Quantifying Errors and Uncertainty in CEM

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At the University of York, research is being conducted to determine possible methods for quantifying the errors and uncertainties that exist in Computational Electromagnetics (CEM) simulations. Standards already exist that require an estimate of the uncertainty in measurements obtained from laboratory Electromagnetic Compatibility (EMC) measurements [1]. Currently no requirement exists for the measurements obtained by CEM simulations. Such error and uncertainty analyses would allow different models to be compared for accuracy. The analyses would help determine quantitatively whether a computationally cheaper, less accurate model, is accurate enough for purpose. Knowledge of the error and uncertainty in the output of a simulation would also provide a quantitative level of confidence in the results.

The 'Guide to the Expression of Uncertainty in Measurement' [2] provides a framework for quantifying uncertainties in measurements. It is currently the internationally accepted master document for quantifying uncertainties [3]. This guide describes the error in a measured value as the difference between the measured value and the true value of the measurand [2]. It describes the uncertainty in the measured value as the quantification of the doubt about the measured value [2]. These descriptions of error and uncertainty are generally applicable to all types of measurement. It is possible to make more explicit definitions when considering the errors and uncertainties in computer modelling.

Currently there has been more work on error and uncertainty analyses in Computational Fluid Dynamics (CFD) [4] - [7] than in CEM, and so this discipline is chosen to provide a formal definition of the errors and uncertainties in computer models. The following definitions come from the American Institute of Aeronautics and Astronautics (AIAA) report on the verification and validation of CFD simulations [5].

Error: *A recognizable deficiency in any phase or activity of modelling and simulation that is not due to lack of knowledge.*

Errors are introduced into our models via the approximations and assumptions that are made in forming the model. Since these approximations and assumptions are generally known, the errors they produce can be analysed [4]. One type of error in conventional FDTD is the staircasing error that arises from modelling a curved surface on a rectangular grid. The modelled surface is *known* to be inaccurate and this inaccuracy will manifest itself as an error in the final measured value.

Uncertainty: *A potential deficiency in any phase or activity of the modelling process that is due to lack of knowledge.*

Uncertainties can be further categorized into two groups. The first is the uncertainty in how well the mathematical model represents the true behaviour of the real physical system [7]. This uncertainty is very difficult to determine [7]. Electromagnetism is mathematically represented by Maxwell's Equations, which have been verified by many people over many years. Thus it may be assumed that the model uncertainty can be ignored in CEM. The second type of uncertainty is the uncertainty that arises due to a lack of precise input parameter data [7]. If there are uncertainties in the input parameter data, then there will be uncertainties in the output. This type of uncertainty is often known as parameter uncertainty [7].

Determining Errors

In order to determine the errors that exist in a model, an error taxonomy must first be formed. This taxonomy is a list of all the possible sources of error that may exist in the simulation. The errors in this list may be quantified by considering the approximations and/or assumptions that have caused them. This is not a trivial task and may be computationally expensive.

In FDTD, one known source of error is the discretisation of space and time. This can be analysed by comparing the results of one simulation with the results of the same simulation, performed on a finer mesh. As the mesh size decreases so does the error in the simulation. Thus an estimate of the error in the less accurate simulation can be formed by comparing the results of this simulation with that of the more accurate simulation.

Determining Parameter Uncertainties

Uncertainty analyses are either possibilistic or probabilistic. The work at York concentrates on the probabilistic methods. Probabilistic methods use the known Probability Density Functions (PDFs) of the input parameters to estimate the mean output value, and the combined uncertainty in this value. Three methods that are currently being investigated are the: Method of Moments (MOM), Monte Carlo Method (MCM) and Polynomial Chaos Method (PCM) [8]-[10].

The MOM is similar to the method outlined in UKAS [1] for the determination of uncertainty in practical EMC measurements. It is the method outlined in the 'Guide to the Expression of Uncertainty in Measurement' [2], for the propagation of uncertainties through a model. This method first calculates the uncertainty in the output due to each of the individual uncertain parameters; these individual output uncertainties are then combined to form the combined output uncertainty. It seems that this method is computationally the cheapest, but not the most accurate.

The MCM involves taking multiple samples of the input parameters from their respective PDFs, performing multiple simulations using these samples, and then combining the multiple outputs to form an output PDF. The combined output uncertainty is taken to be the standard deviation of this PDF. This method is much more expensive computationally, but provides the most rigorous analysis of the uncertainty.

In the PCM, the solution is assumed to be the expansion of certain orthogonal basis polynomials, which represent the individual input uncertainties. These polynomials are propagated through the model and the combined uncertainty is found from the outputs. This method has been found to predict uncertainties that agree with those predicted by the more rigorous MCM. The PCM is slower than the MOM but much faster than the MCM. The memory requirements for the PCM are much bigger than both the other methods described here. This may limit its applicability to more complex simulations. The PCM has already been applied to one area of CEM [11]. Through this work it was shown that the PCM can accurately estimate uncertainties much more quickly than the MCM for a number of applications.

All of the above uncertainty analyses require extra computational expense. The determination of uncertainty in laboratory measurements also requires extra work. At the University of York an uncertainty analysis method is currently being sought which will require minimal computational expense. This method will use a prior knowledge based expert system, informed by economic use of the techniques described above, applied to the current problem.

Conclusion

Error and uncertainty analyses would help us quantitatively compare the accuracy of different models in CEM. The analyses would also provide us with a quantitative level of confidence in the results obtained from these models. The formulation of such analyses is not a trivial task. Errors may be determined by using the results of more accurate simulations. Uncertainties can be determined in many different ways: some methods are more accurate, but other methods are computationally cheaper. All the uncertainty analyses described here require significant extra computational expense; the laboratory derivation of experimental uncertainty also requires a significant expense. Work is currently being carried out to determine whether it is possible to estimate the combined uncertainty in a simulation with only a small amount of extra computational expense. This work is based on using the techniques described above to update a prior knowledge based expert system.

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