Analysis of Thin Microstrip Antennas by Meshless Methods

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Abstract **─** In this paper, different meshless methods are applied to the analysis of microstrip antennas with thin substrate. Comparison is made between the performance of the methods with respect to convergence, CPU time and condition number of final coefficient matrix. The exact modal solution and method of moments are used for validation. The input impedance results of all methods are in agreement with each other.

Index Terms **─** LBIE, meshless, microstrip antenna, MLBE, MLPG.

I. INTRODUCTION

Meshless methods are powerful tools for numerical solution of partial differential equations (PDEs) [1]. Up to now, these methods have been widely used in mechanical engineering, but with limited use in electrical engineering [2]-[17]. Evidently, the purpose of meshless methods is elimination of mesh in discretization of operator equations. This goal necessitates the design of fitting strategies for scattered data in multidimensional spaces. This attempt resulted in the emergence of meshless shape functions with superb fitting capability. Consequently, expanding the unknown field variable over such functions not only makes it possible to solve problems meshfree, but also decreases the number of unknowns in the corresponding system of equations. In general but not necessarily, these two intrinsic benefits are accomplished at the expense of computational cost. In this work, we have applied different meshless methods such as *meshless local Petrov-Galerkin* (MLPG) [18], *local boundary integral equation* (LBIE) [19] and *meshless local boundary equation* (MLBE) [20] to the analysis of microstrip antennas with thin substrate. A rectangular-coax-fed, a square-line-fed and a two-element array antenna are analyzed. The aforementioned methods are compared with each other from the aspects of convergence, CPU time and condition number of the corresponding linear systems. In order to highlight the capabilities of meshless methods, various node arrangements for describing the problem domains are used including uniform, non-uniform and random. Clearly, by imposing a logical irregularity in accordance with the geometry and physical sense of a problem, the number of unknowns could be considerably decreased.

The results are validated by the exact modal solution for the coax-fed antenna case and the method of moments (MoM) for the others. It is observed that all meshless methods have essentially the same convergence rates. However, LBIE is seen to be the most well-conditioned and the MLBE the fastest.

II. MATHEMATICAL STATEMENT OF THE PROBLEM

Microstrip structures with thin substrate are *planar* microwave components [21]. By this assumption, variations normal to the substrate become negligible and consequently, Maxwell's equations simplify to a two-dimensional (2D) scalar Helmholtz equation with homogeneous Neumann boundary conditions. In addition, coax excitation ports can be well modeled by Dirac delta functions. Suppose a planar microstrip structure placed on the *x-y* plane, with the domain and boundary of Ω and $\Gamma(\equiv\partial\Omega)$, respectively.

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Based on these assumptions, the mathematical statement of the problem corresponding to a single feed antenna is:

$$\begin{cases} \left(\nabla^2 + k^2\right) E_z = j\omega\mu_0 \delta\left(\mathbf{x} - \mathbf{x}_p\right), & \mathbf{x} \in \Omega \\ \mathbf{n} \cdot \nabla E_z = 0, & \mathbf{x} \in \partial\Omega \end{cases}, \tag{1}$$

where E_z is the component of the electric field normal to the substrate, k is the propagation constant of the field inside the structure, ω is the working angular frequency, μ_0 is the magnetic permeability, $\mathbf{x}_p$ is the position of the excitation port and $\mathbf{n}$ is the normal vector to the boundary. Clearly, (1) can be equivalently written as:

$$\begin{cases} \left(\nabla^2 + k^2\right) u = 0, & \mathbf{x} \in \Omega \\ \mathbf{n} \cdot \nabla u = \overline{q}, & \mathbf{x} \in \Gamma \end{cases}, \tag{2}$$

with:

$$\begin{cases} u = E_z - j\omega\mu_0 \cdot \dfrac{j}{4} H_0^{(2)}\left(k\left|\mathbf{x} - \mathbf{x}_p\right|\right) \\ \overline{q} = -\mathbf{n} \cdot \nabla j\omega\mu_0 \cdot \dfrac{j}{4} H_0^{(2)}\left(k\left|\mathbf{x} - \mathbf{x}_p\right|\right) \end{cases}. \tag{3}$$

We have found that (2) is more proper for meshless discretization, so it is regarded as the mathematical statement of the problem. Our observations of different situations showed that meshless methods are incapable of handling abrupt changes, in the sense of convergence. In fact, we could not get a convergence solution of (1) unless we approximated the Dirac delta by a sharp bell-shaped function. Even by applying this trick, a series of difficulties could be encountered. For example, the sharpness should not exceed a certain level; otherwise, the number of nodes at the vicinity of the excitation should be sufficiently increased to track the function.

Equation (2) cannot be solved unless k is determined. Since in antenna applications a considerable amount of electromagnetic energy should be radiated in free space, the trivial value of the propagation constant in the substrate, i.e., $\omega(\varepsilon\mu)^{1/2}$, is incapable of modeling the behavior of the structure.

An *effective* value for propagation constant, k_{eff}, can be well estimated by the cavity method [22]. The electric field distribution on the antenna can be computed by solution of (2) based on k_{eff}. Once this is done, the input impedance at the antenna port can be evaluated by:

$$Z_{in} = -h \cdot E_z\left(\mathbf{x}_p\right) / I\left(\mathbf{x}_p\right), \tag{4}$$

where h is the height of substrate and I the source current defined as:

$$I\left(\mathbf{x}_p\right) = \begin{cases} 1, & \mathbf{x} = \mathbf{x}_p \\ 0, & \mathbf{x} \neq \mathbf{x}_p \end{cases}. \tag{5}$$

In general, this procedure is iterative but in most of the cases sufficient accuracy is achieved at the second run.

III. CHOICES OF MESHLESS METHODS

Meshless methods are classified as *weighted residual methods*. Therefore, by changing the form of residual statement and/or kind of weighting function, different meshless methods can be generated [23]. Two equivalent *global* weak statements of (2), neglecting imposition of boundary conditions, are:

$$\begin{cases} \int_\Gamma w u_{,n} d\Gamma - \int_\Omega \nabla w \cdot \nabla u d\Omega + k^2 \int_\Omega w u d\Omega = 0 \\ \int_\Gamma \left(w u_{,n} - u w_{,n}\right) d\Gamma + \int_\Omega \left(\nabla^2 + k^2\right) w u d\Omega = 0 \end{cases}, \tag{6}$$

where w is the weighting function and $_{,n} = \partial/\partial n$. Based on these forms, three meshless methods can be developed:

A. MLPG5

This method is based on the first form of (6) with the Heaviside step function as weighting and leads to:

$$\int_\Gamma u_{,n} d\Gamma + k^2 \int_\Omega u d\Omega = 0. \tag{7}$$

Although this choice of weighting function is the simplest one, an extensive study in [23] showed its better performance in comparison with more complicated weights such as MLS shape functions, in the sense of convergence. In this work, hereafter, by MLPG we mean MLPG5.

B. LBIE

This method is based on the second form of (6) with the Green's function of the PDE as weighting, leading to:

$$\begin{cases} \int_\Gamma \left(w u_{,n} - u w_{,n}\right) d\Gamma + u\left(\mathbf{x}\right) = 0 \\ w(x, y) = \dfrac{j}{4} H_0^{(2)}\left(k\sqrt{x^2 + y^2}\right) \end{cases}. \tag{8}$$

The LBIE method makes it possible to impose the essential boundary conditions by the weak statement of the problem, which is an important capability. Thus, all of the Dirichlet, Neumann and Robin boundary conditions could be directly imposed by this method.

C. MLBE

This one is our developed method and similar to the LBIE is based on the second form of (6) but with a *proper homogeneous solution* of the differential equation as weighting and leads to:

$$\begin{cases} \int_{\Gamma} \left(w u_{,n} - u w_{,n} \right) d\Gamma = 0 \\ w(x, y) = \sin\left[k(x+y)/\sqrt{2} + \pi/4 \right] \end{cases}. \tag{9}$$

In this method, w should be selected such that imposition of boundary conditions becomes possible. Clearly, MLBE preserves the valuable properties of the LBIE while removing singular integrands. Further details of the MLBE method are reported in [20].

It is worth mentioning that all of these three meshless methods have a similar benefit over the others in the sense that, they reduce the computational complexity by transforming domain integrals to boundary integrals. For the problem at hand, this happens completely for the LBIE and MLBE cases, although in the case of MLPG one domain integral remains. On the other hand, LBIE requires singular integration arising from the presence of the Green's function which is in contrast to MLPG and MLBE. Thus, we can expect the MLBE to be the fastest.

IV. MESHLESS DISCRETIZATION AND SOLUTION

Only MLPG method is considered in this section. Generalization to other methods is straightforward. Meshless discretization can be considered as a four stage process.

First, the domain and boundary of the problem is represented by a sufficient number of nodes, e.g., N nodes. A sample 2D domain with its nodal description is depicted in Fig. 1.

Second, the global weak statement of the problem is applied to *local sub-domains*. In the case of (2), this leads to:

$$\int_{L_{si}} u_{,n} d\Gamma + k^2 \int_{\Omega_{si}} u d\Omega = -\int_{\Gamma_{si}} \bar{q} d\Gamma. \tag{10}$$

Third, the unknown field variable, i.e. u, is expanded over a set of meshless shape functions with unknown coefficients. Let $\{\varphi_i\}_{i=1}^{N}$ be the aforementioned set. Therefore:

$$u^h(\mathbf{x}) = \mathbf{\Phi}^T(\mathbf{x}) \cdot \hat{\mathbf{u}} = \sum_{i=1}^{N} \varphi_i(\mathbf{x}) \widehat{u_i}, \tag{11}$$

in which $\hat{\mathbf{u}} = \left[\widehat{u_1} \ \ldots \ \widehat{u_N} \right]^T$ and u^h is the approximated/interpolated value of u.

Fourth, in the local weak statement of the problem, i.e. (10), u is replaced by its equivalent expansion, i.e. (11). This completes the meshless discretization. The immediate result of the last step is formation of the following system of equations:

$$\mathbf{K}\hat{\mathbf{u}} = \mathbf{f}, \tag{12}$$

where $\hat{\mathbf{u}}$ is unknown, and entries of $\mathbf{K}$ and $\mathbf{f}$ are given by:

$$\begin{cases} K_{ij} = \int_{L_{si}} \varphi_{j,n} d\Gamma + k^2 \int_{\Omega_{si}} \varphi_j d\Omega \\ f_i = -\int_{\Gamma_{si}} \bar{q} d\Gamma \end{cases}. \tag{13}$$

Once $\hat{\mathbf{u}}$ is computed, the unknown function u and thus, E_z can be approximated/interpolated at any point in the problem domain and on its boundary.

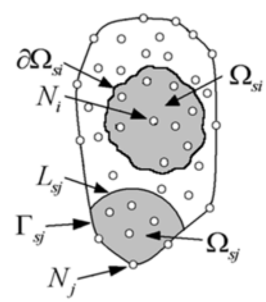


Fig. 1. Nodal geometry description of a sample 2D problem and definitions: N_i: ith node at $\mathbf{x}_i$, Ω_{si}: ith local domain, $\partial\Omega_{si}$: ith local boundary, L_{sj}: non-intersecting part of $\partial\Omega_{sj}$, Γ_{sj}: intersecting part of $\partial\Omega_{sj}$, $\partial\Omega_{sj} = L_{sj} \cup \Gamma_{sj}$.

V. NUMERICAL RESULTS

In this section, the selected meshless methods are applied to the selected microstrip antennas. The single-element antennas are depicted in Fig. 2. The two-element array antenna is constructed from the two coax-fed antennas that are 4 cm apart. In all cases $\varepsilon_r = 2.62$, $h = 1.6$ mm, *loss tangent* = 10^{-3} and $\sigma = 5.8\times10^7$ (S/m).

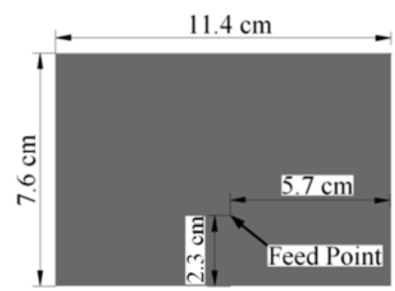
Thin-plate spline (TPS) functions of 9th order are used for construction of meshless shape functions [1]. Local sub-domains are rectangles with side length of d. For all uniform node distributions, $d = 1.0d_r$, where d_r is the radial nodal distance. For non-uniform and random cases, $d = 1.2d_r$, where d_r is computed based on uniform node arrangement.

For error estimate we used the relative error defined as:

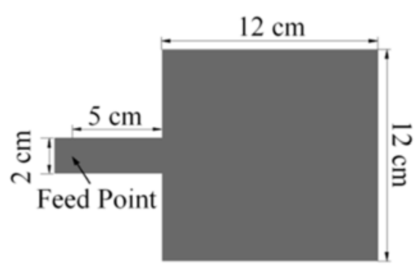
$$r_e\left(u_m, u_{m+1}\right) = \left\|u_m - u_{m+1}\right\| / \left\|u_m\right\|, \tag{14}$$

where u_m is the field variable of the mth pass and

$$\|u\| = \left(\int_{\Omega} |u|^2 \, d\Omega\right)^{1/2}.$$



(a)



(b)

Fig. 2. Geometrical description of (a) coax-fed antenna, (b) line-fed antenna.

For the coax-fed antenna, the problem has an analytical modal solution which is regarded as the exact solution [24]. In this case, convergence curves, CPU time and condition numbers of the coefficient matrices for different methods versus number of unknowns are depicted in Fig. 3, based on uniform node arrangements.

This problem is also solved by random node arrangement via the MLBE method. The nodal description and the corresponding electric field distribution on the patch, reconstructed at 5400 nodes, are represented in Figs. 4(a) and 4(b), respectively. The computed S-parameters based on uniform and random node arrangements are reported in Fig. 4(c).

The input impedance and the corresponding S-parameters are shown in Fig. 5 and Fig. 6 for coax-fed and line-fed antennas, respectively, all based on uniform node arrangements. The line-fed case is also simulated by non-uniform node arrangement via the MLBE method. The nodal description and the corresponding electric field distribution, reconstructed at 7440 nodes, are shown in Figs. 7(a) and 7(b), respectively. In addition, the computed S-parameters based on uniform and non-uniform node arrangements are depicted in Fig. 7(c). It is worth mentioning that by this irregularity, the number of nodes is reduced from 374 to 214, without a considerable effect on S_{11}.

Finally, the S-parameters of the two-element array antenna are reported in Fig. 8. All simulations are performed on an Intel(R) Core(TM)2 CPU with 4 GB RAM.

VI. CONCLUSION

In this paper, MLPG, meshless LBIE and MLBE methods are compared by applying to thin microstrip antennas. The results are validated by the exact solution for the coax-fed antenna and the MoM for line-fed and a two-element array. It is observed that all the meshless methods have essentially the same convergence rate, with LBIE possessing the least condition number and MLBE the least CPU time.

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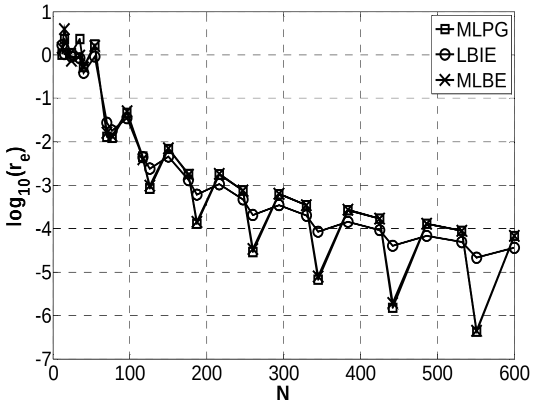
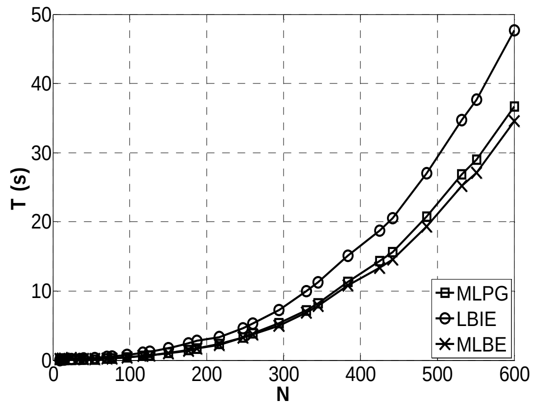


(a)



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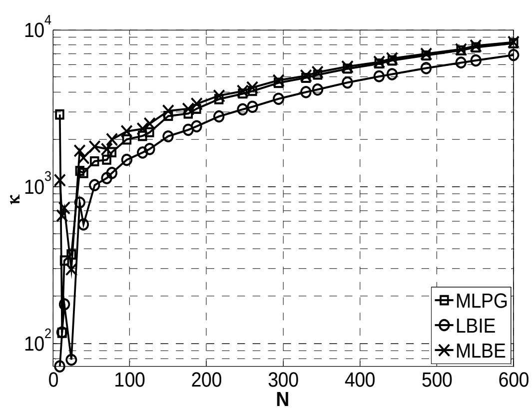
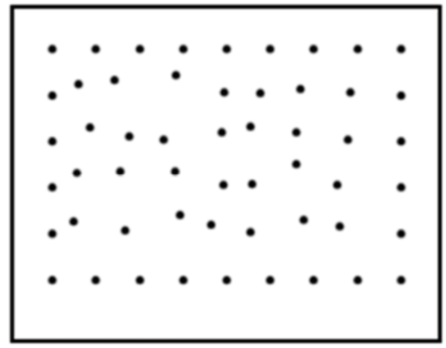
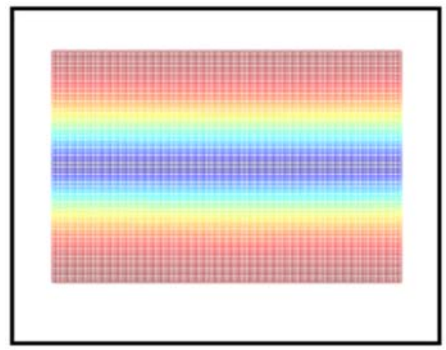


(c)

Fig. 3. Coax-fed antenna: (a) convergence curves, (b) computational complexity, (c) condition numbers.



(a)



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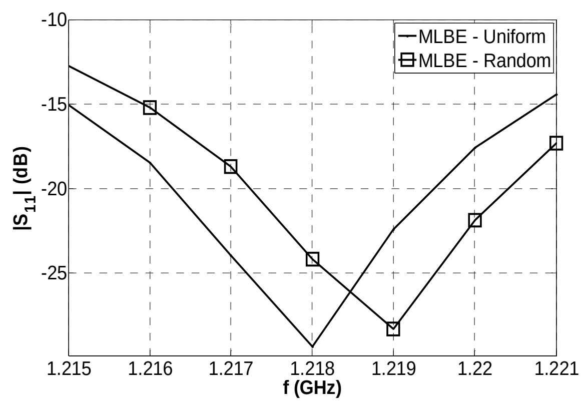


(c)

Fig. 4. Analysis of the coax-fed antenna based on random node distribution by the MLBE method: (a) node arrangement, (b) electric filed on the patch, (c) $|S_{11}|$.

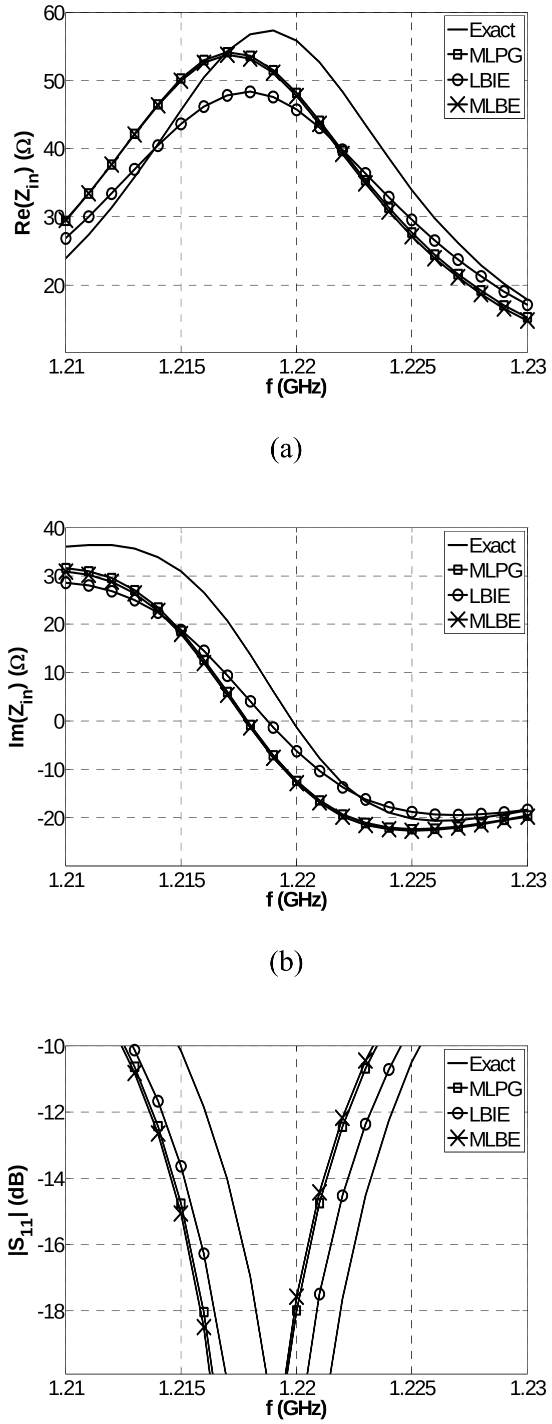


Fig. 5. Coax-fed antenna: (a) real part of input impedance, (b) imaginary part of input impedance (c) $|S_{11}|$.

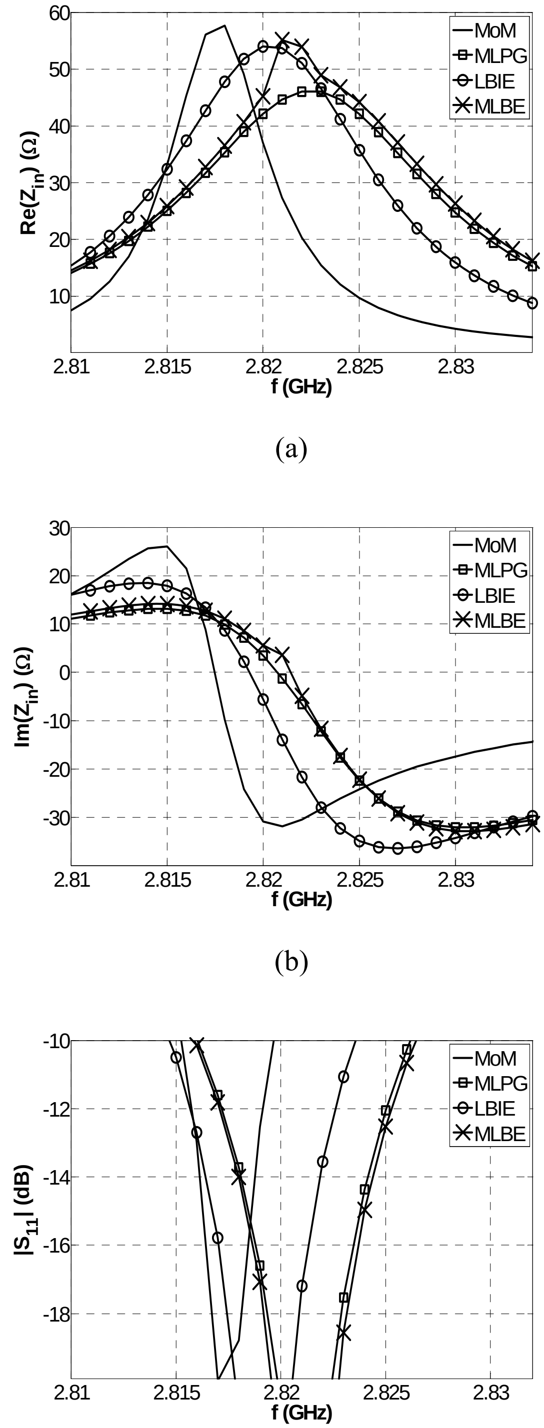
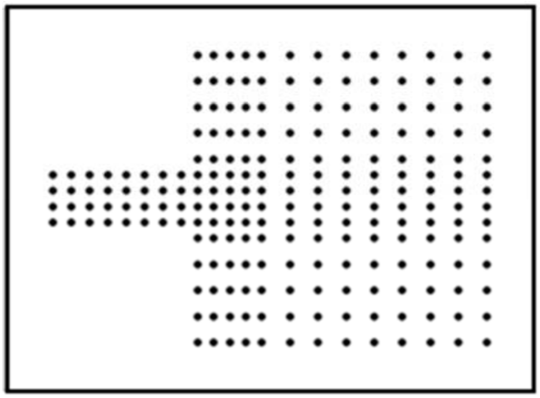
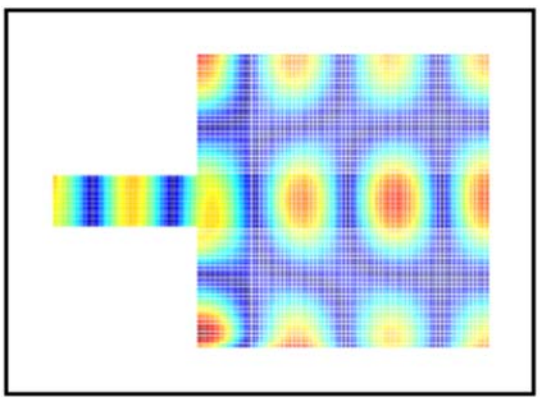


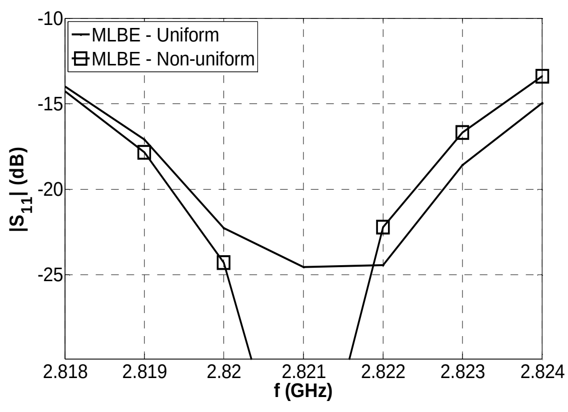
Fig. 6. Line-fed antenna: (a) real part of input impedance, (b) imaginary part of input impedance (c) $|S_{11}|$.



(a)

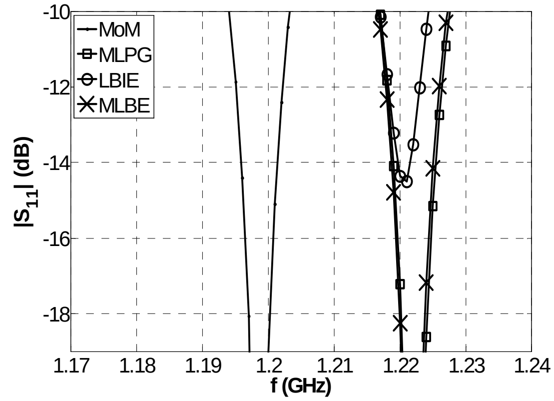


(b)

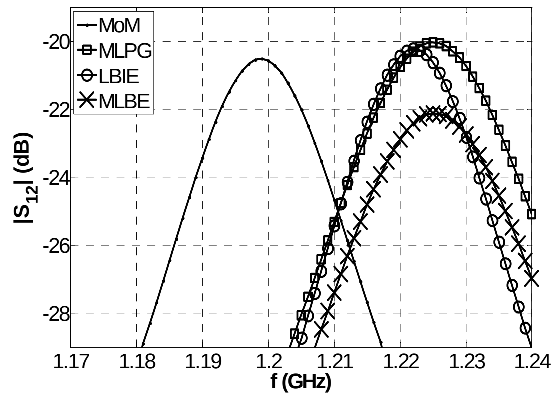


(c)

Fig. 7. Analysis of the line-fed antenna based on non-uniform node distribution by the MLBE method: (a) node arrangement, (b) electric filed on the patch, (c) $|S_{11}|$.



(a)



(b)

Fig. 6. Two-element array antenna: (a) $|S_{11}|$, (b) $|S_{12}|$.

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