Transient scattering from a perfectly conducting cube

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1. Description of problem and method of solution.

We consider the transient scattering of an electromagnetic pulse from a perfectly conducting unit cube. Solutions are obtained by means of time marching methods applied to both the Magnetic Field Integral Equation (MFIE) and the Electric Field Integral Equation (EFIE). The MFIE is solved using the method outlined in [1], with a finite difference discretisation of the time derivative as described in [3] used to ensure stability of the time marching solutions. Two variant techniques for solving the EFIE were used :

(i) EFIE-FD : finite differences on interlacing grids on the faces of the cube;

(ii) EFIE-TS : a triangular scheme on the cube surface using the basis functions developed by Rao et al. [2].

The first method (i) is outlined in detail in [4], whilst the second (ii) is explained in [5]. In both cases the averaging methods described in [4] are used to ensure stability of the solution.

2. Numerical results.

We present some numerical results for the scattering of a Gaussian pulse from a unit cube, located with its centre at the origin and edges parallel to the Cartesian coordinate axes. Throughout, the system of units is chosen so that ε, μ and c take the value unity. Thus results in any particular system of units can be obtained by scaling our results appropriately. The pulse travels along the x axis in the direction of decreasing x. The incident electric and magnetic field profiles are

$$\mathbf{E}^i = -\hat{\mathbf{y}} \exp((x+t)^2) \quad , \quad \mathbf{H}^i = -\hat{\mathbf{x}} \times \mathbf{E}^i = \hat{\mathbf{z}} \exp((x+t)^2).$$

The time histories of two quantities were calculated with the 3 methods stated above:

(i) the y-component $J_y(t)$ of the current at the centre of the face of the cube at which the pulse first impinges (i.e. at point $x = ½, y = z = 0$).

(ii) the z-component $H_z^b(t_f)$ of the backscattered magnetic far-field $\mathbf{H}^f(\hat{\mathbf{x}},t_f)$ (this quantity is defined in [1]. The origin of the far-field time t_f is chosen so that an impulse emitted from the origin at $t = 0$ arrives in the far-field at time $t_f = 0$);

In the case of the MFIE and the EFIE using finite differences (EFIE-FD), each face of the cube was divided into 25 square patches, totalling 150 patches in all. Thus the minimum grid point separation Δ is $\sqrt{2}/10$. On the other hand, with the EFIE and the triangular scheme (EFIE-TS), each face was divided into 16 square patches, and each square subdivided into 2 right angled triangles, totalling 192 triangular patches. The maximum size of the time step Δt is restricted (see [1] and [4]); it was chosen as follows:

(i) MFIE : $\Delta t = .12 < \Delta = 0.1414$;

(ii) EFIE-FD : $\Delta t = 0.09 < \Delta/\sqrt{2} = 0.1$;

(iii) EFIE-TS : $\Delta t = 0.1$.

The time history of the current component $J_y(t)$ is shown on Figure 1. The MFIE calculation is shown as a solid line, the EFIE-FD calculation is displayed with a longer broken line, and the EFIE-TS calculation is shown with a shorter broken line. The results are in good general agreement. The greatest discrepancy occurs at the extrema and is about 6%. The same convention is used to display the backscattered magnetic field component $H_z^b(t_f)$ in Figure 2. Again the results are in good general agreement, though there is a more noticeable discrepancy between the MFIE results and either of the EFIE results at the second maximum. There is a slowly growing instability in the EFIE-FD result which is insignificant in the time window of interest; it can be removed if desired by use of the stronger averaging technique described in [4] without detectable change to the main part of the response.

Figs. 3 and 4 show the absolute value of the Fourier transform of the results in Figs. 1 and 2 respectively, as a function of the wave number k, divided by the absolute value of the Fourier transform of the incident pulse (which we denote by $\tilde{H}^i(k)$). This yields frequency domain results corresponding to unit incident field for those frequencies within the bandwidth of the incident field (up to about $k \approx 2.5$ for our pulse). Again there is good agreement between the figures, particularly for k below 2.5. Above this, the spectral content of the pulse is small so one expects greater discrepancies.

3. Comparison of methods.

The scattering of an electromagnetic pulse has been obtained by means of time marching methods applied to both the MFIE and the EFIE. The grids used in each case were of comparable fineness, so these three different procedures provide a check on the accuracy of the solutions so obtained, and in addition enable us to compare directly the merits of each. The computed surface point current time history and the backscattered response were found to be in good agreement with each other.

The MFIE is considerably simpler to encode than the EFIE-FD method, and much much simpler than the EFIE-TS method. In the above computations its execution time (5 mins CPU time) was faster by a factor of 3 compared to the EFIE-FD method (15 mins CPU time), and a factor of 5 compared to the EFIE-TS method (25 mins CPU time). The computation times are those on a PRIME 6350. This is attributable to (a) the larger time step permitted in the MFIE; (b) the need to use an averaging procedure to stabilise the EFIE which, as implemented in the simple manner described in [4], doubled the computation time required for an unaveraged version of the EFIE. By improving the code for the averaging step this extra time requirement could be nearly eliminated. Also, the difference between the two EFIE methods is almost entirely attributable to the somewhat finer grid used in the triangular scheme rather than the use of the more complicated basis functions. Thus, after discounting the effects of our implementation of the averaging process, we can conclude that the main difference in computation time between the MFIE and the EFIE methods is due to the larger time step permitted by the MFIE. Further optimization of the codes would probably not change these relative factors significantly.

4. References.

[1] A. J. Poggio, and E. K. Miller, "Integral equation solutions of three dimensional scattering problems," In *Computer Techniques for Electromagnetics*. Ed. R. Mittra, Pergamon Press, (1973).

[2] S. M. Rao, D. R. Wilton and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.* AP- **30** (1982), 409-418.

[3] B. P. Rynne, "Stability and convergence of time marching methods in scattering problems," *IMA J. Applied Math.* **35** (1985), 297-310.

[4] B. P. Rynne and P. D. Smith, "Stability of marching in time algorithms based upon the electric field integral equation," *to appear in J. Electromagnetic Waves and Applications.*

[5] B. P. Rynne, "Time domain scattering from arbitrary surfaces using the electric field integral equation," *submitted.*

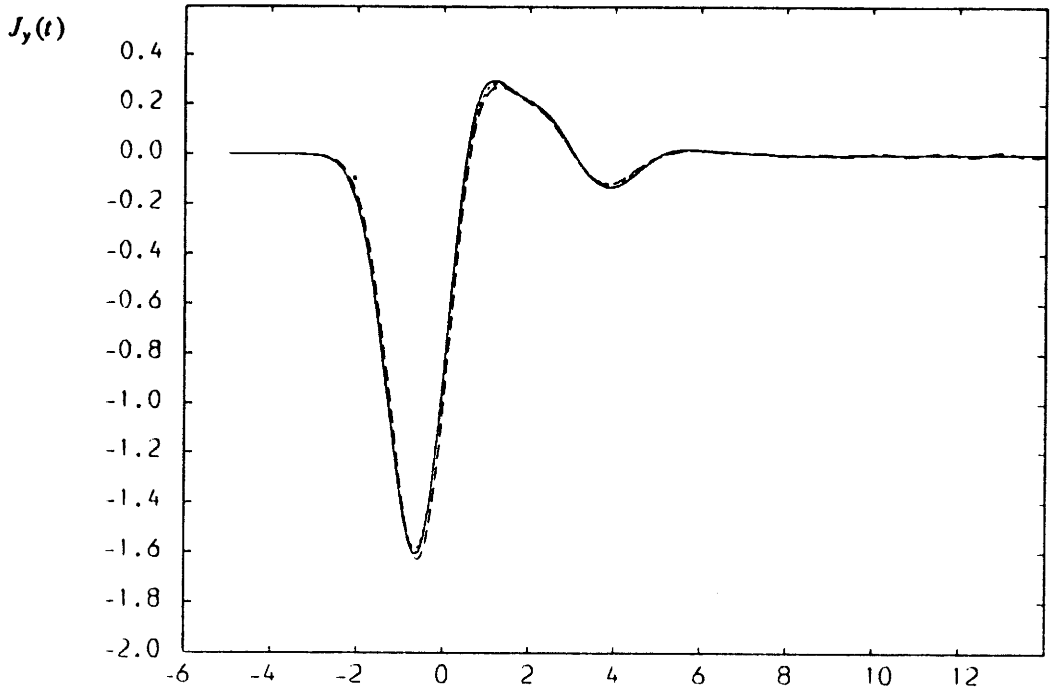

Fig. 1 y-component of current at centre of front face of cube.
MFIE ——, EFIE-FD - - - , EFIE-TS ········ .

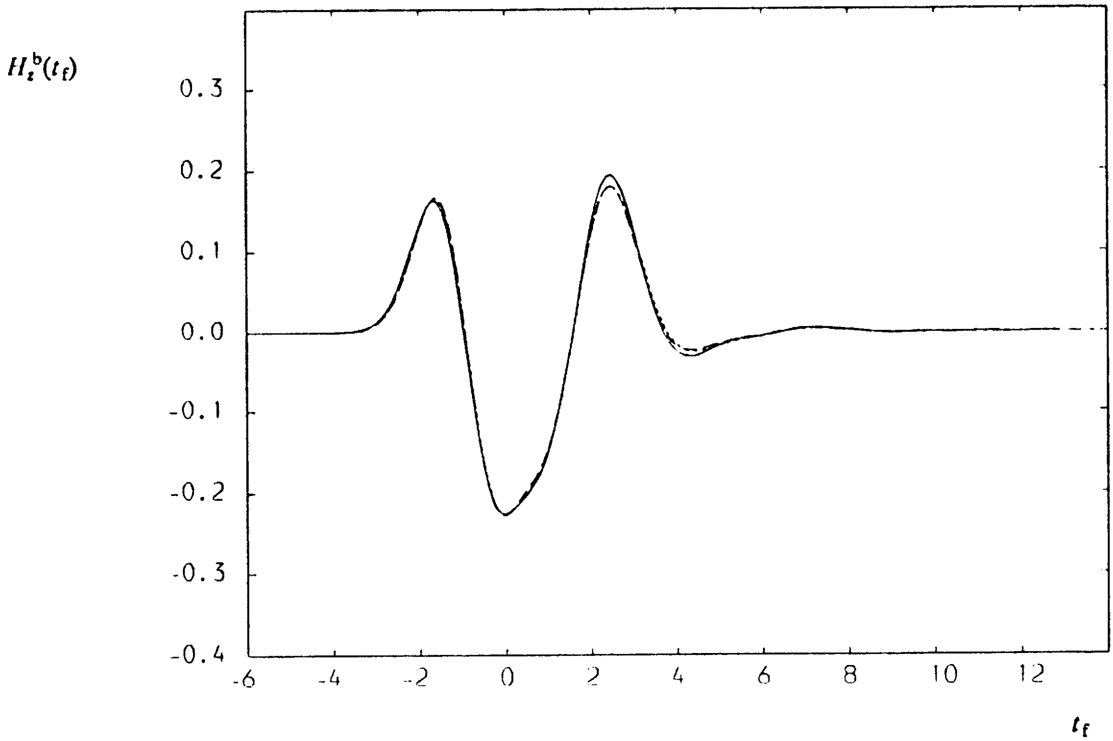

Fig. 2 z-component of far backscattered magnetic H-field.
MFIE ——, EFIE-FD - - - , EFIE-TS ········.

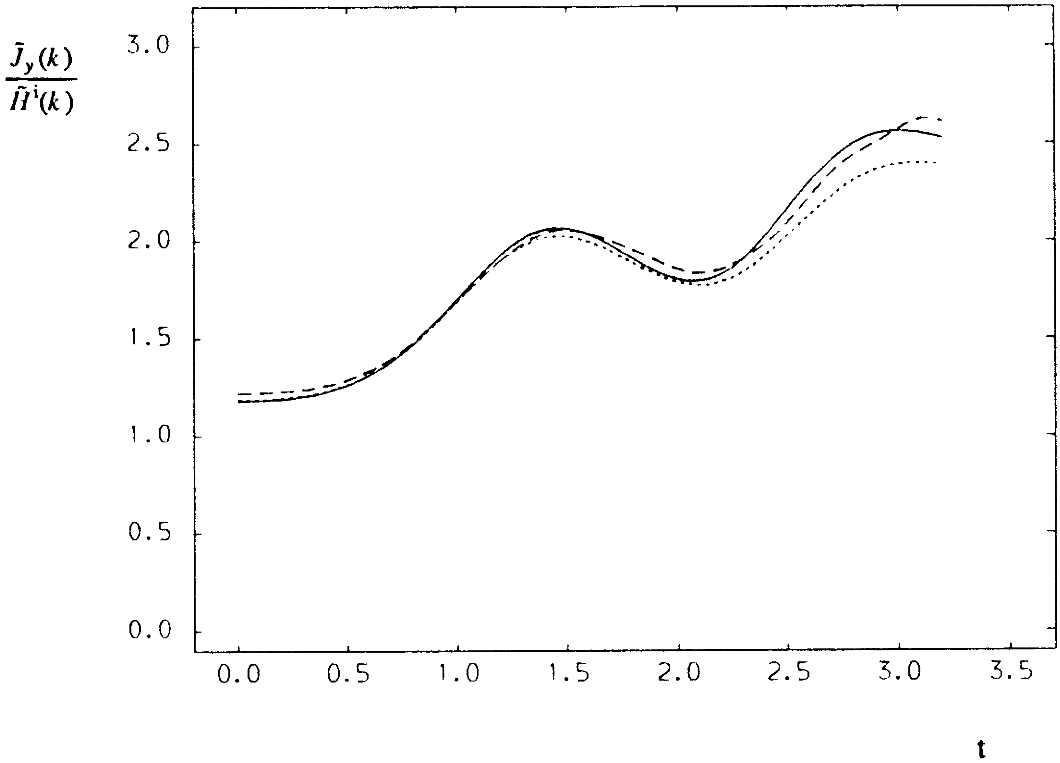

Fig. 3 Fourier transform of current in Fig. 1, scaled by $\tilde{H}^i(k)$.
MFIE ——, EFIE-FD - - -, EFIE-TS ········.

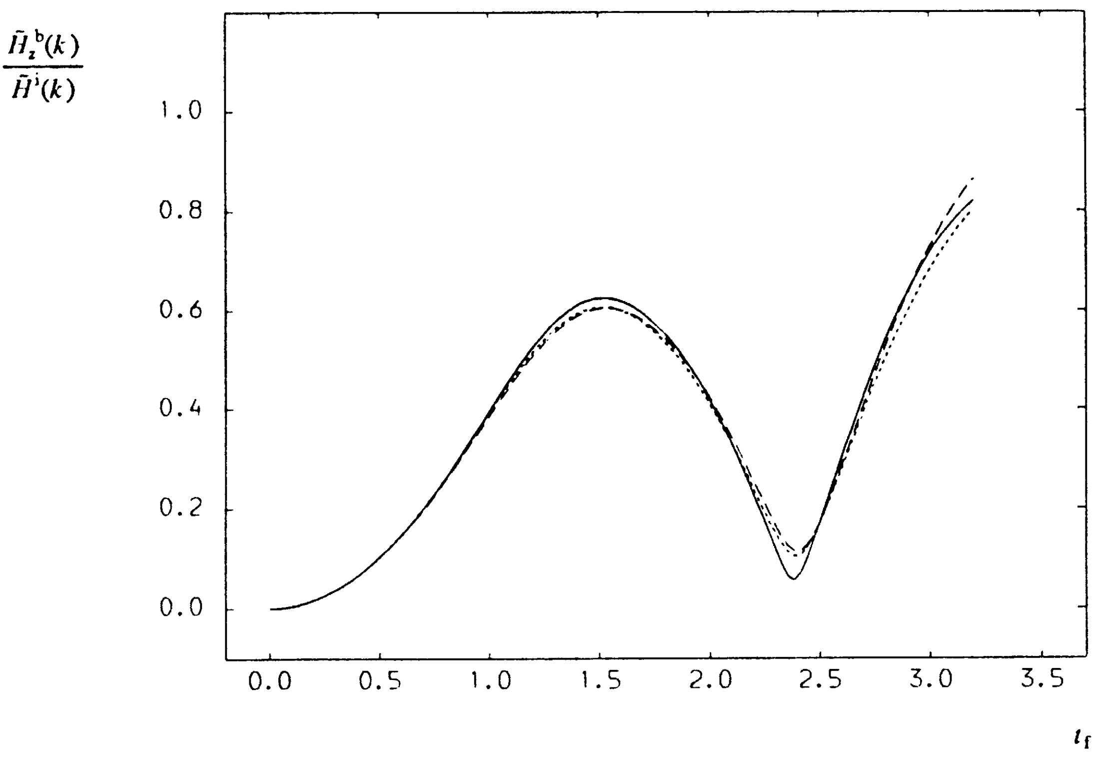

Fig. 4 Fourier transform of backscattered magnetic H-field in Fig. 2, scaled by $\tilde{H}^i(k)$.
MFIE ——, EFIE-FD - - -, EFIE-TS ·········.