A Three-Dimensional Analysis of Magnetic Shielding with Thin Layers

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***Abstract*-This paper describes a numerical method for the analysis of the magnetic shielding with thin layers in three dimensional, time-harmonic magnetic fields. In this method, FEM is employed to solve a couple of differential equations which express the surface impedance of the thin shielding materials. The magnetic fields in air regions are modeled by means of BEM. The above formulations are coupled to evaluate the shielding properties. The resultant matrix equation includes unknowns of two times the number of nodal points. This method can analyze not only a shielding system with closed surfaces but also that with open surfaces without introducing fictitious boundaries in air region. The method is shown to give accurate shielding factors of a spherical shell over tested range of frequencies. The eddy currents on a shielding plate are successfully obtained using the present method.**

I. INTRODUCTION

Magnetic shielding is indispensable for sensitive measurement of biological magnetic fields. It also plays an important role in electron microscope systems which cannot work well in magnetic disturbances above 0.1 [μT]. Moreover, in magnetic levitation cars, which generate strong magnetic fields, magnetic shielding is necessary to reduce possible health hazards due to the magnetic fields. The magnetic shielding system must be optimized to obtain good shielding factors which characterizes the efficiency of the shielding [1]-[3]. The main purpose of this work is to develop a numerical method which effectively analyzes the efficiency of the shielding on the basis of computer-aided design.

Since shielding layers are usually extremely thin compared with the overall size of the shielding system, it may be inefficient to discretize them in the direction of their thickness. So far, special numerical treatments of thin layers have been reported. An FEM which can analyze eddy current problems with a thin conducting plate has been proposed [4]. This method assumes constant distribution of eddy currents in the direction of thickness of the plate. This assumption is, however, not valid when the skin depth is shorter than the thickness.

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Static magnetic fields have been computed by means of an FEM-BEM coupled method [5]. Moreover, the hypersingular integral equation has been used to analyze static fields with thin magnetic materials [6]. The above two methods do not include treatment of eddy currents. To deal with eddy-current problems with thin shielding layers which have high permeability and conductivity, a couple of differential equations which express the relation between the magnetic fields on both sides of the surface have been introduced and solved in conjunction with the integral equations which govern the magnetic fields in vacuum region [7]. The above differential equations have been explained from the view point of so-called surface impedance [8].

In this paper, a numerical method based on the FEM-BEM hybridization is introduced to analyze three dimensional magnetic fields with thin shielding layers on which eddy currents can flow. This approach reduces the number of unknowns by employing the indirect method for the integral equations. Moreover, this method allows us to deal with both closed and open surfaces of shielding layers without introducing fictitious boundaries in air region.

The remainder of this paper is organized as follows. The next section gives a brief derivation of the surface impedance of thin layers, which is expressed in the form of a couple of differential equations. After the present method is described in the third section, numerical examples and discussions on the results are given in the fourth section. Finally, some concluding remarks are given in the last section.

II. SURFACE IMPEDANCE OF THIN SHIELDING LAYERS [7], [8]

Let us consider a thin shielding layer S, with thickness d, permeability μ and conductivity σ, immersed in a time-harmonic magnetic field (see Fig. 1). The thickness of the layer S is here assumed to be small compared to the overall size of the layer. Moreover, the electromagnetic properties of this layer are assumed to be linear.

We first consider Faraday's law

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B}. \tag{1}$$

Taking the inner product of both sides of (1) with the normal unit vector $\mathbf{n}$, and integrating it in the direction n of the thickness which is parallel to $\mathbf{n}$, we have

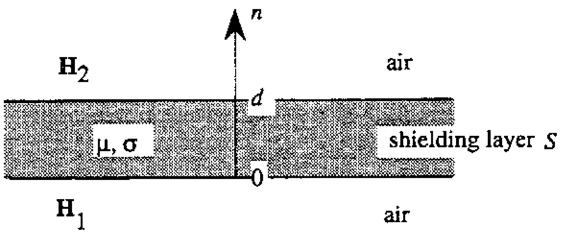

Fig. 1 Thin shielding layer

$$\int_0^d \nabla_t \cdot (\mathbf{E}\times\mathbf{n})dn = -j\omega \int_0^d \mathbf{B}\cdot\mathbf{n}dn, \qquad (2)$$

where ∇_t is the tangential gradient operator which is defined by $\nabla_t \equiv \nabla - \mathbf{n}\partial_n$. The left hand side of (2) can be written in terms of the surface current density **K**, which is the integral of the current density **J** in the direction of the thickness. Moreover, **K** can be expressed in terms of the jump in the tangential magnetic field $\mathbf{H}_t$, that is,

$$\int_0^d \nabla_t \cdot (\mathbf{E}\times\mathbf{n})dn = \nabla_t \cdot (\mathbf{K}\times\mathbf{n}/\sigma)$$
$$= \nabla_t \cdot \left[\left(\mathbf{H}_{2t} - \mathbf{H}_{1t}\right)/\sigma\right]. \qquad (3)$$

On the other hand, the right hand side of (2) can be expressed in terms of the normal components H_n on the surfaces using the fact that **H** in the layer can be written as $\mathbf{H} = \mathbf{h}e^{\gamma n} + \mathbf{h}'e^{-\gamma n}$, where γ is the complex wavenumber defined by $\gamma \equiv (1+j)/\delta$ and δ denotes the skin depth. The result is

$$-j\omega \int_0^d \mathbf{B}\cdot\mathbf{n}dn = -\frac{j\omega\mu_0}{\sigma\varsigma}\left(H_{2n} + H_{1n}\right), \qquad (4)$$

where $\varsigma \equiv \gamma/[\sigma \tanh(\gamma d/2)]$. The insertion of (3) and (4) in (2) yields

$$H_{n2} + H_{n1} = \frac{j\varsigma}{\omega\mu_0}\nabla_t \cdot \left(\mathbf{H}_{t2} - \mathbf{H}_{t1}\right). \qquad (5)$$

We next consider the equation of flux conservation

$$\nabla \cdot \mathbf{B} = 0. \qquad (6)$$

Integrating (6) in the direction of the thickness, we have

$$\mu_0\left(H_{n2} - H_{n1}\right) + \nabla_t \cdot \int_0^d \mathbf{B}_t dn = 0 \qquad (7)$$

Evaluating $\mathbf{B}_t$ again in the layer in the same way as the one used above, we obtain the second equation

$$H_{n2} - H_{n1} = \frac{j\eta}{\omega\mu_0}\nabla_t \cdot \left(\mathbf{H}_{t2} + \mathbf{H}_{t1}\right), \qquad (8)$$

where $\eta \equiv j\omega\mu \tanh(\gamma d/2)/\gamma$. The symmetric differential equations (5) and (8) express the surface impedance of the thin shielding material. Since the variation of the magnetic field along the direction of the thickness has been already evaluated in the above procedure, the discretization of the shielding layer in this direction is not necessary. Of course, when the material is non-linear, this formulation is no longer valid. The equations (5) and (8) can be regarded as the boundary conditions to the equation that governs the magnetic fields encompassing the shielding layer.

III. Numerical Method

In this section, for the analysis of magnetic fields around thin shielding layers, we consider the numerical method based on the surface impedance introduced in the previous section.

Since the couple of differential equations (5) and (8) representing the surface impedance include a differential operator defined on a curved manifold, it is, in general, difficult to find the corresponding fundamental solution necessary for BEM. On the other hand, there is no difficulty in applying the FEM to the solution of the differential equations. The region encompassing the shielding materials, which is usually air and unbounded, can be effectively computed by BEM. For these reasons, we employ FEM and BEM to deal with the equations of surface impedance (5) and (8), and equations of magnetic fields in air region, respectively.

Let us consider the magnetic fields around a thin shielding layer S shown in Fig. 2. The shielding layer S is first assumed

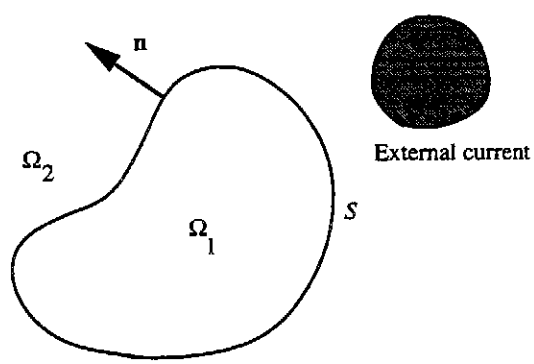

Fig.2. Shielding layer and external current

to be a closed surface. This assumption will be relaxed later. The current source which generates the applied field $\mathbf{H}_0$ is placed outside of S. The inside and outside regions Ω_1 and Ω_2 are air regions.

The magnetic fields in Ω_1 and Ω_2 are expressed by the integral equations as follows:

$$C(x)\varphi_1(x) = -\int_S H_{n1}(x')G(x';x)dS' - \int_S \varphi_1(x')\frac{\partial G(x';x)}{\partial n'}dS', \tag{9a}$$

$$[1-C(x)]\varphi_2(x) = \int_S H_{n2}(x')G(x';x)dS' + \int_S \varphi_2(x')\frac{\partial G(x';x)}{\partial n'}dS' + \varphi_0(x), \tag{9b}$$

where $G = 1/4\pi|\mathbf{x}'-\mathbf{x}|$, $C(x)$ is a coefficient whose value characterizes the solid angle of the point x, and φ_0 denotes the potential corresponding to the source field $\mathbf{H}_0$. We here employ the indirect method, that is, we add (9a) to (9b) to get

$$\frac{\tilde{\psi}(x)}{2} = \int_S \left[h(x')G(x';x) + \psi(x')\frac{\partial G(x';x)}{\partial n'}\right]dS' + [2C(x)-1]\frac{\psi(x)}{2} + \varphi_0(x), \tag{10}$$

where new variables have been introduced as follows :

$$\begin{cases} \tilde{\psi} \equiv \varphi_2 + \varphi_1, & (11a) \\ \psi \equiv \varphi_2 - \varphi_1, & (11b) \end{cases}$$

and

$$\begin{cases} \tilde{h} \equiv H_{n2} + H_{n1}, & (12a) \\ h \equiv H_{n2} - H_{n1}. & (12b) \end{cases}$$

Note that (10) holds not only for closed surfaces but also open surfaces. This can be understood by considering a closed surface S^* which consists of a open surface S corresponding to the shielding layer and a fictitious boundary S_f placed in vacuum region. We can now see that when (10) holds for S^*, it must also hold for S since the surface integrals in the right hand side of (10) vanishes on S_f.

We derive another equation in addition to (10) by taking the derivative of (9) in the normal direction $\mathbf{n}$ of the point x on S and adding them again. The result is

$$\frac{\tilde{h}(x)}{2} = \mathbf{n}\cdot\int_S \left[h(x')\nabla' G(x';x) + \left\{\nabla'_t\psi(x')\times\mathbf{n}'\right\} \times\nabla' G(x';x)\right]dS' + [2C(x)-1]\frac{h(x)}{2} + H_{0n}(x). \tag{13}$$

Equation (13) is also valid for both closed and open boundaries.

The discretization of (10) is easily performed by choosing the nodal points as the collocation points. On the other hand, this collocation is not valid for (13) which includes the normal vector $\mathbf{n}$ unless S is smooth around x. The simplest way to avoid this difficulty may be to put the collocation points at the center of boundary elements, where $\mathbf{n}$ is always well defined. To do so, $\tilde{h}$ is assumed to be constant on each element while the other variables are piecewise linear. Now we obtain the matrix equations from (10) and (13) as

$$\{\tilde{\psi}\} = [\mathrm{G}_1]\{h\} + [\mathrm{H}_1]\{\psi\} + \{\varphi_0\}, \tag{14a}$$

$$\{\tilde{h}\} = [\mathrm{G}_2]\{h\} + [\mathrm{H}_2]\{\psi\} + \{H_{0n}\}, \tag{14b}$$

where $[\mathrm{G}_1]$ and $[\mathrm{H}_1]$ are $N\times N$ matrices while $[\mathrm{G}_2]$ and $[\mathrm{H}_2]$ are $N_e\times N$ matrices.

On the other hand, we discretize (5) and (8) using FEM to get matrix equations of the form

$$[\mathrm{K}_1]\{\psi\} + [\mathrm{M}_1]\{\tilde{h}\} = \{0\}, \tag{15a}$$

$$[\mathrm{K}_2]\{\tilde{\psi}\} + [\mathrm{M}_2]\{h\} = \{0\}, \tag{15b}$$

where $[\mathrm{M}_1]$ is $N\times N_e$, and the others are $N\times N$ matrices. The substitution of (14) into (15) yields a system of equations

$$\begin{bmatrix} [\mathrm{M}_1][\mathrm{G}_2] & [\mathrm{K}_1]+[\mathrm{M}_1][\mathrm{H}_2] \\ [\mathrm{K}_2][\mathrm{G}_1]+[\mathrm{M}_2] & [\mathrm{K}_2][\mathrm{H}_1] \end{bmatrix}\begin{Bmatrix} h \\ \psi \end{Bmatrix} = -\begin{bmatrix} [\mathrm{M}_1] & [0] \\ [0] & [\mathrm{K}_2] \end{bmatrix}\begin{Bmatrix} H_{0n} \\ \varphi_0 \end{Bmatrix}. \tag{16}$$

We see that this formulation allows us to reduce the number of unknowns compared to the original problem; it is now $2N$ where N is the number of nodal points. After the solution of (16) the magnetic fields are computed by

$$\mathbf{H} = \int_S \left[h(x')\nabla' G(x';x) + \left\{\nabla'_t\psi(x')\times\mathbf{n}'\right\}\times\nabla' G(x';x)\right]dS' \tag{17}$$

IV. Numerical Example

We consider a spherical shell, with radius 0.1 [m], thickness 1[mm], conductivity $\sigma = 1.0\times10^7[1/\Omega\mathrm{m}]$, immersed in a uniform, time-harmonic magnetic field. The

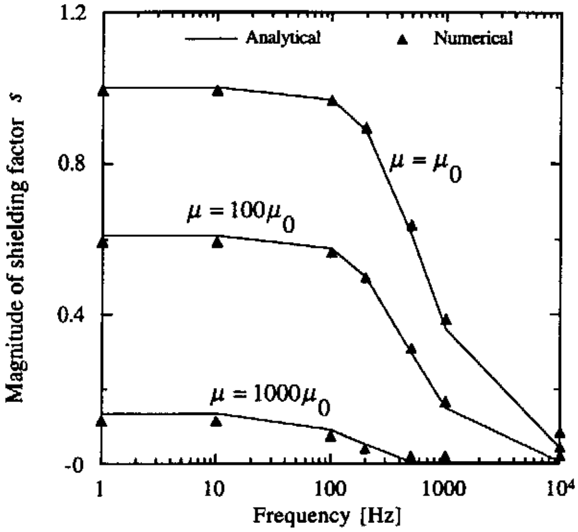

Fig. 3 Shielding factor of a sphericall shell

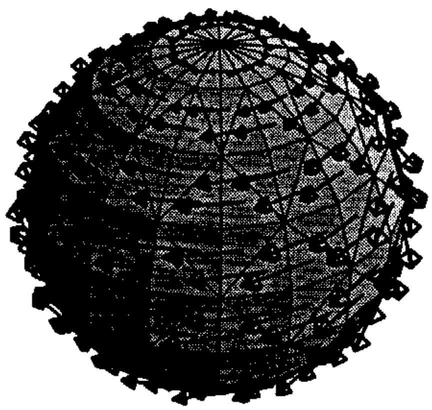

Fig. 4 Eddy currents on a spherical shell immersed in a uniform, time-hamonic magnetic field.

shielding factor s defined by $s=|\mathbf{H}_i|/|\mathbf{H}_0|$, where $\mathbf{H}_i$ and $\mathbf{H}_0$ are the magnetic field at the center of the shpere and applied magnetic field, respectively, is evaluated by the present method. The number of nodes N and elements N_e are taken to be 146 and 288, respectively. In Fig. 3, the computed shielding factors are plotted against the frequency. We see that the present method yields good results over the test range although the accuracy seems to become worse as the permeability increases. Figure 4 shows the eddy current distribution, for $\mu=100\mu_0$ and f=1 [kHz].

As mentioned in the previous section, the present method can also analyze magnetic fields around a shielding material with an open surface. To test this ability, the eddy current which is induced on a plate by a uniform, time-harmonic magnetic field perpendicular to the plate is analyzed by the method. Figure 5 shows the result, which seems reasonable from physical point of view.

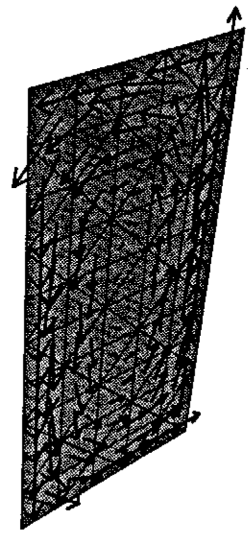

Fig. 5 Eddy currents on a plate immersed in a uniform, time-harmonic magnetic field.

V. Conclusion

In this paper, a numerical method for the analysis of magnetic fields with thin shielding layers have been described. The present method has the reduced set of unknowns, and thus shortens computational times. Moreover, it can be applied to the analysis of fields with open shielding layers. The shielding factors computed by the present method agree well with the analytical values. The method provides reasonable eddy currents on a plate.

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