

# A Non-Focal Rotman Lens Design to Support Cylindrically Conformal Array Antenna

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**Abstract** — Rotman lenses offer broad bandwidth, and render to planar structures making them ideal for a variety of applications. However, a limitation of the Rotman lens is that it is based on the assumption of supporting linear arrays. In this paper, we develop a new design technique to enable the lens to feed a conformal array antenna.

**Index Terms** — Beam Forming Networks (BFNs), conformal array antennas, Particle Swarm Optimization (PSO), Rotman lens.

## I. INTRODUCTION

Rotman lens, which was invented by Rotman and Turner in 1962, is an analog beam former that creates a specific phase taper at its output ports to feed an array antenna. Based on the input parameters (such as the scanning angle ( $\phi$ ), the focal angle ( $\alpha$ ), number of input and output ports; the lens equations can be solved to determine its receive contour, delay lines ( $w_i$ ) and the focal arc to generate a desired phase taper [1]. In the past few decades, numerous Rotman lens designs have been developed [2-5]. However, a limitation of these designs is the assumption of a uniform linear array at the output ports. This prohibits their applications to conformal array antennas.

The ability to feed conformal array antennas with a Rotman lens would be of interest as these systems can be applied to a variety of applications such as antennas residing on the surface of an airplane's wing, the body of a missile or a high-speed train, which are used for communications or navigation purposes. In this paper, we develop a new design inspired by Rotman lens to feed a cylindrically conformal array. We solve for the lens equations to satisfy the constraints of the design by employing the Particle Swarm Optimization (PSO) [4]. Our approach applies to other conformal geometries as well. For the purposes of this paper, we will use our method to design a lens with 3 input ports feeding an 8-element conformal array operating at a center frequency of 10 GHz.

The remainder of the paper is organized as follows. Section II introduces the overview of our system design. Section III gives a brief overview of the PSO algorithm. Section IV discusses the optimization of phase values for the conformal array, and Section V reviews the proposed design procedure of the non-focal lens. The performance of the optimization process is discussed in Section VI. Simulation results are presented in Section VII, followed by the conclusions in Section VIII.

## II. OVERVIEW OF THE SYSTEM DESIGN

In this paper, we demonstrate the design of a Rotman lens inspired, non-focal microwave lens to support an array conforming to a cylindrical surface as shown in Fig. 1.

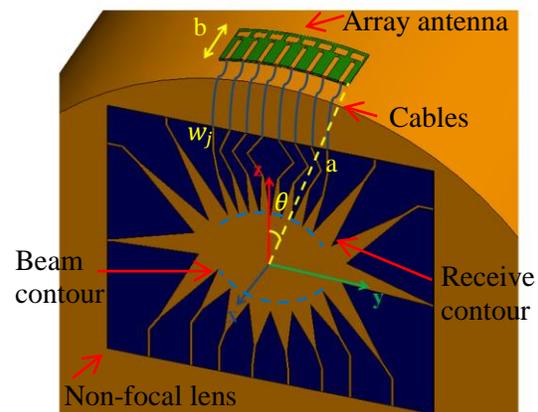


Fig. 1. Overall system design.

Two tasks are considered in order to design the lens. The first task is to optimize the required phase information to scan the conformal array in the desired directions. The second task is to use the phase information obtained in the first task as an objective function to design the non-focal lens that generates the desired phase values identified in the first task. Both tasks require an optimization method for which we choose PSO for its

ease of use and reliable performance. The optimization in the first task is straightforward while the second task can be challenging as it involves numerous variables to be optimized under a set of constraints.

### III. OVERVIEW OF PSO

Particle Swarm Optimization (PSO) is a random search algorithm, which simulates the behavior of bees in their search for the best location in the field [6]. As with all random search algorithms, the objective is to minimize a cost function defined for the specific problem. In the optimization process, the bees sample the optimization space and decide where to go next as they search for the best location based on the collective intelligence of the swarm and their personal experiences. The velocity that determines the next position of an agent is based on these two factors and the user defined constants  $w, c_1, c_2$ , which correspond to the weights of velocity along the original direction of the bee and towards the personal and global best values, respectively. The user also specifies the total number of agents, maximum number of iterations for termination and the boundary conditions for the search space.

### IV. OPTIMIZATION OF PHASE EXCITATION FOR CONFORMAL ARRAY

Our design of the cylindrically conforming array antenna is based on the parameters shown in Fig. 1, where  $a$  is the radius of the cylinder,  $b$  represents the width of the array, and  $\theta$  is the angle measured with respect to the z-axis in radians, and determines the total length  $l$  of the array by the relation:  $l = 2a\theta$ . The phase distribution for each element of the array to acquire the desired scan direction depends on the curvature characteristics of the surface it resides on.

Our paper investigates two different array geometries as depicted in Table 1. Both arrays will be designed at a center frequency of 10 GHz; the center-to-center spacing between the radiating elements is  $\lambda_0/2$  (where  $\lambda_0$  is the wavelength at 10 GHz).

Table 1: Design parameters of array for two cases

Parameters	Case 1	Case 2
$\theta$ (°)	16	32
$a$ (cm)	30	15
$b$ (cm)	5	5
$l$ (cm)	16.8	16.8

In this paper, we first employ PSO to optimize the phase distribution for the 8-element conformal array for a desired scanning angle. The desired scan positions are  $[-30^\circ, 0^\circ, 30^\circ]$ , where the angle is calculated with respect to the z-axis. This results in 8 variables in the interval of  $[0, 2\pi]$  to be optimized for each scan position. We employ 100 agents, and terminate the search after 500 iterations.

The user defined parameters  $c_1 = c_2$  and  $w$  are chosen to be 2 and 0.9, respectively. We minimize the cost function as given in (1) to achieve the desired solution:

$$F(\vec{\alpha}) = w(|SLL|_{dB} - |SLL_{max}|_{dB}) - Gain(\theta, \phi, \vec{\alpha}), \quad (1)$$

where  $Gain$  is the power gain in the  $(\theta, \phi)$  direction,  $\vec{\alpha}$  is a 24 element vector corresponding to the phase distribution for each input port,  $SLL_{dB}$  refers to sidelobe levels achieved from the PSO algorithm and  $SLL_{maxdB}$  is the maximum level allowed. The PSO stops the search when  $F \leq -15$  dB (when  $Gain \geq 15$  dB and  $SLL_{dB}$  achieved is approximately equal  $SLL_{maxdB}$  required) or the maximum number of iterations is reached. The radiation patterns of arrays for both cases are shown in Figs. 2 (a) and 2 (b), respectively. The maximum gain we obtain is 15 dB for each scanning angle while the SLL is about 10 dB down from the peak.

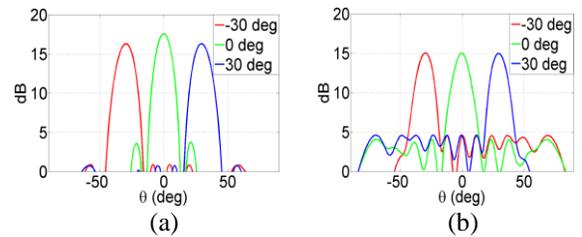


Fig. 2. Radiation pattern of conformal array in: (a) case 1 and (b) case 2.

### V. OPTIMIZATION OF THE NON-FOCAL LENS

Figure 3 shows a general concept of the optimized lens. Due to the curvature dependence of the phase values feeding the conformal array, it is not possible to directly apply the Rotman lens formulation in [1] to solve for the receiving contour, delay lines and beam contour since the Rotman lens formulation assumes a linear array at its output. We modify the general equations in finding the path length from an input port on the beam contour to an output port on the receiving contour. By optimizing the locations of input and output ports as well as the delay lines, we obtain the expected phase value at the input of conformal array as described in (2):

$$\phi_{i,j} = k_r \sqrt{(x_j - x_{fi})^2 + (y_j - y_{fi})^2} + k_{eff} \cdot w_j, \quad (2)$$

where  $(x_{fi}, y_{fi})$  is the coordinate of the input port,  $(x_j, y_j)$  is the coordinate of the output port,  $w_j$  is the length of the delay line corresponding to each output port,  $k_r = \frac{2\pi}{\lambda} \sqrt{\epsilon_r}$  and  $k_{eff} = \frac{2\pi}{\lambda} \sqrt{\epsilon_{eff}}$  where  $\lambda$  is the free space wavelength at the center frequency and  $\epsilon_{eff}$  is the effective dielectric constant of the microstrip line feeding the array.

In our formulation, we assume that the receiving contour lies on an ellipse. This constraint ensures a

smooth curvature at the output. There are a variety of options in choosing the shape of the receiving contour. We choose an elliptical arc because it resembles the shape in a conventional Rotman lens. As shown in Fig. 3, we have 24 variables to optimize, which include the coordinates of input ports ( $x_{f1} \dots x_{f3}$ ,  $y_{f1} \dots y_{f3}$ ), y-coordinates of output ports ( $y_1 \dots y_8$ ), radii of elliptical receiving contour ( $l_1$ ,  $l_2$ ) and corresponding delay line lengths ( $w_1 \dots w_8$ ). For cases with higher number of input or output ports, the number of optimization variables increase, which would inevitably slow down the convergence of the PSO to good solutions.

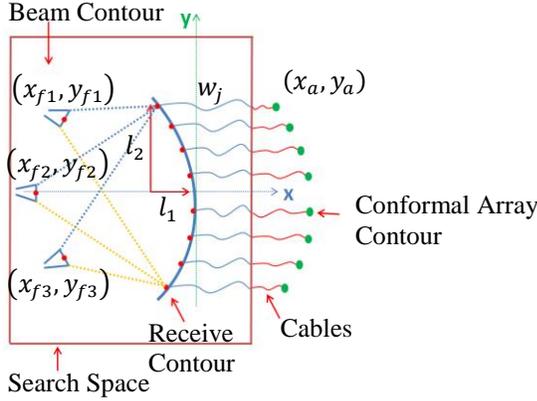


Fig. 3. Concept of non-focal lens design.

The fundamental challenges of this problem are how to design a lens so that it not only supports a conformal array, but is also practical for fabrication. Thus, we must implement a suitable cost function in PSO to guarantee a smooth curvature, and to position all output ports well for maximum power reception. Below are the constraints we choose for the PSO algorithm to design an appropriate lens:

- i. Ports are randomly distributed on the receiving contour with spacing requirement  $\lambda_r/2 \leq S_i \leq 3\lambda_r$  where  $\lambda_r = \lambda/\sqrt{\epsilon_r}$  is the effective wavelength inside the substrate, and  $S_j$  is the arc length between two adjacent output ports  $(x_j, y_j)$  and  $(x_{j+1}, y_{j+1})$ . The minimum spacing between two adjacent ports is  $\lambda_r/2$  to make sure that it is possible for the lens to be fabricated while the maximum spacing is  $3\lambda_r$  constrains the overall dimensions of the lens.
- ii. Beam ports are positioned randomly with a constraint on the minimum distance between each pair, such that  $d_i = \sqrt{(x_{fi} - x_{fi+1})^2 + (y_{fi} - y_{fi+1})^2} \geq \frac{\lambda_r}{4}$ . Unlike the conventional lens, our approach has no limitations for the beam ports to lie on a circular or elliptical arc.
- iii. An important constraint is to make sure beam ports

do not block each other. We define an illuminating region from a beam port to the output ports (the regions are separated by the dashed blue and solid yellow lines as shown in Fig. 3). If the other input ports happen to lie in this region, the PSO algorithm will not accept these cases as solutions.

## VI. PERFORMANCE OF PSO

After all the constraints are satisfied, the phases at the output ports are calculated and compared with the set of desired phase values to assess the cost function for this optimization problem. We use 600 agents and a maximum of 10000 iterations, where the user defined variables are chosen as  $c_1 = c_2 = 2$  and  $w = 0.9$ . The optimization aims to minimize the cost function (3):

$$F = \sqrt{\sum_{i=1}^{N_{input}} \sum_{j=1}^{N_{output}} (\phi_{i,j} - \phi_{i,jref})^2}, \quad (3)$$

where  $\phi_{i,j}$  is calculated in (2) and  $\phi_{i,jref}$  is the desired phase values. To limit the overall dimensions of the lens, we set the search space for input ports, output ports and length of a delay line in appropriate intervals. For example, for a 3-input, 8-output Rotman lens, we would want  $x_{fi} \in [-10\lambda_r, 0]$ ,  $y_{fi}, y_j \in [-10\lambda_r, 10\lambda_r]$ ,  $a, b \in [4\lambda_r, 10\lambda_r]$ ,  $w_j \in [\lambda_r, 10\lambda_r]$ . Figure 4 shows the convergence rate of the optimized lens in case 1 and 2 over 10000 iterations.

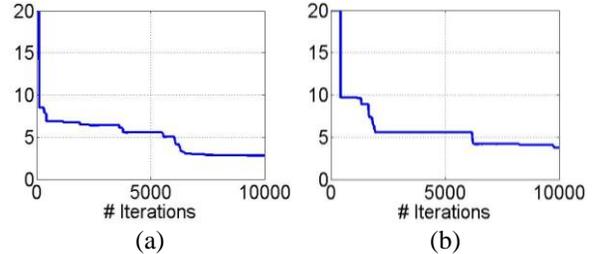


Fig. 4. Performance of PSO for optimized lens in case 1 and case 2.

## VII. RESULTS FOR NON-FOCAL LENS

To verify the robustness of our lens design, we design a 3-input, 8-output used to feed a slightly bent conformal array with parameters specified in case 1 and case 2, Table 1. The substrate we are using is Duroid 5880 with  $\epsilon_r = 2.2$ . Figures 5, 6 show the simulation model of the lens using commercial software package FEKO and its performance over a bandwidth from 9 GHz to 11.5 GHz. Figures 7, 8 show the radiation pattern of the array fed by the output of the lens at 10 GHz.

The overall dimensions of the non-focal lens design in each case are greater compared with the conventional Rotman lens working at 10 GHz and using the same substrate.

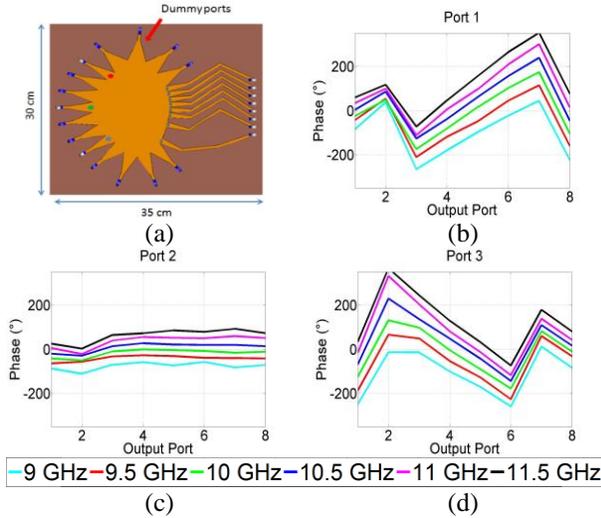


Fig. 5. 3-input, 8-output lens to feed conformal array in case 1: (a) CAD model, phase performance, (b) Port 1, (c) Port 2, and (d) Port 3.

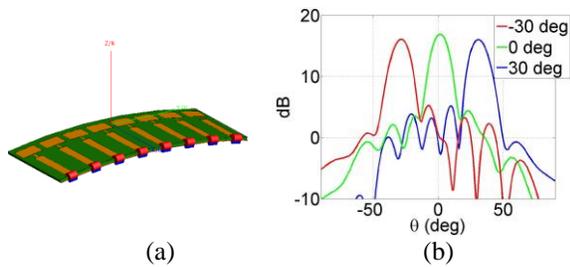


Fig. 6. (a) Bent array in case 1, and (b) radiation pattern of the array at  $-30^\circ$ ,  $0^\circ$ ,  $30^\circ$ .

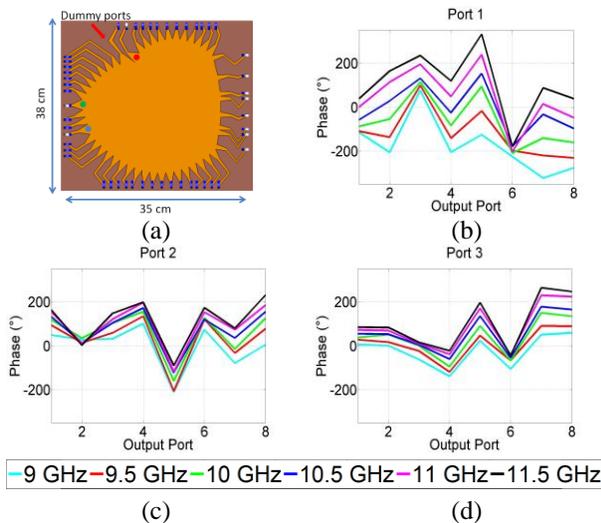


Fig. 7. 3-input, 8-output lens to feed conformal array in case 2: (a) CAD model, phase performance, (b) Port 1, (c) Port 2, and (d) Port 3.

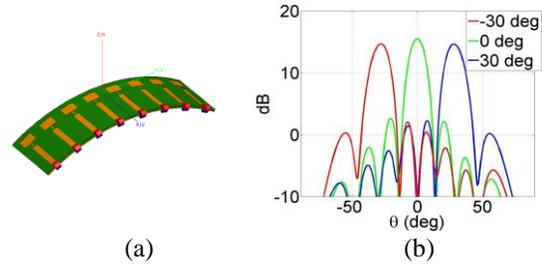


Fig. 8. (a) Bent array in case 2, and (b) radiation pattern of the array at  $-30^\circ$ ,  $0^\circ$ ,  $30^\circ$ .

### VIII. CONCLUSIONS

In this work, we investigated a new design technique to extend the design equations of Rotman lens to feed conformal array antennas. The applicability was shown using a cylindrical curvature, but the method applies to other curvatures as well. The results show that with similar total dimensions compared with conventional Rotman lens working at the same frequency and substrate, this lens design is able to support a cylindrical array. The proposed design works with any array antenna on different singly curved surfaces. While the concept has been shown for a 3-input port lens, the optimization can be run for higher number of input and output ports. However, for such cases, the number of optimization variables increase, which would inevitably slow down the convergence of the PSO algorithm to good solutions.

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