The Equivalent Circuit Extraction and Application for Arbitrary Shape Graphene Sheet

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Abstract **─** In this work, for the first time the electromagnetic features of graphene are characterized by a circuit model derived instead of fitted from the electric field integral equation (EFIE). The atomically thick graphene is equivalently replaced by an impedance surface. When it is magnetized, the impedance surface is anisotropic with a tensor conductivity. Based on EFIE, the graphene's circuit model can be derived by the partial element equivalency circuit (PEEC) concept. The anisotropic resistivity is modeled using a serial resistor with current control voltage sources (CCVSs). From the derived circuit model, electromagnetic properties of graphene can be conveniently analyzed. This work also provides a new characterization method for dispersive and anisotropic materials.

Index Terms **─** Graphene, magnetized, non-magnetized, PEEC.

I. INTRODUCTION

Graphene is an atomically thin nanomaterial with promising application potentials. However, its electromagnetic modeling is usually totally numerical [1] or physically empirical. This fact motivated us, in this paper, to derive an equivalent circuit model based on EFIE for both non-magnetized and magnetized graphene. It employs the PEEC [2-7] process to convert the electromagnetic interactions on the graphene surface into resistive, inductive and capacitive effects on an isotropic or anisotropic impedance surface. By solving the derived circuit model, graphene's electromagnetic properties can be fully predicted conveniently and efficiently.

In this paper, a novel circuit model based on the electric field integral equation (EFIE) is proposed to solve the dispersivity of non-magnetized and magnetized graphene. For the non-magnetized graphene, the conductivity is composed of intraband and interband contributions. The resistive part in the equivalent circuit model is modeled as a resistor, an inductor and they are in series with *Z*inter which accounts for the interband contribution of the surface conductivity of graphene. In the equivalent circuit model for magnetized graphene, the diagonal elements of the surface conductivity tensor intrinsically correspond to the resistance of each inductive branch, which is the same as the unbiased scalar conductivity of graphene. For the off-diagonal elements of the conductivity tensor, a new equivalent circuit model is developed to model the resistive characteristics by utilizing current-controlled voltage sources (CCVSs).

The advantages of the proposed new method are: (i) The model is derived based on EM wave equations. It is not empirical or curve fitted. Hence, it is more reliable and general. Based on our search, this is the first derived model for graphene. (ii) The new method is much more efficient than the volumetric based graphene modeling process. (iii) Compared with the numerical process [8], the derived circuit model of graphene provides a convenient bridge to integrate graphene EM parasitic effects with lumped circuit designs.

II. EQUIVALENT CIRCUIT MODEL DERIVATION FOR GRAPHENE

The 2D atomically thin graphene can be considered as an impedance surface with the dispersive conductivity that is isotropic or anisotropic along tangential directions [9]. Based on the electric field integral equation, we have:

$$\mathbf{E}^{inc}(\mathbf{r},t)=\frac{\mathbf{J}(\mathbf{r},t)}{\sigma}+\mu\int_{v'}G(\mathbf{r},\mathbf{r}')\frac{\partial\mathbf{J}(\mathbf{r}',t)}{\partial t}dv' + \frac{\nabla}{\varepsilon}\int_{v'}G(\mathbf{r},\mathbf{r}')q(\mathbf{r}',t)dv', \quad (1)$$

where $G(\mathbf{r},\mathbf{r}')$ is the full wave Green's function. For one current filament on the graphene sheet, using the partial equivalence element concept, Eq. (1) becomes Kirchhoff's

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Voltage Law:

$$V = RI + Lp\frac{dI}{dt} + Q \cdot Pp, \tag{2}$$

where Lp is the partial inductance, and Pp is the partial coefficient of potential. The Lp between cell α and β and Pp between cell i and j are:

$$Lp_{\alpha\beta} = \frac{\mu}{a_\alpha a_\beta}\int_{v_\alpha}\int_{v_\beta} G(\mathbf{r}_\alpha, \mathbf{r}_\beta) dv_\alpha dv_\beta, \tag{3}$$

$$Pp_{ij} = \frac{1}{\varepsilon S_i S_j}\int_{S_i}\int_{S_j} G(\mathbf{r}_i, \mathbf{r}_j) dS_i dS_j. \tag{4}$$

The 2-dimensional meshing scheme can be represented as Fig. 1 [2].

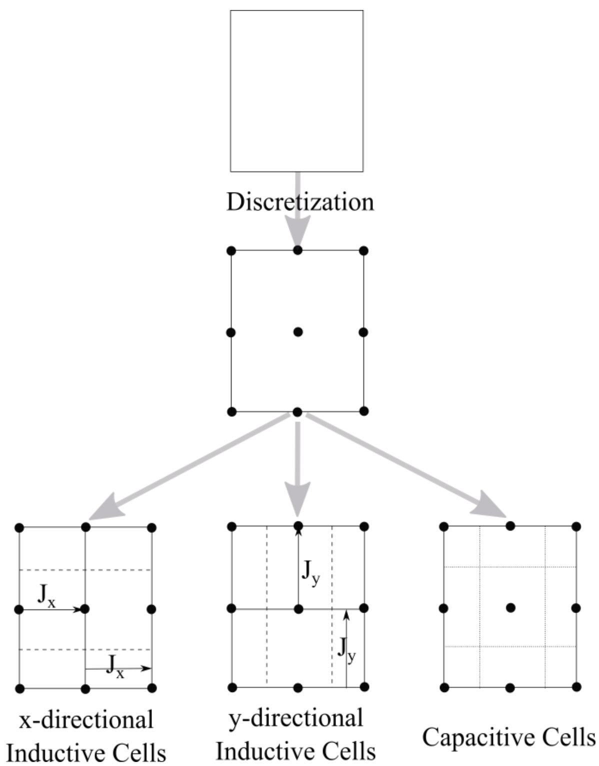

Fig. 1. This is the 2D discretization of thin conductive plate. Dark circles indicate nodes, dashed lines separate inductive cells, and dotted lines separate capacitive cells.

A. Non-magnetized graphene

The graphene dispersive conductivity σ is a summation of both intraband and interband contributions [10]. For a one freestanding rectangular patch with the length l and width w, its surface resistance can be derived from Kubo's formula:

$$R_{surf} = R_r + j\omega L_r + Z_{inter}, \tag{5}$$

where R_r is from the real part of σ, L_r is from the imaginary part of σ, and Z_{inter} is from the inter band part. Combining other parts of the cell model, the new non-magnetized graphene unit model is illustrated in Fig. 2.

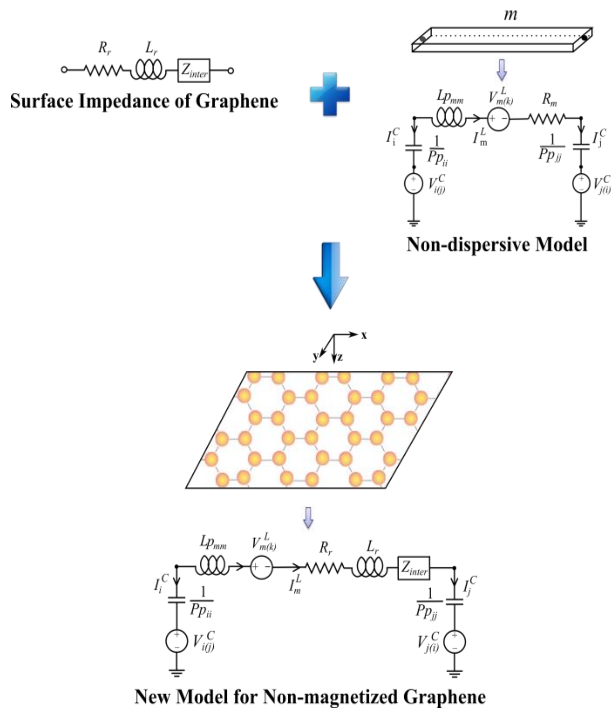

Fig. 2. One cell's model for a non-magnetized graphene patch. The left bottom model is the traditional equivalent model, where Lp_{mm}, Pp_{ii} and Pp_{jj} are self-inductance and self-coefficients of potential, respectively. $V^L_{m(k)}$ is the voltage control voltage source (VCVS) corresponding to mutual inductive couplings. $V^C_{i(j)}$ and $V^C_{j(i)}$ are the voltage control voltage sources (VCVSs) due to mutual capacitive coupling between two capacitors i and j.

B. Magnetized graphene

With the bias by a static magnetic field, the surface conductivity of graphene becomes an anisotropic tensor, which complicates the problem. Hence, the numerical methods have to settle the dispersive and anisotropic properties of graphene simultaneously.

For the magnetized graphene, its surface conductivity becomes an anisotropic and dispersive tensor $\bar{\bar{\sigma}}$ [9]. Hence, the electric field has contributions from orthogonal current components. For example, the x-direction electric field is a function of J_x and J_y. Hence, for off-diagonal elements of the conductivity tensor, a current control voltage source (CCVS) can be used to represent each of these orthogonal contributions. CCVSs are in series with the intrinsic resistances that are derived from diagonal terms of $\bar{\bar{\sigma}}$. In Fig. 3, the new equivalent circuit model for the magnetized graphene is sketched.

$$\mathbf{E}^{tot} = \mathbf{E}^{inc} - \frac{\partial \mathbf{A}}{\partial t} - \nabla\phi = \overline{\overline{\sigma}}^{-1}\mathbf{J} = \overline{\overline{\rho}}\mathbf{J}$$
$$= (\rho_{xx}J_x + \rho_{xy}J_y)\hat{x} + (\rho_{yx}J_x + \rho_{yy}J_y)\hat{y}$$

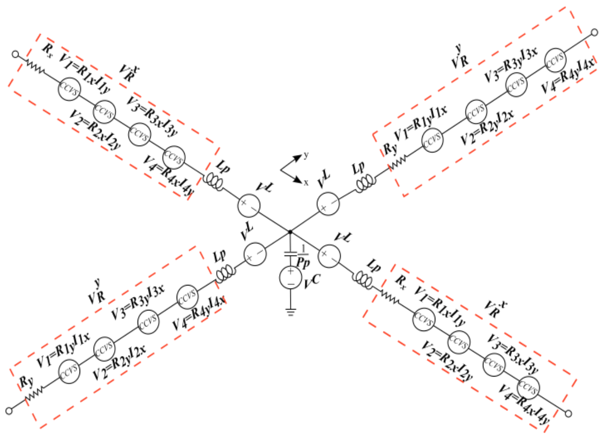

Fig. 3. A schematic diagram of the complete equivalent circuit for magnetized anisotropic conductivity graphene. This circuit model is for four nearby cells which share a common node, and two for x–directional cells and the other two for y–directional cells. The scripts for each cell are omitted for simplicity.

III. NUMERICAL RESULTS

A. Non-magnetized graphene

The $5\times0.5\ \mu m^2$ graphene patch is illuminated by a plane wave linearly polarized along the patch length. The direction of propagation is normal to the surface of graphene. The absorption cross section of the graphene patch is shown in Fig. 4 for different relaxation times. By comparing with results from [10] (represented by circles), it is seen that perfect agreements are achieved including the positions of important resonant frequencies.

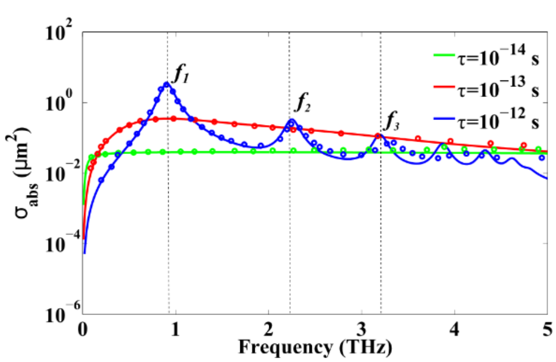

Fig. 4. Absorption cross section (in logarithmic scale) of a non-magnetized graphene patch as a function of frequency, for different relaxation time: 10^{-14} s (green line), 10^{-13} s (red line), 10^{-12} s (blue line). The results are compared with [5] plotted using circles. f_1, f_2 and f_3 are resonant frequencies.

B. Magnetized graphene

To validate the accuracy of the proposed algorithm for the magnetized graphene, a $10\times2\ \mu m^2$ graphene patch is studied first. Relaxation time t = 1.3×10^{-13} s. The magnetic bias B_0=0.25 T. The graphene patch is biased by a *z*–directional static magnetic field and the same excitation plane wave in III.A is used. The absorption cross section and extinction cross section calculated by new method and discontinuous Galerkin method [9] are compared in Fig. 5. The definitions of the absorption cross section and extinction cross section can be found in [6].

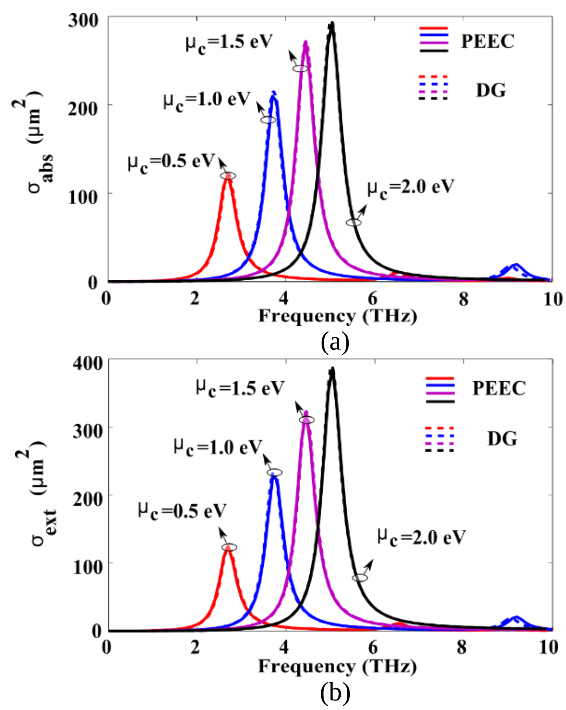

Fig. 5. (a) σ_{abs} (absorption cross section) and (b) σ_{ext} (extinction cross section) of the magnetized graphene patch.

IV. CONCLUSION

In this paper, a novel equivalent circuit model is derived for the general graphene sheet based on EM integral equations. It provides a new bridge between EM parasitic effects and lumped circuit designs for researches on graphene and other dispersive anisotropic media.

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REFERENCES

[1] V. Nayyeri, M. Soleimani, and M. Ramahi, "Wideband modeling of graphene using the finite-difference time-domain metod," *IEEE Trans. Antenna Propag.*, vol. 6, no. 12, pp. 6107-6114, Dec. 2013.

[2] A. E. Ruehli, "Equivalent circuit models for three dimensional multiconductor systems," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-22, no. 3, pp. 216-221, Mar. 1974.

[3] A. E. Ruehli, "Inductance calculations in a complex integrated circuit environment," *IBM J. Res. Develop.*, vol. 16, no. 5, pp. 470-481, Sep. 1972.

[4] A. E. Ruehli and P. A. Brennan, "Efficient capacitance calculations for three-dimensional multiconductor systems," *IEEE Trans. Microw. Theory Tech.*, vol. 21, no. 2, pp. 76-82, Feb. 1973.

[5] Y. S. Cao, L. Jiang, and A. E. Ruehli, "Distributive radiation and transfer characterization based on the PEEC method," *IEEE Trans. Electromag. Compat*., vol. 57, no. 4, pp. 734-742, Aug. 2015.

[6] Y. S. Cao, L. Jiang, and A. E. Ruehli, "An equivalent circuit model for graphene-based terahertz antenna using the PEEC method," *IEEE Trans. Antenna Propag.*, vol. 64, no. 4, pp. 1385-1393, Apr. 2016.

[7] Y. S. Cao, L. Jiang, and A. E. Ruehli, "The derived equivalent circuit model for magnetized anisotropic grapheme," submitted to *IEEE Trans. Antenna Propag*.

[8] O. V. Shapoval, J. S. G.-Diaz, J. P.-Carrier, J. R. Mosig, and A. I. Nosich, "Integral equation analysis of plane wave scattering by coplanar graphenestrip gratings in the THz range," *IEEE Trans. Terahertz Sci. Techn.*, vol. 6, no. 3, pp. 666-674, Sept. 2013.

[9] P. Li and L. J. Jiang, "Modeling of magnetized graphene from microwave to THz range by DGTD with a scalar RBC and an ADE," *IEEE Trans. Antennas Propag.*, vol. 63, no. 10, Oct. 2015.

[10] I. Llatser, C. Kremers, D. N. Chigrin, J. M. Jornet, M. C. Lemme, A. Cabellos-Aparicio, and E. Alarcon, "Radiation characteristics of tunable graphene as in the terahertz band," Radioengineering, vol. 21, no. 4, pp. 946-953, 2012.