Exact Radiation by Isorefractive Slotted Elliptic Cylindrical Antenna

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Abstract:

Exact solution of the electromagnetic radiation from a conducting slotted elliptical cylinder coated by an isorefractive material derived using the separation of variables technique. The fields inside and outside the dielectric layer are expressed in terms of Mathieu functions. No matrix inversion is required after obtaining exact expressions for the radiated field expansion coefficients by imposing the appropriate boundary conditions. Numerical results are plotted for the radiation pattern, aperture conductance and antenna gain. The results show that a slotted antenna coated with isorefractive material has more directive beam and lower side-lobes compared to that coated with conventional dielectric material of comparable physical dimensions.

1. Introduction

Radiation properties of an axially slotted antenna are very important in communications and airplane industries. Numerous authors in the literature have investigated the radiation by dielectric coated slotted circular and elliptical cylinders. For example, Hurd [1] studied the radiation pattern of a dielectric axially-slotted cylinder. The external admittance of an axial slot on a dielectric coated metal cylinder was investigated by Knop [2]. Shafai [3] obtained the radiation properties of an axial slotted antenna coated with a homogenous material. Wong [4,5] investigated the radiation properties of slotted cylinder of elliptical cross section while Richmond [6] studied the radiation from an axial slot antenna on a dielectric- coated elliptic cylinder. The analysis was later extended to the radiation by axial slots on a dielectric coated nonconfocal conducting elliptic cylinder by Ragheb [7]. Hussein and Hamid [8] studied the radiation by N axially slotted cylinders of elliptical cross section coated with a lossy dielectric material.

Recently, Hamid investigated the radiation characteristics of slotted circular or elliptical cylinder coated by lossy and lossless metamaterials [9-10].

Lately, materials possessing both lossy and lossles metamaterials have gained considerable attention by many researches [11-26]. Thus a new artificial class of materials with interesting electromagnetic properties has been recently introduced. Two media separated by an interface are called isorefractive (IR) if they have the same refractive index. Such a relation is maintained when the permittivity and permeability of the two media obey

$$\mu_1\varepsilon_1 = \mu_2\varepsilon_2 \tag{1}$$

So that the propagation constant k and the wavelength λ are the same in both media, where

$$k = \frac{2\pi}{\lambda} = \omega\sqrt{\varepsilon_i\mu_i}\,,\ i = 1,2 \tag{2}$$

In addition, the two media have different intrinsic impedances, i.e.

$$Z_i = \sqrt{\frac{\mu_i}{\varepsilon_i}},\ \ i = 1, 2 \tag{3}$$

In this paper, a theoretical analysis based on a boundary value solution for the case of antenna radiation by an axial slot on a conducting circular or elliptical cylinder coated by an IR metamaterial is presented. The fields inside and outside the IR coating are expressed in terms of radial and angular Mathieu functions. The IR metamaterial elliptic layer allows an exact solution as the boundary conditions lead to one-to-one matching between the field modes on either side of the interface. Thus there is no need for matrix inversion since new expressions are obtained for the radiated field expansion coefficients. Numerical results are presented for the radiation pattern, aperture conductance and antenna gain vs coating thickness as well as compared with uncoated, conventionally dielectric- coated and isorefractive -coated antenna.

2. Problem Formulation

Fig. 1 illustrates the geometry of the problem. The structure is assumed to be infinite along the *z*-axis. The symbols a_c and b_c correspond to the semi-major and semi-minor axes of the conducting cylinder, respectively, while the symbols *a* and *b* are semi-major and semi-minor axes of the dielectric coating. The elliptical coordinate system (*u*,*v*,*z*) is defined in terms of the Cartesian coordinate system (*x*,*y*,*z*) by $x = F\cosh(u)\cos(v)$ and $y = F\sinh(u)\sin(v)$, where *F* is the semifocal length of the elliptical cross section.

The electric fields outside the dielectric layer (region I) for ($\xi > \xi_1$) and inside the dielectric layer (region II) for $\xi < \xi_1$ can be expressed in terms of Mathieu functions as follows

$$E_z^I = \sum_{m=0}^{\infty} C_{em} R_{em}^{(4)}(c,\xi) S_{em}(c,\eta) + \sum_{m=1}^{\infty} C_{om} R_{om}^{(4)}(c,\xi) S_{om}(c,\eta) \tag{4}$$

$$\begin{aligned} E_z^{II} = & \sum_{m=0}^{\infty} [A_{em} R_{em}^{(1)}(c,\xi) + B_{em} R_{em}^{(2)}(c,\xi)] S_{em}(c,\eta) + \\ & \sum_{m=1}^{\infty} [A_{om} R_{om}^{(1)}(c,\xi) + B_{om} R_{om}^{(2)}(c,\xi)] S_{om}(c,\eta) \end{aligned} \tag{5}$$

where $A_{\substack{em\\om}}$, $B_{\substack{em\\om}}$ and $C_{\substack{em\\om}}$ are the unknown expansion coefficients, $R_{\substack{em\\om}}^{(1)}$, $R_{\substack{em\\om}}^{(2)}$ and $R_{\substack{em\\om}}^{(4)}$ are the even and odd modified Mathieu functions of the first, second and fourth kinds, respectively. It should be noted that, $\xi = \cosh u$, $\eta = \cos v$ while $c = kF$. The magnetic field components inside and outside the dielectric layer can be obtained using Maxwell's equations as

$$H_v^I = \frac{-j}{\omega\mu_1 h} \left\{ \sum_{m=0}^{\infty} C_{em} R_{em}^{(4)'}(c,\xi) S_{em}(c,\eta) + \sum_{m=1}^{\infty} C_{om} R_{om}^{(4)'}(c,\xi) S_{om}(c,\eta) \right\} \tag{6}$$

4

$$H_v^{II} = \frac{-j}{\omega\mu_2 h}\Big\{ \sum_{m=0}^{\infty}[A_{em} R_{em}^{(1)'}(c,\xi) + B_{em} R_{em}^{(2)'}(c,\xi)]S_{em}(c,\eta) + \sum_{m=1}^{\infty}[A_{om} R_{om}^{(1)'}(c,\xi) + B_{om} R_{om}^{(2)'}(c,\xi)]S_{om}(c,\eta) \Big\} \quad (7)$$

where $h = F\sqrt{(\cosh^2 u - \cos^2 v}$. The prime in equations (6) and (7) denotes derivative with respect to u while μ_1 and μ_2 are the permeabilities of regions 1 and 2, respectively.

We require the tangential components of the electric and magnetic fields to be continuous across the interface at $\xi = \xi_1$. In region (II), we require also that the tangential electric field on the conducting surface ($\xi = \xi_c$) must vanish except at the slot location. Enforcing these boundary conditions with the help of the orthogonality property of the angular Mathieu functions, we obtain an exact expression of the radiated field as

$$C_{\substack{em\\om}} = \frac{F_{\substack{em\\om}}}{N_{\substack{em\\om}}(c)} \left[\frac{P_{\substack{em\\om}}(\xi_1,\xi_c)\dfrac{R_{\substack{em\\om}}^{(2)}(c,\xi_1)}{R_{\substack{em\\om}}^{(2)}(c,\xi_c)} - \dfrac{R'^{(2)}_{\substack{em\\om}}(c,\xi_1)}{R_{\substack{em\\om}}^{(2)}(c,\xi_c)}}{P_{\substack{em\\om}}(\xi_1,\xi_c) R_{\substack{em\\om}}^{(4)}(c,\xi_1) - \dfrac{\mu_2}{\mu_1} R'^{(4)}_{\substack{em\\om}}(c,\xi_1)} \right] \quad (8)$$

Where

$$P_{\substack{em\\om}}(\xi_1,\xi_c) = \frac{R'^{(1)}_{\substack{em\\om}}(c,\xi_1) R_{\substack{em\\om}}^{(2)}(c,\xi_c) - R_{\substack{em\\om}}^{(1)}(c,\xi_c) R'^{(2)}_{\substack{em\\om}}(c,\xi_1)}{R_{\substack{em\\om}}^{(1)}(c,\xi_1) R_{\substack{em\\om}}^{(2)}(c,\xi_c) - R_{\substack{em\\om}}^{(1)}(c,\xi_c) R_{\substack{em\\om}}^{(2)}(c,\xi_1)} \quad (9)$$

$$F_{en} = E_O \sum_k D_e^k(c,n) \int_{v_1}^{v_2} \cos[\pi(v - v_0)/(2\alpha)] \cos(kv)\, dv \quad (10)$$

$$F_{on} = E_O \sum_k D_o^k(c,n) \int_{v_1}^{v_2} \cos[\pi(v - v_0)/(2\alpha)] \sin(kv)\, dv \quad (11)$$

$$\nu_0 = (\nu_1 + \nu_2)/2 \tag{12}$$

$$\alpha = (\nu_2 - \nu_1)/2 \tag{13}$$

$$N_{\substack{em\\om}}(c) = \int_0^{2\pi} [S_{\substack{em\\om}}(c,\eta)]^2 \, dv \tag{14}$$

and $D_{\substack{e\\o}}^{k}$ are the Fourier series coefficients [27].

3. Numerical Results

Once the unknown expansion coefficients are obtained, quantities of interest such as far field radiation pattern, antenna gain and aperture conductance can be computed. The far field expression of the antenna can be expressed as

$$E_z^I(\rho,\phi) = \sqrt{\frac{j}{k\rho}} e^{-jk\rho} \left[\sum_{m=0}^{\infty} j^m C_{em} S_{em}(c,\cos\phi) + \sum_{m=1}^{\infty} j^m C_{om} S_{om}(c,\cos\phi) \right] \tag{15}$$

where ρ and ϕ denote the polar coordinates in the circular cylindrical system. The antenna gain may be expressed as

$$G(\phi) = \frac{1}{Z_1 k\rho} \left[\left| \sum_{m=0}^{\infty} j^m C_{em} S_{em}(c,\cos\phi) \right|^2 + \left| \sum_{m=1}^{\infty} j^m C_{om} S_{om}(c,\cos\phi) \right|^2 \right] \tag{16}$$

where Z_1 is the intrinsic impedance of region I (in this case it is taken to be free space). The aperture conductance per unit length of the slot antenna is defined as

$$G_a = 2\pi\rho \frac{S_{av}}{|E_0|^2} \tag{17}$$

where S_{av} is the average power density averaged over an imaginary cylinder of radius ρ and is given by

$$S_{av} = \frac{1}{2\pi Z_1 k\rho}\left[\sum_{m=0}^{\infty}|C_{em}|N_{em}(c) + \sum_{m=1}^{\infty}|C_{om}|N_{om}(c)\right] \tag{18}$$

The geometrical parameters used in obtaining the numerical results are $a_c = \lambda$, $b_c = \lambda/2$, $b = b_c + t$, where t is the coating thickness, $\nu_0 = 90^o$ and $\alpha = 2.8657^o$. Figure 2 shows the numerical results for the radiation pattern (gain versus ϕ) obtained for the uncoated antenna and is presented by solid line for comparison ($\varepsilon_r = 1$, $\mu_r = 1$). The results for the antenna coated with conventional dielectric material are presented by dashed line ($\varepsilon_r = 4$, $\mu_r = 1$). The results for the antenna coated with isorefractiare material presented by dotted line ($\varepsilon_r = 4$, $\mu_r = 1/4$) for dielectric thickness $t = 0.15\lambda$. It can be seen that the IR coating makes the beam sharper and more directive with lower side-lobes (of about -40 dB at -90 and 270 degrees) when compared with the conventional dielectric material coating. This may be due to the fact that the radiated field in the IR layer does not suffer reflection since the wave number is the same in all regions. Fig. 3 is similar to Fig. 2 except the coating thickness is increased to 0.25λ. It can be seen that the conventional dielectric coating has diverged the antenna beam while the isorefractive material coating has reduced the side-lobes by 20 dB when it is compared with the uncoated antenna and by 40 dB when it is compared with the conventional dielectric coated antenna especially at -90 and 270 degrees. It is also worth mentioning the pattern of the uncoated and coated antenna with IR material looks like a band pass filter while the conventional coated antenna losses this important feature.

The gain versus electrical coating thickness for a single slotted elliptical antenna with the same geometrical parameters used in Fig. 2 is displayed in Fig. 4. The gain is evaluated at ϕ=90^o since the slot is centered at ν=90^o where the gain is expected to be maximum. For very small coating thickness, the conventional and IR coatings have the same effect on the gain. As t becomes greater than 0.05λ, IR coating increases the antenna gain when compared with the conventional coating until the thickness becomes 0.275λ. Further, the presence of surface waves in the case of conventional coating starts to disappear in the case of IR coating f or higher values of electrical thickness.

The aperture conductance for conventional and IR coatings is shown in Fig. 5 for an elliptical antenna with the same geometrical parameters used in Fig. 2. The antenna appears to have higher conductance values for IR coating thicknesses greater than 0.2λ, and lower conductance values for t less than $0.2\ \lambda$ when compared with conventional dielectric coating.

4. Conclusions

Exact radiation from a slot on a conducting elliptical cylinder coated with IR concentric layer was obtained using the boundary value technique with no matrix inversion required. The application of the boundary conditions has lead to a one-to-one matching between the field modes on either side of the interface, and thus exact expressions were obtained for the radiation expansion coefficients. Finally, the IR coating may be used to enhance the gain and reduce the side-lobes of slotted antenna over a certain coating range.

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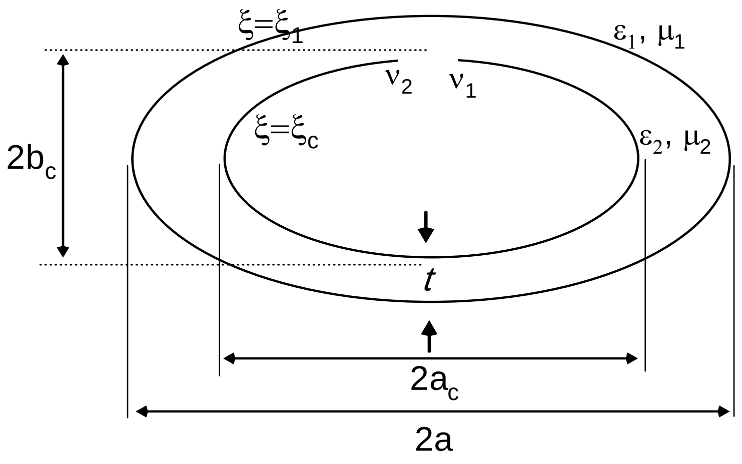

Figure 1: Geometry of axially Slotted antenna on elliptic cylinder with isorefractive layer

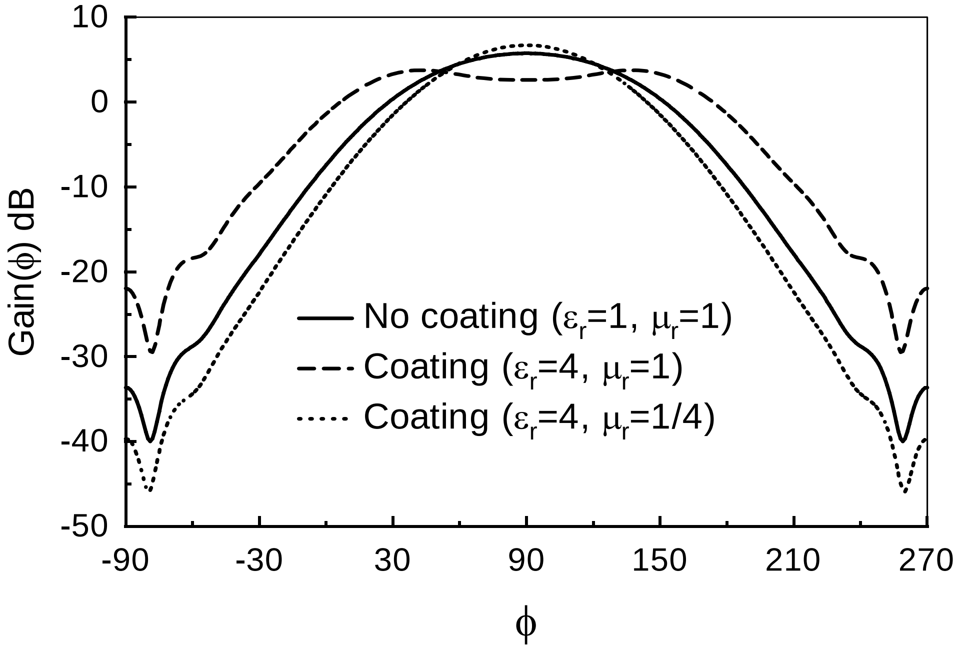

Figure 2: Radiation pattern for an axially slotted elliptic cylinder coated with different types of dielectric materials and coating thickness $t = 0.15\lambda$.

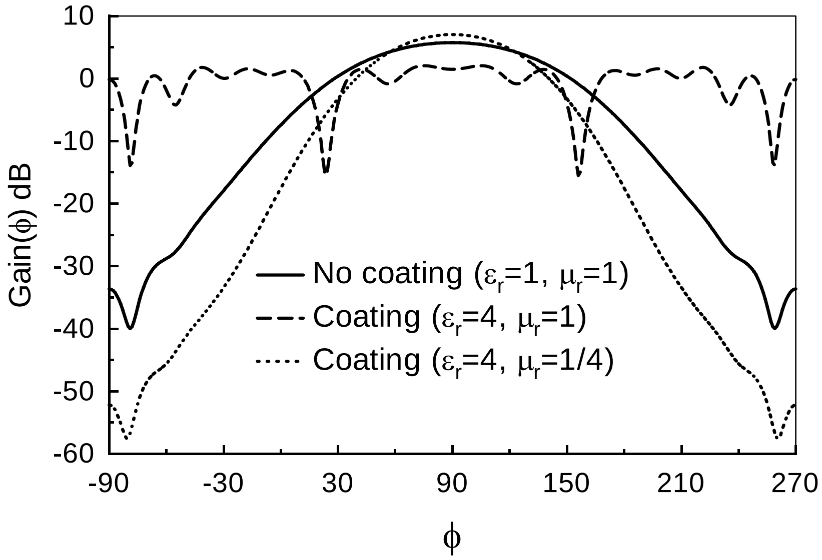

Figure 3: Radiation pattern for an axially slotted elliptic cylinder coated with different types of dielectric materials and coating thickness $t = 0.25\lambda$.

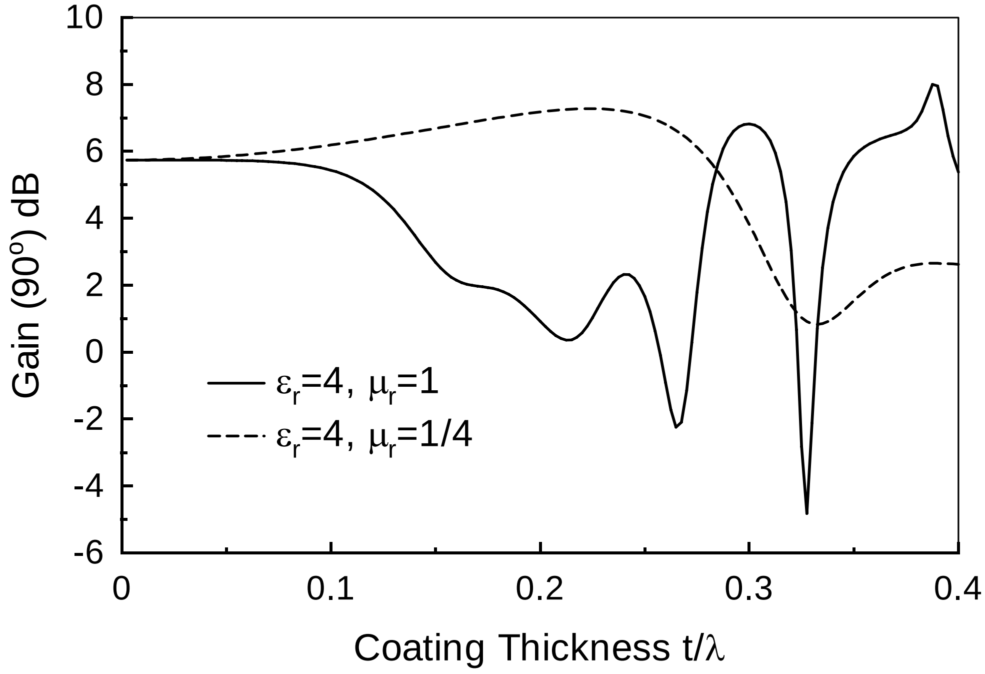

Figure 4: Gain versus coating thickness for an axially slotted elliptic cylinder coated with conventional and isorefractive dielectric materials.

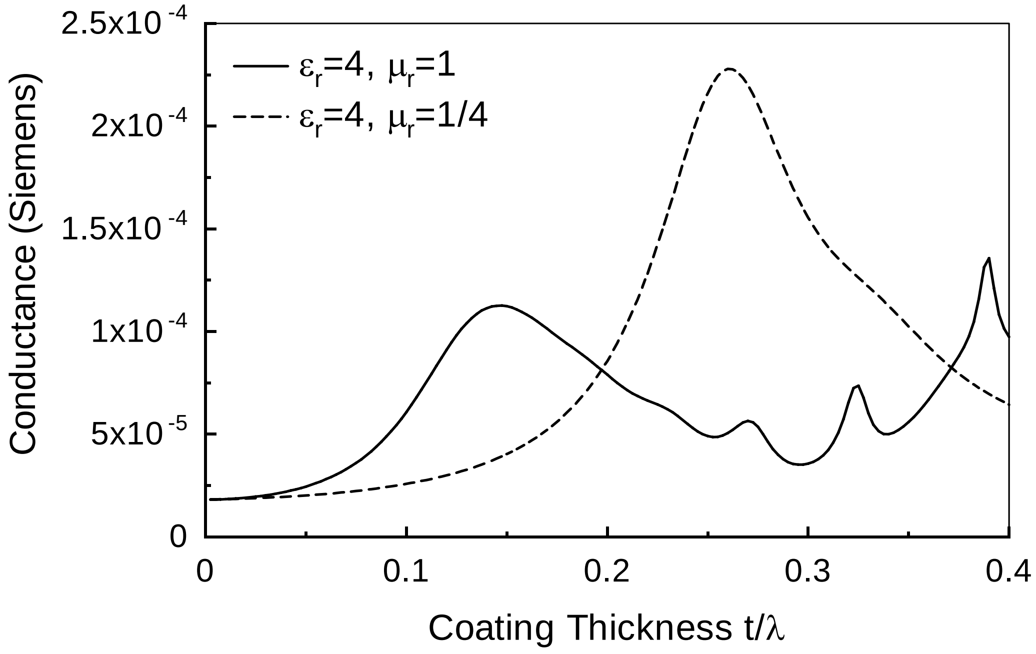

Figure 5: Aperture conductance versus coating thickness for an axially slotted elliptic cylinder coated with conventional and isorefractive materials.