Enhancement of Numerical Computation Methods Useful for Radio Communication Antenna Systems

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Abstract

The radiation from a flanged parallel-plate waveguide is solved efficiently by the moment method, where the expansion functions contain the correct edge behavior of the fields. This computation method can be useful to optimize radiation of microwave transmitters and efficiency of receiver antenna and front end circuits. It is shown that three appropriate expansion functions are sufficient for an excellent accuracy and convergence rate of the solution.

1 Introduction

Several radio communication microwave systems use aperture antennas as waveguide apertures, slot arrays, horns, etc. The moment method (MM) is one of the most popular method for the calculation of the radiation related to this kind of antennas. In this paper we present an efficient selection of the MM expansion functions in order to investigate the accuracy of the required parameters necessary to improve efficiency of such antennas. We achieve a first step for accurate computation by solving the simple example of the radiation from a flanged parallel-plate waveguide. An accurate solution for the waveguide electromagnetic field components and reflection coefficient will enable optimum adaptation and efficiency with minimum of cut and try steps.

The problem of radiation from a flanged parallel-plate waveguide has been solved by many authors [1-15]. An efficient MM solution has been offered by

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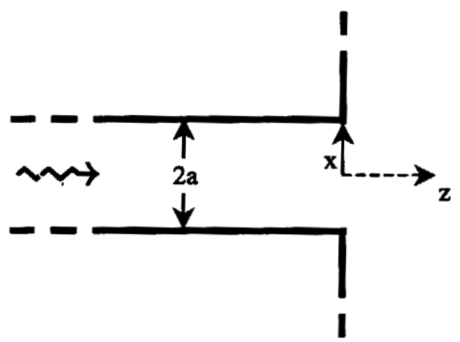

Figure 1: The geometry of the problem. A TEM wave is propagating in the $\hat{z}$ direction inside the flanged parallel-plate waveguide and scattered by the discontinuity.

[15] ,where the dominant edge behavior of the fields is included in the MM expansion functions. It has been shown by [16], [17] that applying the MM with expansion functions that include the *correct* edge behavior of the fields improves the accuracy and convergence rate of the solution. In this paper we solve the problem of the transverse electromagnetic (TEM) case for the radiation from a flanged parallel-plate waveguide by applying the method used in [16], [17]. We compare also the results with the case where only the dominant behavior of the fields near the edges of the waveguide are included in the MM expansion functions [15].

2 Formulation of the Problem

2.1 The Relevant Field Components

The geometry of the waveguide is given in Fig. 1

The distance between the plates is $2a$ and the center of the gap is at the coordinates origin. The TEM incident field is expressed by

$$E_x = \exp(-jk_0 z) \tag{1}$$

$$H_y = \frac{1}{\eta}\exp(-jk_0 z) \tag{2}$$

where $\eta = 120\pi$ ohms and k_0 is the wave number. The TEM and transverse magnetic (TM) backward transverse fields are well known, and can be expressed by [18]

$$E_x = \frac{1}{k_0} \sum_{n=0}^{\infty} \beta_n A_n \cos\left(\frac{n\pi x}{a}\right) \exp(j\beta_n z) \tag{3}$$

$$H_y = -\frac{1}{\eta} \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{a}\right) \exp(j\beta_n z) \tag{4}$$

where

$$\beta_n = \begin{cases} \sqrt{k_0^2 - (n\pi/a)^2}, & k_0 \geq n\pi/a \\ -j\sqrt{(n\pi/a)^2 - k_0^2}, & n\pi/a > k_0 \end{cases} \tag{5}$$

where the $n = 0$ term is connected to the TEM mode. The electric field above the flange is given by

$$E_x^{II} = \int_{-\infty}^{\infty} \tilde{A}(k_x) \exp(-jk_x x) \exp(-j\gamma z) dk_x \tag{6}$$

where

$$\gamma = \begin{cases} \sqrt{k_0^2 - k_x^2}, & k_0 \geq k_x \\ -j\sqrt{k_x^2 - k_0^2}, & k_x > k_0 \end{cases} \tag{7}$$

Using

$$\mathbf{H} = \frac{j}{\omega\mu} \nabla \times \mathbf{E} \tag{8}$$

we get

$$\begin{aligned} H_y^{II} &= \frac{1}{\omega\mu} \int_{-\infty}^{\infty} (\gamma + \frac{k_x^2}{\gamma}) \tilde{A}(k_x) \exp(-jk_x x) \exp(-j\gamma z) dk_x \\ &= \frac{1}{\eta} \int_{-\infty}^{\infty} \frac{k_0}{\gamma} \tilde{A}(k_x) \exp(-jk_x x) \exp(-j\gamma z) dk_x \end{aligned} \tag{9}$$

2.2 Moment Method Solution

The tangential electric field on the gap is given by the moment expansion

$$E_x^{III} = \sum_{j=1}^{N} a_j e_j(x) \tag{10}$$

where N is the number of expansion function taken to the calculation, and

$$e_j(x) = \cos^{\alpha_j}\left(\frac{\pi x}{2a}\right), \qquad \alpha_j = \{-1/3, 1/3, 5/3, ...\} \tag{11}$$

It was shown in [16] ,[17] that the correct edge behavior of E_x near a similar 2-dimensional 90° conducting edge is given by $\alpha_1 r^{-1/3} + \alpha_2 r^{1/3} + \alpha_3 r^{5/3} +$ ...where r is the distance of the field point from the edge and α_i are arbitrary constants. It also has been shown that these $e_j(x)$ expansion functions exactly contain this edge power series behavior, that is, $e_j(x)$ contain all the terms of the edge power series, and do not contain terms which do not belong to the edge power series. We apply now the Galerkin MM version, that is, the set of the test functions the same as the set of the expansion functions.

In order to write the final equation, the A_m's and $\widetilde{A}(k_x)$ terms are extracted in terms of the a_j's , and next we apply the boundary condition for the tangential magnetic field on the gap. Thus, the A_n's terms are extracted by

$$1 + \frac{1}{k_0} \sum_{n=0}^{\infty} \beta_n A_n \cos \frac{n\pi x}{a} = \sum_{j=1}^{N} a_j e_j(x) \tag{12}$$

using the orthogonality relation

$$2a + 2aA_0 = \sum_{j=1}^{N} a_j \overline{e}_{j0} \tag{13}$$

where

$$\overline{e}_{j0} = \int_{-a}^{a} e_j(x) dx \tag{14}$$

hence

$$A_0 = \frac{1}{2a} \sum_{j=1}^{N} a_j \overline{e}_{j0} - 1 \tag{15}$$

and

$$aA_n \frac{\beta_n}{k_0} = \sum_{j=1}^{N} a_j \overline{e}_{jn} \tag{16}$$

where

$$\overline{e}_{jn} = \int_{-a}^{a} e_j(x) \cos\left(\frac{n\pi x}{a}\right) dx \tag{17}$$

For the external field we obtain

$$\int_{-\infty}^{\infty} \widetilde{A}(k_x) \exp(-jk_x x) dk_x = \sum_{j=1}^{N} a_j e_j(x) \tag{18}$$

hence

$$2\pi \widetilde{A}(k_x) = \sum_{j=1}^{N} a_j \widetilde{e}_j(k_x) \tag{19}$$

where

$$\tilde{e}_j(k_x) = \int_{-a}^{a} e_j(x) \exp(jk_x x) dx \tag{20}$$

The MM equation now obtained by equating H_y on the gap

$$1 - \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{a}\right) = \int_{-\infty}^{\infty} \frac{k_0}{\gamma} \tilde{A}(k_x) \exp(-jk_x x) dk_x \tag{21}$$

Multiply each side by $e_i(x)$ and integrate over the aperture, we obtain

$$(1 - A_0)\overline{e}_{i0} - \sum_{n=1}^{\infty} A_n \overline{e}_{in} = k_0 \int_{-\infty}^{\infty} \frac{1}{\gamma} \tilde{A}(k_x) \tilde{e}_i^*(k_x) dk_x, \qquad i = 1, 2, ..N \tag{22}$$

or

$$\left\{1 - \left[\frac{1}{2a}\left(\sum_{j=1}^{N} a_j \overline{e}_{j0}\right) - 1\right]\right\} \overline{e}_{i0} - \sum_{n=1}^{\infty} \left(\frac{k_0}{a\beta_n} \sum_{j=1}^{N} a_j \overline{e}_{jn}\right) \overline{e}_{in} \tag{23}$$

$$= \quad k_0 \int_{-\infty}^{\infty} \frac{1}{\gamma} \left[\frac{1}{2\pi} \sum_{j=1}^{N} a_j \tilde{e}_j(k_x)\right] \tilde{e}_i(k_x) dk_x$$

Finally, the MM linear system of eqautions is given by

$$\sum_{j=1}^{N} A_{ij} \beta_j = B_i, \qquad i = 1, 2, ..N \tag{24}$$

where

$$A_{ij} = \frac{k_0}{\pi} \int_0^{\infty} \frac{1}{\gamma} \tilde{e}_j(k_x) \tilde{e}_i(k_x) dk_x + \frac{k_0}{2a} \sum_{n=0}^{\infty} \frac{\epsilon_n}{\beta_n} \overline{e}_{jn} \overline{e}_{in}, \tag{25}$$

$$\epsilon_n = \begin{cases} 1, & n = 0 \\ 2 & n \neq 0 \end{cases} \tag{26}$$

$$B_i = 2\overline{e}_{i0} \tag{27}$$

3 Main Results

We present here the results for the reflection coefficient

$$\Gamma = \frac{1}{2a} \sum_{j=1}^{N} a_j \overline{e}_{j0} - 1 \tag{28}$$

in comparison with the results of [15]. For a single frequency point $2a/\lambda = 0.8$ we present the following table:

# funcs.	$\|\Gamma\|$ [15]	arg (Γ) [deg][15]	$\|\Gamma\|$ (*)	arg (Γ) [deg] (*)
1	0.1463	112.01	0.167271	109.801
2	0.0643	75.82	0.0644	75.7013
3	0.06433	75.73	0.064322	75.7072
4	0.06433	75.71	0.064322	75.7072
5	0.06433	75.71		

Table 1: Comparison between the results of [15] (columns 2,3) for the amplitude and phase of the reflection coefficient, for $2a/\lambda = 0.8$, and our results (*) in columns 4,5. Column 1 is for the number of the expansion functions taken to the calculation.

It is shown that the rate of convergence of the amplitude of the reflection coefficient in [15] is slightly better, but the rate of convergence of the phase and the overall rate of convergence is slightly better in our results.

Now we show a graph (Fig. 2) of the amplitude of the reflection coefficient for a/λ from zero to one, applying 3 expansion functions, in comparison to the results of [15]. It is shown that both results are in very good agreement.

Finally a graph of the phase of the amplitude of the reflection coefficient is presented (Fig. 3), for a/λ from zero to one, 3 expansion functions where applyed, and compared to the results of [15]. A very good agreement is observed.

4 Conclusions

The problem of the scattering of a TEM wave from the end of a flanged parallel-plate waveguide has been solved, where the MM expansion functions obey the correct edge behavior near the waveguide corners. It has been shown that the specific choice of the MM expansion functions in our work improves the accuracy and the rate of convergence of the solution, when compared to MM solutions for which the expansion functions only contain the dominant behavior of the field near the edges. The investigated computation method enable us to compute with accuracy the field components and the reflection coefficient required for antenna adaptation.

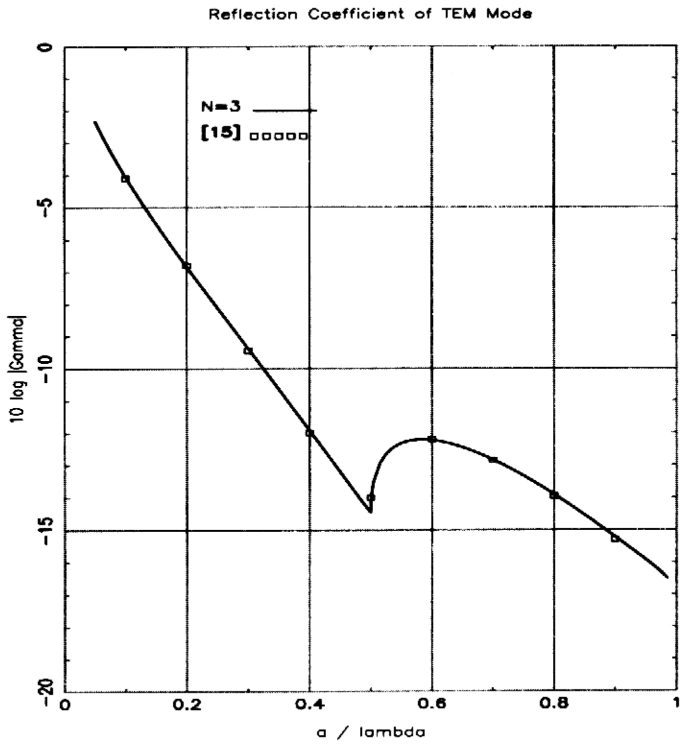

Figure 2: Amplitude of the reflection coefficient [dB] as a function of a/λ, where $2a$ is the distance between the planes of the waveguide and λ is the wavelength. Solid line - our results with 3 MM expansion functions, squares - result of ref. [15].

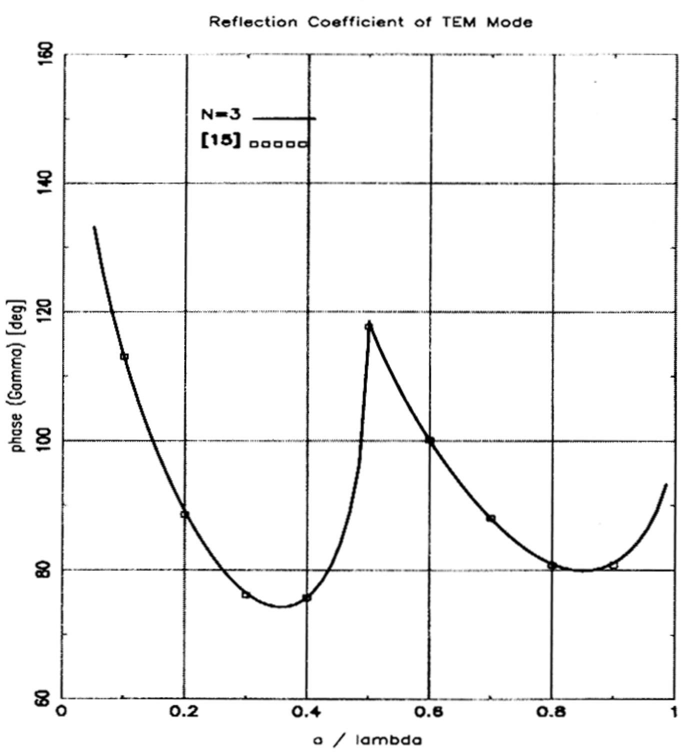

Figure 3: Phase of the reflection coefficient [deg] as a function of a/λ, where $2a$ is the distance between the planes of the waveguide and λ is the wavelength. Solid line - our results with 3 MM expansion functions, squares - result of ref. [15].

5 Appendix

e_{j0} is given by

$$e_{j0} = \int_{-a}^{a} e_j(x)dx = 2\int_0^a \cos^{\alpha_j}(\frac{\pi x}{2a})dx = 2\frac{a\Gamma\left[(1+\alpha_j)/2\right]}{\sqrt{\pi}\Gamma\left(1+\alpha_j/2\right)} \quad (29)$$

$\overline{e}_{jm}$ is given by

$$\begin{aligned} \overline{e}_{jn} &= \int_{-a}^{a} e_j(x)\cos\frac{n\pi x}{a}dx = 2\int_0^a \cos^{\alpha_j}(\frac{\pi x}{2a})\cos\frac{n\pi x}{a}dx \quad (30) \\ &= 4\frac{a}{\pi}\int_0^{\pi/2}\cos^{\alpha_j}(t)\cos(2nt)dt = 4\frac{a}{\pi}I_c(\alpha_j, 2n) \end{aligned}$$

$\tilde{e}_j(k_x)$ is given by

$$\begin{aligned} \tilde{e}_j(k_x) &= \int_{-a}^{a} e_j(x)\exp(-jk_x x)dx = 2\int_0^a \cos^{\alpha_j}(\frac{\pi x}{2a})\cos(k_x x)dx \quad (31) \\ &= 4\frac{a}{\pi}\int_0^{\pi/2}\cos^{\alpha_j}(t)\cos(k_x\frac{2a}{\pi}t)dt = 4\frac{a}{\pi}I_c(\alpha_j, k_x\frac{2a}{\pi}) \end{aligned}$$

and $I_c(a,k)$ is defined by

$$I_c(a,k) = \int_0^{\pi/2}\cos^a(t)\cos(kt)dt = \frac{2^{-1-a}\pi\Gamma(1+a)}{\Gamma\left(\frac{2+a-k}{2}\right)\Gamma\left(\frac{2+a+k}{2}\right)} \quad (32)$$

[19].

6 Acknowledgements

We thank Prof. S. Shtrikman from the Electronics Department of the Weizmann Institute of Science for his great help and fruitfull discussions.

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