Combination of Asymptotic Phase Basis Functions and Matrix Interpolation Method for Fast Analysis of Monostatic RCS

Yuejin Zhang [1], Dechang Huang [1], Jiaqi Chen [2]

[1] School of Information Engineering
East China Jiaotong University, Nanchang, 330013, China
zyjecjtu@foxmail.com, huangdechang2006@126.com

[2] College of Computer and Information
Hohai University, Nanjing, 210098, China
cjq19840130@163.com

Abstract — The combination of asymptotic phase basis functions and matrix impedance method is proposed and used for fast computation of monostatic scattering from electrically large object. Since asymptotic phase (AP) basis function can be defined on large patches, less number of unknowns is required than that when using traditional Rao-Wilton-Glisson (RWG) vector basis function. In order to efficiently compute electromagnetic scattering, the flexible general minimal residual (FGMRES) iterative solver is applied to compute the coefficients of the basis functions and the sparse approximate inversion (SAI) preconditioning technique is used to accelerate the iterative solver. However, the impedance matrix varies with incident angles, resulting in significant computation time cost for construction of impedance and SAI preconditioning matrices. This difficulty can be alleviated by using the model-based parameter estimation (MBPE) technique. Both the impedance and SAI preconditioning matrices are interpolated at intermediate angles over a relatively large angular band with rational function interpolation method. Numerical results demonstrate that this method is efficient for monostatic RCS calculation with high accuracy.

Index terms — Interpolation, linear phase basis function, preconditioning technique, monostatic RCS, electromagnetic scattering

I. INTRODUCTION

Electromagnetic wave scattering problems address the physical issue of detecting the diffraction pattern of the electromagnetic radiation scattered from a large and complex body when illuminated by an incident incoming wave. A good understanding of these phenomena is crucial to radar cross section (RCS) calculation, antenna design, electromagnetic compatibility, and so on. All these simulations are very demanding in terms of computer resources, and require efficient numerical methods to compute an approximate solution of Maxwell's equations. Using the equivalence principle, Maxwell's equations can be recast in the form of integral equations that relate the electric and magnetic fields to the equivalent electric and magnetic currents on the surface of the object. Amongst integral formulations, the surface integral equation (SIE) is widely used for electromagnetic wave scattering problems as it can handle the most general geometries. The matrix associated with the resulting linear systems is large, dense, complex and non-Hermitian [1]. It is basically impractical to solve SIE matrix equations using direct methods because they have a memory requirement of $O(N^2)$, where N refers to the number of unknowns. This difficulty can be circumvented by use of iterative methods, and the required matrix-vector product operation can be efficiently evaluated by multilevel fast multipole algorithm (MLFMA) [2, 3]. The use of MLFMA reduces the memory requirement to $O(N\log N)$ and the computational complexity of per-iteration to $O(N\log N)$.

Generally, the scattering of arbitrary metallic object can be accurately computed by Rao-Wilton-Glisson (RWG) basis function and MoM-MLFMA. Using traditional RWG basis functions, the required number of unknowns is on the order of 100 per square wavelength making electrically large problems impractical [20]. For large smooth objects, the rapid spatial

Submitted On: Apr. 27, 2012
Accepted On: Dec. 25, 2012

variation in the current is due to phase variations rather than magnitude variations. By using the asymptotic phase (AP) basis functions [5-8], drastically computation time can be reduced for large and smooth bodies. However, the AP based impedance matrix varies with incident angles, resulting in significant computation time cost for impedance matrix construction for monostatic calculations. This can be computationally prohibitive despite the increased power of the present generation of computers.

Since the LP-RWG based impedance matrix is not constant for monostatic RCS computation, traditional current interpolation techniques [15-19] are not suitable for fast angular sweep. To efficiently obtain the monostatic RCS using AP-RWG basis function, the impedance matrix interpolation method can be applied to avoid the construction of impedance matrices repeatedly [11-13]. MBPE is the abbreviation of model-based parameter estimation and the rational function approximation is used in MBPE. Using integral equation and moment method to compute the scattering, the elements of impedance matrix are calculated by integral of Green's function. Since the Green's function takes the form of exponential function which is easily to be approximated by rational function, the MBPE could be able to perform good results in impedance matrix interpolation.

Interpolating impedance matrix is able to save much time for constructing impedance matrix but can do nothing for iterative solution repeatedly. Using SAI preconditioning method [9,10] can accelerate iterative solution but increases large time for constructing SAI matrices. Thus, new method is required to circumvent this difficulty. Due to SAI matrix is an approximate inverse of impedance matrix, it is still a continuous function of angle. Moreover, inaccurate preconditioning matrix can not impact the precise of linear system. Consequently, using interpolation technique is a good way to accelerate the construction of SAI matrices. In this paper, the combination of the impedance matrix interpolation and the preconditioning matrix interpolation is proposed to efficient computation of monostatic RCS over broad angular band.

The remainder of this paper is organized as follows. Section II demonstrates the theory and formulation of asymptotic phase basis function. Impedance and SAI preconditioning matrix interpolation technique is discussed in section III. Numerical experiments of several geometries are presented to demonstrate the efficiency of this proposed method in Section IV. Conclusions are provided in Section V.

II. FORMULATIONS OF INTEGRAL EQUATIONS WITH ASYMPTOTIC PHASE BASIS

For electromagnetic scattering from perfect electrical conductor (PEC), the SIE includes electric field integral equation (EFIE) and magnetic field integral equation (MFIE). In order for avoiding resonance problem, the combination form of EFIE and MFIE which names combined field integral equations (CFIE) is widely used for closed structure [4]. The CFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [20]. The resulting linear systems from CFIE formulation after Galerkin's testing are briefly outlined as follows:

$$\sum_{n=1}^{N} Z_{mn} a_n = V_m, \qquad \mathrm{m} = 1,2,\ldots,N \tag{1}$$

where Z_{mn} is the element of the impedance matrix. V_m is the element of the right hand side.

$$Z_{mn}^{EFIE} = jk\eta \int_{S_m} \mathbf{\Lambda}_m \int_{S_n} \left(\overline{\overline{\mathbf{I}}} + \frac{\nabla\nabla}{k^2} \right) G(\mathbf{r},\mathbf{r}')\,\mathbf{\Lambda}_n d\mathbf{r}' d\mathbf{r}$$

$$Z_{mn}^{MFIE} = \frac{1}{2}\int_{S_m} \mathbf{\Lambda}_m \mathbf{\Lambda}_n d\mathbf{r} - \int_{S_m} \mathbf{\Lambda}_m \cdot \hat{\mathbf{n}} \times \nabla \times \int_{S_n} G(\mathbf{r},\mathbf{r}')\,\mathbf{\Lambda}_n d\mathbf{r}' d\mathbf{r}$$

$$V_m = \int_{S_m} \mathbf{\Lambda}_m \cdot \left[\alpha \frac{\mathbf{E}^{inc}}{\eta} + (1-\alpha)\,\hat{\mathbf{n}} \times \mathbf{H}^{inc} \right] d\mathbf{r}$$

Here G($\mathbf{r}$, $\mathbf{r}'$) refers to the Green's function in free space and $\{a_n\}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. Also, as usual, $\mathbf{r}$ and $\mathbf{r}'$ denote the observation and source point locations. $\mathbf{E}^{inc}$(r) and $\mathbf{H}^{inc}$(r) is the incident excitation plane wave, and η and k denote the free space impendence and wave number, respectively. Once the matrix equation (1) is solved by numerical matrix equation solvers, the expansion coefficients $\{a_n\}$ can be used to calculate the scattered field and RCS. In the following, we use $\mathbf{A}$ to denote the coefficient matrix in equation (1), $\mathbf{x} = \{a_n\}$, and $\mathbf{b} = \{V_m\}$ for simplicity. Then, the CFIE matrix equation (1) can be symbolically rewritten as:

$$\mathbf{Ax} = \mathbf{b} \tag{2}$$

Following the conventional MoM formulation, the induced current $\mathbf{J}$ is expanded in terms of subsectional basis functions $\mathbf{f}_n$. On the smooth regions of S, where the induced surface currents present an asymptotic behaviour, the current density is expanded in terms of the so-called linearly phased Rao–Wilton–Glisson (LP-RWG) vector basis functions proposed in [5-8], whose formulation is included here for the sake of completeness:

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \mathbf{\Lambda}_n^+ e^{-j\mathbf{k}_n^+\cdot(\boldsymbol{\rho}_n^+ - \boldsymbol{\rho}_{nc}^+)} & \mathbf{r} \in T_n^+ \\ \mathbf{\Lambda}_n^- e^{-j\mathbf{k}_n^-\cdot(\boldsymbol{\rho}_n^- - \boldsymbol{\rho}_{nc}^-)} & \mathbf{r} \in T_n^- \\ 0 & otherwise \end{cases} \quad (3)$$

where $\mathbf{\Lambda}_n^{\pm} = \pm\dfrac{l_n}{2A_n^{\pm}}\boldsymbol{\rho}_n^{\pm}$ and $\mathbf{\Lambda}$ is the RWG basis function. l_n is the length of the common edge to the triangles $T_n^{\pm}$ conforming the basis function, $A_n^{\pm}$ is the area of each triangle, $\boldsymbol{\rho}_n^{\pm}$ is the corresponding vector from the free vertex of $T_n^{\pm}$ to a point $\mathbf{r}$ on the triangle, and $\boldsymbol{\rho}_{nc}^{\pm}$ is the vector from the free vertex of triangle $T_n^{\pm}$ to the midpoint of the common edge $\mathbf{r}_{nc}$. Finally, $\mathbf{k}_n$ is the vector wavenumber associated to the phase of the current density on the function. Compared with traditional RWG basis functions, drastic reduction of the required number of unknowns can be achieved by using the linearly-phased RWG basis functions.

To solve the equation (2) by an iterative method, the matrix-vector products are needed at each iteration step. Physically, a matrix-vector product corresponds to one cycle of iterations between the basis functions. The basic idea of the fast multipole method (FMM) is to convert the interaction of element-to-element to the interaction of group-to-group. Here a group includes the elements residing in a spatial box. The mathematical foundation of the FMM is the addition theorem for the scalar Green's function in free space. Using the FMM, the matrix-vector product $\mathbf{Ax}$ can be written as:

$$\mathbf{Ax} = \mathbf{A}_N\mathbf{x} + \mathbf{A}_F\mathbf{x} \quad (4)$$

Here $\mathbf{A}_N$ is the near part of $\mathbf{A}$ and $\mathbf{A}_F$ is the far part of $\mathbf{A}$.

In the FMM, the calculation of matrix elements in $\mathbf{A}_N$ remains the same as in the MoM procedure. However, those elements in $\mathbf{A}_F$ are not explicitly computed and stored. Hence they are not numerically available in the FMM. It has been shown that the operation complexity of FMM to perform $\mathbf{Ax}$ is $O(N^{1.5})$. If the FMM is implemented in multilevel, the total cost can be reduced further to $O(N\log N)$ [2,3].

III. IMPEDANCE AND PRECONDITIONING MATRIX INTERPOLATION METHOD

The methodology on how to efficient calculation of monostatic scattering with asymptotic phase basis function is discussed in this section. When asymptotic phase basis is applied for construction of the impedance matrix, each element of the matrix is not constant over the interested angular band. Repeated impedance matrix construction cost plenty of time. Accordingly, interpolation method is used to accelerate monostatic scattering calculation. First of all, the impedance matrix interpolation method is introduced. Then SAI preconditioning matrix interpolation method is proposed. Finally, a hybrid method combines both of the two interpolation methods is discussed, which make a good way to the efficient analysis of wide-band scattering.

Using method of moment, the current density at certain angle can be obtained by solving equation (2). For a wide angular band, we have to repeat this procedure at a set of discrete frequencies to get the monostatic response. For structures with a large electrically scale, the required solution is highly computationally expensive. In order to reduce the matrix filling time of equation (2), the MBPE interpolation is employed to obtain the impedance matrix over a wide band.

$$Z^{ij}(f) = \frac{c_0 + c_1 f + \cdots + c_p f^p}{1 + d_1 f + \cdots + d_q f^q} \quad (5)$$

where Z^{ij} denotes the element of the impedance matrix $\mathbf{Z}$, the superscripts i and j are the serial number of row and column, respectively. c_0, …, c_p and d_1, …, d_q are coefficients determined by the solution of following linear equations:

$$\begin{bmatrix} 1 & \cdots & f_1^p & -Z_1^{ij} f_1 & \cdots & -Z_1^{ij} f_1^q \\ \vdots & & \vdots & \vdots & & \vdots \\ 1 & \cdots & f_{p+1}^p & -Z_{p+1}^{ij} f_{p+1} & \cdots & -Z_{p+1}^{ij} f_{p+1}^q \\ 1 & \cdots & f_{p+2}^p & -Z_{p+2}^{ij} f_{p+2} & \cdots & -Z_{p+2}^{ij} f_{p+2}^q \\ \vdots & & \vdots & \vdots & & \vdots \\ 1 & \cdots & f_{p+q+1}^p & -Z_{p+q+1}^{ij} f_{p+q+1} & \cdots & -Z_{p+q+1}^{ij} f_{p+q+1}^q \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ \vdots \\ c_p \\ d_1 \\ \vdots \\ d_q \end{bmatrix} = \begin{bmatrix} Z_1^{ij} \\ \vdots \\ Z_{p+1}^{ij} \\ Z_{p+2}^{ij} \\ \vdots \\ Z_{p+q+1}^{ij} \end{bmatrix} \quad (6)$$

Equation (6) can be solved by a direct matrix inversion, since the order of the matrix $p + q + 1$ is low in this case. To accelerate the solution of (2), the octree structure based fast multiple method [2,3] is applied to MoM. Then equation (2) can be rewritten as

$$(\mathbf{Z}_{\text{near}} + \mathbf{Z}_{\text{far}})\cdot\mathbf{I} = \mathbf{V} \quad (7)$$

where $\mathbf{Z}_{\text{near}}$ is the near field impedance matrix evaluated by the MoM and $\mathbf{Z}_{\text{far}}$ is the far field part evaluated by the MLFMA. The set of $\mathbf{Z}_{\text{near}}$

are interpolated by the MBPE while $\mathbf{Z}_{far}$ are evaluated by the MLFMA method efficiently.

Although the impedance matrix interpolation method can avoid filling impedance matrix repeatedly, iterative solution of matrix equations is still required at each angular point. Thus, computational efficiency is challenged by ill-conditioned linear equations. Preconditioning technique, such as SAI, can greatly improve condition number of the system so as to accelerate the convergence of the iterative solver. The formulation of preconditioning technique can be described by

$$\mathbf{M}\cdot\mathbf{ZI} = \mathbf{M}\cdot\mathbf{V} \tag{8}$$

where $\mathbf{M}$ is the SAI preconditioning matrix in this paper, the purpose of preconditioning is to make the preconditioned matrix $\mathbf{MZ}$ better conditioned than matrix $\mathbf{Z}$. Generally, $\mathbf{Z}_{near}$ is used as the basis for constructing preconditioner. Thus, it suffices to solve a single problem for each minimum group at the lowest level as in [9, 10]. Since the operation on all edges of the same group is done at a time, it can reduce the construction of SAI significantly. However, it is still time-consuming to construct SAI preconditioning matrix repeatedly at each frequency point. According to the theory of SAI, it is apparent that preconditioning matrix is a sparse matrix for computation and storage, which makes the utilization of interpolation method possible. Therefore, matrix interpolation by the MBPE can be transplanted to interpolate SAI preconditioning matrix.

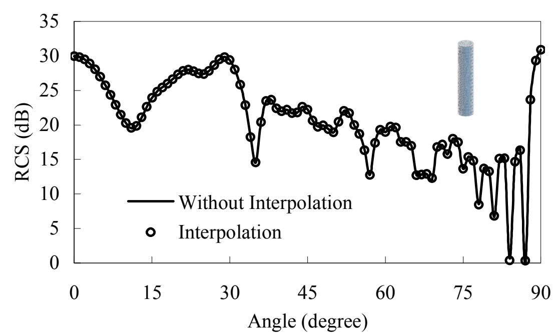
$$M^{ij}(f) = \frac{c_0 + c_1 f + \cdots + c_p f^p}{1 + d_1 f + \cdots + d_q f^q} \tag{9}$$

Also the unknown coefficients of the numerator and denominator are uniquely determined by matching the $p + q + 1$ sampling M^{ij} as equation (6).

IV. NUMERICAL RESULTS

In this section, a number of numerical results are presented to demonstrate the accuracy and efficiency of the preconditioning matrix interpolation method for fast calculation of RCS over wide band. The flexible general minimal residual (FGMRES) [21,22] algorithm is applied to solve linear systems. The dimension size of Krylov subspace is set to be 30 for outer iteration and the dimension is set to be 10 for inner iteration. The tolerance of inner iteration is 0.1 in this paper. All experiments are conducted on an Intel Core(TM) II Duo with 3.45 GB local memory and run at 2.40 GHz in single precision. The iteration process is terminated when the 2-norm residual error is reduced by 10^{-3}, and the limit of the maximum number of iterations is set as 1000.

As well known, the impedance matrix with traditional RWG basis is constant for monostatic scattering computation. Using traditional RWG basis functions, the required number of unknowns is on the order of 100 per square wavelength making electrically large problems impractical. In order to alleviate this difficulty, the asymptotic phase RWG basis is used for construction impedance matrix. However, the impedance matrix is not constant over the angular band. Fortunately, the element of impedance matrix is a trigonometric function of the incident angle and can be interpolated by MBPE successfully. Three geometries are applied to illustrate the performance of our method. They consist of a metallic cylinder (10λ ×4λ) with 5279 unknowns, a PEC Cube (15λ× 15λ×15λ) with 7137 unknowns, and a metallic plane with 10968 unknowns. The incident wave is the plane wave with vertical polarization. That is, if the incident angle is θ and φ, the vector of incident direction is (-sinφ, cosφ). The frequency of incident wave is 300 MHz for both cylinder and cube. The frequency is 600MHz for PEC plane.



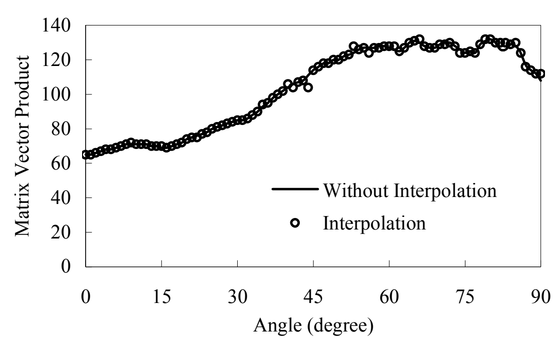


Fig. 1. Cylinder: (a) RCS for VV-polarization, 300 MHz; (b) Number of matrix vector products.

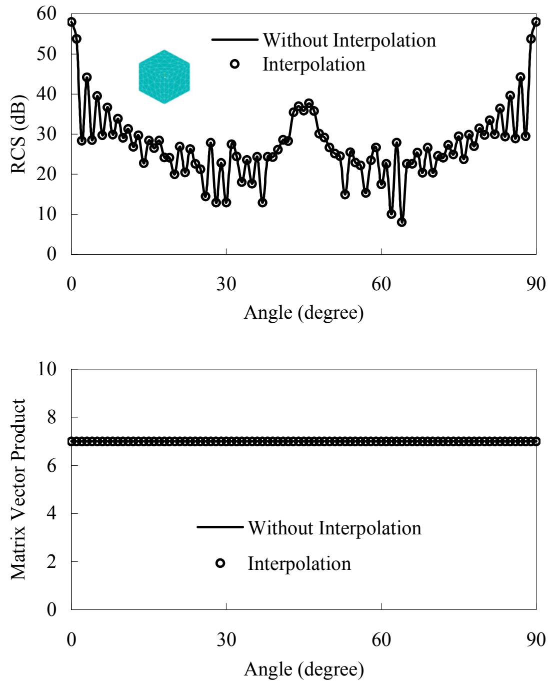


Fig. 2. Cube: (a) RCS for VV-polarization, 300 MHz; (b) Number of matrix vector products.

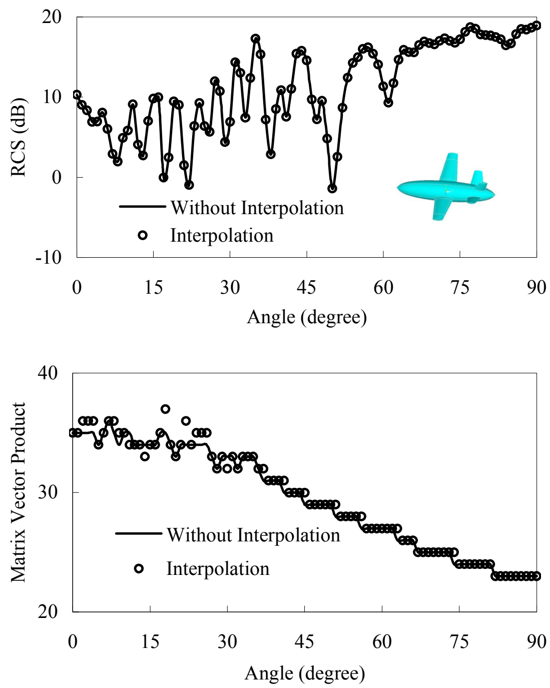


Fig. 3. Plane: (a) RCS for VV-polarization, 600 MHz; (b) Number of matrix vector products.

In our simulations, 6 uniform samples are required in the impedance matrix interpolation method for these three examples. As shown in Fig. 1(a), Fig. 2(a) and Fig. 3(a), it can be seen that the impedance matrix interpolation method is an accurate method and the impedance matrix interpolation method is more efficient than the traditional method. As shown in Fig. 1(b), Fig. 2(b) and Fig. 3(b), there is no difference for the number of the matrix-vector production when the SAI matrices are interpolated. It can be concluded that almost the same convergence can be obtained whether the SAI matrix is constructed by interpolation method or not.

Since the elements of the impedance matrix is a simple function of the angle, only few sampling angles are needed for a wide angular band. That is, only few solution processes of the linear system constructed by method of moment are needed for a wide angular band. This property is also valid for frequency sweep. In this paper, the angular sweep is focused on and only 6 uniform samples are computed for every example. The number 6 is an experience parameter. Generally speaking, interpolation results are inaccurate. In this paper, the interpolation method is used to interpolating the impedance matrix and preconditioning matrix. Definitely, there is some difference between the exact results and interpolation results. The difference will influence the surface electrical current distribution. However, the RCS is the logarithmic function of current. Accordingly, the difference will not impact the RCS greatly.

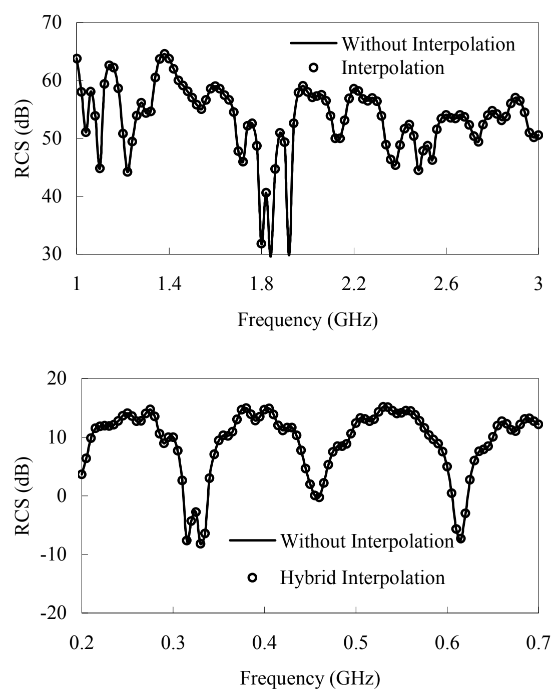


Fig. 4. RCS for frequency sweep: (a) Cube, 1 GHz ~ 3 GHz; (b) Plane, 200 MHz ~ 700 MHz.

In order to better understanding the proposed method, the results of frequency sweep are given from Fig. 4. Since this paper focuses on the monostatic RCS, only the RCS of the last two examples (Cube and Plane) are computed with respect to frequency.

From Fig. 4, it is concluded that the proposed method can also be used for frequency sweep. The interpolation results are almost the same as the reference results. Either angle sweep or frequency sweep, the elements of the impedance matrix are calculated by integral of Green's function. Since the Green's function takes the form of exponential function which is easily to be approximated by rational function, the MBPE could be able to perform good results in impedance matrix interpolation.

When the method of moment is used for the computation of radar cross section, the main problem is to solve the linear system $\mathbf{Ax} = \mathbf{b}$. For computation of monostatic RCS, especially asymptotic phase basis is used, the impedance matrix $\mathbf{A}$ will be modified according to the incident angle. The interpolation method can not be applied for induced current $\mathbf{x}$. A good way for better efficient simulation is to interpolating the impedance matrix. That is, cost more memories to achieve less computation time. From the results of this paper, MBPE performs well for interpolating both impedance matrix and pre-conditioner matrix.

Table 1: Construction time for impedance matrix (Time: second)

Object	**Unknown**	**Impedance without Interpolation**	**Impedance with Interpolation**
Cylinder	4279	7211	628
Cube	7137	9980	1041
Plane	10968	63742	7734

Table 2: Construction time for SAI preconditioning matrix (Time: second)

Object	**Unknown**	**SAI without Interpolation**	**SAI with Interpolation**
Cylinder	4279	1527	143
Cube	7137	543	58
Plane	10968	19080	2045

Table 3: Total solution time for fast frequency sweep (Time: second)

Object	**Unknown**	**Frequency**	**Angular Band**	**Without Interpolation**	**Hybrid Interpolation**
Cylinder	4279	300 MHz	0~90°	10423	3954
Cube	7137	300 MHz	0~90°	12654	3872
Plane	10968	600 MHz	0~90°	92718	19877

As shown in Tab.1, the construction time of near field impedance matrices are compared between traditional method and interpolation method for these three examples. As shown in Tab.2, the construction time of SAI matrices are compared between traditional method and interpolation method for these three examples. It can be found that the computational cost of the interpolation method is much less. The main cost of impedance and SAI interpolation method is the construction time and memory requirement for those angular sampling points. The memory requirement to save samples of near-field impedance matrices and preconditioning matrices is 147 MB for the first example, 254 MB for the second example and 1.34GB for these three examples. As shown in Tab. 3, the total computation time is compared for the frequency sweep. "*Without Interpolation*" means impedance matrix constructed directly and SAI pre-conditioner constructed directly. "*Hybrid Interpolation*" means impedance matrix interpolation and SAI preconditioning method interpolation with the rational interpolation method. It can be also found by comparison that the large calculation time can be saved when the hybrid interpolation technique is used.

V. CONCLUSION

In this paper, the asymptotic phase basis function and impedance matrix interpolation method is combined together to analyze the monostatic scattering from electrically large objects over a wide angular band. The impedance matrix is approximately constructed by MBPE method at each incident angle. The MLFMA and Krylov subspace iterative solver are used and the SAI is used to accelerate the convergence. In order to further reduce the computation time of constructing SAI preconditioning matrix, the MBPE technique is used for construction of SAI matrices at each angle. Numerical experiments demonstrate that our proposed hybrid interpolation method is more efficient when compared with the traditional method for electromagnetic scattering from the electrically large objects.

ACKNOWLEDGEMENTS

The authors would like to thank the support of National Science Foundation of China (No: 61261005, No: 61061002), Youth Science Foundation of Jiangxi (No: 20122BAB211018), Youth Science Foundation of Education Department of Jiangxi (No: GJJ11112) and East China Jiaotong University Foundation (No: 11XX01).

References

[1] R. F. Harrington, *Field Computation by Moment Methods*, Malabar, Fla.: R. E. Krieger, 1968.

[2] W. C. Chew, J. M. Jin, E. Michielssen, and J. M. Song, *Fast and Efficient Algorithms in Computational Electromagnetics*. Boston, MA: Artech House, 2001.

[3] J. M. Song, C. C. Lu and W. C. Chew, “Multilevel Fast Multipole Algorithm for Electromagnetic Scattering by Large Complex Objects,” *IEEE Trans. Antennas Propagat.*, vol. 45, no. 10, pp. 1488-1493, 1997.

[4] J. R. Mautz and R. F. Harrington, "H-field, E-field, and Combined Field Solution for Conducting Bodies of Revolution," Arch. Elecktron. Übertragungstech (AEÜ), vol. 32, pp. 157-164, 1978.

[5] Z. Nie, S. Yan, S. He, and J. Hu, “On the Basis Functions with Travelling Wave Phase Factor for Efficient Analysis of Scattering from Electrically Large Targets,” *Progress In Electromagnetics Research*, PIER 85, pp. 83-114, 2008.

[6] Su Yan, Shiquan He, Zaiping Nie, and Jun Hu, “Calculating the Wide Band Responses from Metallic Objects by Employing the Phase Extracted Basis Functions,” *IEEE International Symposium on Antennas and Propagations*, 2008.

[7] John Robert Gulick, “A Combination of Rao-Wilton-Glisson and Asymptotic Phase Basic Functions to Solve the Electric and Magnetic Field Integral Equations,” Master of science Department of Electrical and Computer Engineering, 2001.

[8] I. G.-T., J. M. Taboada, J. L. R, F. Obelleiro, and L. Landesa, “Efficient Asymptotic-Phase Modeling of the Induced Currents in the Fast Multipole Method,” *Microwave and Optical Technology Letters*, vol. 48, no. 8, Aug. 2006.

[9] M. Benzi and M. Tuma, “A Sparse Approximate Inverse Preconditioner for Nonsymmetric Linear Systems,” *SIAM Journal on Scientific Computing*, vol. 19, pp. 968-994, 1998.

[10] P. L. Rui and R. S. Chen, “An Efficient Sparse Approximate Inverse Preconditioning for FMM Implementation,” *Micro. Opt. Tech. Lett.*, vol. 49, no. 7, pp. 1746-1750, 2007.

[11] E. H. Newman, “Generation of Wide-band Data from the Method of Moments by Interpolating the Impedance Matrix,” *IEEE Trans. Antennas Propagat*, vol. 36, no. 12, pp. 1820-1824, 1988.

[12] K. L. Virga and Y. R. Samii, “Efficient Wide-Band Evaluation of Mobile Communications Antennas using Z or Y Matrix Interpolation with the Method of Moments,” *IEEE Trans. Antennas Propagat*, vol. 47, no. 1, pp. 65-76, 1999.

[13] X. C. Wei, E. P. Li, “Wide-band EMC Analysis of On-platform Antennas using Impedance-Matrix Interpolation with the Moment of Method-Physical Optics Method,” *IEEE Trans. Electromagnetic Compatibility*, vol. 45, no. 3, pp. 552-556, 2003.

[14] G. J. Burke, E. K. Miller, S. Chakrabarthi, and K. Demarest, “Using Model-Based Parameter Estimation to Increase the Efficiency of Computing Electromagnetic Transfer Functions,” *IEEE Trans. Magn.*, vol. 25, pp. 2807-2809, July 1989.

[15] C. J. Reddy, M. D. Deshpande, C. R. Cockresll, and F. B. Beck, “Fast RCS Computation Over a Frequency Band using Method of Moments in Conjunction with Asymptotic Waveform Evaluation Technique,” *IEEE Trans. Antennas Propagat*, vol. 46, no. 8, pp. 1229-1233, August 1998.

[16] Y. E. Erdemli, J. Gong, C. J. Reddy, and J. L.Volakis, “Fast RCS Pattern Fill Using AWE Technique,” *IEEE Trans. Antennas Propagat*, vol. 46, no. 11, pp. 1752-1753, November 1998.

[17] R. D. Slong, R. Lee, and J. F. Lee. “Multipoint Galerkin Asymptotic Waveform Evaluation for Model Order Reduction of Frequency Domain

FEM Electromagnetic Radiation Problems," *IEEE Trans. Antennas Propagat*, vol. 49, no. 10, pp. 1504-1513, October 2001.
[18] M. S. Chen, X. L. Wu, W. Sha and Z. X. Huang, "Fast and Accurate Radar Cross-Section Computation over a Broad Frequency Band using the Best Uniform Rational Approximation," *IET Micro. Antennas Propag.*, vol. 2, no. 2, pp. 200-204, 2008.
[19] Z. W. Liu, D. Z. Ding, Z. H. Fan, and R. S. Chen, "Adaptive Sampling Bicubic Spline Interpolation Method for Fast Computation of Monstatic RCS," *Micro. Opt. Tech. Lett.* vol. 50, no. 7, pp. 1851-1857, July 2008.
[20] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic Scattering by Surfaces of Arbitrary Shape," *IEEE Trans. Antennas Propagat*, vol. 30, no. 3, pp. 409-418, 1982.
[21] Y. Saad and M. Schultz, "GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems," *SIAM J. Sci. Stat. Comput.*, vol. 7, no. 3, pp. 856-869, 1986.
[22] Valeria Simoncini, and Daniel B. Szyld, "Flexible Inner-Outer Krylov Subspace Methods," *SIAM J. Numer. Anal.*, vol. 40, no. 6, pp. 2219-2239, 2003.
[23] Z. H. Fan, Z. W. Liu, D. Z. Ding, R. S. Chen, "Preconditioning Matrix Interpolation Technique for Fast Analysis of Scattering over Broad Frequency Band," *IEEE Trans. Antenna and Propagation*, vol. 58, no. 7, pp. 2484-2487, July 2010.
[24] B. M. Kolundzija, M. S. Tasic, D. I. Olcan, D. P. Zoric, S. M. Stevanetic, "Advanced Techniques for Efficient Modeling of Electrically Large Structures on Desktop PCs," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 27, no. 2, pp. 123-131, February 2012.
[25] Z. N. Jiang, R. S. Chen, Z. H. Fan, S. G. Shen, X. Q. Hu, "Efficient Multilevel Compressed Block Decomposition for Large-Scale Electromagnetic Problems Using Asymptotic Phasefront Extraction," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 11, pp. 876-885, November 2011.
[26] Z. Liu, R. Chen, J. Chen, Z. Fan, "Using Adaptive Cross Approximation for Efficient Calculation of Monostatic Scattering with Multiple Incident Angles," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 4, pp. 325-333, April 2011.
[27] Z. N. Jiang, Z. H. Fan, D. Z. Ding, R. S. Chen, K. W. Leung, "Preconditioned MDA-SVD-MLFMA for Analysis of Multi-scale Problems," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 25, no. 11, pp. 914-925, November 2010.
[28] R. J. Burkholder, Ç. Tokgöz, C. J. Reddy, W. O. Coburn, "Iterative Physical Optics for Radar Scattering Predictions," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 24, no. 2, pp. 241-258, April 2009.
[29] R. Mittra, "Characteristic Basis Function Method (CBFM)—an Iteration-Free Domain Decomposition Approach in Computational Electromagnetics," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 24, no. 2, pp. 204-223, April 2009.

Yuejin Zhang was born in Hubei, China in 1978. He received B.S. degree in Communication Engineering from PLA Second Artillery Command College in 2002, M.S. degree in Computer Application Technology from East China Jiaotong University in 2008, respectively. He is currently working at School of Information Engineering, East China Jiaotong University. His research interests focus on the theory of electromagnetic scattering and inverse scattering.

Dechang Huang was born in Jiangxi, China in 1982. He received B. S. and M. S. degree in Communication Engineering from East China Jiaotong University in 2006 and 2011, respectively. He is currently working at School of Information Engineering, East China Jiaotong University. His research interests focus on the high frequency circuit and electromagnetics.

Jiaqi Chen was born in Gansu, China. He received the B.S. degrees in communication engineering from Nanjing University of Science and Technology (NUST), in 2005, and Ph.D. degree in Nanjing University of Science & Technology in 2012, respectively. He was with the Center for Sensor systems (ZESS), University of Siegen, Siegen, Germany, as a visiting scholar in 2009. He is currently working at Computer and Information College, Hohai University. His research interests include computational electromagnetics, SAR imaging and precipitation radar.