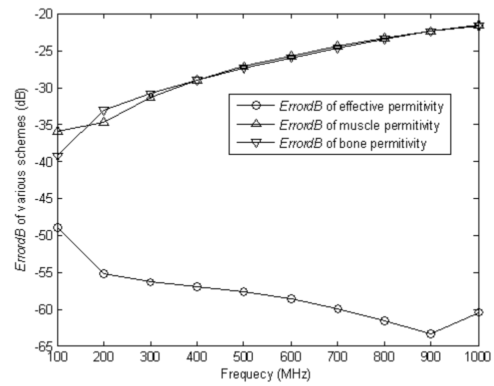

Fig. 4. Comparison of Error (dB) of various schemes.

Figure 4 plots *ErrordB* of the three schemes mentioned above and it can be found that this effective permittivity scheme in ADI-FDTD successfully avoids the significant error due to improper assignment of media permittivity.

IV. STABILITY ANALYSIS

In order to analyze the stability of the updating equations for the ADI-FDTD method at the interface of dispersive media discussed in the previous section by the von Neumann method [14], the trial solutions of the field components in the form of equation (15) are substituted into the updating equations,

$$V\big|_{I,J}^{n} = V^n e^{-j(k_x I\Delta x + k_y J\Delta y)} \quad (15)$$

where V represents various field components, I and J are the spatial indexes. Then the updating equations can be written in a matrix form as,

$$\mathbf{M}_{1\mathrm{L}} \vec{u}^{\,n+1/2} = \mathbf{M}_{1\mathrm{R}} \vec{u}^{\,n}, \quad (16a)$$

$$\mathbf{M}_{2\mathrm{L}} \vec{u}^{\,n+1} = \mathbf{M}_{2\mathrm{R}} \vec{u}^{\,n+1/2} \quad (16b)$$

where

$$\vec{u}^{\,n} = \left[\tilde{D}_x^n \quad E_x^n \quad \tilde{D}_x^{n-1/2} \quad E_x^{n-1/2} \quad \tilde{D}_y^n \quad E_y^n \quad \tilde{D}_x^{n-1/2} \quad E_x^{n-1/2} \quad H_z^n\right]^T,$$

$$\mathbf{M}_{1\mathrm{L}}=\begin{bmatrix}1&0&0&0&0&0&0&0&0\\-q_0&p_0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&1&0&0&0&jbP_x\\0&0&0&0&-q_0&p_0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1&0\\0&0&0&0&0&jdP_x&0&0&1\end{bmatrix},$$

$$\mathbf{M}_{1\mathrm{R}}=\begin{bmatrix}1&0&0&0&0&0&0&0&jbP_y\\q_1&-p_1&q_2&-p_2&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&q_1&-p_1&q_2&-p_2&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&0&1&0&0&0\\0&jdP_y&0&0&0&0&0&0&1\end{bmatrix},$$

$$\mathbf{M}_{2\mathrm{L}}=\begin{bmatrix}1&0&0&0&0&0&0&0&-jbP_y\\-q_0&p_0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&-q_0&p_0&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1&0\\0&-jdP_y&0&0&0&0&0&0&1\end{bmatrix},$$

$$\mathbf{M}_{2\mathrm{R}}=\begin{bmatrix}1&0&0&0&0&0&0&0&0\\q_1&-p_1&q_2&-p_2&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&-jbP_x\\0&0&0&0&q_1&-p_1&q_2&-p_2&0\\0&0&0&0&1&0&0&0&0\\0&0&0&0&0&1&0&0&0\\0&0&0&0&0&-jdP_x&0&0&1\end{bmatrix},$$

$$b=\frac{\Delta t}{2\varepsilon_0},\ d=\frac{\Delta t}{2\mu},\ P_\xi=-\frac{2\sin\left(k_\xi\Delta\xi/2\right)}{\Delta\xi}$$

where $\widetilde{D}_\xi=\varepsilon_r E_\xi$ and ξ can be x or y.

According to the von Neumann method, the magnitudes of all the eigenvalues of the updating matrix **M** in equation (17) need to be no greater than 1 to make sure the updating equations to be stable, as shown

$$\vec{u}^{\,n+1}=\mathbf{M}\vec{u}^{\,n}=\left(\mathbf{M}_{2\mathrm{L}}\right)^{-1}\mathbf{M}_{2\mathrm{R}}\left(\mathbf{M}_{1\mathrm{L}}\right)^{-1}\mathbf{M}_{1\mathrm{R}}\vec{u}^{\,n}. \tag{17}$$

However, for the case of dispersive media, it is difficult to achieve the analytical solutions of the eigenvalues as in [6]. Therefore the combination of von Neumann method and Routh-Hurwitz criterion [15] is adopted here to investigate the stability. The reduced-order model of the dispersive media permittivity in equation (13) needs to be substituted into equation (17) to analyze the stability. If the bilinear transformation in equation (3) of [15] is applied to the eigenpolynomial of **M** and then the steps in [15] are followed to build the Routh table, it can be found that all the entries of the first column of the Routh table are non-negative quantities regardless of the time step size, which means the updating equation is still unconditionally stable for this case. One can analyze the stability of the updating equations by this approach.

The detailed procedures and the result of the Routh table will not be presented here due to the limit of paper length since they are very lengthy. But it is not difficult to work it out with the help of some mathematical software such as Matlab.

V. CONCLUSION

This paper has presented an effective permittivity scheme to treat the dispersive media interfaces in ADI-FDTD method. The approach to analyze its stability has been discussed as well. This scheme is also applicable to the cases of the dispersive and non dispersive media interfaces. To reduce the extra memory storage and computation operation generally required, a reduced-order modeling method for discrete system is applied to deal with the effective permittivity at the interface of two neighboring dispersive media, which can also simplify the programming work and therefore has a significant practical advantage. The numerical experiment, which is about wave propagation in different biological tissues for the demonstration of the case of dispersive-dispersive media interfaces, has been performed to illustrate the procedure and effect of our scheme. One can find that this scheme works well for avoiding the significant error due to improper assignment of media permittivity, as well as reducing the extra memory storage and computation operation required. Meanwhile the programming work is simplified. So the investigation on the treatment of media interfaces in ADI-FDTD method is meaningful since it will be useful for many practical problems.

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