# Vector Parabolic Equation Method for the EM Scattering from PEC Objects in Half-Space 

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#### Abstract

The vector parabolic equation method was widely used to analyze the electromagnetic scattering from electrically large PEC objects in free space. In this paper, it is applied for the analysis of the electromagnetic scattering from PEC objects in halfspace. By introducing both the incident field and the reflected field to illuminate the objects, the vector parabolic equation method can be used to efficiently calculate the radar cross-section of PEC objects with electrically large size in half-space. The numerical results demonstrate that the proposed method can efficiently give reasonably accurate results.


Index Terms - Electromagnetic scattering, half-space, parabolic equation method.

## I. INTRODUCTION

The differential methods such as the finite differential method (FD) and the finite element method (FEM) in frequency domain [1-2] were widely used for computing the radar cross-section (RCS) of complex objects in free space. However, a large number of unknowns were needed to analyze three dimensional electromagnetic scattering from electrically large objects for these methods. The parabolic equation (PE) method was an approximation of the wave equation with marching gradually from one plane to another along the paraxial direction [3-5]. By this way, the three-dimensional problem could be converted into a series of two-dimensional problems to be solved by the PE method. Therefore, the electromagnetic scattering from electrically large PEC objects could be analyzed efficiently. However, the PE method could not provide the full bistatic scattering pattern of an object because of intrinsic paraxial limitations [6]. And it has not been reported that the electrically large conductive objects in half-space were calculated by the parabolic equation method so far.

With the development of the electromagnetism, more and more attention was paid to the electromagnetic scattering from electrically large objects above the earth or the sea [7-10]. The
electromagnetic scattering from the electrically large objects in half-space could be analyzed by both the FDTD [11] and the FEM [12], but there were difficulties in the procedures of modeling and computing. Moreover, the half-space Green's function was used to be combined with the method of moments $(\mathrm{MoM})$ for the electromagnetic scattering from PEC objects in half-space [13-15]. And the multilevel fast multipole method (MLFMM) was applied to accelerate the calculation [8]. But memory requirements and computation time were still excessive by this method, which led to bad computational efficiency.

In this paper, the vector PE method is used for the analysis of the electromagnetic scattering from PEC objects in half-space. Both the incident field and the reflected field from the ground plane are considered to illuminate the objects through the inhomogeneous boundary conditions. Then the scattered electric fields can be calculated from one plane to another with the parabolic equations. Moreover, the scattered magnetic fields can be gotten from the scattered electric fields through the Maxwell equations with the finite difference scheme. As a result, both the scattered electric current and the scattered magnetic current of the last transverse plane can be computed by them. At last, the RCS can be obtained by the reciprocal theory with the scattered electric current and the scattered magnetic current of the last transverse plane.

The remainder of this paper is organized as follows. In Section 2, the theory and the formulations are given. Two numerical experiments are presented in Section 3 to show the efficiency of the proposed method. Section 4 concludes this paper.

## II. THEORY AND FORMULATIONS

## A. PE framework for scattering problems

The three-dimensional electromagnetic scattering from PEC objects in free space can be analyzed by the three-dimensional wave equation. The scattered field components $\vec{E}_{\mathrm{x}}^{s}, \vec{E}_{y}^{s}, \vec{E}_{z}^{s}$ satisfy scalar wave equation as follows:

$$
\begin{align*}
& \frac{\partial^{2} \vec{E}_{x}^{s}}{\partial x^{2}}+\frac{\partial^{2} \vec{E}_{x}^{s}}{\partial y^{2}}+\frac{\partial^{2} \vec{E}_{x}^{s}}{\partial z^{2}}+k^{2} \vec{E}_{x}^{s}=0 \\
& \frac{\partial^{2} \vec{E}_{y}^{s}}{\partial x^{2}}+\frac{\partial^{2} \vec{E}_{y}^{s}}{\partial y^{2}}+\frac{\partial^{2} \vec{E}_{y}^{s}}{\partial z^{2}}+k^{2} \vec{E}_{y}^{s}=0  \tag{1}\\
& \frac{\partial^{2} \vec{E}_{z}^{s}}{\partial x^{2}}+\frac{\partial^{2} \vec{E}_{z}^{s}}{\partial y^{2}}+\frac{\partial^{2} \vec{E}_{z}^{s}}{\partial z^{2}}+k^{2} \vec{E}_{z}^{s}=0,
\end{align*}
$$

where $k$ is the wave number.
The reduced scattered fields $u_{x}^{s}, u_{y}^{s}, u_{z}^{s}$ are introduced with the scattered field components:

$$
\begin{equation*}
u_{\xi}^{s}(x, y, z)=e^{-j / x} \vec{E}_{\xi}^{s}(x, y, z) \quad \xi=x, y, z \tag{2}
\end{equation*}
$$

Define the pseudo-differential operator $Q$ as the following format:

$$
\begin{equation*}
Q=\frac{1}{k^{2}}\left(\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}\right) . \tag{3}
\end{equation*}
$$

Substitute equation (1) with the equations (2) and (3), the following equations can be gotten:

$$
\begin{array}{r}
{\left[\frac{\partial}{\partial x}+j k(1-\sqrt{1+Q})\right]\left[\frac{\partial}{\partial x}+j k(1+\sqrt{1+Q})\right] u_{\xi}^{s}}
\end{array}=0, ~\left(\begin{array}{rl} 
 \tag{4}\\
\xi & =x, y, z .
\end{array}\right.
$$

The two parts of the equation (4) correspond to forward and backward propagating waves, respectively. The first order Taylor expansions of the square root and the exponential are used in this paper. This yields the well-known standard vector parabolic equations:

$$
\begin{array}{r}
\frac{\partial^{2} u_{\zeta}^{s}}{\partial y^{2}}(x, y, z)+\frac{\partial^{2} u_{\xi}^{s}}{\partial z^{2}}(x, y, z)+2 j k \frac{\partial u_{\xi}^{s}}{\partial x}(x, y, z)=0  \tag{5}\\
\xi=x, y, z .
\end{array}
$$

When the FD scheme of the Crank-Nicolson type is used to the equation (5), the forward vector parabolic wave equations can be written as follows:

$$
\begin{align*}
& \frac{\Delta x}{2 j k(\Delta y)^{2}} u_{\xi}^{s}(x+\Delta x, y+\Delta y, z)+ \\
& \frac{\Delta x}{2 j k(\Delta z)^{2}} u_{\xi}^{s}(x+\Delta x, y, z+\Delta z)+ \\
& \left(1-\frac{\Delta x}{j k(\Delta y)^{2}}-\frac{\Delta x}{j k(\Delta z)^{2}}\right) u_{\xi}^{s}(x+\Delta x, y, z)+  \tag{6}\\
& \frac{\Delta x}{2 j k(\Delta y)^{2}} u_{\xi}^{s}(x+\Delta x, y-\Delta y, z)+ \\
& \frac{\Delta x}{2 j k(\Delta z)^{2}} u_{\xi}^{s}(x+\Delta x, y, z-\Delta z) \\
& \quad=u_{\xi}^{s}(x, y, z) \quad \xi=x, y, z .
\end{align*}
$$

The reduced scattered fields at $(x+\Delta x)$ plane can be calculated from those at $x$ plane by the use of the equation (6). As shown in Fig. 1, the calculation starts in the plane before the scatterer and stops in the plane beyond the scatterer. The initial fields are set to be zero in the first transverse plane. In order to truncate the computational domain, the perfectly matched layer (PML) is selected to truncate each transverse plane [1617]. The radar cross section (RCS) can be obtained by the reciprocal theory with the scattered electric current
and the scattered magnetic current of the last transverse plane.


Fig. 1. The marching strategy of the PE method.

## B. Boundary conditions for scattering problems in half-space

As shown in Fig. 2, the scattered fields $\vec{E}^{s}$ of the electrically large conductive objects in half space can be composed of three parts approximately [18-19]. They are the reflected fields of the ground, the reflected fields of the object and the reflected fields of both the ground and the object. And the PEC object in halfspace is illuminated by both the incident field $\vec{E}^{i}$ and the reflected field in the ground plane $\vec{E}^{r}$. The intersection angle between the incident field and the reflected field is $2 \alpha$. As shown in Fig. 2, the marching direction of the PE method is given for backward scattering from a PEC object.


Fig. 2. The object in half-space.
The reduced scattered fields in equation (6) are coupled through boundary conditions on the PEC objects. For a PEC, the tangential electric field must be zero on the surface of the scatterer:

$$
\begin{align*}
& n_{x} u_{y}^{s}-n_{y} u_{x}^{s}=-e^{-j k x}\left[n_{x}\left(E_{y}^{i}+E_{y}^{r}\right)-n_{y}\left(E_{x}^{i}+E_{x}^{r}\right)\right] \\
& n_{x} u_{z}^{s}-n_{z} u_{x}^{s}=-e^{-j k x}\left[n_{x}\left(E_{z}^{i}+E_{z}^{r}\right)-n_{z}\left(E_{x}^{i}+E_{x}^{r}\right)\right]  \tag{7}\\
& n_{y} u_{z}^{s}-n_{z} u_{y}^{s}=-e^{-j k x}\left[n_{y}\left(E_{z}^{i}+E_{z}^{r}\right)-n_{z}\left(E_{y}^{i}+E_{y}^{r}\right)\right],
\end{align*}
$$

in which $\left(E_{x}^{i}, E_{y}^{i}, E_{z}^{i}\right)$ are the components of the incident field for the directions $\vec{x}, \vec{y}, \vec{z},\left(E_{x}^{r}, E_{y}^{r}, E_{z}^{r}\right)$ are the ones of the reflected field, and $\left(n_{x}, n_{y}, n_{z}\right)$ denote the surface outward pointing unit normal.

Another equation is needed because the equations in (7) are not independent. As a result, the divergencefree condition is added to the equation for guaranteeing the unicity of the solution [6]:

$$
\begin{equation*}
\frac{j}{2 k}\left(\frac{\partial^{2} u_{x}^{s}}{\partial y^{2}}+\frac{\partial^{2} u_{x}^{s}}{\partial z^{2}}\right)+j k u_{x}^{s}+\frac{\partial u_{y}^{s}}{\partial y}+\frac{\partial u_{z}^{s}}{\partial z}=0 . \tag{8}
\end{equation*}
$$

The incident field and the reflected field can be expressed by the following equations, respectively:

$$
\begin{align*}
& \vec{E}^{i}=\left(\hat{\theta}_{i} \cos \beta+\hat{\varphi}_{i} \sin \beta\right) e^{-j \vec{k}^{i} \cdot \vec{r}} \\
& \text { for } \quad \vec{k}^{i}=-k\left(\cos \theta^{i} \hat{x}+\sin \theta^{i} \cos \varphi^{i} \hat{y}+\sin \theta^{i} \sin \varphi^{i} \hat{z}\right) \\
& \qquad \hat{\theta}_{i}=-\sin \theta^{i} \hat{x}+\cos \theta^{i} \cos \varphi^{i} \hat{y}+\cos \theta^{i} \sin \varphi^{i} \hat{z}  \tag{9}\\
& \hat{\varphi}_{i}=-\sin \varphi^{i} \hat{y}+\cos \varphi^{i} \hat{z}, \\
& \vec{E}^{r}=\left(\hat{\theta}_{t} R^{T M} \cos \beta+\hat{\varphi}_{t} R^{T E} \sin \beta\right) e^{-j \vec{k}^{t} \cdot \vec{r}} \\
& \text { for } \quad \vec{k}^{t}=-k\left(\cos \theta^{t} \hat{x}+\sin \theta^{t} \cos \varphi^{t} \hat{y}+\sin \theta^{t} \sin \varphi^{t} \hat{z}\right) \\
& \hat{\theta}_{t}=-\sin \theta^{t} \hat{x}+\cos \theta^{t} \cos \varphi^{t} \hat{y}+\cos \theta^{t} \sin \varphi^{t} \hat{z} \\
& \hat{\varphi}_{t}=-\sin \varphi^{t} \hat{y}+\cos \varphi^{t} \hat{z} . \tag{10}
\end{align*}
$$

In equations (9) and (10), $\beta=0$ represents vertical polarization while $\beta=\frac{\pi}{2}$ represents horizontal polarization, $\left(\theta^{i}, \varphi^{i}\right)$ is the angle of the incident field and $\left(\theta^{t}, \varphi^{t}\right)$ is the angle of the reflected field. The reflection coefficients $R^{T M}, R^{T E}$ can be expressed as follows, respectively [20]:

$$
\begin{align*}
R^{T M} & =\frac{\left(\varepsilon_{2} / \varepsilon_{1}\right) \cos \theta^{i}-\sqrt{\varepsilon_{2} / \varepsilon_{1}-\sin ^{2} \theta^{i}}}{\left(\varepsilon_{2} / \varepsilon_{1}\right) \cos \theta^{i}+\sqrt{\varepsilon_{2} / \varepsilon_{1}-\sin ^{2} \theta^{i}}} \\
R^{T E} & =\frac{\cos \theta^{i}-\sqrt{\varepsilon_{2} / \varepsilon_{1}-\sin ^{2} \theta^{i}}}{\cos \theta^{i}+\sqrt{\varepsilon_{2} / \varepsilon_{1}-\sin ^{2} \theta^{i}}} . \tag{11}
\end{align*}
$$

## C. Far-field calculations

Combining the equations (6), (7) and (8) with the finite difference scheme of the Crank-Nicolson type, the scattered electric fields $\vec{E}^{s}(x, y, z)$ can be computed. And the scattered magnetic fields $\vec{H}^{s}(x, y, z)$ of the last transverse $(y, z)$ plane can be gotten by the following equation with the scattered electric fields $\vec{E}^{s}(x, y, z)$ of the last two transverse $(y, z)$ planes:

$$
\begin{aligned}
& \vec{H}^{s}=\frac{j}{\omega \mu_{1}} \nabla \times \vec{E}^{s} \\
& =\frac{j}{\omega \mu_{1}}\left[\vec{e}_{x}\left(\frac{\partial E_{z}^{s}}{\partial y}-\frac{\partial E_{y}^{s}}{\partial z}\right)+\right. \\
& \left.\vec{e}_{y}\left(\frac{\partial E_{x}^{s}}{\partial z}-\frac{\partial E_{z}^{s}}{\partial x}\right)+\vec{e}_{z}\left(\frac{\partial E_{y}^{s}}{\partial x}-\frac{\partial E_{x}^{s}}{\partial y}\right)\right] .
\end{aligned}
$$

Moreover, the scattered electric current and the scattered magnetic current of the last transverse plane can be computed by the scattered electric fields and the scattered magnetic fields in the last transverse $(y, z)$ planes:

$$
\begin{align*}
\vec{J}^{s}= & \vec{n}_{p} \times \vec{H}^{s} \\
= & \vec{e}_{x}\left(n_{y, p} H_{z}^{s}-n_{z, p} H_{y}^{s}\right)+\vec{e}_{y}\left(n_{z, p} H_{x}^{s}-n_{x, p} H_{z}^{s}\right)  \tag{13}\\
& +\vec{e}_{z}\left(n_{x, p} H_{y}^{s}-n_{y, p} H_{x}^{s}\right), \\
\vec{M}^{s}= & \vec{E}^{s} \times \vec{n}_{p} \\
= & \vec{e}_{x}\left(E_{y}^{s} n_{z, p}-E_{z}^{s} n_{y, p}\right)+\vec{e}_{y}\left(E_{z}^{s} n_{x, p}-E_{x}^{s} n_{z, p}\right)  \tag{14}\\
& +\vec{e}_{z}\left(E_{x}^{s} n_{y, p}-E_{y}^{s} n_{x, p}\right),
\end{align*}
$$

where $\vec{n}_{p}=\left(n_{x, p}, n_{y, p}, n_{z, p}\right)$ denotes the unit normal of the last transverse $(y, z)$ plane. Once both the scattered electric current and the scattered magnetic current are obtained in the last transverse plane, the far field components can be calculated by them with the reciprocal theory. And the RCS in direction $(\theta, \varphi)$ is given by:

$$
\begin{align*}
& \sigma(\theta, \varphi)=\lim _{r \rightarrow \infty} 4 \pi r^{2} \frac{\left|\vec{E}^{S}(x, y, z)\right|^{2}}{\left|\vec{E}^{i}(x, y, z)\right|^{2}}= \\
& \frac{k^{2} \cos ^{2} \theta}{\pi}\left|\int_{-\infty}^{\infty} \int_{-\infty}^{\infty}\left(\vec{E}^{s}(x, y, z) \cdot \vec{t}\right) \cdot e^{-j k \sin \theta(y \cos \varphi+z \sin \varphi)} d y d z\right|^{2}, \tag{15}
\end{align*}
$$

where $\vec{t}$ is the receiver polarization.

## III. NUMERICAL RESULTS

In this section, a series of examples are presented to demonstrate the efficiency of the proposed method.

## A. The bistatic RCS for a PEC cylinder

Firstly, the electromagnetic scattering from a PEC cylinder is considered at the frequency of 300 MHz with the radius 2 m and the height 1 m . The model of the PEC cylinder is shown in Fig. 3. The incident wave propagates along the x -axis. The PEC cylinder is placed $\lambda / 5$ above the half space with $\varepsilon_{r}=4.0$, $\mu_{r}=1.0$ and $\sigma=0.01$, where $\lambda$ represents the wavelength in the vacuum. As shown in Fig. 4, the bistatic RCS curves above the half space are compared between the proposed method and software FEKO. It can be seen that there is a good agreement between them and it should be noted that seven rotating PE runs are used to obtain the full bistatic RCS [6]. Moreover, comparisons of the memory requirement, the matrix size, the matrix-filling time and the CPU time are made between the proposed method and the MLFMM, as shown in Table 1. It can be found that memory requirement and CPU time can be reduced $40.1 \%$ and
88.3\% for the proposed method, respectively. At last, the bistatic RCS values of different spacing values between the cylinder and the soil are computed in Fig. 5.


Fig. 3. The cylinder model in half space.


Fig. 4. Bistatic RCS of a PEC cylinder at the frequency of 300 MHz .


Fig. 5. Bistatic RCS values of different spacing values for the PEC cylinder.

Table 1: Comparisons of the memory requirement, the matrix size, the matrix-filling time and the CPU time for the PEC cylinder

| Methods | Matrix <br> Size | Matrix- <br> Filling <br> Time (s) | CPU <br> Time <br> $(\mathrm{s})$ | Memory <br> Requirement <br> $(\mathrm{MB})$ |
| :---: | :---: | :---: | :---: | :---: |
| Proposed <br> Method | 10000 | 23 | 97 | 225.3 |
| MLFMM | 25506 | 511 | 826 | 376.2 |

## B. The monostatic RCS for a PEC polygon

Secondly, the analysis of monostatic RCS is taken for a PEC polygon at the frequency of 0.3 GHz . The PEC polygon is placed $\lambda / 2$ above the soil with $\varepsilon_{r}=2.3, \mu_{r}=1.0$ and $\sigma=0.01$. The size of the polygon is shown in Fig. 6. It should be noted that there are different numbers of transverse planes to be computed for each observation angle, since the length of the scatterer along the x -axis (the paraxial direction) is changing for different angles of the incident wave. As shown in Fig. 7, the monostatic RCS curves are compared between the proposed method and FEKO. As shown in Table 2, the comparisons of the memory requirement, the matrix size, the matrix-filling time and the CPU time are made between the proposed method and the MLFMM. It can be seen that memory requirement and CPU time can be reduced $65.4 \%$ and $34.2 \%$ for the proposed method, respectively. Therefore, the vector parabolic equation method can be utilized for the efficient analysis of the electromagnetic scattering from the conductive objects with electrically large size in half-space.


Fig. 6. Model of the PEC polygon in half space.
Table 2: Comparisons of the memory requirement, the matrix size, the matrix-filling time and the CPU time for the PEC polygon

| Methods | Matrix <br> Size | Matrix- <br> Filling <br> Time (s) | CPU <br> Time <br> (s) | Memory <br> Requirement <br> $(\mathrm{MB})$ |
| :---: | :---: | :---: | :---: | :---: |
| Proposed <br> Method | 10000 | 4391 | 10978 | 339.7 |
| MLFMM | 34245 | 10007 | 16679 | 980.4 |



Fig. 7. Monostatic RCS of a PEC polygon at the frequency of 300 MHz in half space.

## C. Comparisons of the bistatic RCS curves in free space and in half space for an aircraft

At last, we consider the scattering from an aircraft both in the free space and in the half space at the frequency of 1 GHz . Its maximum sizes in $\mathrm{x}, \mathrm{y}$ and z directions are $4.8 \mathrm{~m}, 3.3 \mathrm{~m}, 1.06 \mathrm{~m}$. The full bistatic RCS curves in free space and in half space are given in Fig. 8 with 7 PE runs. The aircraft is placed $2 \lambda$ above the half space with $\varepsilon_{r}=4.5, \mu_{r}=1.0$ and $\sigma=0.01$.


Fig. 8. Bistatic RCS of an aircraft at the frequency of 1 GHz.

## IV. CONCLUSION

In this paper, the vector parabolic equation method is used to analyze the electromagnetic scattering from electrically large PEC objects in half-space. Both the incident field and the reflected field from the ground plane illuminate the objects through the inhomogeneous boundary conditions. The numerical results are
compared between the proposed method and the MLFMM. It can be found that there is a good agreement between them for the three-dimensional electromagnetic scatterings in half-space. Moreover, the proposed method can give encouraging results with less CPU time and smaller memory requirement than the MLFMM.

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