HF Fractal Wire Antenna Case Study

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***Abstract*—A HF fractal wire antenna case study is reported here. SWR and radiation pattern results for two realizations of a particular fractal geometry, plus a plain bowtie implementation for comparison, are presented. Sufficient quantitative results are shown to effectively aid a radio communicator contemplating the potential merits of deploying a fractal wire dipole to significantly lower the fundamental resonant frequency for a specified antenna length from that of a classical single-wire $\frac{\lambda}{2}$ dipole.**

I. INTRODUCTION

There is always considerable interest in the amateur radio community in compact wire antenna candidates for the 1.8 - 2.0 MHz (160 m) and 3.5 - 4.0 MHz (80 m) bands because the long lengths associated with classical dipole antennas for these bands are prohibitive for many prospective users. Also, there are considerable numbers of other practical communicators who use the HF spectrum for their radio communication systems. First thought may go to antenna length, but close behind in priority come considerations of gain, radiation pattern, and overall competitiveness with full-sized classical dipoles.

An overview or tutorial on fractal antennas is outside the scope of this work. There are many references available to the interested reader, including references [1]-[8] cited at the end of this paper. Additional information is also available from a number of Web sites, including www.fractenna.com.

The fractal shape selected for this engineering study is illustrated in Figure 3 of [1]. The corresponding basic "building block" version, constructed entirely with wires, is shown in Figure 1. In Figure 1, the individual wires are numbered, the antenna is in the y-z plane, the center of the wire is at $z = 22$ feet, the antenna length is 33 feet (32 feet plus a 1-foot wide feed-point wire at the center, wire #31), and the respective maximum and minimum heights of the antenna ends are 30 feet and 14 feet.

To obtain SWR and radiation patterns for all antenna variations in this paper, Roy Lewallen's EZNEC version 4.0 [9] provided the numerical analysis. For all EZNEC results reported here, real/high accuracy ground was selected with $\sigma = 3$ mS/m and $\epsilon_r = 12$, typical of west central Alabama soil conditions. Also, "copper" wire loss was selected, so the results here include conductor loss. In all cases, these planar antennas are placed in the $y - z$ plane at $x = 0$, with +y corresponding to the compass direction North, and +x corresponding to the compass direction East. Therefore, in visualizing the radiation from these example antennas in the real world, azimuth angle $\varphi = 0°$ is toward the East, $\varphi = 90°$ is toward the North, and so forth.

II. BASIC FRACTAL DIPOLE CHARACTERIZATION

For the basic fractal dipole shown in Figure 1 and described above, the SWR plot obtained by use of the EZNEC code for 1.75 to 30 MHz in steps of 0.25 MHz is given in Figure 2. Note that a feeding transmission line characteristic impedance of $Z_0 = 50\ \Omega$ was used as reference for the SWR calculations.

It would be expected that qualitative features of the plot will change if an alternative Z_0 is used and, for illustration, Figure 3 shows the corresponding plot with the alternate $Z_0 = 25\ \Omega$ applied.

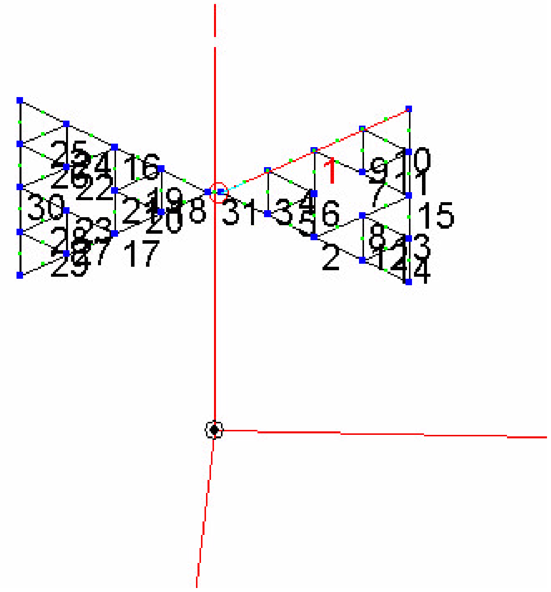
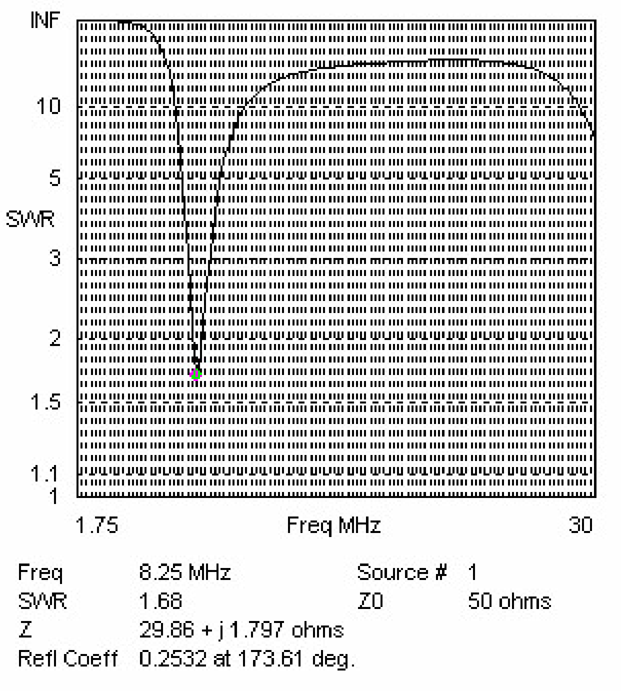


Fig. 1. Basic fractal dipole geometry.

The fundamental resonant frequency for the basic fractal dipole is close to 8.25 MHz. This may be compared to the resonant frequency of a single-wire dipole of the same length (33 feet) obtained from

$$f_{0d} = \frac{468}{\ell}$$

where f_0 is half-wave resonant frequency in MHz, ℓ is the antenna length in feet, and the formula takes into account end effect. By this formula, f_0 comes out to be 14.2 MHz, and so the fractal dipole geometry reduces the fundamental resonant frequency by approximately 42%.



```
Freq         8.25 MHz               Source #   1
SWR          1.68                   Z0         50 ohms
Z            29.86 + j 1.797 ohms
Refl Coeff   0.2532 at 173.61 deg.
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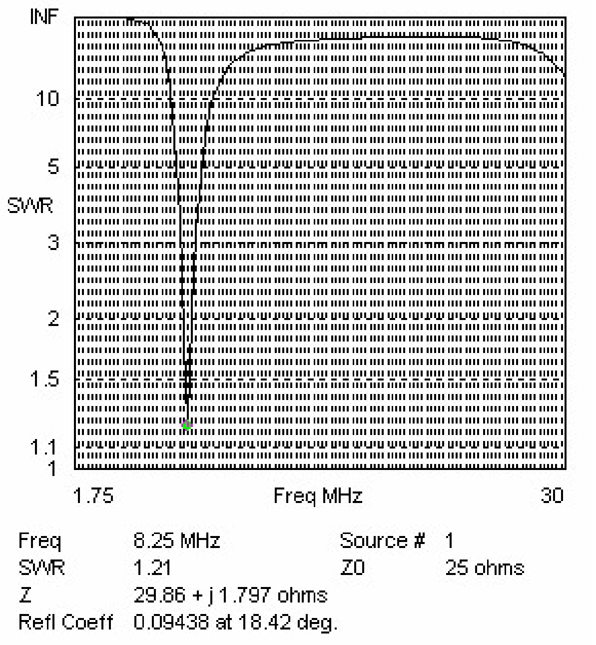
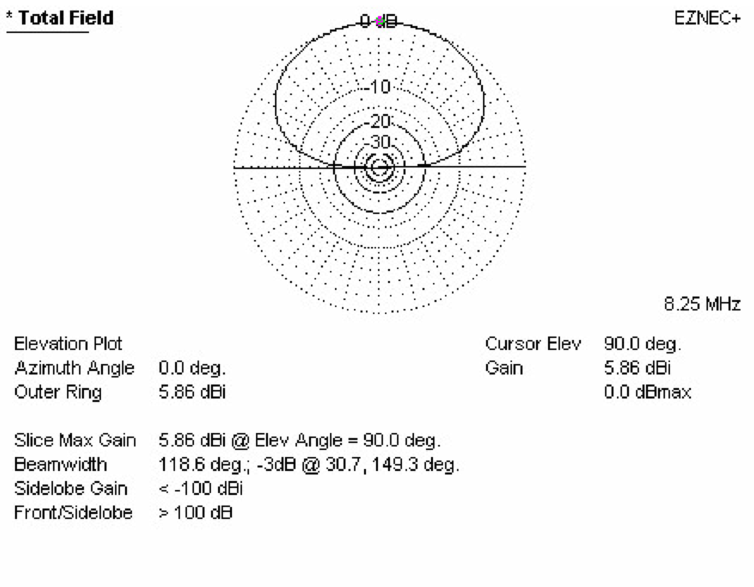
Fig. 2. Basic fractal dipole SWR, Z0 = 50.



```
Freq         8.25 MHz               Source #   1
SWR          1.21                   Z0         25 ohms
Z            29.86 + j 1.797 ohms
Refl Coeff   0.09438 at 18.42 deg.
```

Fig. 3. Basic fractal dipole SWR, Z0 = 25.

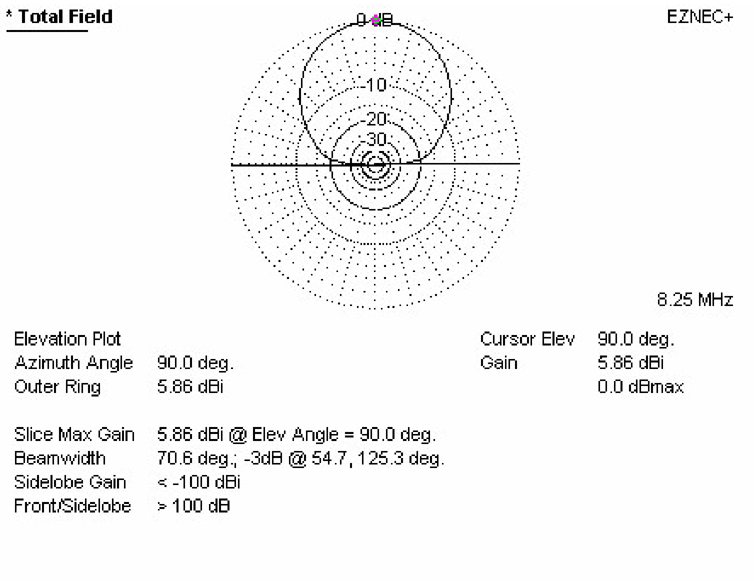
The major prices paid for this reduction in resonant frequency are that the dipole ends are now 16 feet tall, and some 31 wires are now involved in the place of one. As for radiation efficiency, an elevation plot in the East-West plane (recall the antenna is oriented on a North-South line) is shown in Figure 4 and the corresponding North-South pattern is in Figure 5.



```
Elevation Plot                                       Cursor Elev   90.0 deg.
Azimuth Angle     0.0 deg.                           Gain          5.86 dBi
Outer Ring        5.86 dBi                                         0.0 dBmax

Slice Max Gain    5.86 dBi @ Elev Angle = 90.0 deg.
Beamwidth         118.6 deg.; -3dB @ 30.7, 149.3 deg.
Sidelobe Gain     < -100 dBi
Front/Sidelobe    > 100 dB
```

Fig. 4. East-West elevation plot, 8.25 MHz.



```
Elevation Plot                                       Cursor Elev   90.0 deg.
Azimuth Angle     90.0 deg.                          Gain          5.86 dBi
Outer Ring        5.86 dBi                                         0.0 dBmax

Slice Max Gain    5.86 dBi @ Elev Angle = 90.0 deg.
Beamwidth         70.6 deg.; -3dB @ 54.7, 125.3 deg.
Sidelobe Gain     < -100 dBi
Front/Sidelobe    > 100 dB
```

Fig. 5. North-South elevation plot, 8.25 MHz.

III. COMPOUND FRACTAL DIPOLE

The second illustrative analysis to be presented is that of a "compound fractal dipole," comprising a total of six of the building block geometries used to make the basic fractal dipole antenna considered above. The EZNEC view of the antenna geometry is seen in Figure 6.

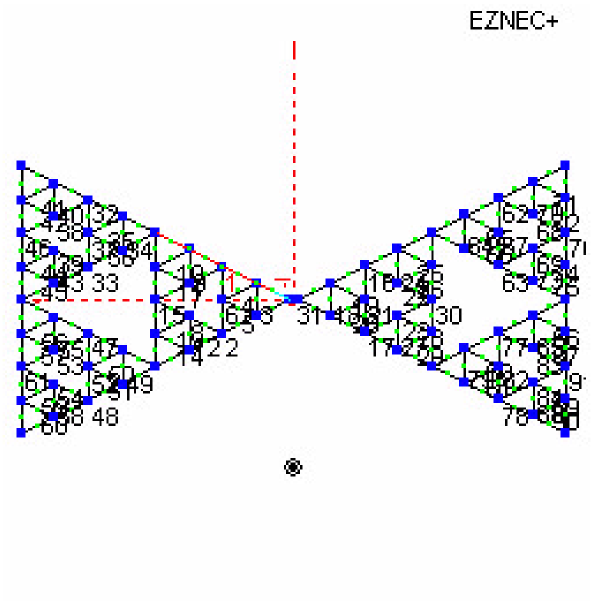


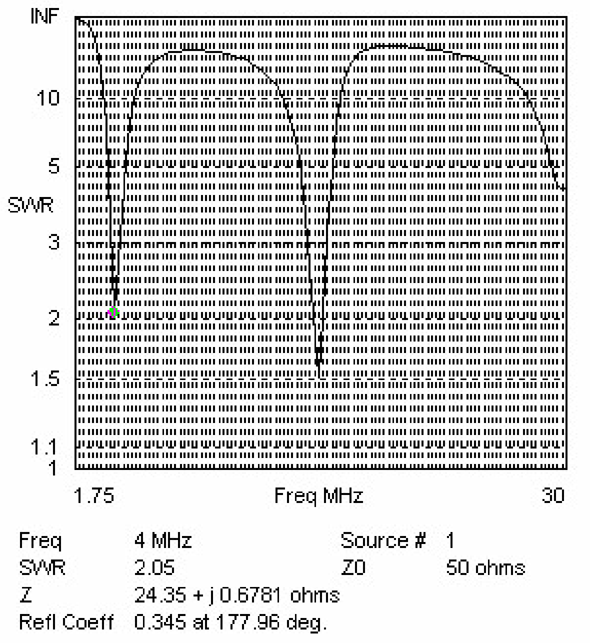
Fig. 6. Compound fractal dipole geometry.

Here, the length of the dipole is 65 feet, and the height of the ends is 32 feet. The composite stucture is comprised of some 91 wires. Again, the antenna is placed on a North-South line, and the center is at height $z = 22$ feet, so the maximum height of the ends is 38 feet and the minimum height of the ends is 6 feet above ground. The HF SWR plot, again in 0.25 MHz steps and for $Z_0 = 50\ \Omega$, may be seen in Figure 7. In this case, an interesting alternative plot results from chosing $Z_0 = 600\ \Omega$ (Figure 8), of practical interest because many HF dipoles are fed with $600\ \Omega$ ladder line. Note in Figure 8 that the fundamental resonant frequency has become obscured, but that the SWR exhibits a favorable characteristic curve over most higher frequencies in the HF spectrum.

The fundamental resonant frequency is about 4 MHz, in contrast to a classical wire dipole resonant frequency of about

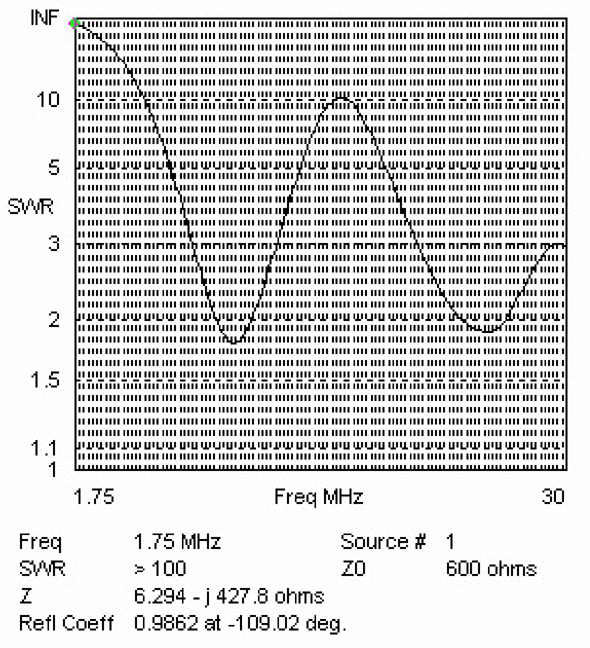
$$f_{0d} = \frac{468}{65} = 7.2 \text{ MHz}.$$

Therefore, in this case, the resonant frequency has been lowered by approximately 44.4%. Clearly the resonant frequency has been cut nearly in half compared to the single-wire dipole, but dealing with ends that are now 32 feet tall becomes a mechanical issue of increasing concern and implementation difficulty. On the other hand, since $600\ \Omega$ ladder line feed at HF has insignificant loss properties and 10:1 SWR is not considered problematic with ladder line feed, it is apparent that this variation of the fractal dipole antenna exhibits favorable SWR characteristics over most of the HF spectrum. Unfortunately, in the $Z_0 = 600$ case the low frequency SWR becomes elevated to values considerably above 10:1 so that only the amateur bands 40m - 10m benefit significantly from the broadband low SWR behavior.



Freq 4 MHz Source # 1
SWR 2.05 Z0 50 ohms
Z 24.35 + j 0.6781 ohms
Refl Coeff 0.345 at 177.96 deg.

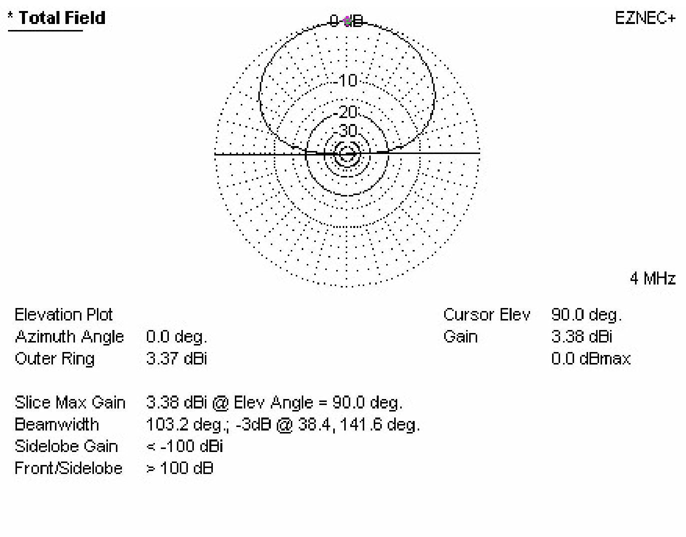
Fig. 7. Compound fractal dipole SWR, Z0 = 50.



Freq 1.75 MHz Source # 1
SWR > 100 Z0 600 ohms
Z 6.294 - j 427.8 ohms
Refl Coeff 0.9862 at -109.02 deg.

Fig. 8. Compound dipole SWR, Z0 = 600.

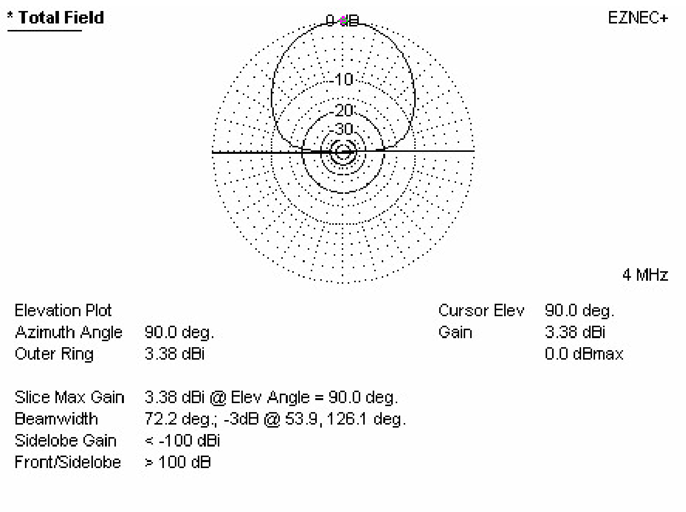
An East-West elevation plot (broadside to the antenna deployment) and North-South elevation pattern plot (in the plane of the antenna) follow, as Figures 9 and 10, respectively.



```
Elevation Plot                          Cursor Elev   90.0 deg.
Azimuth Angle    0.0 deg.               Gain          3.38 dBi
Outer Ring       3.37 dBi                             0.0 dBmax

Slice Max Gain   3.38 dBi @ Elev Angle = 90.0 deg.
Beamwidth        103.2 deg.; -3dB @ 38.4, 141.6 deg.
Sidelobe Gain    < -100 dBi
Front/Sidelobe   > 100 dB
```

Fig. 9. East-West elevation plot, 4 MHz.



```
Elevation Plot                          Cursor Elev   90.0 deg.
Azimuth Angle    90.0 deg.              Gain          3.38 dBi
Outer Ring       3.38 dBi                             0.0 dBmax

Slice Max Gain   3.38 dBi @ Elev Angle = 90.0 deg.
Beamwidth        72.2 deg.; -3dB @ 53.9, 126.1 deg.
Sidelobe Gain    < -100 dBi
Front/Sidelobe   > 100 dB
```

Fig. 10. North-South elevation plot, 4 MHz.

The gain of 3.4 dBi and the radiation patterns, computed above real ground as noted earlier, compare quite favorably with those of a classical dipole at the same height as the compound fractal dipole's (center) feed height.

IV. PLAIN HF BOWTIE

A natural question is to ask how much the "frill" associated with the fractal geometry contributes to lowering the resonant frequency for a dipole of given length. Some initial insight into the matter is gained by an analysis of the bowtie skeleton associated with the compound fractal dipole analyzed and discussed above. The internal (fractal geometry) wires were removed, leaving a frame of seven wires (again including a one-foot connector wire at the center for applying the rf feed) as shown in Figure 11 below.

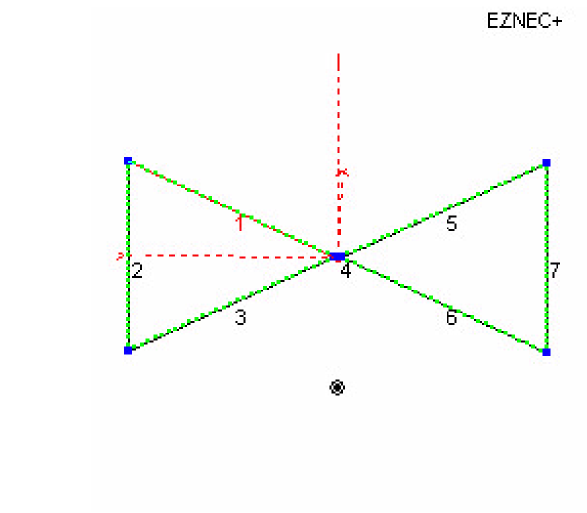
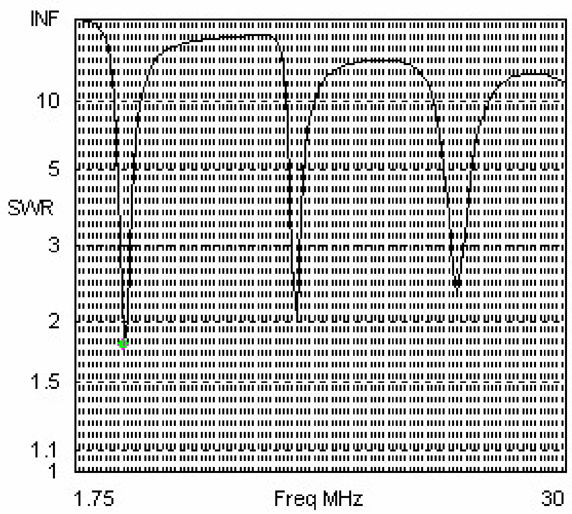


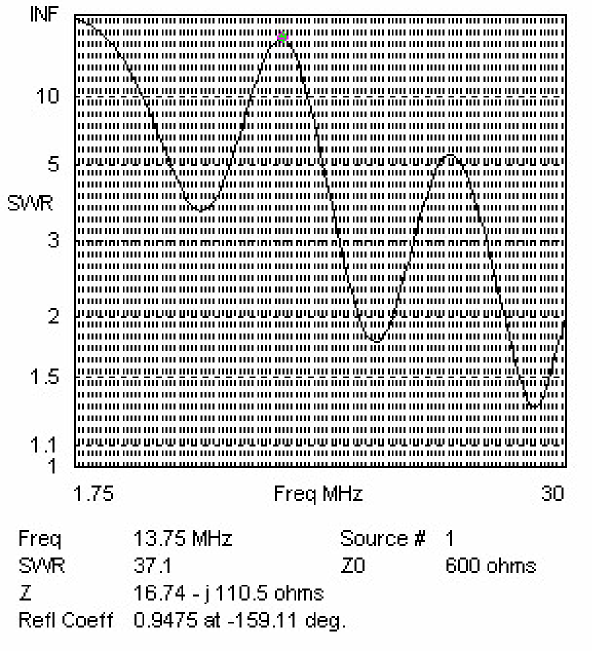
Fig. 11. Plain HF bowtie antenna.

To facilitate a fair comparison, this antenna is the same length as the compound fractal dipole (65 feet), has its center at the same height above ground (at $z = 22$ feet) and has ends that are the same height (maximum elevation at $z = 38$ and minimum elevation at $z = 6$ feet). The EZNEC analysis was made using real ground parameters of $\sigma = 3$ mS/m and $\epsilon_r = 12$, the consistent practice throughout this study. The resulting HF SWR plots for $Z_0 = 50$ and $Z_0 = 600$ Ω may be seen in Figures 12 and 13. As indicated in Figure 12, the fundamental resonant frequency is now about 4.5 MHz, approximately 37.5% lower than that of a classical single-wire dipole of the same length.



```
Freq         4.5 MHz                  Source #   1
SWR          1.78                     Z0         50 ohms
Z            30.9 - j 12.89 ohms
Refl Coeff   0.2813 at -136.95 deg.
```

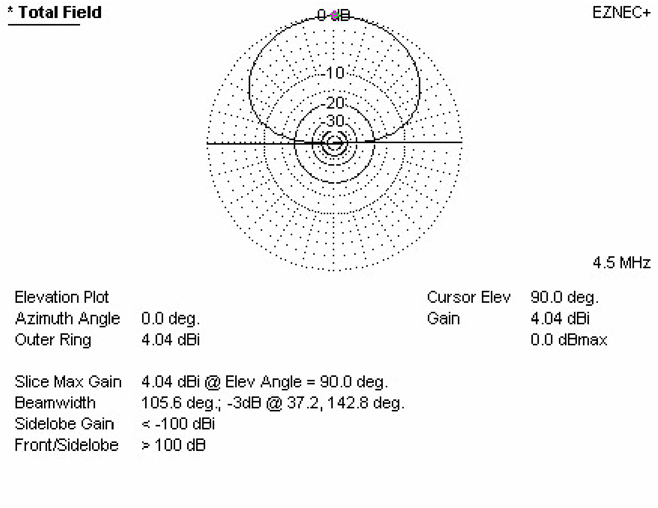
Fig. 12. HF bowtie SWR, Z0 = 50.



```
Freq         13.75 MHz          Source #   1
SWR          37.1               Z0         600 ohms
Z            16.74 - j 110.5 ohms
Refl Coeff   0.9475 at -159.11 deg.
```

Fig. 13. HF bowtie SWR, Z0 = 600.

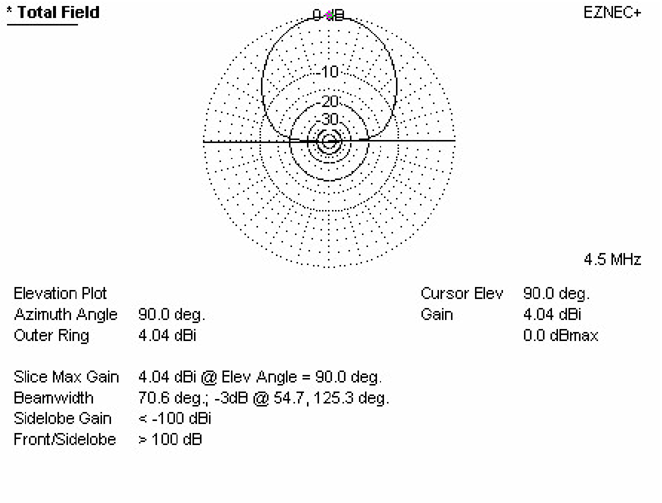
Elevation plots at $\varphi = 0°$ (East-West) and $\varphi = 90°$ (North-South) are presented in Figures 14 and 15.



```
Elevation Plot                          Cursor Elev   90.0 deg.
Azimuth Angle    0.0 deg.               Gain          4.04 dBi
Outer Ring       4.04 dBi                             0.0 dBmax

Slice Max Gain   4.04 dBi @ Elev Angle = 90.0 deg.
Beamwidth        105.6 deg.; -3dB @ 37.2, 142.8 deg.
Sidelobe Gain    < -100 dBi
Front/Sidelobe   > 100 dB
```

Fig. 14. East-West elevation plot, 4.5 MHz.



```
Elevation Plot                          Cursor Elev   90.0 deg.
Azimuth Angle    90.0 deg.              Gain          4.04 dBi
Outer Ring       4.04 dBi                             0.0 dBmax

Slice Max Gain   4.04 dBi @ Elev Angle = 90.0 deg.
Beamwidth        70.6 deg.; -3dB @ 54.7, 125.3 deg.
Sidelobe Gain    < -100 dBi
Front/Sidelobe   > 100 dB
```

Fig. 15. North-South elevation plot, 4.5 MHz.

Notable differences from those for the compound fractal dipole are that the fundamental resonance has shifted up by about 0.5 MHz, from the vicinity of 4.0 to about 4.5 MHz, and the maximum gain for the plain bowtie at its resonant frequency is greater by more than 0.5 dB.

V. CONCLUDING REMARKS

From this engineering study of limited scope, it appears that reduction in resonant frequency follows mostly from flaring out the two dipole sides, and here the plain HF bowtie was rather effective is achieving a lower resonance for a given dipole length. Essentially, if the length decreases, the governing fundamental physics seems to require an associated breakout in the other dimension (width) of appropriate extent in order to maintain rough parity with the classical full length single-wire dipole. The further reduction in resonance achieved by adding the detailed frill of a fractal geometry interior to the bowtie skeleton may be second-order, but it is nonetheless significant and potentially worthwhile.

Where the available antenna deployment space is limited, but adequate, a 7-wire bowtie has some attractive SWR and radiation characteristics for 80m and 160m band use. In cases where the available length is insufficient for the plain frame bowtie, adding the fractal wire geometry inside the frame both lowers the antenna's resonant frequency further and provides an interesting conversation piece for its owner.

A 160m (1.9 MHz) extension of the shaping geometry considered here would require an available length of about 154 feet for the bowtie, with ends that are approximately 76 feet high, compared with a length requirement of about 137 feet and ends that are approximately 68 feet in height for the compound fractal variation. There well may be instances where the available supports or length preclude the extra 17 feet of length and/or 8 feet of height required by the plain bowtie, but would accommodate the smaller dimensions of the compound fractal dipole geometry.

This report is merely one particular case study, and does not make any general claims with respect to electrical properties, performance, and overall merit of fractal versus classical antenna realizations. Further, the figures reported in this paper are strictly from computer-based numerical modeling and no experimental data is available for these antennas.

The interested reader is encouraged to further explore the emerging world of fractal antennas by studying readily available references treating their background, theory, and desirable properties. Finally, all readers should be made aware that certain commercial interests in the manufacture and sale of fractal antennas are protected by a number of patents that have already been granted (see [1], for example).

REFERENCES

[1] Cohen, Nathan, "Fractals' new era in military antenna design," *Defense Electronics*, August 2005, pp. 12-17.

[2] Barnsley, M., Fractals Everywhere, Academic Press, London: 1988.

[3] Cohen, N.L., "Fractal antennas Part I," *Communications Quarterly*, Summer 1995, pp. 5-23.

[4] Hohlfeld, R.G. and Cohen, N.L., "Self similarity and the geometric requirements for frequency independence in antenna," *Fractals*, **7** (1999), pp. 79-84.

[5] J. Anguera et al., "The Fractal Hilbert Monopole: A Two-Dimensional Wire," *Microwaveand Optical Tech. Lett.* **36**, 2102 (2003).

[6] Best, S.R.and Morrow, J.D.,"The effectiveness of space-filling fractal geometry in lowering resonant frequency," *Antennas and Wireless Propagation Letters* , Volume: 1 , Issue: 5 , 2002, p 112 - 115.

[7] Vinoy, K.J., Jose, K.A., Varadan, V.K., and Varadan, V.V., "Resonant frequency of Hilbert curve fractal antennas," *Antennas and Propagation Society International Symposium*, 2001. IEEE , Volume: 3 , 8-13 July 2001, p 648 - 651.

[8] Vinoy, K.J., Abraham, J.K., and Varadan, V.K., "Fractal dimension and frequency response of fractal shaped antennas," *Antennas and Propagation Society International Symposium*, 2003. IEEE , Volume: 4 , 22-27 June 2003, p 222 - 225.

[9] EZNEC is a software product of Roy Lewallen, as described at http://www.eznec.com/.