Accurate Modeling of Stepped-Radii Antennas

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ABSTRACT

The LP8, eight element, log periodic antenna made by M^2 Enterprises has recently been modeled at Penn State University using both Version 2 and 4 of the Numerical Electromagnetics Code (NEC). The antenna has a stepped-radii element construction and operates from 10 to 30 MHz. The stepped-radii element construction is difficult to model using NEC Version 2. To model a stepped-radii antenna with this version, equivalent lengths of constant radii are calculated using another method of moments program called ELNEC. These equivalent lengths replaced the original stepped radii elements of the log periodic antenna during the modeling. Fortunately, NEC Version 4 can simulate the stepped-radii elements directly, which makes the modeling procedure much easier and accurate. Both NEC models show good agreement with the measurements when compared to VSWR measurements. The authors hope that this paper will help others with similar modeling problems who cannot yet obtain NEC Version 4.

I. Introduction

Accurate modeling of complex wire antennas has been a problem for some time. Two widely used programs which can model general wire antenna structures are: The Numerical Electromagnetics Code (NEC) and The Mini-Numerical Electromagnetic Code (MININEC). The formulation of MININEC is a method of moment technique using the Galerkin approach to solve the Electric Field Integral Equation (EFIE) for thin wires [1]. The NEC formulation is also a method of moment technique, but a combination of sinusoidal and constant basis functions with point matching are used to solve the EFIE [2,3]. NEC Version 4 (NEC4) uses a different solution approach than NEC Version 2 (NEC2) such that the current is no longer a filament along the axis, but it is distributed on the surface of the wire [3].

ELNEC [1] is a user-friendly antenna modeling program that uses the MININEC formulation and adds a graphical user interface, therefore, MININEC and ELNEC are considered synonymous in this paper. ELNEC, which is written in BASIC, has trouble handling large complex antennas due to internal limitations of the compilers. When modeling log periodic antennas in ELNEC, transmission lines must be modeled as separate wires that takes away from the already limited number of segments available (260 for Version 3 with MaxP option). Also, it has been shown that ELNEC has a frequency shift in impedance compared to measurements for radii greater than about 10^{-3} wavelengths [4,5]. Due to these limitations we will not model the entire log periodic antenna with ELNEC.

It has also been shown that accurate modeling results cannot be obtained for antennas with stepped radii using NEC Version 2 [6,7]. Today, most log periodic antennas in the HF band use stepped radii elements to prevent excessive weight and allow ease of assembly [8]. To model the antenna properly with NEC Version 2, an equivalent length with a constant radius is calculated to replace each stepped-radii element from the original log-periodic antenna. The equivalent length is calculated using ELNEC, which models stepped-radii elements properly, assuming the radius is less than 10^{-3} wavelengths. In contrast, NEC Version 4 has incorporated a technique for determining the charge distributions on the wires that corrects the stepped-radii problem [9].

This paper is concerned with the accurate modeling of antennas with stepped radii elements using both Versions 2 (NEC2) and 4 (NEC4) of NEC and the comparison of the modeling results with measured data.

II. Log-Periodic Model

A. General Model

Model dimensions are shown on the M^2 *Enterprises* drawing dated 4-17-91 in Figure 1. Note that the element sections overlap 3 inches, thus, the total exposed length of a 60 inch section is 57 inches. Referring to Figure 1, the director (bottom of the figure) has only one 7/8" x 30" element section that connects the two 3/4" x 60" sections. The antenna is modeled at a height of 55 feet above real ground using the reflection coefficient option with relative permittivity, $\epsilon_r = 12$ and conductivity, $\sigma = 6x10^{-4}$ S/m.

The transmission line of the log-periodic antenna is contorted such that reconstruction using wire segments would be difficult. Therefore, the antenna transmission line is modeled using the TL option. The characteristic impedance, Z_0, of a two-wire transmission line is given by [8] as:

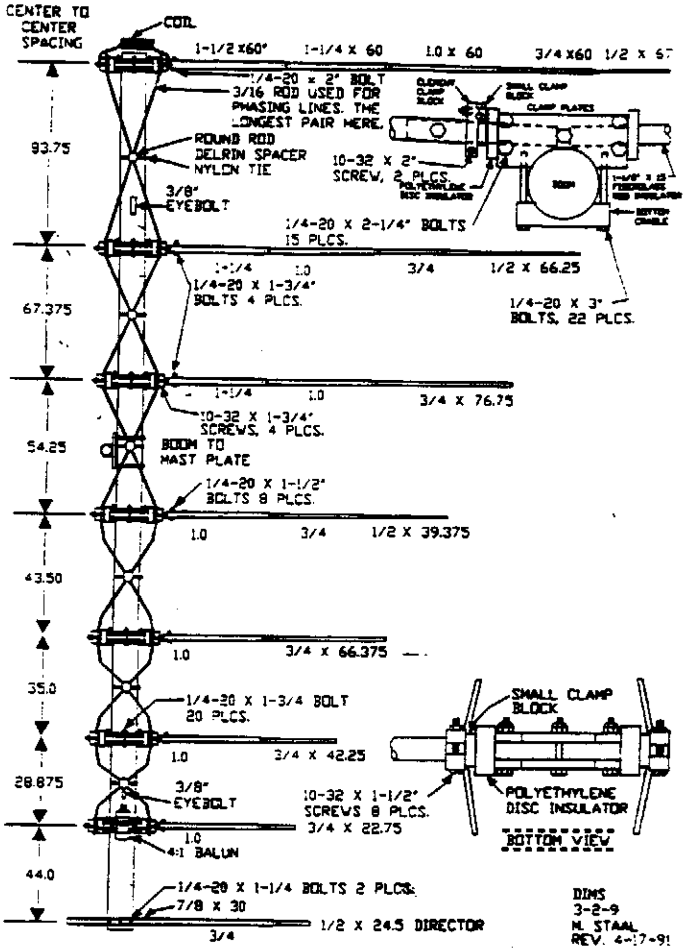

Figure 1. HF 10-30 MHz Antenna Dimensions.

$$Z_o = 276 \log\left(\frac{2s}{d}\right) \ \Omega \qquad (1)$$

where s is the separation distance and d is the wire diameter, where s >> d. Using seven inches as the separation distance between two 3/16" diameter wires, the theoretical characteristic impedance is 517 Ω from (1). Since the transmission line separation distance is not constant along the line, in fact it narrows to 1/2" between elements, the characteristic impedance of the line is an unknown variable less than 517 Ω. The transmission line characteristic impedance was varied by hand and the value that gave closest agreement with measurements is 350 Ω.

The antenna feed system consists of an RG-213 coaxial cable with a characteristic impedance of 50 Ω connected to a 4:1 balun. Thus, a line characteristic impedance of 200 Ω is used for all VSWR calculations.

The value of the inductive load on the longest element can be found [10] using the equation relating the coil length (l), coil diameter (D), and number of turns (n) as follows:

$$L(\mu H) = \frac{D^2 n^2}{(18D + 40l)} \qquad (2)$$

For use in the NEC models, an inductance of approximately 1.14 μH is found for a length of 4.5" with a 3/4" coil diameter and 16 turns of No. 10 wire.

B. NEC2 Model

As stated earlier, the NEC2 code does not accurately model stepped radii segments. Figure 2 shows the impedance of the longest LP8 stepped-radius dipole calculated using both NEC2 and NEC4. Note that the reactance of the dipole element is in disagreement by 20 Ω across the band. This impedance offset will lead to erroneous results in the modeling of a stepped-radii antenna.

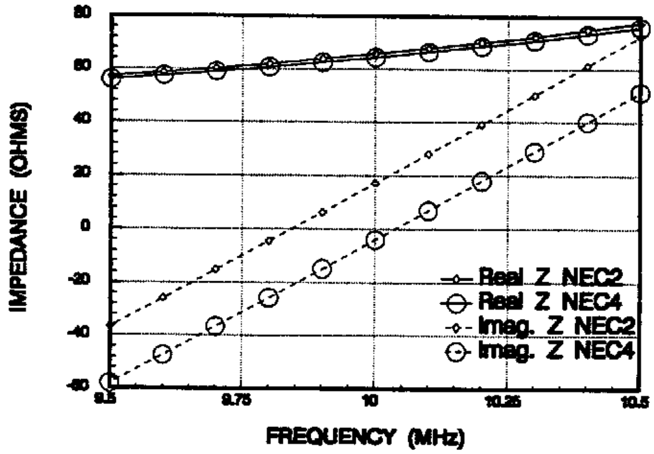

Figure 2. Impedance of a Stepped Radius Dipole.

If an equivalent length of constant radii is not calculated for the NEC2 model the results will be incorrect [7]. Equivalent lengths of a constant radius can be found for each stepped-radii element using ELNEC. The transmission line width, which in the LP8 case is seven inches, should be included in the largest radii section of each element for the ELNEC model. The procedure used to find the equivalent length is more detailed than in the references [6,11] and follows:

1. Find the average radius, R_{av}, of each element.

2. Using stepped-radii wires in ELNEC (including the transmission line width in the largest radii section), find the resonant frequency, F_r, for each element. Segment according to guidelines given in [12].

   NOTE: Model in free space with no wire loss.

3. Now model each element in ELNEC at frequency, F_r, with the average radius, R_{av}, using the same number of segments as in step 2. Adjust the length, L_{eq}, until the resonant frequency matches the resonant frequency of the stepped-radii elements.

   NOTE: Linear interpolation can be used to find an approximate equivalent length, L_{eq}, by obtaining the reactance for two different lengths and then solving for the length that gives zero reactance at the frequency, F_r.

These equivalent lengths (shown in Table 1) which are independent of frequency are used in the NEC2 model. An odd number of segments are used for each element in the NEC model so the transmission lines can be connected to the center segment of each element. The method used here of including the transmission line width in the equivalent length gives better results than using seven inch segments in the center of each element.

A segment length of about 0.5 meters (20") is used, which is 1/20 of a wavelength at 30 MHz. The elements are connected using the NEC Transmission Line (TL) option. The inductive load is modeled in parallel with the transmission line using the NEC NeTwork (NT) option.

Table 1 - HF Antenna Modeling Equivalent Lengths

ELEMENT	R_{av} (in.)	L_{eq} (in.)	F_r (MHz)
1	0.5	561.00	10.102
2	0.437	453.88	12.485
3	0.5	370.12	15.27
4	0.375	296.78	19.07
5	0.437	242.88	23.245
6	0.437	195.5	28.85
7	0.437	157.86	35.687
8	0.354	182.18	31.004

C. NEC4 Model

For the NEC4 model, wire dimensions are directly taken from Figure 1. A segment length of about 0.5 meters (20 inches) is used, which is 1/20 of a wavelength at 30 MHz. The elements are connected using NEC Transmission Line, TL, options. The width of the transmission line is not included in the largest radii section of a given element. The inductive load is modeled in parallel with the transmission line using the NEC NeTwork, NT, option.

D. Model Results

The agreement between the equivalent lengths modeled in NEC2 and the NEC4 model is shown by comparing calculations of impedance, elevation patterns and VSWR. The calculated input impedance of the antenna has a periodic behavior characteristic of a log-periodic antenna as shown in Figure 3. The calculated elevation patterns from NEC2 and NEC4 for 10 to 29 MHz in 1 MHz intervals are shown in Figures 4 through 8. The pattern differences are indistinguishable for most of the frequencies even in the sidelobe region. Note that the side lobes encountered at the higher frequencies are due to the antenna height and can be suppressed by mounting the antenna on a higher tower, but their presence reveals how close the two models match. At a frequency of 24 MHz, the models show a discrepancy which can be noted in the pattern shape of Figure 7 and the VSWR plot of Figure 9. The authors believe this is due to an anomalous resonance in the NEC2 model caused by replacing all the stepped radii elements with equivalent constant radius elements. The VSWR, Gain and Input Impedance calculated by NEC2 and NEC4 for the frequencies of interest in 1 MHz steps are shown in Table 2. The VSWR deviation between the two models is about $\pm .1$, accepting the 24 MHz anomaly.

The models are compared to VSWR field measurements as described in Section IV. The calculated VSWR for NEC2 and NEC4 is plotted versus the measured VSWR in Figure 9 with excellent agreement. Both versions of NEC match the trend of the measurements, they even overlap along some portions of the curve.

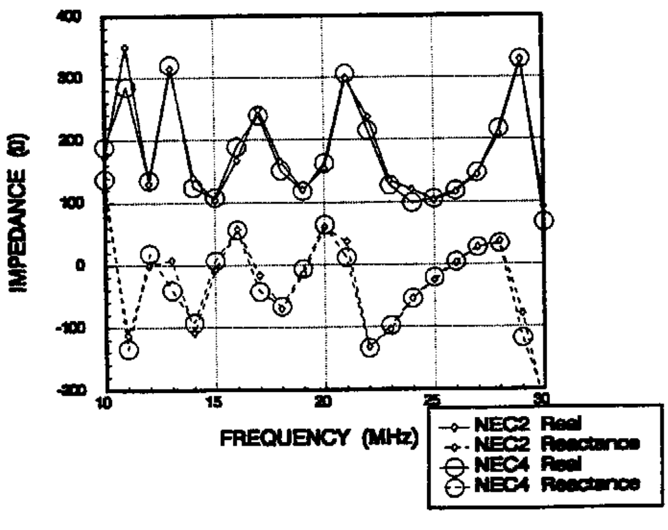

Figure 3. LP8 Antenna Input Impedance.

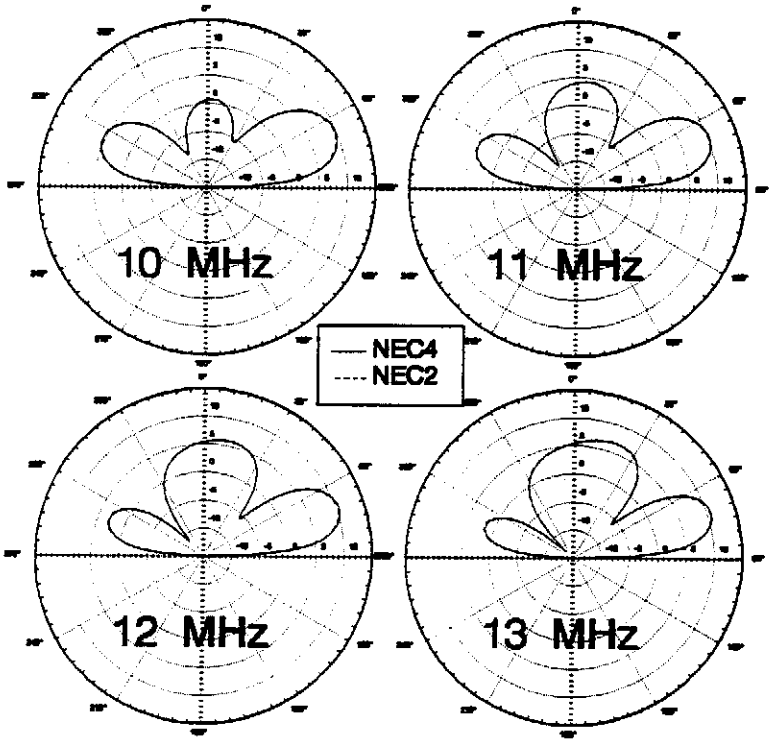

Figure 4. LP8 Antenna Elevation Pattern at 10, 11, 12 and 13 MHz.

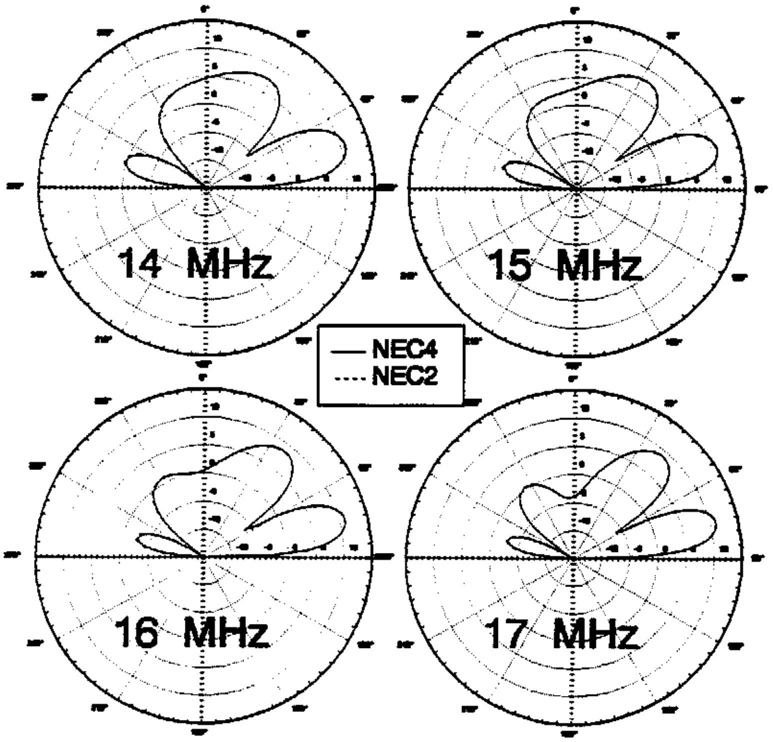

Figure 5. LP8 Antenna Elevation Pattern at 14, 15, 16 and 17 MHz.

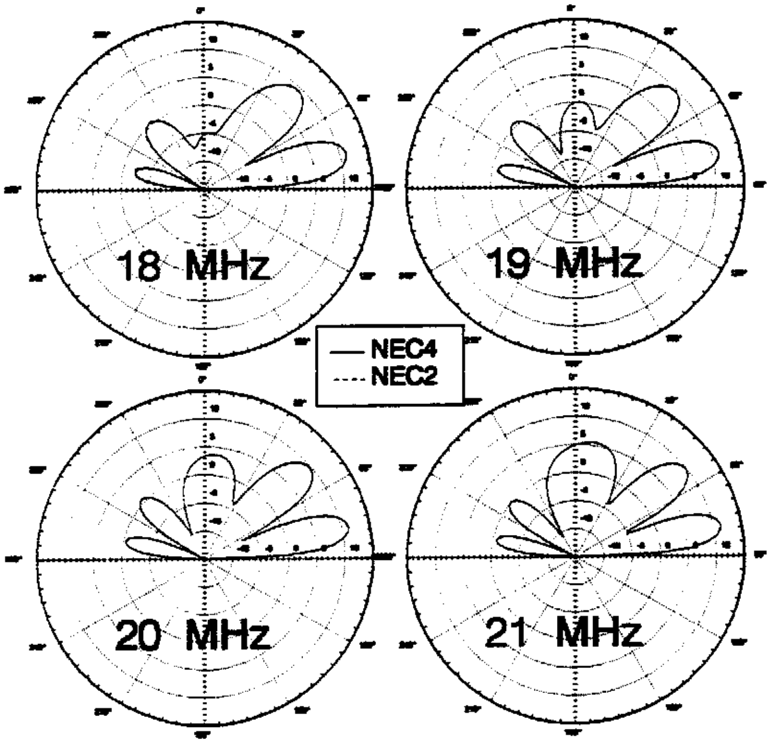

Figure 6. LP8 Antenna Elevation Pattern at 18, 19, 20 and 21 MHz.

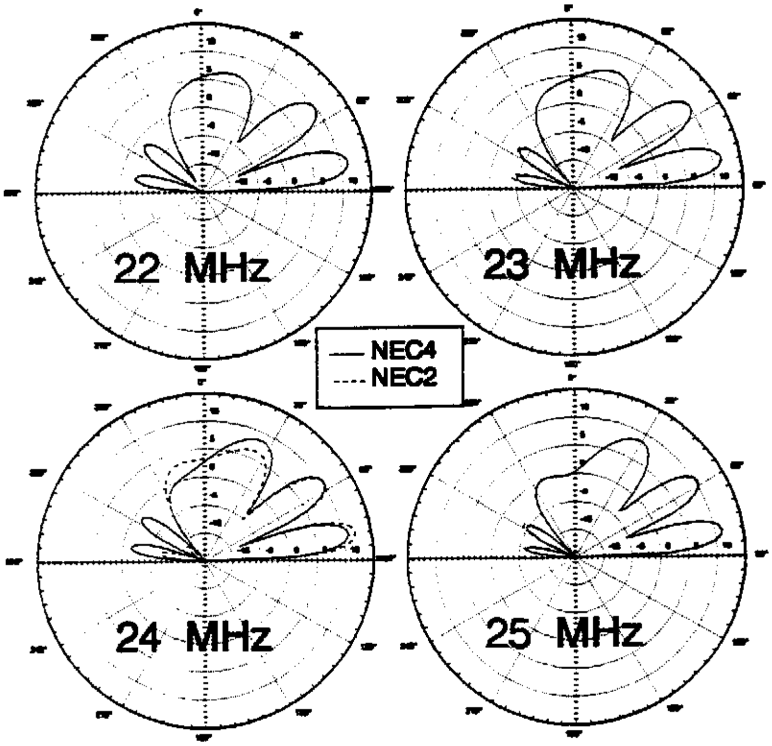

Figure 7. LP8 Antenna Elevation Pattern at 22, 23, 24 and 25 MHz.

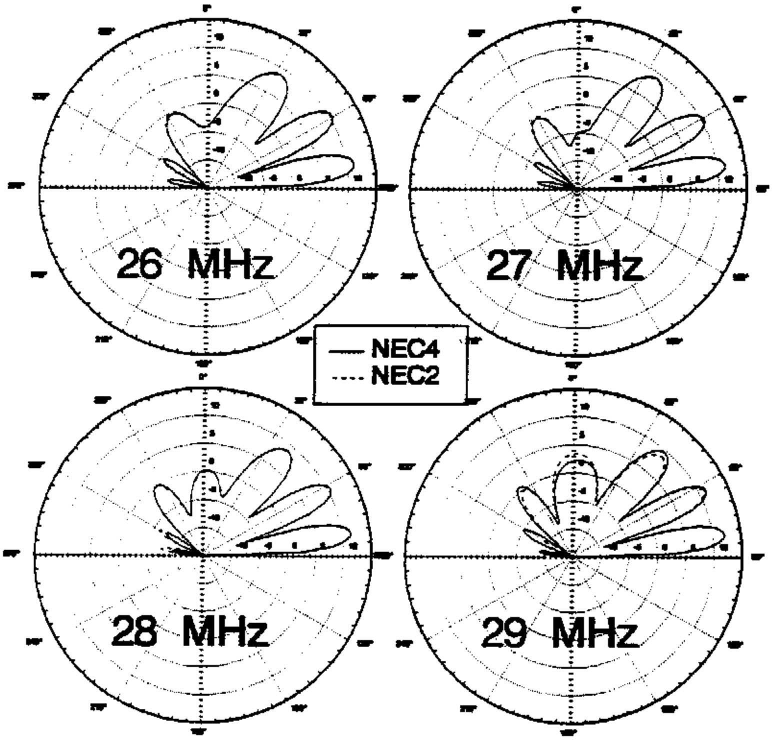

Figure 8. LP8 Antenna Elevation Pattern at 26, 27, 28 and 29 MHz.

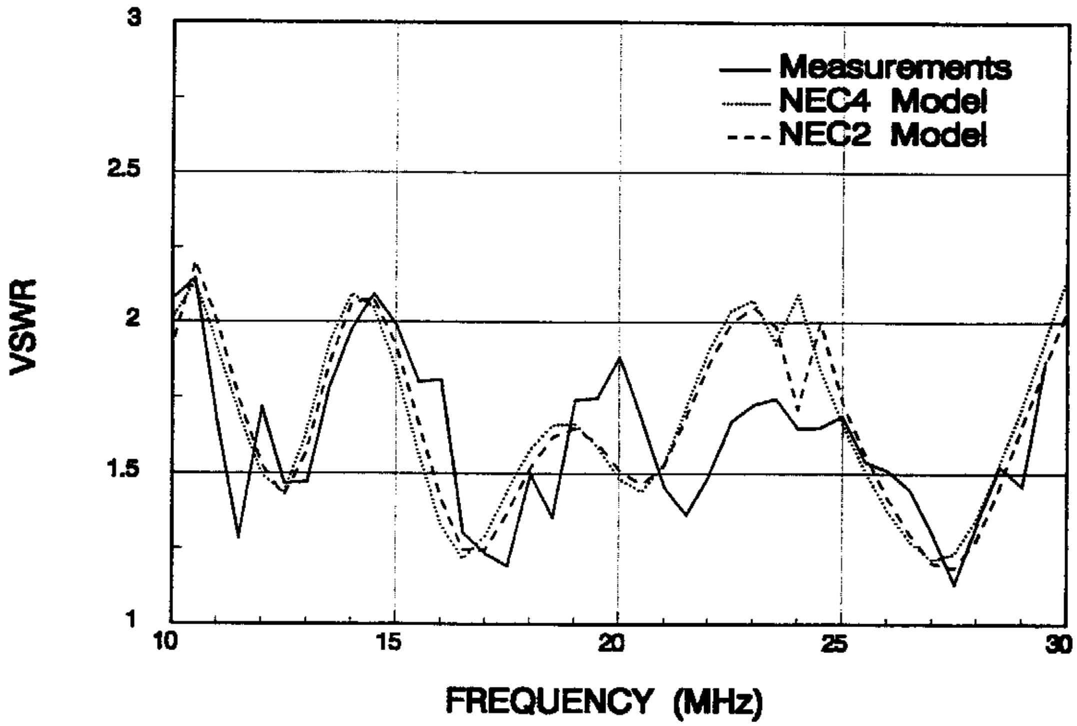

Figure 9. LP8 Antenna VSWR.

Table 2 - HF Antenna Modeling Results

Freq. (MHz)	VSWR		Max. Gain (dB)		$Z_R(\Omega)$		$Z_I(\Omega)$	
	NEC2	NEC4	NEC2	NEC4	NEC2	NEC4	NEC2	NEC4
10	1.92	2.01	9.41	9.76	172	193	120	122
11	2.00	1.92	9.97	10.25	350.	285.	-112.	-134.
12	1.53	1.50	10.34	10.53	130.	135.	-31.	19.
13	1.58	1.64	10.59	10.72	314.	320.	8.	-40.
14	2.07	2.09	10.73	10.88	139.	125.	-108.	-93.
15	1.92	1.84	10.83	11.00	103.	108.	-7.	7.
16	1.42	1.33	10.91	11.08	167.	189.	58.	56.
17	1.25	1.30	11.00	11.15	248.	239.	-17.	-42.
18	1.52	1.58	11.11	11.22	165.	151.	-70.	-66.
19	1.65	1.66	11.22	11.33	120.	118.	-14.	-6.
20	1.51	1.48	11.33	11.32	153.	163.	60.	64.
21	1.53	1.53	11.48	11.45	298.	306.	37.	12.
22	1.86	1.90	11.69	11.59	235.	214.	-131.	-134.
23	2.05	2.08	10.75	11.77	134.	127.	-104.	-97.
24	1.71	2.09	11.63	12.01	120.	99.	-55.	-54.
25	1.74	1.67	11.82	10.41	103.	106.	-25.	-21.
26	1.42	1.38	11.98	11.77	116.	118.	3.	5.
27	1.20	1.21	12.23	12.09	143.	147.	28.	28.
28	1.28	1.35	12.61	12.58	208.	216.	37.	34.
29	1.66	1.72	12.85	13.34	327.	329.	-79.	-118.
30	2.04	2.15	14.14	14.19	92.	67.	-239.	-219.

IV. Measurements

A. Procedure

Forward (P_f) and reflected (P_r) power were measured from 9 MHz to 29.5 MHz at 0.5 MHz intervals using a Model 43 Bird Watt Meter. The *VSWR* levels were calculated from these power measurements. Another set of data was taken at 10, 15, 20, 25 and 29 MHz, with an open circuit at the antenna (load) end and is used to determine the attenuation of the cable.

The experimental *VSWR* values were taken at the point where the transmitter is located. In order to compare the model predictions with the measurements, the *VSWR* values at the load end, or at the antenna, must be determined by including the effects of cable loss. A 225 foot Heliax 1/2" Hardline cable (50 Ω) connects the transmitter to the base of the tower, and a 70 ft RG-213 cable (50 Ω) extends from the tower base to the antenna feed-point. The attenuation in both cables is considered in the *VSWR* analysis.

By measuring the forward (P_f) and reflected (P_r) power at the transmitter the *VSWR* is easily calculated at this end of the transmission line by [13] as

$$VSWR = \frac{1 + |\Gamma_o|}{1 - |\Gamma_o|} \tag{3}$$

where

$$|\Gamma_o| = \sqrt{\frac{P_r}{P_f}} \tag{4}$$

The magnitude of the reflection coefficient, along a lossy line is expressed in terms of position, z, by [13] as

$$|\Gamma(z)| = |\Gamma_o| e^{+2 \; z} \qquad (5)$$

The normalized voltage standing wave ratio, $VSWR_n$, at any point, z, is then defined as

$$VSWR_n = \frac{1 + |\Gamma(z)|}{1 - |\Gamma(z)|} \qquad (6)$$

By knowing the attenuation constant in Nepers/m and the length of each cable, the $VSWR_n$ can be calculated from equation (5) and (6) as follows

$$VSWR_n = \frac{1 + |\Gamma_o| e^{2(\;_{213} l_{213} + \;_H l_H)}}{1 - |\Gamma_o| e^{2(\;_{213} l_{213} + \;_H l_H)}} \qquad (7)$$

where α_{213} and α_H are the attenuation constant for the RG-213 and the Heliax cable, respectively. The length of each cable is given in meters by l_{213} and l_H.

The attenuation in Nepers/m for a typical Heliax and an RG-213 cable is found from [8] as

$$\alpha_H = \frac{f^{0.55}}{4197.1} \; Np/m \qquad (8a)$$

$$\alpha_{213} = \frac{f^{0.5693}}{1455.3} \; Np/m \qquad (8b)$$

The attenuation constant for the Heliax cable given by (8a) was compared to experimental results shown below. The second set of measurements, performed with the open circuit was used to verify the validity of α_H experimentally. For this case, $|\Gamma(l_H)| = 1$, and after solving for α in (5) and using (4), an expression for the measured α_H was obtained as

$$\alpha_{H_{EXP}} = \frac{\ln\left(\frac{Pf}{Pr}\right)}{4 l_H} \; Np/m \qquad (9)$$

B. Results

The experimental values for α_H obtained with (9) are shown with the typical values from (8a) in Figure 11 after converting both to dB/100ft. The figure only shows the values for α_H, because the RG-213 cable was not connected when the open circuit measurements were made.

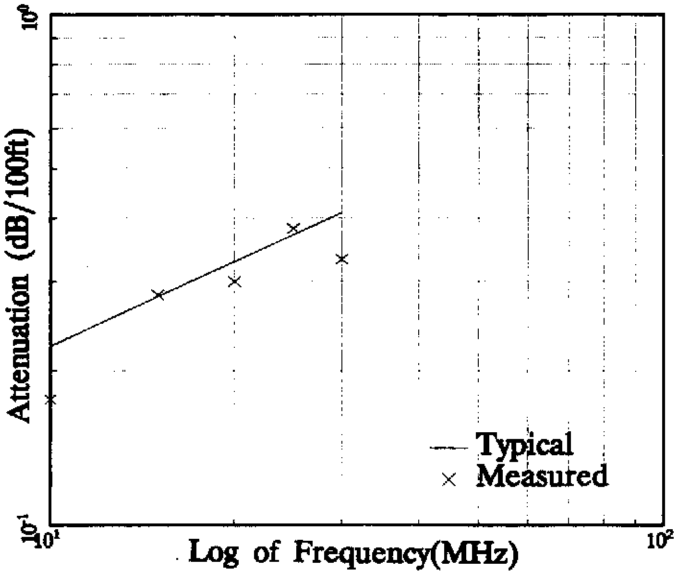

Figure 10. Heliax Cable Attenuation Typical vs. Measured.

The $VSWR_n$ is shown in Table 3 compared to the direct *VSWR* measurements. Figure 9 compares the modeling results of NEC2 and NEC4 versus the normalized measured values. Note that the results for both of the NEC2 and NEC4 models are in good agreement with the measured values. This agreement between measurements and model calculations is considered excellent for full-sized HF antennas in the field.

IV. Conclusions

It has been shown that the NEC Version 2 and 4 antenna modeling programs are appropriate tools for the modeling of antennas with a stepped radii construction. NEC4 is simpler to use because it has the capability to simulate the stepped radii elements of the antenna directly. To model the antenna with NEC2, the stepped radii elements were replaced by elements with constant radius and an equivalent length. The equivalent lengths for each element were calculated using ELNEC. The whole procedure of completing the modeling with NEC2 was much more time consuming than with NEC4. The results for both models are shown to be in good agreement with the measurements. The attenuation for the Heliax cable was also measured and compared to typical values, and the results are also in agreement. Therefore, it is possible for those without access to NEC Version 4 to accurately model stepped radii antenna by using the method of equivalent length.

Table 3 - HF *VSWR* Measurements

Freq.	P_f	P_r	*VSWR*	$VSWR_n$
9.	25	12.3	5.7	12.94
9.5	50	3.1	1.66	1.88
10.	50	4	1.79	2.07
10.5	50	4.3	1.83	2.15
11.	50	2	1.5	1.67
11.5	50	.5	1.22	1.28
12.	50	2.2	1.53	1.72
12.5	50	1.1	1.35	1.47
13.	50	1.1	1.35	1.47
13.5	50	2.4	1.56	1.78
14.	50	3.2	1.68	1.97
14.5	50	3.7	1.75	2.10
15.	50	3.2	1.68	1.99
15.5	50	2.4	1.56	1.80
16.	50	2.4	1.56	1.81
16.5	50	.5	1.22	1.30
17.	50	.3	1.17	1.23
17.5	50	.2	1.14	1.19
18.	50	1.1	1.35	1.50
18.5	50	.6	1.25	1.35
19.	50	2.	1.5	1.74
19.5	50	2.	1.5	1.75
20.	50	2.5	1.58	1.89
20.5	50	1.7	1.45	1.67
21.	50	.9	1.31	1.45
21.5	50	.6	1.25	1.36
22.	50	1.0	1.33	1.49
22.5	50	1.6	1.44	1.67
23.	50	1.8	1.47	1.73
23.5	50	1.9	1.48	1.75
24.	50	1.5	1.42	1.65
24.5	50	1.5	1.42	1.65
25.	50	1.6	1.44	1.69

TABLE 3. (Cont.)

Freq. (MHz)	P_f (Watts)	P_r (Watts)	*VSWR* (meas.)	$VSWR_n$
25.5	50	1.1	1.35	1.54
26.	50	1.0	1.33	1.51
26.5	50	.8	1.29	1.45
27.	50	.4	1.2	1.30
27.5	50	.0	1.09	1.13
28.	50	.5	1.22	1.34
28.5	50	1.	1.33	1.52
29.	50	.8	1.29	1.46
29.5	50	2.1	1.52	1.87

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References

[1] J. C. Logan and J. W. Rockway, "The New MININEC (Version 3): A Mini-Numerical Electromagnetics Code," Naval Ocean Systems Center, NOSC Technical Document TD 938, San Diego, CA, September 1986.

[2] G. J. Burke and A. J. Pogio, "Numerical Electromagnetic Code (NEC) - Method of Moments," NOSC Technical Document 116, Naval Ocean Systems Center, San Diego, CA, January 1981.

[3] G. J. Burke "Numerical Electromagnetics Code - NEC-4 - Method of Moments," Lawrence Livermore National Laboratory, Report UCRL-109338, January 1992.

[4] R. J. Bauerle, "A comparison Between NEC and MININEC Calculation of Monopole Antenna Feed Point Impedance," Master's Thesis, Pennsylvania State University, December 1993.

[5] M. G. Knedeison, "An Investigation into the Properties of the Exact Uniform Current Vector Potential and its Application to the Numerical Modeling of Cylindrical Wire Antennas," Master's Thesis, Pennsylvania State University, May 1993.

[6] J. K. Breakall and R. W. Adler, "A Comparison of NEC and MININEC on the Stepped Radius Problem," *Applied Computational Electromagnetics Society Journal & Newsletter*, Vol. 2 No. 2, Fall 1987.

[7] Jae Yim, "An experimental and computer modeling study of stepped radius monopole antennas," Master's Thesis, Naval Postgraduate School, December 1988.

[8] The ARRL Antenna Book, The American Radio Relay League, Newington, CT, 1988, Ch. 10, 26.

[9] G. J. Burke, "Recent Improvements to the Model for Wire Antennas in the Code NEC," IEEE International Symposium Antennas and Propagation, Vol. 1, June 1989, pp. 240-243.

[10] The ARRL Handbook, The American Radio Relay League, Newington, CT, 1984, pp. 2-11.

[11] P. E. Blais, "Validation of the NEC code for a high frequency monopole/log periodic antenna system," Master's Thesis, Naval Postgraduate School, September 1990, Appendix B.

[12] ELNEC, Version 3.0, Copyright 1990-93, Roy Lewallen, W7EL, P.O. Box 6658, Beaverton, OR, 97007, April 1993.

[13] M. N. O. Sadiku, *Elements of Electromagnetics*, Saunders College Publishing, Orlando, Fl, 1989, pp. 506-507.