An Overview of Field-to-Transmission Line Interaction

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Abstract

In this paper, we discuss the Transmission Line (TL) theory and its application to the problem of external electromagnetic field coupling to transmission lines. After a short discussion on the underlying assumptions of the TL theory, we start with the derivation of field-to-transmission line coupling equations for the case of a single wire line above a perfectly conducting ground. We also describe three seemingly different but completely equivalent approaches that have been proposed to describe the coupling of electromagnetic field coupling to transmission lines. The derived equations are extended to deal with the presence of losses and multiple conductors. The time-domain representation of field-to-transmission line coupling equations which allows a straightforward treatment of non linear phenomena as well as the variation in the line topology is also described. Finally, solution methods in frequency domain and time domain are presented.

1 Transmission Line (TL) Approximation

The problem of an external electromagnetic field coupling to an overhead line can be solved using a number of approaches. One such approach makes use of antenna theory, a general methodology based on Maxwell's equations[1] [1]. When electrically long lines are involved, however, the antenna theory approach implies prohibitively long computational times and high computer resources. On the other hand, the less resource hungry quasi-static approximation [1], in which propagation is neglected and coupling is described by means of lumped elements, can be adopted only when the overall dimensions of the circuit are smaller than the minimum significant wavelength of the electromagnetic field. For many practical cases, however, this condition is not satisfied. As an example, let us consider the case of power lines illuminated by a lightning electromagnetic pulse (LEMP). Power networks extend, in general, over distances of several kilometres, much larger than the minimum wavelengths associated with LEMP. Indeed, significant portions of the frequency spectrum of LEMP extend to frequencies up to of a few MHz and beyond, which corresponds to minimum wavelengths of about 100 m or less (e.g. [2]).

A third approach is known as transmission line (TL) theory. The main assumptions for this approach are::

1) Propagation occurs along the line axis.

[1] Different methods based on this approach generally assume that the wire's cross section is smaller than the minimum significant wavelength (thin-wire approximation).

2) The sum of the line currents at any cross-section of the line is zero. In other words, the ground – the reference conductor – is the return path for the currents in the *n* overhead conductors.
3) The response of the line to the coupled electromagnetic fields is quasi transverse electromagnetic (quasi-TEM) or, in other words, the electromagnetic field produced by the electric charges and currents along the line is confined in the transverse plane and perpendicular to the line axis.

If the cross-sectional dimensions of the line are electrically small, propagation can indeed be assumed to occur essentially along the line axis only and the first assumption can be considered to be a good approximation.
The second condition is satisfied if the ground plane exhibits infinite conductivity since, in that case, the currents and voltages can be obtained making use of the method of images, which guarantees currents of equal amplitude and opposite direction in the ground.
The condition that the response of the line is quasi-TEM is satisfied only up to a threshold frequency above which higher-order modes begin to appear [1]. For some cases, such as infinite parallel plates or coaxial lines, it is possible to derive an exact expression for the cutoff frequency below which only the TEM mode exists [3]. For other line structures (i.e. multiple conductors above a ground plane), the TEM mode response is generally satisfied as long as the line cross section is electrically small [3].
Under these conditions, the line can be represented by a distributed-parameter structure along its axis.
For uniform transmission lines with electrically-small cross-sectional dimensions (not exceeding about one tenth of the minimum significant wavelength of the exciting electromagnetic field), a number of theoretical and experimental studies have shown a fairly good agreement between results obtained using the TL approximation and results obtained either by means of antenna theory or experiments (see for example [4]). A detailed discussion of the validity of the basic assumptions of the TL theory is beyond the scope of this paper. However, it is worth noting that, by assuming that the sum of all the currents is equal to zero, we are considering only 'transmission line mode' currents and neglecting the so-called 'antenna-mode' currents [1]. If we wish to compute the load responses of the line, this assumption is adequate, because the antenna mode current response is small near the ends of the line. Along the line, however, and even for electrically small line cross sections, the presence of antenna-mode currents implies that the sum of the currents at a cross section is not necessarily equal to zero [1, 3]. However, the quasi-symmetry due to the presence of the ground plane, if present, results in a very small contribution of antenna mode currents and, consequently, the predominant mode on the line will be transmission line [1].

2 Single-Wire Line Above a Perfectly-Conducting Ground

We will consider first the case of a lossless single-wire line above a perfectly conducting ground. This simple case will allow us to introduce various coupling models and to discuss a number of concepts essential to the understanding of the electromagnetic field coupling phenomenon. Later in this paper (Sections 4 and 5), we will cover the cases of lossy and multiconductor lines. The transmission line is defined by its geometrical parameters (wire radius *a* and height above ground *h*) and its terminations Z_A and Z_B, as illustrated in Fig. 1, where the line is illuminated by an external electromagnetic field. The problem of interest is the calculation of the induced voltages and currents along the line and at the terminations.

It is worth noting that the external exciting electric and magnetic fields $\vec{E}^e$ and $\vec{B}^e$ are defined as the sum of the incident fields, $\vec{E}^i$ and $\vec{B}^i$, and the ground-reflected fields, $\vec{E}^r$ and $\vec{B}^r$, determined in absence of the line conductor. The total fields $\vec{E}$ and $\vec{B}$ at a given point in space are given by the sum of the excitation fields and the scattered fields from the line, the latter being denoted as $\vec{E}^s$ and $\vec{B}^s$. The scattered fields are created by the currents and charges flowing in the line conductor and by the corresponding currents and charges induced in the ground.

Three seemingly different but completely equivalent approaches have been proposed to describe the coupling of electromagnetic fields to transmission lines. In what follows, we will present each one of them in turn. We will first derive the field-to-transmission line coupling equations[2] following the development of Taylor et al. [5].

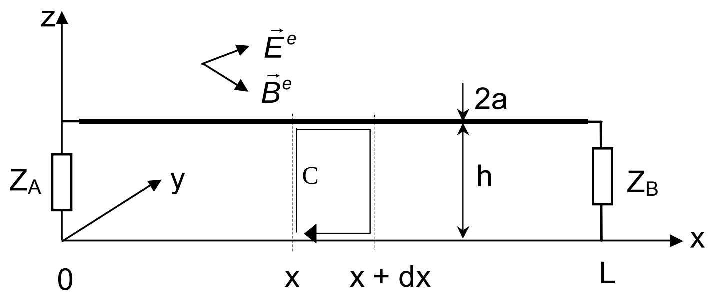

Figure 1: Geometry of the problem.

2.1 Taylor, Satterwhite and Harrison Model

2.1.1 Derivation of the First Field-to-Transmission Line Coupling (Generalized Telegrapher's) Equation

Consider the single conductor transmission line of height h in Figure 1. Applying Stokes' theorem to Maxwell's equation $\nabla \vec{E} = -j\omega\vec{B}$ for the area enclosed by the closed contour C yields

$$\oint_C \vec{E} \cdot dl = -j\omega \iint_S \vec{B} \cdot \vec{e}_y dS \tag{1}$$

[2] The field-to-transmission line coupling equations are sometimes referred to as generalized telegrapher's equations.

Since the contour has a differential width Δx, Equation (1) can be written as[3]

$$\int_0^h \left[E_z(x+\Delta x,z)-E_z(x,z)\right]dz+\int_x^{x+\Delta x}\left[E_x(x,h)-E_x(x,0)\right]dx$$
$$=-j\omega\int_0^h\int_x^{x+\Delta x} B_y(x,z)\,dx\,dz \tag{2}$$

Dividing by Δx and taking the limit as Δx approaches zero yields

$$\frac{\partial}{\partial x}\int_0^h E_z(x,z)dz+E_x(x,h)-E_x(x,0)=-j\omega\int_0^h B_y(x,z)dz \tag{3}$$

Since the wire and the ground are assumed to be perfect conductors, the total tangential electric fields, $E_x(x,h)$ and $E_x(x,0)$, are zero. Defining also the total transverse voltage $V(x)$ in the quasistatic sense (since $h<<\lambda$) as

$$V(x)=-\int_0^h E_z(x,z)dz \tag{4}$$

equation (3) becomes

$$\frac{dV(x)}{dx}=-j\omega\int_0^h B_y(x,z)dz=-j\omega\int_0^h B_y^e(x,z)dz-j\omega\int_0^h B_y^s(x,z)dz \tag{5}$$

where we have decomposed the B-field into the excitation and scattered components.

The last integral in (5) represents the magnetic flux between the conductor and the ground produced by the current $I(x)$ flowing in the conductor.

Now, Ampère-Maxwell's equation in integral form is given by

$$\oint_{C'} \vec{B}^S\cdot d\vec{l}=I+j\omega\iint \vec{D}\cdot d\vec{s} \tag{6}$$

If we use a path *C'* in the transverse plane, defined by a constant *x* in such a manner that the conductor goes through it, Equation (6) can be rewritten as

$$\oint_{C'} \vec{B}_T^S(x,y,z)\cdot d\vec{l}=I(x)+j\omega\iint \vec{D}_X(x,y,z)\cdot\vec{a}_X ds \tag{7}$$

where the subindex *T* is used to indicate that the field is in the transverse direction, $\vec{a}_x$ is the unit vector in the x direction, and where we have explicitly included the dependence of the fields on the three Cartesian coordinates.

If the response of the wire is TEM, the electric flux density *D* in the *x* direction is zero and Equation (7) can be written as

$$\oint_{C'} \vec{B}_T^S(x,y,z)\cdot d\vec{l}=I(x) \tag{8}$$

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[3] The coordinate *y* will be implicitly assumed to be 0 and for the sake of clarity, we will omit the *y*-dependency unless the explicit inclusion is important for the discussion.

Clearly, *I(x)* is the only source of $\vec{B}_T^S(x,y,z)$. Further, it is apparent from Equation (8) that $\vec{B}_T^S(x,y,z)$ is directly proportional to *I(x)*. Indeed, if *I(x)* is multiplied by a constant multiplicative factor which, in general, can be complex, $\vec{B}_T^S(x)$ too will be multiplied by that factor. Further, the proportionality factor for a uniform cross-section line must be independent of *x*.

Let us now concentrate on the *y* component of $\vec{B}_T^S(x,y,z)$ for points in the plane *y=0*. Using the facts we just established that *I(x)* and $\vec{B}_T^S(x)$ are proportional and that the proportionality factor is independent of x, we can now write

$$B_y^S(x, y=0, z) = k(y=0, z)I(x) \tag{9}$$

where *k(y,z)* is the proportionality constant.

With this result, we now go back to the last integral in Equation (5),

$$\int_0^h B_y^S(x,z)dz$$

Note that, although the value of y is not explicitly given, *y=0*. The integral represents the per unit length magnetic flux under the line. Substituting (9) into it, we obtain

$$\int_0^h B_y^S(x,z)dz = \int_0^h k(y=0,z)I(x)dz \tag{10}$$

We ca rewrite (10) as follows

$$\int_0^h B_y^S(x,z)dz = I(x)\int_0^h k(y=0,z)dz \tag{11}$$

Equation (11) implies that the per-unit-length scattered magnetic flux under the line at any point along it is proportional to the current at that point. The proportionality constant, given by $\int_0^h k(y=0,z)dz$, is the per-unit-length inductance of the line.

This results in the well-known linear relationship between the magnetic flux and the line current, the proportionality constant being the line per-unit-length inductance:

$$\int_0^h B_y^S(x,z)dz = L'I(x) \tag{12}$$

Assuming that the transverse dimension of the line is much greater than the height of the line, (*a*<<*h*), the magnetic flux density can be calculated using Ampere's Law and the integral can be evaluated analytically [1]. For *h*>>*a*, $L' \cong \frac{\mu_o}{2\pi}\ln\left(\frac{2h}{a}\right)$.

Inserting (12) into (5), we obtain the first generalized telegrapher's equation

$$\frac{dV(x)}{dx} + j\omega L' I(x) = -j\omega \int_0^h B_y^e(x,z)dz \qquad (13)$$

Note that, unlike the classical telegrapher's equations in which no external excitation is considered, the presence of an external field results in a forcing function expressed in terms of the exciting magnetic flux. This forcing function can be viewed as a distributed voltage source along the line.

Attention must paid to the fact that the voltage *V(x)* in (13) depends on the integration path since it is obtained by integration of an electric field whose curl is not necessarily zero (Equation (4)).

2.1.2 Derivation of the Second Field-to-Transmission Line Coupling Equation

To derive the second telegrapher's equation, we will assume that the medium surrounding the line is air ($\varepsilon = \varepsilon_o$) and we will start from the second Maxwell's equation $\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon_o \vec{E}$. Rearranging the terms and writing it in Cartesian coordinates for the *z*-component:

$$j\omega E_z(x,z) = \frac{1}{\varepsilon_o \mu_o}\left[\frac{\partial B_y(x,z)}{\partial x} - \frac{\partial B_x(x,z)}{\partial y}\right] - \frac{J_z}{\varepsilon_o} \qquad (14)$$

The current density can be related to the E-field using Ohm's law, $\vec{J} = \sigma_{air}\vec{E}$, where σ_{air} is the air conductivity. Since the air conductivity is generally low, we will assume here that σ_{air} =0 and will therefore neglect this term[4].

Integrating (14) along the *z* axis from 0 to *h*, and making use of (4), we obtain

$$\begin{aligned} -j\omega V(x) = & \frac{1}{\varepsilon_o \mu_o}\int_0^h \left[\frac{\partial B_y^e(x,z)}{\partial x} - \frac{\partial B_x^e(x,z)}{\partial y}\right]dz \\ & + \frac{1}{\varepsilon_o \mu_o}\int_0^h \left[\frac{\partial B_y^s(x,z)}{\partial x} - \frac{\partial B_x^s(x,z)}{\partial y}\right]dz \end{aligned} \qquad (15)$$

in which we have decomposed the magnetic flux density field into the excitation and scattered components.

Since the excitation fields are the fields that would exist if the line were not present, they must satisfy Maxwell's equations. Applying Maxwell's equation (14) to the components of the excitation electromagnetic field and integrating along *z* from 0 to *h* along a straight line directly under the line yields

$$\frac{1}{\varepsilon_o \mu_o}\int_0^h \left[\frac{\partial B_y^e}{\partial x} - \frac{\partial B_x^e}{\partial y}\right]dz = j\omega \int_0^h E_z^e dz \qquad (16)$$

Using (12), (16) and given that $B_x^s = 0$ by virtue of the assumed TEM nature of the line response, Equation (15) becomes

[4] This term will eventually result in an equivalent parallel conductance in the coupling equation (see Section 5).

$$\frac{dI(x)}{dx} + j\omega C'V(x) = -j\omega C'\int_0^h E_z^e(x,z)dz \tag{17}$$

where *C'* is the per-unit-length line capacitance related to the per-unit-length inductance through $\varepsilon_o \mu_o = L'C'$. Equation (17) is the second field-to-transmission line coupling equation.

For a line of finite length, such as the one represented in Fig. 1, the boundary conditions for the load currents and voltages must be enforced. They are simply given by

$$V(0) = -Z_A I(0) \tag{18}$$

$$V(L) = Z_B I(L) \tag{19}$$

2.1.3 Equivalent Circuit

Equations (13) and (17) are referred to as the *Taylor* et al. model. They can be represented using an equivalent circuit, as shown in Fig. 2. The forcing functions (source terms) in (13) and (17) are included as a set of distributed series voltage and parallel current sources along the line.

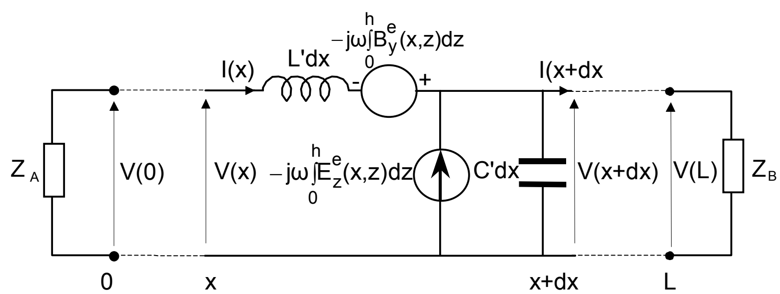

Figure 2: Equivalent circuit of a lossless single-wire overhead line excited by an electromagnetic field. *Taylor* et al. model.

2.2 Agrawal, Price and Gurbaxani Model

An equivalent formulation of field-to-transmission line coupling equations was proposed in 1980 by *Agrawal*, *Price* and *Gurbaxani* [6]. This model is commonly referred to as the Agrawal model. We will call it the *model of Agrawal et al.* or *the Agrawal et al. model* hereafter.

The basis for the derivation of the *Agrawal* et al. model can be described as follows: The excitation fields produce a line response that is TEM. This response is expressed in terms of a scattered voltage $V^s(x)$, which is defined in terms of the line integral of the scattered electric field from the ground to the line, and a scattered current $I^s(x)$ which flows in the line. The total voltage *V(x)* and the total current *I(x)* (the quantities that are actually measurable) are computed as the sum of the excitation and the scattered voltages and currents.

The coupling equations in the model of *Agrawal et al.* are used to obtain the scattered voltage and the scattered current only. Specific components of the incident fields appear either as a source term in the coupling equations or are used to compute the total voltage $V(x)$, which corresponds to that used in the *model of Taylor et al.* In the *model of Agrawal et al.*, the total current $I(x)$ is identical to the scattered current and it is obtained directly from the coupling equations. As we will see in the next section when we present the Rachidi's model [7], it is possible to define a distinct excitation current $I_e(x)$.

In the *model of Agrawal et al.*, the rational behind the writing of the telegrapher's equations in terms of the scattered quantities only is that, whereas the incident fields are arbitrary (they are of course constrained to satisfy Maxwell's equations and the ground boundary conditions), the scattered response is TEM, which allows for them to be calculated using TL theory.

The total voltage can be obtained from the scattered voltage through

$$V(x) = V^s(x) + V^e(x) = V^s(x) - \int_0^h E_z^e(x,z)dz \tag{20}$$

The field-to-transmission line coupling equations as derived by *Agrawal et al.* [6] are given by

$$\frac{dV^s(x)}{dx} + j\omega L' I(x) = E_x^e(x,h) \tag{21}$$

$$\frac{dI(x)}{dx} + j\omega C' V^s(x) = 0 \tag{22}$$

Note that in this model, only one source term is present (in the first equation) and is simply expressed in terms of the exciting electric field tangential to the line conductor $E_x^e(x,h)$.

The boundary conditions in terms of the scattered voltage and the total current as used in (21) and (22), are given by

$$V^s(0) = -Z_A I(0) + \int_0^h E_z^e(0,z)dz \tag{23}$$

$$V^s(L) = Z_B I(L) + \int_0^h E_z^e(L,z)dz \tag{24}$$

The equivalent circuit representation of this model (equations (21)-(24)) is shown in Fig. 3. For this model, the forcing function (the exciting electric field tangential to the line conductor) is represented by distributed voltage sources along the line. In accordance with boundary conditions (23) and (24), two lumped voltage sources (equal to the line integral of the exciting vertical electric field) are inserted at the line terminations.

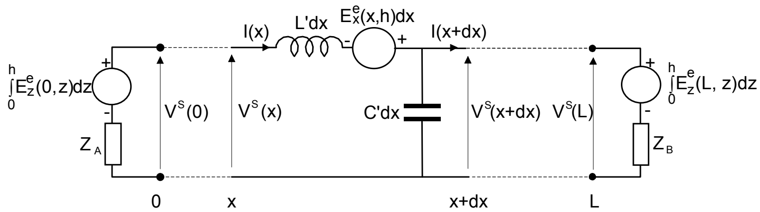

Figure 3: Equivalent circuit of a lossless single-wire overhead line excited by an electromagnetic field. *Agrawal et al. model*.

It is also interesting to note that this model involves only electric field components of the exciting field and the exciting magnetic field does not appear explicitly as a source term in the coupling equations.

2.3 Rachidi Model

Another form of coupling equations, equivalent to the *Agrawal et al.* and to the *Taylor et al.* models, has been derived by *Rachidi* [7]. In this model, only the exciting magnetic field components appear explicitly as forcing functions in the equations:

$$\frac{dV(x)}{dx} + j\omega L' I^s(x) = 0 \tag{25}$$

$$\frac{dI^s(x)}{dx} + j\omega C' V(x) = \frac{1}{L'}\int_0^h \frac{\partial B_x^e(x,z)}{\partial y} dz \tag{26}$$

in which $I^s(x)$ is the so-called scattered current related to the total current by

$$I(x) = I^s(x) + I^e(x) \tag{27}$$

where the excitation current $I^e(x)$ is defined as

$$I^e(x) = -\frac{1}{L'}\int_0^h B_y^e(x,z)dz \tag{28}$$

The boundary conditions corresponding to this formulation are

$$I^s(0) = -\frac{V(0)}{Z_A} + \frac{1}{L'}\int_0^h B_y^e(0,z)dz \tag{29}$$

$$I^s(L) = \frac{V(L)}{Z_B} + \frac{1}{L'}\int_0^h B_y^e(L,z)dz \tag{30}$$

The equivalent circuit corresponding to the above equivalent set of coupling equations is shown in Fig. 4. Note that the equivalent circuit associated with the *Rachidi model* could be seen as the dual circuit - in the sense of electrical network theory - of the one corresponding to the *Agrawal et al. model* (Fig. 3).

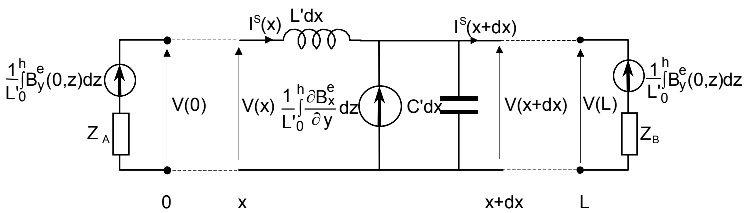

Figure 4: Equivalent circuit of a lossless single-wire overhead line excited by an electromagnetic field. *Rachidi model*.

3 Contribution of the Different Electromagnetic Field Components

Nucci and *Rachidi* [8] have shown, on the basis of a specific numerical example that, as predicted theoretically, the total induced voltage waveforms obtained using the three coupling models presented in Sections 2.1, 2.2 and 2.3 are identical. However, the contribution of a given component of the exciting electromagnetic field to the total induced voltage and current varies depending on the adopted coupling model. Indeed, the three coupling models are different but fully equivalent approaches that predict identical results in terms of total voltages and total currents, in spite of the fact that they take into account the electromagnetic coupling in different ways. In other words, the three models are different expressions of the same equations, cast in terms of different combinations of the various electromagnetic field components, which are related through Maxwell's equations.

4 Inclusion of Losses

In the calculation of lightning-induced voltages, losses are, in principle, to be taken into account both in the wire and in the ground. Losses due to the finite ground conductivity are the most important ones, and they affect both the electromagnetic field and the surge propagation along the line [9].

Let us make reference to the same geometry of Fig. 1, and let us now take into account losses both in the wire and in the ground plane. The wire conductivity and relative permittivity will be denoted σ_w and ε_{rw}, respectively, and the ground, assumed to be homogeneous, is characterized by its conductivity σ_g and its relative permittivity ε_{rg}. The *Agrawal* et al. coupling equations extended to the present case of a wire above an imperfectly conducting ground can be written as (for a step by step derivation see [1])

$$\frac{dV^s(x)}{dx} + Z'I(x) = E_x^e(x,h) \tag{31}$$

$$\frac{dI(x)}{dx} + Y'V^s(x) = 0 \tag{32}$$

where Z' and Y' are the longitudinal and transverse per-unit-length impedance and admittance, respectively, given by [1, 9][5]

$$Z' = j\omega L' + Z'_w + Z'_g \tag{33}$$

$$Y' = \frac{(G' + j\omega C')Y'_g}{G' + j\omega C' + Y'_g} \tag{34}$$

in which

- L', C' and G' are the per-unit-length longitudinal inductance, transverse capacitance and transverse conductance, respectively, calculated for a lossless wire above a perfectly conducting ground:

$$L' = \frac{\mu_o}{2\pi}\cosh^{-1}\left(\frac{h}{a}\right) \cong \frac{\mu_o}{2\pi}\ln\left(\frac{2h}{a}\right) \qquad \text{for } h >> a \tag{35}$$

$$C' = \frac{2\pi\varepsilon_o}{\cosh^{-1}(h/a)} \cong \frac{2\pi\varepsilon_o}{\ln(2h/a)} \qquad \text{for } h >> a \tag{36}$$

$$G' = \frac{\sigma_{\mathrm{air}}}{\varepsilon_o}C' \tag{37}$$

- Z'_w is the per-unit-length internal impedance of the wire; assuming a round wire and an axial symmetry for the current, the following expression can be derived for the wire internal impedance (e.g. [10]):

$$Z'_w = \frac{\gamma_w \mathrm{I}_o(\gamma_w a)}{2\pi a \sigma_w \mathrm{I}_1(\gamma_w a)} \tag{38}$$

where $\gamma_w = \sqrt{j\omega\mu_o(\sigma_w + j\omega\varepsilon_o\varepsilon_{rw})}$ is the propagation constant in the wire and I_o and I_1 are the modified Bessel functions of zero and first order, respectively;

- Z'_g is the per-unit-length ground impedance, which is defined as [11, 12]

$$Z'_g = \frac{j\omega\int_{-\infty}^{h} B_y^s(x,z)dx}{I} - j\omega L' \tag{39}$$

where B_y^s is the y-component of the scattered magnetic induction field.

Several expressions for the ground impedance have been proposed in the literature (e.g. [13]). *Sunde* [14] derived a general expression for the ground impedance, which is given by

$$Z'_g = \frac{j\omega\mu_o}{\pi}\int_0^{\infty}\frac{\mathrm{e}^{-2hx}}{\sqrt{x^2 + \gamma_g^2} + x}dx \tag{40}$$

[5] In [1] the per unit length transverse conductance has been disregarded.

where $\gamma_g = \sqrt{j\omega\mu_o(\sigma_g + j\omega\varepsilon_o\varepsilon_{rg})}$ is the propagation constant in the ground.

As noted in [13], *Sunde*'s expression (40) is directly connected to the general expressions obtained from scattering theory. Indeed, it is shown in [1] that the general expression for the ground impedance derived using scattering theory reduces to the *Sunde* approximation when considering the transmission line approximation. Also, the results obtained using (40) are shown to be accurate within the limits of the transmission line approximation [1].

The general expression (40) is not suitable for a numerical evaluation since it involves an integral over an infinitely long interval. Several approximations for the ground impedance of a single-wire line have been proposed in the literature (see [11] for a survey). One of the simplest and most accurate was proposed by *Sunde* himself and is given by the following logarithmic function

$$Z'_g \cong \frac{j\omega\mu_o}{2\pi}\ln\left(\frac{1+\gamma_g h}{\gamma_g h}\right) \tag{41}$$

It has been shown [11] that the above logarithmic expression represents an excellent approximation to the general expression (40) over the frequency range of interest.

Finally, Y'_g is the so-called ground admittance, given by [1]

$$Y'_g \cong \frac{\gamma_g^2}{Z'_g} \tag{42}$$

5 Case of Multiconductor Lines

Making reference to the geometry of Fig. 5, the field-to-transmission line coupling equations for the case of a multi-wire system along the *x*-axis above an imperfectly conducting ground and in presence of an external electromagnetic excitation are given by [1, 4, 15]

$$\frac{d}{dx}[V_i^s(x)] + j\omega[L'_{ij}][I_i(x)] + [Z'_{g_{ij}}][I_i(x)] = [E_x^e(x,h_i)] \tag{43}$$

$$\frac{d}{dx}[I_i(x)] + [G'_{ij}][V_i^s(x)] + j\omega[C'_{ij}][V_i^s(x)] = [0] \tag{44}$$

in which

- $[V_i^s(x)]$ and $[I_i(x)]$ are frequency-domain vectors of the scattered voltage and the current along the line;

- $[E_x^e(x,h_i)]$ is the vector of the exciting electric field tangential to the line conductors;

- [0] is the zero-matrix (all elements are equal to zero);

- $[L'_{ij}]$ is the per-unit-length line inductance matrix. Assuming that the distances between conductors are much larger than their radii, the general expression for the mutual inductance between two conductors *i* and *j* is given by [1]

$$L'_{ij} = \frac{\mu_o}{2\pi} \ln\left(\frac{r_{ij}^2 + (h_i + h_j)^2}{r_{ij}^2 + (h_i - h_j)^2} \right) \tag{45}$$

The self inductance for conductor *i* is given by

$$L'_{ii} = \frac{\mu_o}{2\pi} \ln\left(\frac{2h_i}{r_{ii}} \right) \tag{46}$$

- $[C'_{ij}]$ is the per-unit-length line capacitance matrix, which can be evaluated directly from the inductance matrix using the following expression [1]

$$\left[C'_{ij}\right] = \varepsilon_o \mu_o \left[L'_{ij}\right]^{-1} \tag{47}$$

- $[G'_{ij}]$ is the per-unit-length transverse conductance matrix. The transverse conductance matrix elements can be evaluated starting either from the capacitance matrix or the inductance matrix using the following relations

$$\left[G'_{ij}\right] = \frac{\sigma_{air}}{\varepsilon_o} \left[C'_{ij}\right] = \sigma_{air} \mu_o \left[L'_{ij}\right]^{-1} \tag{48}$$

In most practical cases, the transverse conductance matrix elements G'_{ij} are much smaller than $j\omega C'_{ij}$ [3] and can therefore be neglected in the computation.

- Finally, $[Z'_{g_{ij}}]$ is the ground impedance matrix. The general expression for the mutual ground impedance between two conductors *i* and *j* derived by Sunde is given by [14]

$$Z'_{g_{ij}} = \frac{j\omega\mu_o}{\pi} \int_0^\infty \frac{e^{-(h_i + h_j)x}}{\sqrt{x^2 + \gamma_g^2} + x} \cos(r_{ij} x) dx \tag{49}$$

In a similar way as for the case of a single-wire line, an accurate logarithmic approximation is proposed by *Rachidi* et al. [15] which is given by

$$Z'_{g_{ij}} \cong \frac{j\omega\mu_o}{4\pi} \ln\left[\frac{\left(1 + \gamma_g \left(\frac{h_i + h_j}{2}\right)\right)^2 + \left(\gamma_g \frac{r_{ij}}{2}\right)^2}{\left(\gamma_g \frac{h_i + h_j}{2}\right)^2 + \left(\gamma_g \frac{r_{ij}}{2}\right)^2} \right] \tag{50}$$

Note that in (43) and (44), the terms corresponding to the wire impedance and the so-called ground admittance have been neglected. This approximation is valid for typical overhead power lines [9].

The boundary conditions for the two line terminations are given by

$$[V_i^s(0)] = -[Z_A][I_i(0)] + [\int_0^{h_i} E_z^e(0,z)dz] \quad (51)$$

$$[V_i^s(L)] = [Z_B][I_i(L)] + [\int_0^{h_i} E_z^e(L,z)dz] \quad (52)$$

in which $[Z_A]$ and $[Z_B]$ are the impedance matrices at the two line terminations.

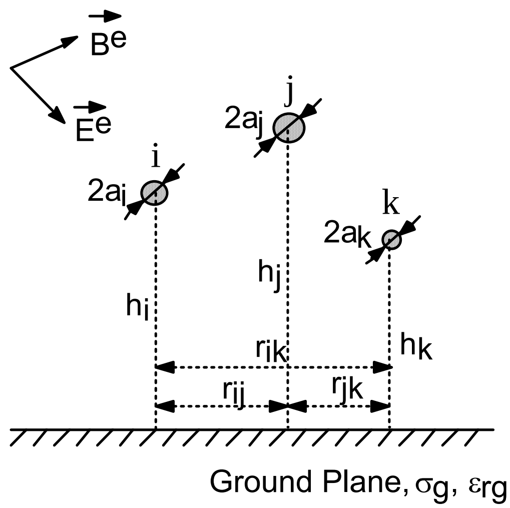

Figure 5: Cross-sectional geometry of a multiconductor line in presence of an external electromagnetic field.

6 Time-Domain Representation of the Coupling Equations

A time domain representation of the field-to-transmission line coupling equations is sometimes preferable because it allows the straightforward treatment of non linear phenomena as well as the variation in the line topology [4]. On the other hand, frequency-dependent parameters, such as the ground impedance, need to be represented using convolution integrals.

The field-to-transmission line coupling equations (43) and (44) can be converted into the time domain to obtain the following expressions

$$\frac{\partial}{\partial x}\left[v_i^s(x,t)\right] + \left[L'_{ij}\right]\frac{\partial}{\partial t}\left[i_i(x,t)\right] + \left[\xi'_{g_{ij}}\right] \otimes \frac{\partial}{\partial t}\left[i_i(x,t)\right] = \left[E_x^e(x,h_i,t)\right] \quad (53)$$

$$\frac{\partial}{\partial x}\left[i_i(x,t)\right] + \left[G'_{ij}\right]\left[v_i^s(x,t)\right] + \left[C'_{ij}\right]\frac{\partial}{\partial t}\left[v_i^s(x,t)\right] = 0 \quad (54)$$

in which $\otimes$ denotes convolution product and the matrix $\left[\xi'_{g_{ij}}\right]$ is called the transient ground resistance matrix; its elements are defined as

$$\left[\xi'_{g_{ij}}\right] = \mathrm{F}^{-1}\left\{\frac{\left[Z'_{g_{ij}}\right]}{j\omega}\right\} \tag{55}$$

The inverse Fourier transforms of the boundary conditions written, for simplicity, for resistive terminal loads read

$$\left[v_i(0,t)\right] = -\left[R_A\right]\left[i_i(0,t)\right] + \left[\int_0^{h_i} E_z^e(0,z,t)dz\right] \tag{56}$$

$$\left[v_i(L)\right] = \left[R_B\right]\left[i_i(0)\right] + \left[\int_0^{h_i} E_z^e(L,z,t)dz\right] \tag{57}$$

where $\left[R_A\right]$ and $\left[R_B\right]$ are the matrices of the resistive loads at the two line terminals.

The general expression for the ground impedance matrix terms in the frequency domain (49) does not have an analytical inverse Fourier transform. Thus, the elements of the transient ground resistance matrix in the time domain are to be, in general, determined using a numerical inverse Fourier transform algorithm. However, analytical expressions have been derived which are shown to be reasonable approximations to the numerical values obtained using an inverse FFT [13].

7 Frequency-Domain Solutions

Different approaches can be employed to find solutions to the presented coupling equations. This section and Section 8 present some solution methods in the frequency domain and in the time domain, respectively.

To solve the coupling equations in the frequency domain, it is convenient to use Green's functions that relate, as a function of frequency, the individual coupling sources to the scattered or the total voltages and currents at any point along the line. Green's functions solutions require integration over the length of the line, where the distributed sources are located. This approach is the subject of section 7.1.

Under special conditions, it is possible to obtain more compact solutions or even analytical expressions. In particular, if the solutions are required at the load terminations only, it is possible to write the load voltages and currents in a compact manner, where the complexity is essentially hidden in the source terms. This formulation, termed the BLT equations, will be presented in Section 7.2.

7.1 Green's Functions

The field-to-transmission line coupling equations, together with the boundary conditions, can be solved using Green's functions, which represent the solutions for line current and voltage due to a point voltage and/or current source [1]. In this section, we will present the solutions, using the *Agrawal at al. model* for the case of a single-conductor line. Similar solutions can be found for the case of a multiconductor line (see, for instance, [1], [3]).

Considering a voltage source of unit amplitude at a location x_s along the line[6], the Green's functions for the current and the voltage along the line read, respectively [1],

$$G_I(x;x_S) = \frac{e^{-\gamma L}}{2Z_c\left(1-\rho_1\rho_2 e^{-2\gamma L}\right)}\left(e^{-\gamma(x_> - L)} - \rho_2 e^{\gamma(x_> - L)}\right)\left(e^{\gamma x_<} - \rho_1 e^{-\gamma x_<}\right) \tag{58}$$

and

$$G_V(x;x_S) = \frac{\delta e^{-\gamma L}}{2\left(1-\rho_1\rho_2 e^{-2\gamma L}\right)}\left(e^{-\gamma(x_> - L)} + \delta\rho_2 e^{\gamma(x_> - L)}\right)\left(e^{\gamma x_<} - \delta\rho_1 e^{-\gamma x_<}\right) \tag{59}$$

where

- $x_<$ represents the smaller of x or x_S, and $x_>$ represents the larger of x or x_S.

– δ=1 for $x > x_S$ and δ= -1 for $x < x_S$.

- $\gamma = \sqrt{Z'Y'}$ is the complex propagation constant along the transmission line,

- $Z_c = \sqrt{Z'/Y'}$ is the line's characteristic impedance.

- ρ_1 and ρ_2 are the voltage reflection coefficients at the loads of the transmission line given by

$$\rho_1 = \frac{Z_A - Z_c}{Z_A + Z_c} \qquad \rho_2 = \frac{Z_B - Z_c}{Z_B + Z_c} \tag{60}$$

The solutions in terms of the total line current and *scattered* voltage can be written as the following integrals of the Green's functions [1]

$$\begin{aligned} I(x) = & \int_0^L G_I(x;x_S)V'_S dx_S + \\ & G_I(x;0)\int_0^h E_z^e(0,z)dz - G_I(x;L)\int_0^h E_z^e(L,z)dz \end{aligned} \tag{61}$$

$$\begin{aligned} V^S(x) = & \int_0^L G_V(x;x_S)V'_S dx_S + \\ & G_V(x;0)\int_0^h E_z^e(0,z)dz - G_V(x;L)\int_0^h E_z^e(0,z)dz \end{aligned} \tag{62}$$

Note that the second and the third terms on the right hand side of (61) and (62) are due to the contribution of equivalent lumped sources at the line ends (see Fig. 3).

The total voltage can be determined from the scattered voltage by adding the contribution from the exciting field as

[6] Since only distributed series voltage sources are present in the model of *Agrawal et al.*, it is not necessary to consider a parallel unitary current source.

$$V(x) = V^s(x) - \int_0^d E_z^e(x,z)dz \quad (63)$$

7.2 BLT Equations

If we are interested in the transmission line response at its terminal loads, the solutions can be expressed in a compact way by using the so-called BLT (Baum, Liu, Tesche) equations [1],

$$\begin{bmatrix} I(0) \\ I(L) \end{bmatrix} = 1/Z_c \begin{bmatrix} 1-\rho_1 & 0 \\ 0 & 1-\rho_2 \end{bmatrix} \begin{bmatrix} -\rho_1 & e^{\gamma L} \\ e^{\gamma L} & -\rho_2 \end{bmatrix}^{-1} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad (64)$$

$$\begin{bmatrix} V(0) \\ V(L) \end{bmatrix} = \begin{bmatrix} 1+\rho_1 & 0 \\ 0 & 1+\rho_2 \end{bmatrix} \begin{bmatrix} -\rho_1 & e^{\gamma L} \\ e^{\gamma L} & -\rho_2 \end{bmatrix}^{-1} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad (65)$$

where the source vector is given by

$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\int_0^L e^{\gamma x_s} E_x^e(x_s,h)dx_s + \frac{1}{2}\int_0^h E_z^e(0,z)dz - \frac{e^{\gamma L}}{2}\int_0^h E_z^e(L,z)dz \\ \frac{-1}{2}\int_0^L e^{\gamma(L-x_s)} E_x^e(x_s,h)dx_s - \frac{e^{\gamma L}}{2}\int_0^h E_z^e(0,z)dz + \frac{1}{2}\int_0^h E_z^e(L,z)dz \end{pmatrix} \quad (66)$$

Note that, in the BLT equations, the solutions are directly given for the total voltage and not for the scattered voltage.

For an arbitrary excitation field, the integrals in Equation (66) cannot be performed analytically. However, for the special case of a plane wave excitation field, the integrations can be performed analytically and closed-form expressions can be obtained for the load responses. General solutions for vertical and horizontal field polarizations are given in [1].

8 Time-Domain Solutions

Several approaches can be used to solve the coupling equations in the time domain ([1, 3]). We will present here simple analytical expressions that can be obtained for the case of a lossless line involving infinite summations.

Under the assumption of a lossless line, it is possible to obtain analytical solutions for the transient response of a transmission line to an external field excitation [1]. In this case, the propagation constant becomes purely imaginary $\gamma = j\omega / c$ and the characteristic impedance is purely real $Z_c = \sqrt{L'/C'}$. If we assume further that the termination impedances are purely resistive, the reflection coefficients ρ_1 and

ρ_2, too, become real. For $\left|\rho_1\rho_2 e^{-2\gamma L}\right| < 1$, the denominator in Green's functions (58) and (59) can be expanded to[7]

$$\frac{1}{\left(1-\rho_1\rho_2 e^{-2\gamma L}\right)} = \sum_{n=0}^{\infty}\left(\rho_1\rho_2 e^{-j\omega 2L/c}\right)^n \tag{67}$$

With the above transformation, it is easy to show that all the frequency dependences in (64) and (65) will be in the form $e^{-j\omega\tau}$, τ being a constant. Therefore, it is possible to convert the frequency domain solutions analytically and to obtain the following transient responses for the load voltages (for details, see [1])

$$v(0,t) = (1+\rho_1)\sum_{n=0}^{\infty}(\rho_1\rho_2)^n \frac{1}{2}\left(\rho_2 v_s(t-\frac{2(n+1)L-x_s}{c}) - v_s(t-\frac{2nL+x_s}{c})\right) \tag{68}$$

$$v(L,t) = (1+\rho_2)\sum_{n=0}^{\infty}(\rho_1\rho_2)^n \frac{1}{2}\left(v_s(t-\frac{2(n+1)L-x_s}{c}) - \rho_1 v_s(t-\frac{2(L+1)+x_s}{c})\right) \tag{69}$$

where

$$v_s(t) = \int_0^L E_x^e(x_s,h,t)dx_s + \int_0^h E_z^e(0,z,t)dz - \int_0^h E_z^e(L,z,t)dz \tag{70}$$

Note that $E_x^e(x_s,h,t)$, $E_z^e(0,z,t)$ and $E_z^e(L,z,t)$ are time-domain components of the exciting field.

9 Conclusions

We discussed the Transmission Line (TL) theory and its application to the problem of external electromagnetic field coupling to transmission lines. After a short discussion on the underlying assumptions of the TL theory, the field-to-transmission line coupling equations were derived for the case of a single wire line above a perfectly conducting ground. Three different but completely equivalent approaches that have been proposed to describe the coupling of electromagnetic field coupling to transmission lines were also presented and discussed. The derived equations were extended to deal with the presence of losses and multiple conductors. The time-domain representation of field-to-transmission line coupling equations which allows a straightforward treatment of non linear phenomena as well as the variation in the line topology was also described. Finally, solution methods in frequency domain and time domain were presented.

[7] For a lossless line with reflection coefficients of magnitude 1, the condition $\rho_1\rho_2 e^{-2\gamma L} = 1$ will be met at a number of resonance frequencies causing the solutions to be unbounded.

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