$$k_y = \frac{\pi}{b}\left(\frac{\pi n_1^{2} b\sqrt{n_1^{2}-n_2^{2}}}{\pi n_1^{2} b\sqrt{n_1^{2}-n_2^{2}}+n_2^{2}\lambda}\right). \tag{7}$$

The total bending loss Λ_T can therefore be determined by including the additional loss found in (6) with the loss in (2), i.e., $\Lambda_T = \Lambda + \Delta\Lambda$.

III. RESULTS AND DISCUSSION

We compute the loss in a 2.4 × 1.3 mm^2 silicon rectangular waveguide, with bending radius R = 1 mm. The conductivities of silicon and the surrounding medium are given as 4.33×10^{-4} S/m and 8.0×10^{-15} S/m, respectively. To validate the closed-form formulations presented here, we compare the computed results with the S21 parameters found from the Finite Element Method (FEM). The results from FEM are simulated from Ansoft's High Frequency Structure Simulator HFSS. Since S21 accounts for the total loss in the waveguide, we have incorporated the total dielectric loss α_z, i.e., the imaginary component of (3) together with the bending loss in (2) during comparison. When calculating the loss, we have set m = 1 and n = 0 for the dominant TE mode. It is worthwhile noting that, the loss in a practical waveguide may also be contributed from the imperfection of the waveguide structure. Since the work presented here is a theoretical exercise, such loss has therefore been neglected.

Figure 2 depicts the comparison of loss between our computed result and that obtained from HFSS. It can be seen from the figure that although the curves agree somewhat with each other, the loss from the computed result has clearly been underestimated. The average error with reference to the FEM result ε_{ave} = 60.17%. Since Marcuse has neglected the presence of the electric field in the x direction E_x, the loss of the E^x mode has not been taken into account. As shown in [6], the modes propagating in a dielectric waveguide are degenerate – both E^y and E^x modes exist concurrently and that the propagation constants of both modes are similar to each other. Figure 3 shows the total loss (i.e., the addition of dielectric and bending losses) when both E^x and E^y modes are taken into account. It can be observed from Fig. 4 that the electric fields of the E^x and E^y modes are orthogonal to each other. Despite their direction of polarizations, however, the profiles exhibited by both modes are qualitatively similar to each other [3]. Here, we have taken the bending loss exhibited by the E^x mode to be identical with that by E^y. The result turns out to be in closer agreement with that obtained from the FEM method, although discrepancy between the results is still apparent (ε_{ave} = 44.53%). Figure 5 shows the final result when the corner field correction factor $\Delta\Lambda$ has been included into our calculation. By considering the loss at the four edges of the waveguide, it can be observed from the figure that the result improves significantly, with the computed result approaches that of the simulation (ε_{ave} = 21.27%).

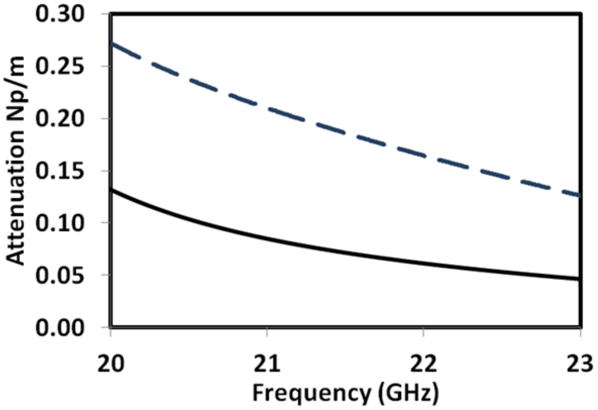


Fig. 2. Loss of a bent rectangular silicon waveguide, obtained from the analytical method proposed here (solid line) and the FEM (dashed line). The analytical method has only considered the dielectric loss and the bending loss from the E^y mode (loss at the corner regions has been neglected).

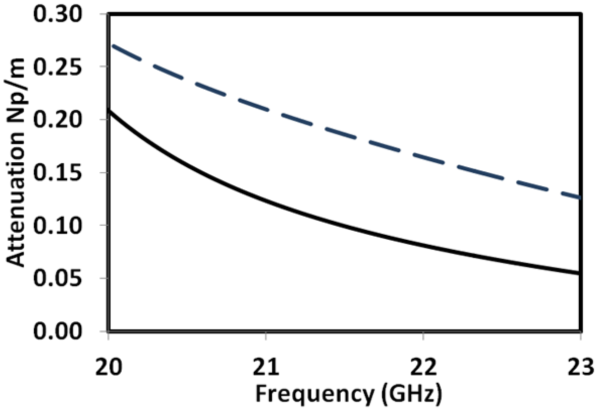
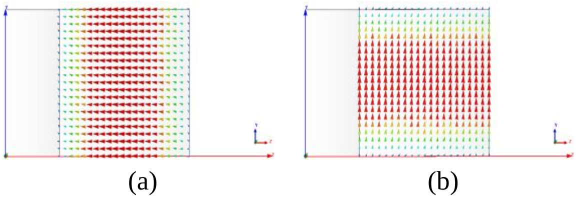


Fig. 3. Loss of a bent rectangular silicon waveguide, obtained from the analytical method proposed here (solid line) and the FEM (dashed line). The analytical method has only considered the dielectric loss and the bending loss from the E^y and E^x modes (loss at the corner regions has been neglected).



(a) (b)

Fig. 4. Electric field lines of: (a) E^x and (b) E^y modes at the cross section of the rectangular waveguide.

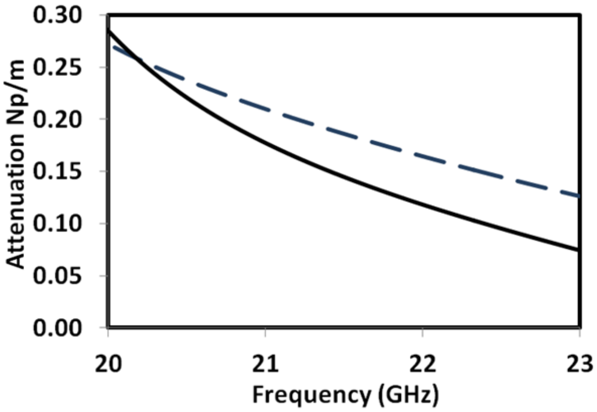


Fig. 5. Loss of a bent rectangular silicon waveguide, obtained from the analytical method proposed here (solid line) and the FEM (dashed line). The analytical method has taken into account the dielectric loss, as well as, the bending loss from the E^y and E^x modes (loss at the corner regions has been included).

IV. CONCLUSION

We have presented a closed-form analytical method to predict the attenuation in a bent dielectric rectangular waveguide. The dielectric loss in the waveguide can be extracted from the propagation constant obtained from a straight waveguide; whereas, the bending loss in the waveguide is determined from Marcuse' approximate method [12]. To enhance the accuracy of Marcuse' method, the correction factor in [13] has been applied to account for the loss at the corner regions. By including the bending loss exhibited by both E^y and E^x modes and the dielectric loss, the result is found to agree closely with that computed using the rigorous computational method. Since the formulations presented here are all in closed-form, it is not necessary to rely on computational intensive machines, such as a computer to calculate them. Besides being straight forward, the method also produces results which can be easily found; while at the same time, sufficiently accurate.

ACKNOWLEDGMENT

Part of this work has been supported by the FRGS Grant (Project: FRGS/2/2013/SG02/UTAR/02/1).

REFERENCES

[1] D. K. Cheng, *Field and Wave Electromagnetics*. 2nd ed., Addison Wesley Inc., 1989.

[2] R. E. Collin, *Field Theory of Guided Waves*. 2nd ed., IEEE Press, New York, 1991.

[3] E. A. J. Marcatili, "Bends in optical dielectric guides," *The Bell System Technical Journal*, vol. 48, no. 9, pp. 2103-2132, 1969.

[4] Y. Cai, T. Mizumoto, and Y. Naito, "Improved perturbation feedback method for the analysis of rectangular dielectric waveguides," *Journal of Lightwave Technology*, vol. 9, no. 10, pp. 1231-1237, 1991.

[5] P. R. Young and R. J. Collier, "Solution of lossy dielectric waveguides using dual effective-index method," *Electronics Letters*, vol. 33, no. 21, pp. 1788-1789, 1997.

[6] E. A. J. Marcatili, "Dielectric rectangular waveguide and directional coupler for integrated optics," *The Bell System Technical Journal*, vol. 48, no. 9, pp. 2071-2102, 1969.

[7] K. H. Yeap, K. H. Teh, K. C. Yeong, K. C. Lai, and M. C. Loh, "Propagation in dielectric rectangular waveguides," *Optica Applicata*, vol. 46, no. 2, pp. 317-330, 2016.

[8] D.-P. Cai, S.-C. Nien, H.-K. Chiu, C.-C. Chen, and C.-C. Lee, "Electrically tuneable liquid crystal waveguide attenuators," *Optics Express*, vol. 19, no. 12, pp. 11890-11896, 2011.

[9] H.-K. Chiu, F.-L. Hsiao, C.-H. Chan, and C.-C. Chen, "Compact and low-loss bent hollow waveguides with distributed Bragg reflector," *Optics Express*, vol. 16, no. 19, pp. 15069-15073, 2008.

[10] S.-S. Lo, C.-C. Chen, S.-C. Hsu, and C.-Y. Liu, "Fabricating a hollow optical waveguide for optical communication applications," *Journal of Microelectromechanical Systems*, vol. 15, no. 3, pp. 584-587, 2006.

[11] S.-S. Lo, M.-S. Wang, and C.-C. Chen, "Semiconductor hollow optical waveguides formed by omni-directional reflectors," *Optics Express*, vol. 22, no. 26, pp. 6589-6593, 2004.

[12] D. Marcuse, "Bending losses of the asymmetric slab waveguide," *The Bell System Technical Journal*, vol. 50, no. 8, pp. 2551-2563, 1971.

[13] R. T. Deck, M. Mirkov, and B. G. Bagley, "Determination of bending losses in rectangular waveguides," *Journal of Lightwave Technology*, vol. 16, no. 9, pp. 1703-1714, 1998.

[14] P. I. Somlo and J. D. Hunter, "On the TE10 mode cutoff frequency in lossy-walled rectangular waveguides," *IEEE Transactions on Instrumentation and Measurement*, vol. 45, pp. 301-304, 1996.

[15] T. K. Hong, "Analysis of Loss in Dielectric Waveguides," *Universiti Tunku Abdul Rahman*, Malaysia, B.Eng., 2014.

[16] B. M. Kolundžija and A. R. Djordjević, *Field Integral Equations*. In: Electromagnetic Modeling of Composite Metallic and Dielectric Structure, Artech House, Massachusetts, pp. 181-182, 2002.