Analysis of EM Scattering of Precipitation Particles in Dual-Band

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Abstract **─** Simulation of EM scattering from complex precipitation particles shows great importance in both theoretical researches and practical applications. In order to analyze the scattering from raindrops in the Ku-band and Ka-band, the T-matrix method is used in this paper. Firstly, a detailed analysis has been illustrated for a single homogeneous rainfall particle model in Ku-band and Ka-band. Then, scattering properties of double-layer rainfall media, such as ice-water or water-ice model, are quantitative analysis. Finally, scattering characteristics of a rainfall group by particle size distribution in a certain region is discussed. Numerical results are given to demonstrate the results of the EM scattering of precipitation particles. It is shown that studying the EM scattering of precipitation particles in Ku-band and Ka-band will lead to great significance for the development of precipitation radar in the future.

Index Terms **─** Dual–band, electromagnetic scattering, precipitation particles, precipitation radar, T-matrix method.

I. INTRODUCTION

With the development of the tropical rainfall measuring mission (TRMM) satellite [1], meteorological radar applications are extended to a global scale. This system is a useful tool to provide a three-dimensional overview of the weather in real time. TRMM PR [2] works on the center frequency of 13.8GHz (Ku band), which can detect the minimum rainfall of about 0.6mm/h, when it is not sensitive to the smaller raindrop backscattering. In order to enhance the measurement sensitivity of light rain and snow, a new space-borne dual-frequency precipitation radar (DPR) [3] will be launched in the near future, which is called PR-2. PR-2 increases higher frequency measurement band (Ka), which will give more accurate measurement of small rainfall.

This article studied on the scattering properties of precipitation particles in both Ku and Ka band. As electromagnetic wave propagates to clouds or rainfall, part of the energy is scattered, while the other part is absorbed and transformed into heat or other forms of energy [4-5]. It is known that the scattering of precipitation particles on the shorter wavelength (Ka band) is stronger, when attenuation of rain decreases rapidly on longer wavelength (Ku band). In order to quantitatively analyze the different scattering character from raindrops in the Ku-band and Ka-band, the transfer matrix (T-matrix) method is applied in this paper. Firstly, it briefly describes the basic principle of the T-matrix method. Secondly, scattering characteristics of both single particle and coated precipitation particles in Ku and Ka band are analyzed. Finally, scattering properties of rainfall group by particle size distribution in a certain region are discussed.

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II. T-matrix method

Waterman [6] introduced the T-matrix method as a technique for computing electromagnetic scattering by single, homogeneous, arbitrarily shaped particles. More details about the T-matrix algorithm are demonstrated below.

We consider scattering of a plane electromagnetic wave having wave-number k by a single homogeneous particle of irregular shape with refractive index m. In spherical coordinates, any point in this system is given by the radius-vector r or three coordinates $\boldsymbol{r} = \{r, \theta, \varphi\}$, where r is the distance from the coordinate system origin, θ is the polar angle, and φ is the azimuth angle. Scattering properties of particles of different sizes are characterized by the so-called size parameter, which can be defined as $x_i = 2\pi r_i / \lambda$, where r_i is the radius of the sphere with a volume equal to that of the particle and λ is the wavelength of incident light. We choose the coordinate system origin inside the particle, and the shape of irregular particles may be described by some function $R = R(\theta, \varphi)$ in this coordinate system. Such an approach limits the applicability of the method to star-shaped particles.

In the exterior region, one can describe the total electric field $\boldsymbol{E}$ as a sum of incident and scattering field which can be expanded in terms of the regular and outgoing spherical vector waves, respectively, referring to the common origin:

$$\boldsymbol{E}_i(\boldsymbol{r}) = Rg\boldsymbol{\Psi}^{\mathrm{T}}(\boldsymbol{r})\,\boldsymbol{a}\,, \qquad (1)$$

$$\boldsymbol{E}_s(\boldsymbol{r}) = \boldsymbol{\Psi}^{\mathrm{T}}(\boldsymbol{r})\,\boldsymbol{f}\,. \qquad (2)$$

Full details concerning the spherical vector waves $\mathrm{Rg}\psi^{\mathrm{T}}(\boldsymbol{r})$ and $\psi^{\mathrm{T}}(\boldsymbol{r})$ can be found e.g. in [6]. Then, the scattering problem can be solved by determining the transition matrix T which describes the linear relation between the scattered field amplitude f and the incident field amplitude a in

$$\boldsymbol{f} = \boldsymbol{T}\,\boldsymbol{a}\,. \qquad (3)$$

The discussion of the T-matrix calculation in details can be found in reference [7-8].

III. PRECIPITATION PARTICLE MODEL AND CALCULATIONS

In this paper, the raindrops are approximated as ellipsoids. We assume z-axis is the rotation axis, with the corresponding axis length b, when the axis length in x, y-axis direction is a. Then an ellipsoid in the Cartesian coordinate system equation can be written as:

$$\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1\,. \qquad (4)$$

The axial ratio ε is a/b. When $a>b$, we call it oblate spheroid particle. Otherwise, if $a<b$, it is prolate spheroid particle [9]. The shape of a spheroid in the spherical coordinate system is governed by the equation:

$$r = r(\theta) = \left(\tfrac{\sin^2\theta}{a^2} + \tfrac{\cos^2\theta}{b^2}\right)^{-\frac{1}{2}}\,, \qquad (5)$$

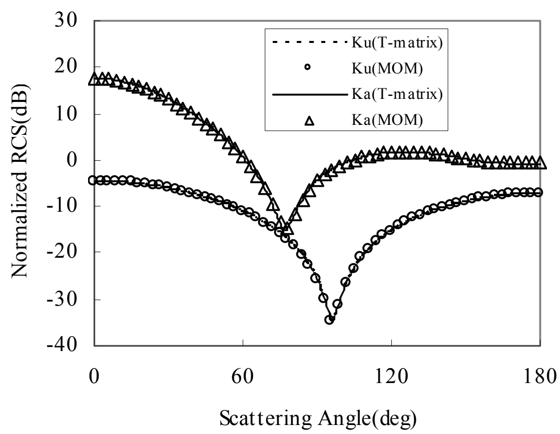
$$\sigma_\theta = \frac{r_\theta}{r} = r^2 \sin\theta \left(\frac{1}{a^2} - \frac{1}{b^2}\right)\frac{\delta y}{\delta x}\,. \qquad (6)$$

T matrix of non-spherical particles can be obtained by equations (5), (6) and T Matrix of spherical particles [10].

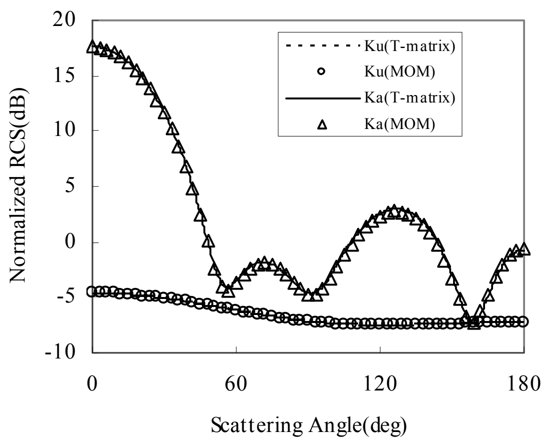
The existing TRMM precipitation radar works at Ku-band when the new generation precipitation radar — PR2 will measure the electromagnetic characteristics of rainfall in dual-band (Ka and Ku). When the wavelength of incident wave is in the Ka-band, it is approximate to the radius of precipitation particles. As a result, the scattering properties of particles will change a lot at Ka band, while the particle size changes. In this paper, electromagnetic characteristics of precipitation particle in both Ku and Ka band are analyzed. By using the data provided by Rothman et al., the complex refractive index of water/ice in Ka-band is range from $1.784+0.0013i$ to $4.954+2.79i$, when in Ku-band the complex refractive index of water/ice is range from $1.784+0.00075i$ to $6.850+2.890i$.

A. Scattering properties of precipitation particles in a fixed orientation

In the first part of the numerical simulation, the fixed-orientation scattering properties of particles are considered. Particle parameter is set as follows: the actual particle size in both Ku-band and Ka-band are the same, with the axial ratio $\varepsilon = 2$. In Ku-band, we assume the complex refractive index of the precipitation particle is $m=1.814+0.0012i$, with an equivalent-volume size parameter $x = 2\pi r_{eff}/\lambda = 1.186$. In Ka-band, we set the complex refractive index $m=1.785+0.001i$, with an equivalent-volume size $x = 2\pi r_{eff}/\lambda = 3.1728$. Particle orientation is $\theta_p=0^\circ$, which is the angle between rotation symmetry axis and the z axis in the Cartesian coordinate system. The separation angle between the incident direction and the scattering direction θ is 180°. In this simulation, the scattering azimuth angle is only considered in the x-z plane.



(a)



(b)

Fig. 1. Scattering properties of precipitation particles in a fixed orientation in dual-band: (a) for the TM wave (b) for the TE wave.

Table 1: Comparison of solution time (in seconds) with T-matrix and MOM on example III.A

Radar Frequency	T-matrix	MOM
Ku	0.03	174.45
Ka	0.03	151.09

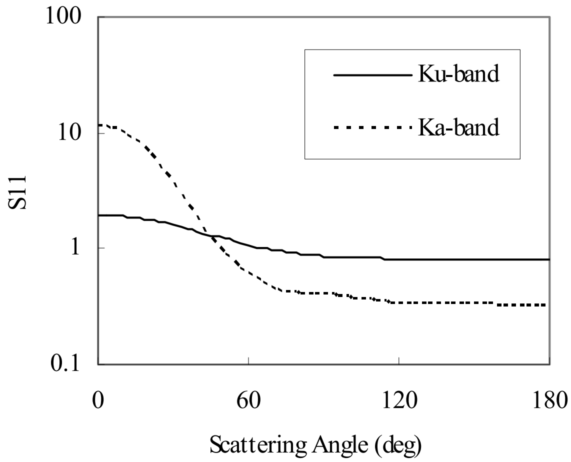
As shown in Figure 1, it can be found that there is an excellent agreement between the T-matrix results and MOM results. Scattering intensity in Ka-band is greater comparing with the case in Ku-band, whether incidence wave is horizontal or vertical polarization. At the same time, volatility of the curve in Ka-band is more significant than the situation in Ku-band. Table 1 lists the solution time of T-matrix and MOM on example III.A. It is obvious that the efficiency of the T-matrix method is much better than MOM for this example.

B. Scattering and attenuation properties of precipitation particles in random orientation

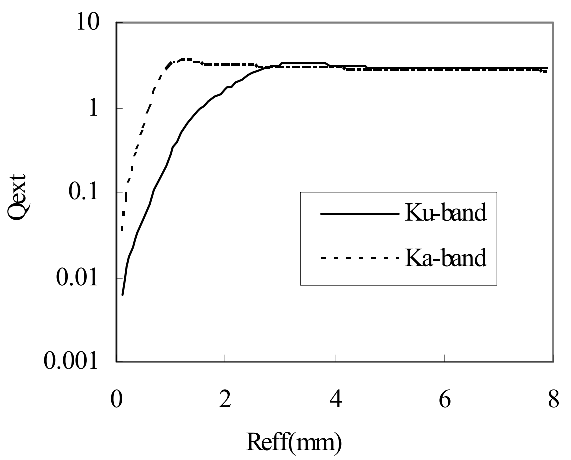
In reality, precipitation particles are usually in random orientation [11]. The scattering field of rainfall can be obtained by computing the average value of a large number of scattering particles in fixed orientation at that time, but this method requires a lot of computing time and memory. As an attempt for a possible remedy, M. I. Mishchenko proposed a new method to avoid solving a lot of scattered fields in a fixed orientation, and more details can be found in [12].

Particle parameter for simulation in this part is set as follows: the actual particle size in both Ku-band and Ka-band are the same, with the axial ratio ε=2. In Ku-band, we assume the complex refractive index of precipitation particle is m=6.800+2.850i, with an equivalent-volume size parameter $x = 2\pi r_{eff}/\lambda = 1.2$. In Ka-band, the complex refractive index of precipitation particle is m=4.854+2.674i, with an equivalent-volume size parameter $x = 2\pi r_{eff}/\lambda = 3.2$, where r_{eff} means the radius of the equal-surface-area sphere.

It can been seen in Figure 2 (a) that the relative scattering intensity of the same size particles in the Ka-band is larger than that in the Ku-band in the 0-45° range of scattering angle, and it is opposite in the 45-180° range. As is shown in Figure 2 (b), the attenuation coefficient in Ku-band is much smaller than that in Ka-band while the particle size is small, and it tends to be a constant when particle size increases. In the real rainfall, as the equivalent-volume size is less than 3mm, the attenuation in Ka-band is much greater than that in Ku-band. However, also as shown in Figure 2 (b), when the equivalent-volume size is larger than 3mm (usually means the bad weather, such as rainstorm or hailstone), rain attenuation of particles in dual-band are basically similar.



(a)



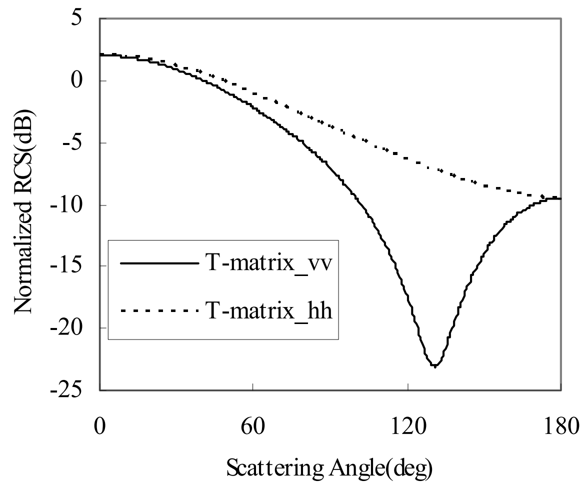
(b)

Fig. 2. Scattering and attenuation properties of precipitation particles in random orientation: (a) the relative scattering intensity (b) the efficiency factors for extinction (Qext).

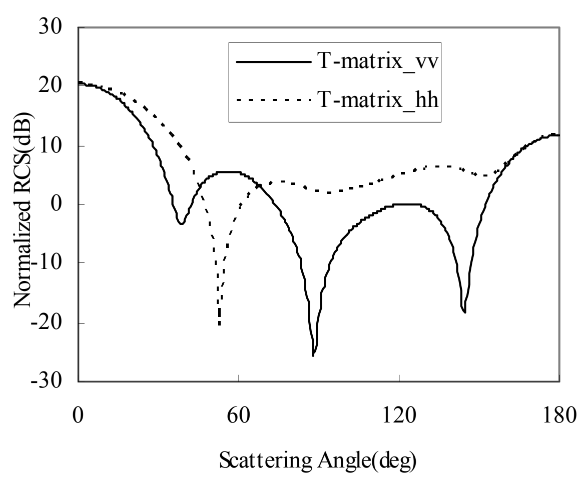
C. Scattering properties of two-layered precipitation particles

As the real rainfall is complex, it is probably that composition of precipitation particles is not a single particle medium but two mediums with the coated structure (for example, ice around water or water around ice) [13-14]. In the following text, the scattering properties of the two-layered precipitation particles are analyzed.

In this section, we assume particles with refractive index 1.784+0.00075i for core and 2.35+0.0015i for shell in Ku-band, that is, ice coated with water in the surface. The particles are oriented spheroids with an equal-volume size parameter x $=2\pi r_{eff}/\lambda$ =0.9026 for core and 1.354 for shell, when the axial ratio is ε=2. In Ka-band, the refractive index of the particles is assumed 1.784+0.0013i for core and 2.245+0.0026i for shell. The particles are oriented spheroids with an equal-volume size x $=2\pi r_{eff}/\lambda$ =2.42 for core, and 3.64 for shell, with the axial ratio ε=2. Particle orientation is θ_p=0°, which is the angle between the rotation symmetry axis and the z axis in the Cartesian coordinate system. The separation angle between the incident direction and the scattering direction θ is 180°. In this simulation, the scattering azimuth angle is only considered in the x-z plane.



(a)



(b)

Fig. 3. Horizontal polarization and vertical polarization of two-layered rainfall particle model in dual-band: (a) Ku-band (b) Ka-band.

From Figure 3(a), we can find the horizontal polarization curve is quite smooth and the vertical polarization curve has minimum value at the scattering angle of 130°. In Figure 3(b), the curves fluctuate much larger than that in Ku-band. This result infers when wavelength of incidence is close to the size of raindrops, corresponding RCS is more sensitive to the changes of the angle.

D. Scattering properties of rainfall group under Gamma distribution

In order to understand the scattering characteristics of the rain group, This section analyzes the scattering properties of some special size distributions of raindrop particles in Ku-band and Ka-band by calculating the average distributions of raindrop particles. Hansen and Travis [15-16] have shown that scattering properties of distributions of spherical particles depend primarily on only two characteristics of the distribution, the effective radius r_{eff} and the effective variance v_{eff}, the particular shape of the distribution being of secondary importance. The effective radius is defined as the cross-sectional-area-weighted mean radius,

$$r_{eff} = \tfrac{1}{G}\int_0^\infty r\pi r^2 n(r)\mathrm{d}r\,. \qquad (7)$$

Similarly, the width of the distribution is characterized by the dimensionless effective variance v_{eff}, which is defined as

$$v_{eff} = \tfrac{1}{Gr_{eff}^2}\int_0^\infty (r - r_{eff})\pi r^2 n(r)\mathrm{d}r\,, \qquad (8)$$

where

$$G = \int_0^\infty \pi r^2 n(r)\mathrm{d}r\,, \qquad (9)$$

is the average geometric cross-sectional area and $n(r)dr$ is the fraction of the particles with radius between r and $r+dr$. Note that

$$\int_0^\infty n(r)\mathrm{d}r = 1\,. \qquad (10)$$

Thus, the result of Hansen and Travis demonstrate that if different size distributions of spherical particles have the same values of the effective radius and effective variance, their scattering properties are practically identical. In the following, we computed the scattering properties of size distributions of randomly oriented oblate spheroids, namely, gamma distribution given by

$$n(r) = \frac{1}{ab\Gamma\left(\frac{1-2b}{b}\right)}\left(\frac{r}{ab}\right)^{(1-3b)/b}\exp\left(-\frac{r}{ab}\right). \qquad (11)$$

Then, we give the contrast of rainfall groups in some size range.

Rainfall group parameter is given as follows: Gamma distribution with size range $2\pi a/\lambda$ is taken as (1, 4). The complex refractive index of water in Ku, Ka-band is set, respectively: 6.800+2.850i and 4.854+2.674i. The axial ratio ε equals two. Incident plane wave is along the z-axis, and the scattering azimuth angle is only considered in the x-z plane.

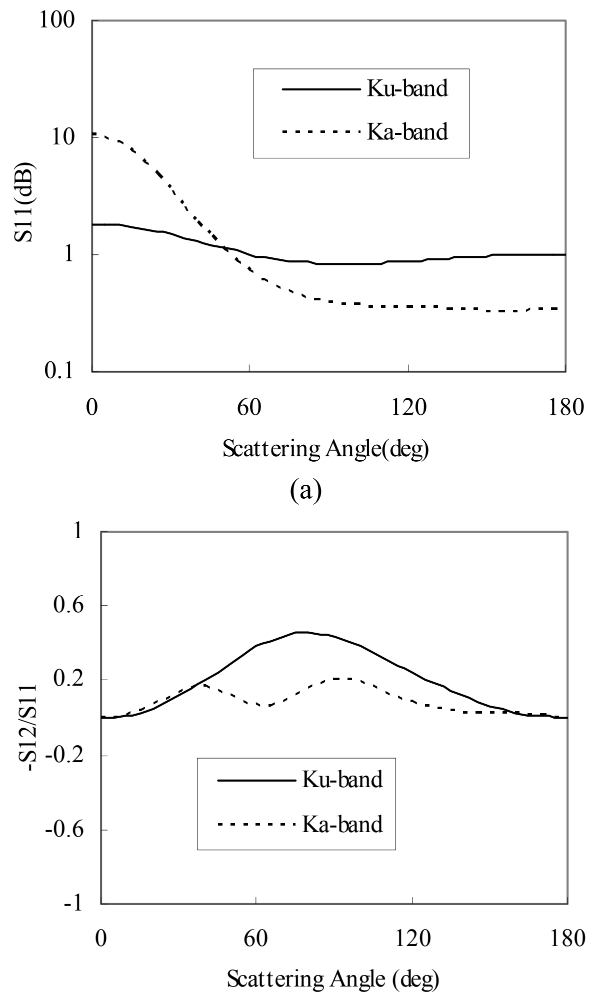


Fig. 4. Scattering properties of rainfall groups under the Gamma distribution in dual-band: (a) the relative scattering intensity (b) linear polarization.

Figure 4 (a) shows the distribution curves of scattering intensity are flat. In Figure 4 (b), the linear polarization curves of rainfall groups in Ku-band become very smooth, and there is only one main peak curve, whose value is always larger than zero. It is also indicated the oscillation of the curve in Ka-band is greater than that in Ku-band.

IV. CONCLUSION

In this paper, the scattering and attenuation properties of precipitation particles in Ku-band and Ka-band are studied. First of all, scattering properties of precipitation particles in a fixed orientation is investigated. Secondly, scattering and attenuation properties of rainfall particles in random orientation are studied. Finally, in order to better simulate the actual rainfall, the scattering properties of two-layered precipitation particles as well as the rainfall group under some size distribution in a certain region are analyzed. Simulation results demonstrate the scattering intensity of precipitation particles in Ka-band is stronger than Ku-band, which means radar in Ka-band is more sensitive to the echo signal of small raindrops. However, considering the situation of rain attenuation, Ku-band is still a suitable frequency band to measure large raindrops. Therefore, the dual-band measurements can retain the advantages of the original system, while increasing the detection capability of small rainfall.

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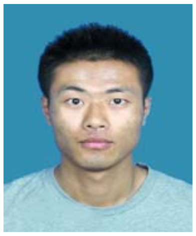

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