Geometrical Scale Modeling of Gain and Echo Area: Simulations, Measurements and Comparisons

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Abstract – Geometrical scale modeling is often necessary to perform measurements of parameters and figures-of-merit of antennas and radar targets with large physical dimensions that cannot be accommodated in indoor and controlled experimental facilities. The measured and simulated parameters and figures-of-merit of the scaled-models can then be translated to represent, if transformed properly, those of the full-scale models. In this paper, the basic theory is summarized which relates the gain and the echo area (RCS) of scaled models to those of their full-scale counterparts. Simulations and measurements are performed on scaled models, for both gain and RCS, and compared with those of full-scale models to verify the geometrical scaling. For the gain, a quarter-wavelength monopole on a scaled helicopter airframe, and for the RCS, a flat plate of complex configuration, are considered for simulations, measurements and comparisons. A very good agreement has been obtained for both gain and RCS between both sets of data.

Index Terms – Antenna, gain, measurement, radar cross section, scale model.

I. INTRODUCTION

In many applications (such as antennas on ships, aircraft, large spacecraft, etc.), antennas and their supporting structure are so immense in weight and/or size that they cannot be accommodated by indoor experimental facilities to measure their radiation characteristics. To overcome some of the challenges presented by physically large structures, a technique that can be used to perform antenna simulations, measurements and comparisons of fundamental parameters and figures-of-merit of antennas and scattering is *geometrical scale modeling* [1], [2]. Geometrical scale modeling is employed to:

- Physically accommodate, within small antenna ranges or enclosures, measurements on relatively small physical scaled models that can be referred to those of large structures.
- Allow experimental, environmental and security control over the measurements.
- Minimize costs and time associated with measurements of physically large structures and corresponding experimental parametric studies.

While [1] laid the foundation for scale modeling, it did not present any predictions, simulations, measurements or comparisons. This paper specifically focuses on Gain (Amplitude) and Echo Area (RCS), and illustrates both concepts with simulations and measurements of an antenna on a scale model helicopter and scattering from square and irregular-shaped metallic plates, for which full-scale and scale model data are compared. The scaling of other antenna and scattering parameters and figures-of-merit can be found in [1], [2]. The theory of geometrical scale modeling is based on the development of *absolute scale modeling* [1], of which the *geometrical scale modeling* is a special case when the ratio of the scale factor of the electric field (α) to that of the magnetic field (β) is unity ($\alpha/\beta = 1$), and the ratio of the scale factor of time (γ) and the geometrical scale factor of linear dimensions (n) is also unity (γ/n = 1). These two ratios are satisfied when the permittivity (ε) and permeability (μ) of the full-scale and scaled models are identical [1]. Both the absolute and geometrical scale modeling are based on Maxwell's equations.

II. GAIN (AMPLITUDE): SIMULATION, MEASUREMENTS AND COMPARISONS

Using Maxwell's equations and a geometrical scaling factor of n, the relationship between the antenna gain G_o (of the full-scale model) to the antenna gain G_o' (of the scaled model) are developed based on the definition of antenna gain between the two scale models. A summary of the derivation is outlined below. The scaled-model parameters are indicated by a prime. It can be shown that:

$$G_o(\text{Gain}) = \frac{U}{U_o} = \frac{4\pi U}{P_{rad}} = 4\pi\left(\frac{U}{P_{rad}}\right) = 4\pi r^2\left(\frac{W}{P_{rad}}\right), \quad (1a)$$

$$G_o' = 4\pi\left(\frac{U'}{P_{rad}'}\right) \overset{U'=(r')^2W'}{=} 4\pi\left(\frac{(r')^2 W'}{P_{rad}'}\right) = 4\pi\left(r'\right)^2\left(\frac{W'}{P_{rad}'}\right), \quad (1b)$$

Submitted On: August 28, 2015
Accepted On: January 24, 2016

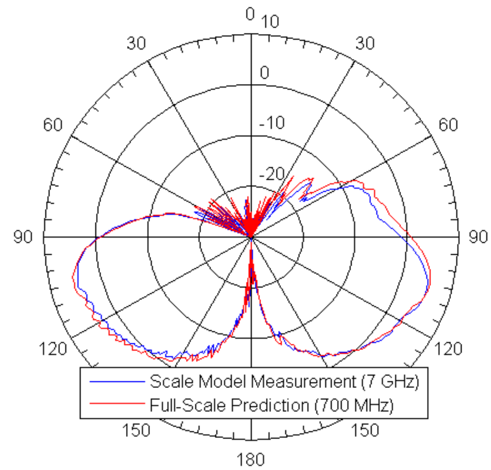
$$G_o^{'} = 4\pi\left(\frac{r}{n}\right)^2 \frac{W/(\alpha\beta)}{P_{rad}/(\alpha\beta n^2)} = 4\pi r^2\left(\frac{W}{P_{rad}}\right) = G_o, \quad (1c)$$

$$G_o^{'} = G_o, \quad (1d)$$

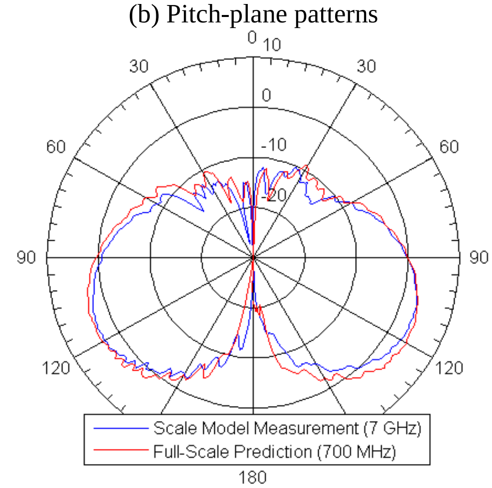
where U represents radiation intensity (W/steradian), W represents power density (W/m^2) and P_{rad} is the power (W) radiated by the antenna. Thus, the gain $G_o^{'}$ (scaled model) = gain G_o (full-scale).

For geometrical scale modeling of the antenna gain, the absolute amplitude radiation patterns of a $\lambda/4$ monopole located at the belly (bottom side) of a generic scale model helicopter [see Fig. 1 (a)], having dimensions that are about 1/10 the size of a full-scale helicopter, were simulated, measured and compared. The absolute amplitude patterns of the $\lambda/4$ monopole on the scale model generic helicopter were measured at 7 GHz along the three principal planes; *pitch*, *roll* and *yaw*. In addition, the same patterns were simulated on a full-scale helicopter (10 times larger) but at a frequency of 700 MHz (1/10 the measured frequency) of the same helicopter geometry. The pitch-plane patterns, simulated and measured, are shown in Fig. 1 (b); a very good agreement is indicated. The corresponding roll- and yaw-plane patterns are shown in Figs. 1 (c) and 1 (d), respectively. It is evident that there is, as expected, a correct scaling between the measured amplitude (gain) patterns on the 1/10 scale model but at a frequency of 7 GHz (increased by a factor of 10 since the size of the scale model was 1/10 of the full-scale) and the simulated patterns at 700 MHz (a factor of 1/10 of the measured frequency) but on a full-scale model (larger by a factor of 10). The maximum gain is about 6 dB, which is basically what is expected from a $\lambda/4$ monopole. In addition, there is an excellent comparison between the respective two sets of patterns (simulated and measured), considering the complexity of the airframe.

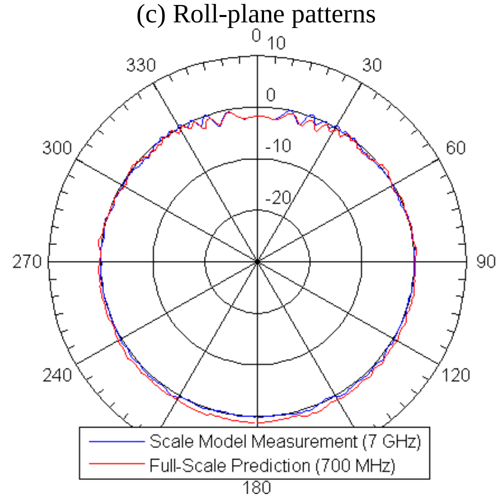
(a) Scale model helicopter (10:1 scale)



(b) Pitch-plane patterns



(c) Roll-plane patterns



(d) Yaw-plane patterns

Fig. 1. (a) Scale model helicopter. (b) Pitch-plane scale model measurements (7 GHz) and full-scale model simulations (700 MHz) of a $\lambda/4$ monopole on the belly of the helicopter airframe. (c) Roll-plane patterns. (d) Yaw-plane patterns.

III. ECHO AREA (RCS): SIMULATIONS, MEASUREMENTS AND COMPARISONS

Using Maxwell's equations and a geometrical scaling factor of linear dimensions of n, the relationship between the echo area (RCS) A_e (of the full-scale model) to the echo area (RCS) $A_e^{'}$ (of the scaled model) are developed based on the definition of the echo area (RCS) between the two scale models. A summary of the derivation is outlined below. The scaled model parameters are indicated by a prime. It can be shown that:

$$A_e = \lim_{r\to\infty}\left[4\pi r^2\left(\frac{W_s}{W_i}\right)\right], \tag{2a}$$

$$A_e^{'} = 4\pi (r')^2\left(\frac{W_s^{'}}{W_i^{'}}\right) = 4\pi\left(\frac{r}{n}\right)^2 \frac{W_s/(\alpha\beta)}{W_i/(\alpha\beta)}, \tag{2b}$$

$$A_e^{'} = \frac{1}{n^2}\left[4\pi r^2\left(\frac{W_s}{W_i}\right)\right] = \frac{1}{n^2}A_e, \tag{2c}$$

$$A_e = n^2 A_e^{'}, \tag{2d}$$

where both A_e and $A_e^{'}$ represent the echo areas (m^2) of the full-scale and scaled models, respectively. Thus, the echo area (full-scale) $A_e = n^2 A_e^{'}$ echo area (scaled model).

The simulated, using the commercial software CST [3], and measured echo area (RCS) monostatic patterns of a scaled and a full-scale odd shaped flat PEC plate, whose geometry is displayed in Fig. 2, were performed and compared. The odd shape of the plate was chosen so that the target will not represent a canonical surface. The dimensions of the full-scale and scaled models are indicated in centimeters; the first number in each axis represents the dimensions (in cm) of the full-scale (large) model, while the second numbers represents the dimensions (in cm) of the scaled (small) model. The overall areas of each are 319.5 cm^2 and 35.5 cm^2, respectively. The scale factor is n = 3 for the linear dimensions while the scale factor is $n^2 = (3)^2 = 9$ for the areas. The frequencies for the simulations and measurements were 15 GHz (scaled) and 5 GHz (full-scale): a scaling factor of 3, which is the same as that of the linear dimensions.

The parallel (hard) polarization, simulated and measured, monostatic RCS patterns (in dBsm) of the scaled and full-scale models along the principal plane are displayed in Fig. 3 (a), while those for the perpendicular (soft) polarization are displayed in Fig. 3 (b). An excellent agreement is indicated between the simulated, using CST [3], and measured patterns, for both the full-scale (large) and scaled (small) plates. The shape of the perpendicular (soft) polarization RCS patterns of Fig. 3 (b) follow, as they should, a nearly sin(q)/q distribution based on physical optics [4] and due to the very weak first-order diffractions from the edges of the plate for this polarization [4]. However, for the parallel polarization, the shape of RCS patterns does not follow the sin(q)/q distribution, especially at the far minor lobes, because the diffractions for the vertical (hard) polarization are more intense [4] and impact the overall distribution to be different from nearly sin(q)/q.

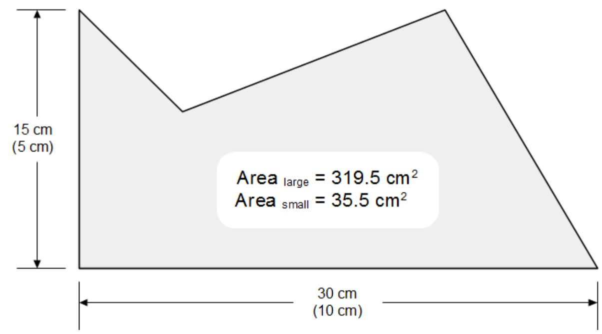


Fig. 2. Shape and dimensions (in cm) of PEC odd shape full-scale (large), and scale (small) plate for echo area (RCS) simulations and measurements.

The maximum monostatic RCS of a flat plate of any geometry, for either parallel or perpendicular polarization (both are identical based on physical optics, PO), occurs at normal incidence and is represented by [4]:

$$\text{RCS}_{\max}(\text{PO}) = 4\pi\left(\frac{\text{Area of plate}}{\lambda}\right)^2. \tag{3}$$

Based on the physical optics (PO) RCS of (3), the maximum monostatic RCS occurs at normal incidence ($\theta_i = 0^o$), and for the full-scale (large) plate of 319.5 cm^2 at 5 GHz is (the same for both the parallel and perpendicular polarizations):

$$\text{RCS(full-scale)}_{\max} = 4\pi\left[\frac{319.5}{6(100)}\right]^2 = 3.563\ \text{m}^2 = 5.52\ \text{dBsm}, \tag{4}$$

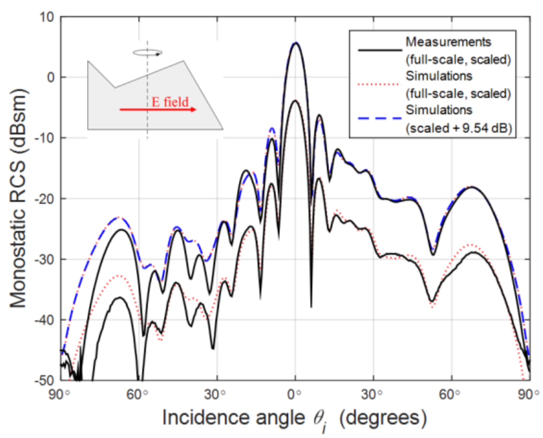
while the simulated maximum of Figs. 3 (a) and 3.(b) is 5.74 dBsm (parallel polarization) and 5.85 dBsm (perpendicular polarization). The measured one for both polarizations is nearly 5.6 dBsm; thus the predicted (based on PO), simulated (using CST) and measured are within 0.3 dB.

For the scaled (small) plate 35.5 cm^2 at 15 GHz, the maximum monostatic RCS based on (3) is:

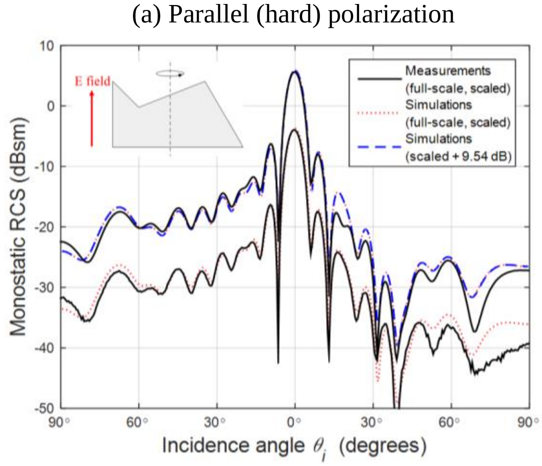
$$\text{RCS(scaled)}_{\max} = 4\pi\left[\frac{35.5}{2(100)}\right]^2 = 0.396\ \text{m}^2 = -4.02\ \text{dBsm}, \tag{5}$$

while the simulated maximum of Figs. 3 (a) and 3 (b) is -3.79 dBsm (parallel polarization) and -3.68 dBsm (perpendicular polarization). The measured one for both polarizations is nearly -3.9 dBsm; thus the predicted (based on PO), simulated (using CST) and measured are within 0.3 dB.

It is also apparent from the parallel polarization monostatic RCS patterns in Fig. 3 (a) and the perpendicular polarization of Fig. 3 (b) that, there is a difference of $n^2=3^2=9$ (dimensionless) or $10\log_{10}(9)$ = 9.54 dB, between the scaled and full-scale (both measured and simulated) RCS patterns; i.e., the full-scale measured and simulated monostatic RCS patterns are 9.54 dB greater than those of the scaled, as they should be according to (2d). In fact, if 9.54 dB is added to the measured and simulated monostatic RCS patterns of the scaled (small) plate monostatic RCS patterns, the adjusted (by + 9.54 dB) RCS patterns match those of the full-scale (large) plate, as shown in Figs. 3 (a) and 3 (b). Again, the agreement is so good that it is difficult to distinguish any differences between any of the patterns for the full-scale and scaled plates. Such comparisons and agreements illustrate and validate the scaling principle for echo area (RCS).



(a) Parallel (hard) polarization



(b) Perpendicular (soft) polarization

Fig. 3. Parallel (hard) and perpendicular (soft) polarizations simulated and measured monostatic echo area (RCS) patterns for full-scale (large) and scaled (small) odd shape flat PEC plate of Fig. 2.

IV. CONCLUSIONS

Geometrical scale model measurements are relatively inexpensive, convenient, and a quick alternative to full-scale measurements. Scale models enable one to perform measurements that otherwise may be impractical or impossible, and they can be related to those of full-scale models using the appropriate geometrical scaling. This has been demonstrated in this paper for both gain (of an antenna) and echo area (of scattering). An excellent agreement has been illustrated between scaled and full-scale simulations and measurements, using the appropriate geometrical scaling factor for gain and echo area of scaled and full-scale models. Measurements of geometrical scale models are recommended; they provide an alternative and effective process that otherwise may be impractical or not cost effective for full-scale model measurements.

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