A Finite Difference Time Domain Technique for the Simulation of Moving Objects

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Abstract - Normally in the finite difference time domain (FDTD) technique, static objects are modeled in a time domain simulation as a field propagates around them, and possibly inside these objects. This paper illustrates a method in which EM fields and a moving object can be modeled using the FDTD technique. By using a technique of dielectric approximation and intermediate step field movement, it is possible to model the movement of objects by the FDTD technique. This paper illustrates these principles in a one-dimensional domain.

1. Introduction

The finite difference time domain technique is a well-known method for modeling Maxwell's equations in time. The technique divides the intended geometry into a spatial grid and solves for the electric and magnetic field components at discrete points. The material at each point on the spatial grid is characterized by its properties (permittivity, permeability, conductivity) in each of the Cartesian directions. From these material characteristics, the coefficients needed to update the fields at each time step can be computed. The technique used in this paper is derived from the algorithm defined by Yee [1]. This approach places all of the magnetic components one-half spatial step in front of the electric components, as well as computing the magnetic fields at one half time step after the electric fields.

In this example using a dielectric material, each spacial cell extends from one E_z point to the next. Component properties are calculated at these points and are used to update the appropriate fields. This spatial setup lends itself well to the approximations needed in order to simulate moving objects.

2. Physical Approximations

In FDTD the geometry is divided into a spatial mesh, which at each point on this mesh, has its own set of coefficients depending upon the material the point lies within. For example a point that lies solely within the interior of an object would have coefficients related to the conductivity and permittivity of that object. However, when the point represents the boundary of that object, the point is assigned an average of the properties of the two objects. Likewise if the object is a magnetic material, the same ideas would apply.

In order to model an object moving at a constant speed, the coefficients on the boundaries must be constantly updated as the object moves. On the forward boundary cell of the moving object, coefficients are modified at each step until it becomes an inside cell. Likewise on the back boundary cell, coefficients are modified at each step until it becomes free space (or the background material at that time). When the coefficients on the front and back edges have been modified until they are completely dielectric or free space, the boundaries have moved one spatial cell in the direction of movement as shown in Figure 1.

This is a simple example showing the fundamental approach. Characteristics such as conductivity can be approximated in the same way.

3. Field Approximations

Modifying the coefficients as the object moves provides a good foundation, however, it neglects the propensity of the fields to move with the object. This tendency of the fields to move depends directly on the material of the moving object itself. In order to model this, an assumption is made to separate the field in the region of the moving object. For non-magnetic material, only the E fields need to be split into two components, $E_{background}$ and E_{object}. The E_{object} is the portion that exists within the moving object and moves with it, such that:

$$E_{object} = E_{total} - E_{background} \qquad (1)$$

While E_{object} exists within the object itself, $E_{background}$ exists everywhere including inside of the object region. The two fields combined must be the same as the total field, these two fields can be related as:

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$$E_{background} = \frac{1}{\varepsilon_r} E_{total} \quad (2a) \quad , \quad E_{object} = \frac{\varepsilon_r - 1}{\varepsilon_r} E_{total} \quad (2b)$$

By splitting the field into these two components, the contributions due to the moving field can be calculated. Then by re-interpolating the field at the grid points as it is moved, a better representation for the actual E field can be used for the updating of the H fields. The fields for E_{object} are shifted in space by the appropriate amount for one time step and are re-interpolated at the grid points. So that the total E field at time step n used when updating the H field becomes:

$$E_{total}^n = E_{background}^n + \left[\frac{E_{object}^n + E_{shifted-object}^n}{2} \right] \quad (3)$$

The averaging provides a good balance for what the E field is when updating H and what it will be when updating E at the next time step. The $E_{shifted-object}^n$ field at each grid point is calculated using a first order LaGrange approximation where *m* is the distance the object moves per time step.

$$E_{shifted-object_p}^n = \left(\frac{\Delta x - m}{\Delta x} \right) E_{object_p}^n + \left(\frac{m}{\Delta x} \right) E_{object_{p-1}}^n \quad (4)$$

This equation assumes movement in only one cardinal direction and in the direction of increasing index for the grid points (denoted by "p"). In a 2D or 3D environment the same equation would be applied to all components of E, separately.
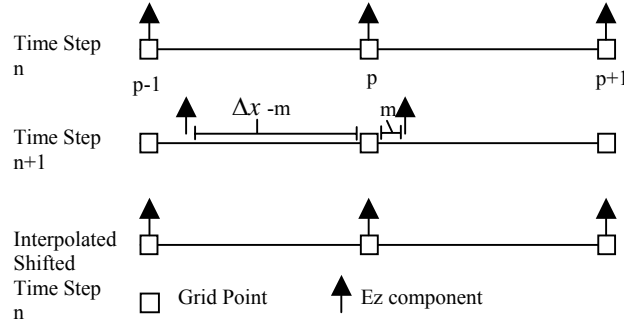

Figure 2. Interpolation of shifted fields.

Since all the materials in this paper are assumed to have a permeability of 1, it is not necessary to split or adjust the H fields inside the object. If the objects used contained materials with magnetic susceptibility ($\mu_r > 1$), a similar procedure to that outlined for the E fields and coefficients would be used.

4. Computational Implementation

Since the movement of the object is assumed to be constant, computational time can be saved by pre-calculating constants, such as the change per time step of each coefficient. With the object moving in one direction, only the coefficients at the front and rear boundaries of the object will change. By keeping track of where these boundaries are and knowing how much they change per time step, modification of the coefficient array becomes an almost trivial process. The standard 1D updating equations used here are [2]:

$$E_{zi}^{n+1} = Ceze(i)E_{zi}^n + Cezh(i)(H_{xi}^{n+\frac{1}{2}} - H_{xi-1}^{n+\frac{1}{2}}) \quad (5a) \qquad H_{xi}^{n+\frac{1}{2}} = Chxh(i)H_{xi}^{n-\frac{1}{2}} + Chxe(i)(E_{zi}^n - E_{zi-1}^n) \quad (5b)$$

For the background E field, no change in either of the updating equation coefficients is required for outside the object region. However, since the field in the region where object exists is represented by the combination of the object and background fields, a new set of coefficients is required for both fields. There are two coefficient arrays that have to be dealt with for the object E field, internal to the object. These coefficients at node "*i*" are given by:

$$Cezh = \frac{\Delta t}{\varepsilon_0 \left(\varepsilon_r + \frac{\Delta t \sigma_e}{2\varepsilon_0} \right) \Delta x} \quad (6a) \quad , \quad Ceze = \frac{\varepsilon_r - \frac{\Delta t \sigma_e}{2\varepsilon_0}}{\varepsilon_r + \frac{\Delta t \sigma_e}{2\varepsilon_0}} \frac{1}{\Delta x} \quad (6b)$$

In order to account for the split fields, E_{object} and $E_{background}$ will be updated separately. In these updating equations, the E field used is not the total field at that point, but rather the calculated object or background field at the same point in the last time step. Since there are two E fields, but only one H field, the H field used in the updating equations represents the total H field at the spatial location. Since we only need the H field contribution to either one of the split E fields, a second multiplier is added to the *Cezh* coefficient to find its contribution to either E field. For $E_{background}$ this coefficient is 1 for all areas not containing the object. Likewise for the E_{object} field calculation, this coefficient is zero outside the object. For points inside the object this coefficient is a function of the object relative permittivity. For $E_{background}$ inside the object it is $(1/\varepsilon_r)$, and for E_{object} the coefficient becomes $(\varepsilon_r-1)/\varepsilon_r$.

On the forward and backward edges of the moving object the coefficients are not as straightforward. In order to model the object as it moves, dielectric coefficients are slowly built up and removed from the edges. The simplest way to implement this is to pre-calculate the differential change in the coefficients on the edges and simply add or subtract this amount from the appropriate places [3].

Once the object and background E fields are updated, the approximation (discussed in section 3) for the field as it moves can be calculated based on the splitting of the fields and the dielectric approximation. The formula given by (3,4) actually compresses or dilates the waves emanating from the object. This effect represents a Doppler shift in the reflected wave from the moving object.

The H field updating is performed to complete the cycle by using the coefficients at node "i" [2]:

$$Chxh = \frac{\mu_r - \dfrac{\Delta t \sigma_m}{2\mu_0}}{\mu_r + \dfrac{\Delta t \sigma_m}{2\mu_0}}\frac{1}{\Delta x} \ (7a), \quad Chxe = \frac{\Delta t}{\mu_0 \left(\mu_r + \dfrac{\Delta t \sigma_m}{2\mu_0}\right)\Delta x} \ (7b)$$

The next additional step needed is the actual modification of the coefficients on the leading and trailing edges. This is done for both the object and background coefficient arrays due to movement during one time step. After an appropriate number of time steps, it is time to advance the location of the starting and stopping edges of the structure one cell in the direction of movement.

5. Results

In order to verify the proper operation of the method, several simple simulations were examined. For simplicity, the method was implemented in a one-dimensional FDTD code. The domain (shown in Figure 3) consisted of 3000 points with Δx of 0.1m. An object was defined from point 300 to point 700 and was programmed to move in the negative x direction. A plane wave was initially set up in the domain with the position of the leading edge of the Gaussian waveform at 150m and made to propagate towards the object. The total E field was sampled at a point close to the center. The time step used in the simulation was 300 ps in order to improve the stability of the system.

To prove that the technique works effectively, a simple system of measuring the Doppler shift of the reflected wave is used. The simulation excites the domain using a derivative of a Gaussian wave pulse. This type of pulse is used because of its smooth on and off operation and because it has a peak value at a non-zero frequency. By measuring the shift of this peak location, an estimate for the speed of the object can be calculated using a simple Doppler shift formula. As the speed of the object increases towards the speed of light, the relativistic Doppler formula (8) should be used for higher accuracy [4]. For the results shown, the relativistic Doppler formula was used.

$$\frac{\text{shift in wavelength}}{\text{wavelength}} = \sqrt{\frac{1+\frac{V}{C}}{1-\frac{V}{C}}} - 1 \quad (8)$$

Examining the simplest case of a perfectly conducting (PEC) object, the effects can be clearly seen. In order to model the object as a PEC, the coefficients for the object are calculated with a high electric conductivity. Through a process of 4 adaptive Fourier transforms, each increasing in resolution, the location of the respective peaks for the incident and reflected waves are located. Then using the Doppler shift formula and solving for the velocity (V), confirmation that the object is moving at the programmed speed can be provided. Figure 4 shows the relationship between the speed programmed in the simulation and the computed speed via Doppler shift for a moving conductor. The object was modeled as a conductor by assigning it a conductivity of 10^7 Siemens. The reference line refers to where the computed speed would be equal to the programmed speed. Some oscillation around the actual speed begins to happen as the speed decreases due to the finite maximal resolution of the discrete Fourier transform with the given time step. Because of this finite limit of resolution, this will set the lowest speed in which the object can be accurately modeled with the current FDTD simulation parameters.
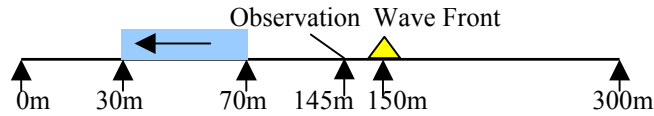
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Figure 3. Domain configuration.

Running the same simulation again for a dielectric object with a relative permittivity of 7.2, as shown in Figure 5, similar results are observed. The measured Doppler shift agrees closely with the actual value in the upper range, while in the lower range below the limit defined by the resolution of the discrete Fourier transform it oscillates around the programmed reference velocity.

6. Conclusions

Through the use of this approach, it is possible to model moving objects using the FDTD technique. The use of both coefficient approximations and field movements to model both the movements effect on objects and on its related fields have proved its ability to predict expected results. There are still many subjects needing to be addressed for the analysis of the data in order to extend this concept to 2D and 3D FDTD analysis.
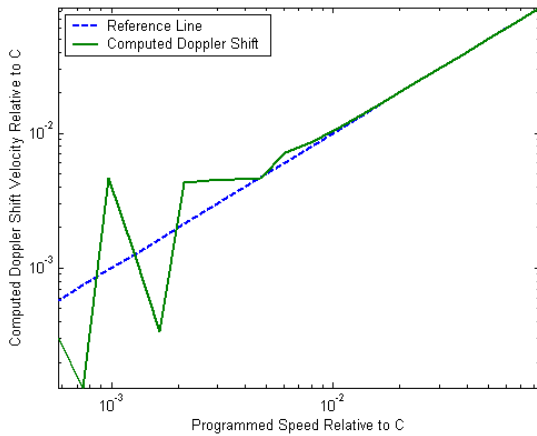

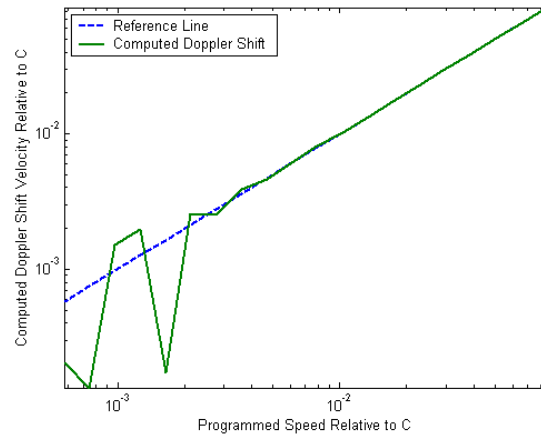Figure 4. Computed Doppler shift of moving conductor.

Figure 5. Computed Doppler shift of moving dielectric.

References

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[3] M. J. Inman, A. Z. Elsherbeni, and C. E. Smith, "Finite Difference Time Domain Analysis of Moving Objects", *The 18th Annual Review of Progress in Applied Computational Electromagnetics,* ACES'02, Monterey, California, March 2002.
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Book Review
By Dr. Ji Chen

Title: **Grid Computing For Electromagnetics**
by Luciano Tarricone, Alessandra Esposito
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The design of advanced electronic systems, such as wireless products, often requires several iterations of prototyping. Due to the complex nature of these products, accurate electromagnetic (EM) modeling/simulations have become necessary. However, most EM simulation tools are very CPU intensive. Often time, simulation time may go beyond one day. This makes some modeling/simulation tools not very applicable for practical designs.

With the recent advancement in parallel computing, it is now feasible to use "supercomputer" to alleviate such CPU intensive simulations. Among various parallel computing approaches, two distinct methods for computational electromagnetic society are the Open specifications for Multi Processing (OpenMP) based scheme and the message passing interface (MPI) based technique. Each paradigm has its own advantages and disadvantages. For practical electromagnetic simulations, it comes down to the affordability of computer platform vs high performance computing. In terms of Gflops per dollar, the MPI based approach typically has a clear advantage. For example, eight-node OpenMP UNIX workstations could cost around $100K, while an eight-node LINUX cluster, with similar performance and comparable amount of memory, can be assembled under $10K. Furthermore, if one can use available computer resources across the Internet, the cost for MPI computing will be even lower.

"Grid computing for electromagnetics" is one of the first books that give the background, implementation, and examples on how to develop your own parallel electromagnetic designs on grid environment. It is a book on parallel computing and beyond. It starts with the parallel high performance computing (HPC) and goes beyond it by introducing cooperative engineering and real time data management on grid environment. The core of this book is Chapters 4-6 where three EM applications, viz the parallel MPI finite-difference time-domain (FDTD) method, the cooperative CAE of rectangular aperture array antenna, and the planning, managing and monitoring of wireless radio base station network, are discussed under the GC framework. Before these applications are elaborated, the authors prepare the readers with some fundamental knowledge of GC by introducing what is a grid environment, explaining the enabling technologies and dedicated tools related to the GC, and describing a step-by-step procedure on how to build you own cluster in Chapters 1-3. Although Chapters 1-3 may not be interested to some researchers who have excellent computer administrative support, it is still recommended that readers go through these chapters since understanding the limitations of the grid environment will help us to design better scalable GC algorithms.

This book consists of seven chapters and four appendixes. The four appendixes describe some required knowledge related to GC. These include basics of UNIX/LINUX operating systems, some foundations of cryptography and security for grid environment, and some fundamental electromagnetics related to the examples in the books.