Robust Adaptive Array Beamforming Based on Modified Norm Constraint Algorithm

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Abstract **─** In order to further improve the performance of the Norm Constrained Capon Beamforming (NCCB) algorithm, a Modified Norm Constraint Capon algorithm (MNCCB) is proposed and investigated in detail. The proposed MNCCB algorithm is realized by exerting an orthogonal projection on the array weight vector and restricting the norm constraint to enhance the array weight vector constraint when Array Steering Vector (ASV) mismatch is large. The simulation results show that the proposed MNCCB can provide stronger robustness against ASV mismatches and can achieve higher output Signal to Interference plus Noise Ratio (SINR), compared with existing adaptive beamforming algorithms.

Index Terms **─** Norm constraint, orthogonal projection, robust adaptive beamforming.

I. INTRODUCTION

Array signal processing has been widely used in radar, mobile communications, sonar and microphone array speech processing. Adaptive beamforming is one of the hottest topics in array signal processing. As for adaptive beamformer, it can adaptively adjust weight vector to achieve maximum gain at the direction of desired signal and suppress interferences by forming nulls at the directions of interferences [1-3]. To meet these applications, many beamformers have been proposed, such as Standard Capon Beamformer (SCB), Diagonal Loading SCB (DL-SCB) and NCCB [4-14]. However, the SCB is very sensitive to the ASV mismatch and may suppress the signal of interest, which might reduce the array output SINR [4-6]. As for the DL-SCB algorithm, although it can improve the robustness of the SCB, it is difficult to choose the optimal diagonal loading factor and it may increase the power noise [7]. Another effective beamformer is the Robust Capon Beamforming algorithm (RCB) [9], which can enhance the robustness of the DL-SCB. It is proved that RCB is equivalent and belongs to the class of diagonal loading. The RCB may lose its interference suppression capability when the mismatch is large. Recently, a popular beamformer named as NCCB is studied to achieve higher performance compared with the basic SCB algorithm for small ASV mismatch [11,13], while its performance is not good for large ASV mismatch.

In this paper, an MNCCB is proposed to further improve the performance of NCCB for large ASV mismatch by using the orthogonal projection and norm constraint techniques. The proposed MNCCB algorithm can ensure approximate orthogonality between the weight vector and noise subspace, which significantly improves the robustness performance with respect to ASV mismatch. The detailed theoretical analysis and analytical expression of the proposed MNCCB algorithm is provided in detail. The simulation results demonstrate that the proposed MNCCB algorithm has excellent performance against the ASV mismatches.

II. SIGNAL MODEL

We consider an N elements omnidirectional array, spaced with element distance of d, and M far field narrow band signals are incident on

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this antenna array. A signal $s_k(t)$ is incident from angle θ_k, and the received data $\mathbf{X}$ can be expressed as follows:

$$\mathbf{X}(t)=\mathbf{A}\mathbf{S}(t)+\mathbf{N}(t), \quad (1)$$

where $\mathbf{X}(t)=[x_1(t),x_2(t),\cdots,x_N(t)]^T$ is $N\times 1$ snap data vector. $\mathbf{S}(t)=[s_0(t),s_1(t),\cdots,s_{M-1}(t)]^T$ is a vector, which contains the complex signal envelops from M narrow-band signal sources. $\mathbf{N}(t)=[n_1(t),n_2(t),\cdots,n_N(t)]^T$ is a vector of zero-mean white Gaussian noise with variance of σ_n^2 and $\mathbf{A}$ is an array manifold matrix that can be written as $\mathbf{A}=[\mathbf{a}(\theta_0),\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_{M-1})]$. Here, $\mathbf{a}(\theta_1)$=$[1,e^{j\beta_k},\cdots,e^{j(N\text{-}1)\beta_k}]^T$, $k=1,2,\cdots,M$, represents an ASV in the direction of θ_k, and β_k is the phase difference that can be expressed as:

$$\beta_k=\frac{2\pi}{\lambda}d\sin(\theta_k). \quad (2)$$

Assume that the signal and noise are statistically independent, and the covariance of the received data can be written as:

$$\mathbf{R}=E\left\{\mathbf{X}(t)\mathbf{X}^H(t)\right\}=\mathbf{A}\mathbf{R}_s\mathbf{A}^H+\sigma_n^2\mathbf{I}, \quad (3)$$

where0 $E\{\ \}$ is an expectation operator; $\mathbf{R}_s=E\left\{\mathbf{S}(t)\mathbf{S}^H(t)\right\}$ represents the autocorrelation matrix of the complex signal envelops. $\mathbf{I}$ is the unit matrix and $(\cdot)^H$ denotes the Hermitian transpose.

On the basis of the previous researches on the SCB, we assume that the ASV of the desired signal $\mathbf{a}(\theta_0)$ is known precisely. Then, the Capon beamformer can be expressed as:

$$\begin{cases}\min\limits_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{R}}_{i+n}\mathbf{w} \\ \text{s } t\ \ \mathbf{w}^H\mathbf{a}(\theta_0)=1\end{cases}, \quad (4)$$

where $\mathbf{w}$ is the beamformer weight vector. $\hat{\mathbf{R}}_{i+n}$ is the inference-plus-noise covariance matrix, which is commonly replaced by the sampled covariance matrix in the practical applications and it can be written as:

$$\hat{\mathbf{R}}=\frac{1}{K}\sum_{i=1}^{K}\mathbf{X}(i)\mathbf{X}^H(i), \quad (5)$$

where K is the number of snapshots collected by the beamformer. The optimal solution of (4) is given by:

$$\mathbf{W}_{opt}=\frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)}. \quad (6)$$

The array output power is:

$$P_{out}=\mathbf{W}^H\mathbf{R}_{i+n}^{-1}\mathbf{W}=\frac{1}{\mathbf{a}^H(\theta_0)\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)}. \quad (7)$$

The array output SINR is expressed as:

$$\begin{aligned}SINR_{opt}&=\frac{\sigma_s^2\left|\mathbf{W}_{opt}^H\mathbf{a}(\theta_0)\right|^2}{\mathbf{W}_{opt}^H\mathbf{R}_{i+n}\mathbf{W}_{opt}}\\&=\frac{\sigma_s^2\left|\mathbf{a}^H(\theta_0)\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)\right|^2}{\mathbf{a}^H(\theta_0)\mathbf{R}_{i+n}^{-1}\mathbf{R}_{i+n}\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)},\\&=\sigma_s^2\mathbf{a}^H(\theta_0)\hat{\mathbf{R}}_{i+n}^{-1}\mathbf{a}(\theta_0)\end{aligned} \quad (8)$$

where $\sigma_s^2=E\left[\left|s_0(t)\right|^2\right]$ is the desired signal power. The SCB algorithm can obtain high output SINR when the ASV of the desired signal is known accurately. However, in practical applications, there often exist differences between the assumed signal arrival angle and true arrival angle. Therefore, the ASV may be imprecise, resulting in steering vector mismatches [12]. It is found that the SCB cannot provide good robustness against ASV errors between the presumed and actual ASVs. We assume that $\bar{\mathbf{a}}$ denotes the actual ASV of the desired signal. We can get:

$$\bar{\mathbf{a}}=\mathbf{a}+\Delta, \quad (9)$$

where Δ is an unknown complex vector which describes the effect of steering vector distortion. In this case, the mismatch of the ASV may result in desired signal suppression and get poor output SINR. Thus, robust adaptive beamforming is necessary in practical applications.

In order to improve the robustness of the SCB, an effective NCCB algorithm is widely investigated, which is realized by using a norm constraint on the weight vector. Thus, the NCCB is formulated as follows:

$$\begin{cases}\min\limits_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{R}}\mathbf{w} \\ \text{s } t\ \ \mathbf{w}^H\bar{\mathbf{a}}(\theta_0)=1 \\ \|\mathbf{w}\|^2\le\xi\end{cases}, \quad (10)$$

where $\bar{\mathbf{a}}(\theta_0)$ is the presumed signal steering vector, $\|\cdot\|$ denotes the l_2 norm and ξ is the norm constraint parameter. From the analysis of [13], we can see that the NCCB can enhance the robustness of the SCB. However, the analysis and simulation results show that its efficiency is not good enough when the ASV mismatch is large. Although, the performance of the NCCB can be controlled by ξ, the optimal method for obtaining ξ is difficult.

III. MODIFIED NORM CONSTRAINT ROBUST BEAMFORMING

In this section, we develop an MNCCB to improve the robustness performance, which is realized by adding an orthomodular constraint on the NCCB. Here, we first discuss the formulation of the MNCCB and then give the detailed theoretical analysis.

Define the projection matrix as $\hat{\mathbf{R}}^{-m}$, where m is positive integer. By projecting the matrix $\hat{\mathbf{R}}^{-m}$ to the weight vector, we can get $\hat{\mathbf{w}}=\hat{\mathbf{R}}^{-m}\mathbf{w}$. Thus, the MNCCB algorithm can be described as:

$$\begin{cases} \min\limits_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w} \\ \text{s } t \quad \mathbf{w}^H \bar{\mathbf{a}}(\theta_0) = 1 \\ \quad \| \hat{\mathbf{R}}^{-m} \mathbf{w} \|^2 \le \zeta \end{cases}, \tag{11}$$

where ζ is a constraint parameter with a small value. It found that the optimization of (11) is a convex problem, which can be solved by using inter-point method [15]. In this paper, Lagrange multiplier method is employed to find the solution of (11).

Firstly, let S be the set that is defined by the constraints in (11):

$$\text{S}=\{\mathbf{w} \mid \mathbf{w}^H \bar{\mathbf{a}}(\theta_0) = 1, \| \hat{\mathbf{R}}^{-m} \mathbf{w} \|^2 \le \zeta \}. \tag{12}$$

Define:

$$f_1(\mathbf{w}, \lambda, \mu) = \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w} + \lambda(\| \hat{\mathbf{R}}^{-m} \mathbf{w} \|^2 - \zeta) + \mu(-\mathbf{w}^H \bar{\mathbf{a}} - \bar{\mathbf{a}}\mathbf{w} + 2), \tag{13}$$

where λ is the real-valued Lagrange multiplier and $\lambda \ge 0$ satisfying $\hat{\mathbf{R}}+\lambda\mathbf{I}>0$. By minimizing $f_1(\mathbf{w}, \lambda, \mu)$ with respect to $\mathbf{w}$, we have:

$$f_1(\mathbf{w}, \lambda, \mu) \le \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w} \quad \forall \mathbf{w} \in S. \tag{14}$$

From the discussion of the SCB algorithm shown in (6), $\mathbf{a}$ is replaced by $\bar{\mathbf{a}}$. Then, we get:

$$\hat{\mathbf{w}} = \frac{\hat{\mathbf{R}}^{-1} \bar{\mathbf{a}}}{\bar{\mathbf{a}}^{-1} \hat{\mathbf{R}}^{-1} \bar{\mathbf{a}}}. \tag{15}$$

Consider the condition:

$$\zeta < \frac{\bar{\mathbf{a}}^H \hat{\mathbf{R}}^{-2} \bar{\mathbf{a}}}{[\bar{\mathbf{a}}^H \hat{\mathbf{R}}^{-1} \bar{\mathbf{a}}]^2}. \tag{16}$$

We can rewrite $f_1(\mathrm{w}, \lambda, \mu)$ as follows:

$$\begin{aligned} f_1(\mathbf{w}, \lambda, \mu) = {} & [\mathbf{w} - \mu(\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}]^H \\ & (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m}) \\ & [\mathbf{w} - \mu(\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}] \\ & - \mu^2 \bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}} \\ & - \lambda\zeta + 2\mu. \end{aligned} \tag{17}$$

Therefore, for the fixed parameters λ and μ, the unconstrained minimizer of $f_1(\mathbf{w}, \lambda, \mu)$ is given by:

$$\hat{\mathbf{w}}_{\lambda,\mu} = \mu(\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}. \tag{18}$$

Substituting (18) to (17), we can rewrite (17) as:

$$\begin{aligned} f_2(\lambda, \mu) = {} & -\mu^2 \bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}} \\ & - \lambda\zeta + 2\mu \le \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w}. \end{aligned} \tag{19}$$

By considering the maximization of $f_2(\lambda, \mu)$ with respect to μ, we have:

$$\hat{\mu} = \frac{1}{\bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}}. \tag{20}$$

and we can get:

$$\begin{aligned} f_3(\lambda) = f_2(\lambda, \hat{\mu}) = {} & -\lambda\zeta \\ & + \frac{1}{\bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}}. \end{aligned} \tag{21}$$

Thus, the maximization of $f_3(\lambda)$ with respect to λ can be expressed as:

$$\frac{\bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-2} \bar{\mathbf{a}}}{[\bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}]^2} = \zeta. \tag{22}$$

Hence, the optimal Lagrange multiplier $\hat{\lambda}$ can be efficiently obtained by using a Newton's method.

By introducing $\hat{\mu}$ into $\hat{\mathbf{w}}_{\lambda,\mu}$, we get:

$$\hat{\mathbf{w}} = \frac{(\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}}{\bar{\mathbf{a}}^H (\hat{\mathbf{R}} + \lambda(\hat{\mathbf{R}}^{-m})^H \hat{\mathbf{R}}^{-m})^{-1} \bar{\mathbf{a}}}, \tag{23}$$

which satisfies:

$$\hat{\mathbf{w}}^H \bar{\mathbf{a}} = 1, \tag{24}$$

and

$$\| \hat{\mathbf{R}}^{-m} \hat{\mathbf{w}} \|^2 \le \zeta. \tag{25}$$

In addition, it is observed that the proposed MNCCB is also an improved DL-SCB algorithm and the λ is the diagonal loading factor. The optimal diagonal loading can be precisely calculated by solving the constrained quadratic optimization problem.

Let us pay attention to the constraint $\|\hat{\mathbf{R}}^{-m}\hat{\mathbf{w}}\|^2 \le \zeta$. According to Schmidt's orthogonal subspace theory, $\hat{\mathbf{R}}$ can be decomposed as [16]:

$$\begin{aligned}\hat{\mathbf{R}} &= \begin{bmatrix}\mathbf{U}_s & \mathbf{U}_n\end{bmatrix}\begin{bmatrix}\mathbf{\Lambda}_s & \mathbf{0}\\ \mathbf{0} & \sigma_n^2\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{U}_s^H\\ \mathbf{U}_n^H\end{bmatrix}\\ &= \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \sigma_n^2\mathbf{U}_n\mathbf{U}_n^H,\end{aligned} \quad (26)$$

where $\mathbf{U}_s$ represents the desired signal-plus-interference subspace, which is formed by P interferences and one desired signal. $\mathbf{U}_n$ is the noise subspace. $\Lambda_s = diag\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ is the big eigenvalues corresponding to signal and interferences. On the basis of the above discussion, we have:

$$\hat{\mathbf{R}}^{-m} = \frac{1}{\sigma_n^{2m}}\left\{\mathbf{U}_s diag\left\{(\frac{\sigma_n^2}{\lambda_i})^m\right\}\mathbf{U}_s^H + \mathbf{U}_n\mathbf{U}_n^H\right\}. \quad (27)$$

We can clearly see that $\frac{\sigma_n^2}{\lambda_i} < 1$. Thereby, $(\frac{\sigma_n^2}{\lambda_i})^m$ converges to zero when m is large. Taking above discussions into consideration, we can get $\lim_{m\to\infty} diag\left\{(\frac{\sigma_n^2}{\lambda_i})^m\right\} = 0$, which means:

$$\lim_{m\to\infty} \sigma_n^{2m}\hat{\mathbf{R}}^{-m} = \mathbf{U}_n\mathbf{U}_n^H. \quad (28)$$

In practical applications, m is usually a fixed and finite integer, and hence the equation (28) can be satisfied, which means that the noise subspace $\mathbf{U}_n\mathbf{U}_n^H$ can be obtained without decomposition of the covariance matrix $\hat{\mathbf{R}}$. Furthermore, it is indicted that the number of incident signals is not necessary for the estimation.

From equations (26) and (28), we can see that $\hat{\mathbf{R}}^{-m}$ is an orthogonal projection matrix. The constraint $\|\hat{\mathbf{R}}^{-m}\hat{\mathbf{w}}\|^2 \le \zeta$ not only imposes norm constraint on the weight vector, but also ensures approximate orthogonality between the weight vector and noise subspace. Thus, the MNCCB algorithm can effectively improve the robustness of the SCB against large ASV mismatch.

The proposed MNCCB algorithm can be summarized as follows:

The MNCCB algorithm

Step 1) Compute the covariance matrix $\hat{\mathbf{R}}$;

Step 2) Compute the power of covariance matrix $\hat{\mathbf{R}}^{-m}$;

Step 3) Project $\hat{\mathbf{R}}^{-m}$ to the weight vector;

Step 4) Solve (22) to obtain $\hat{\lambda}$;

Step 5) Substitute $\hat{\lambda}$ to (23) to get $\hat{\mathbf{w}}$.

IV. NUMERICAL EXAMPLES

In this section, we will discuss the performance of the proposed MNCCB algorithm. A Uniform Line Array (ULA) with 10 omnidirectional antennas spaced half a wave length uniformly. The Direction-of-Arrival (DOA) of the desired signal is 0°. The DOAs of the two independent interferences are 30° and 50°, respectively, while the Interference to Noise Ratio (INR) is INR $= 30$ dB. The number of snapshots is $K = 100$. The other key parameters for the proposed MNCCB algorithm are $m = 2$ and $\zeta = 0.03$. The simulation results of the proposed MNCCB are obtained in comparison with SCB, RCB [9], ESB [8], NCCB [11], DL-SCB [7] and SQP [10]. $\varepsilon = 0.3N$ is used for the RCB, and the diagonal loading factor in [7] is twice as great as the noise power, and $\xi = 0.11$ is used for NCCB. In all experiments, 100 Monte Carlo runs are used to obtain each simulation point.

Example 1: *Exactly known signal steering vector*

In this example, we assume the presumed DOA of desired signal is also set as 0°. The normalized beampattern plots of the mentioned beamformers with the SNR $= 0$ dB are shown in Fig. 1 and the output SINR of these beamformers are shown in Fig. 2. It can be seen from the Fig. 1, that the SCB, NCCB and MNCCB perform well with the exactly known ASV. The peak response of these algorithms are well agreed with the actual direction of desired signal, while the nulls are located at the directions of interferences, which can help to suppress unwanted interferences. In addition, it is observed that the MNCCB algorithm has the lowest sidelobe level. From Fig. 2, we can see that the MNCCB algorithm has the highest output SINR because it exhibits deeper null than other methods. Thus, we can conclude that our proposed MNCCB algorithm outperforms previously reported algorithms.

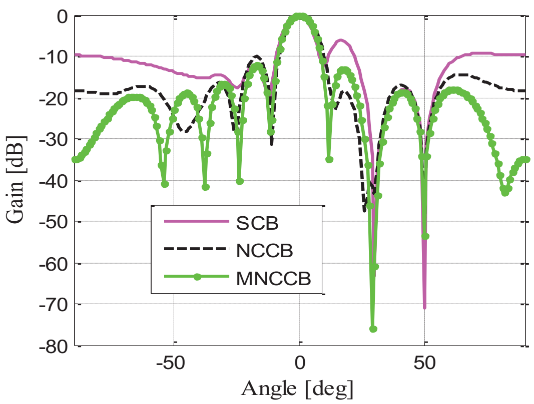

Fig. 1. Normalized beampatterns at zero pointing error.

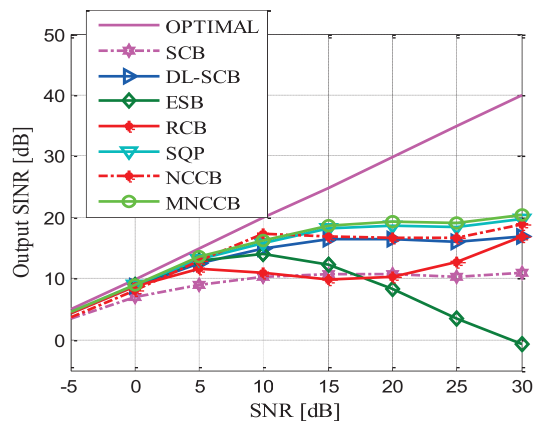

Fig. 2. Output SINR versus SNR at zero pointing error.

Example 2: *Signal look direction with 5° mismatch*

In the second experiment, the presumed DOA is 5° of the desired signal. The normalized beampattern plot of the proposed MNCCB beamformer with the input SNR=0 dB is shown in Fig. 3 in comparison with SCB and NCCB algorithms. It can be seen from Fig. 3, that the SCB with 5° ASV mismatch completely fails and its main lobe departs from the actual signal direction, which means that it cannot distinguish the desired signal and interferences, and hence will suppress the desired signal. Additionally, although the performance of the NCCB is better than SCB, its main lobe also departs from the actual signal direction. However, our proposed MNCCB algorithm shows excellent performance in interference suppression, which gives deep nulls at the directions of inferences. Thus, the response peaks for the MNCCB estimation algorithm is located at the actual direction of desired signal without target signal cancellation.

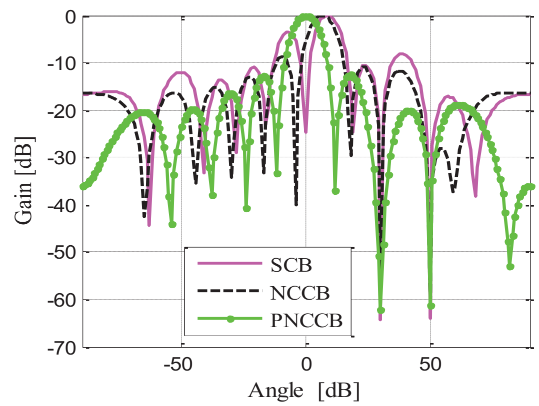

Fig. 3. Normalized beampatterns at 5° pointing error.

Figure 4 demonstrates the output SINR performance of the above mentioned beamformers versus the input SNR. We can see that the proposed MNCCB algorithm has better performance compared to other beamformers when the ASV mismatch is 5°. The SCB and DL-SCB algorithms suffer from severe degradation when SNR increases from 0 dB to 30 dB, while the performance of the NCCB significantly degraded at low SNR. In other words, the proposed MNCCB algorithm is superior to other beamformers.

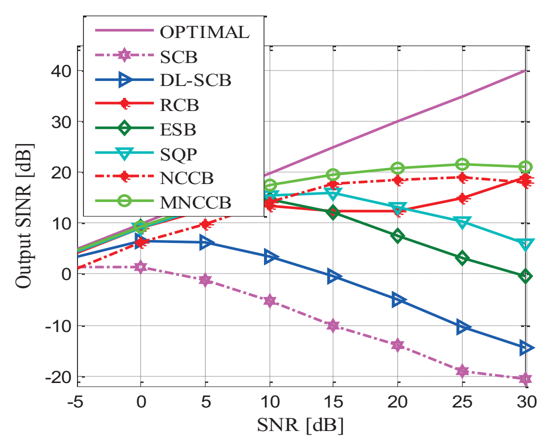

Fig. 4. Output SINR versus SNR at 5° pointing error.

Figure 5 exhibits the output SINR performance with respect to the number of training snapshots K at SNR=10 dB. It can be seen that the proposed MNCCB algorithm still has highest robustness against the ASV mismatch and better performance than other methods.

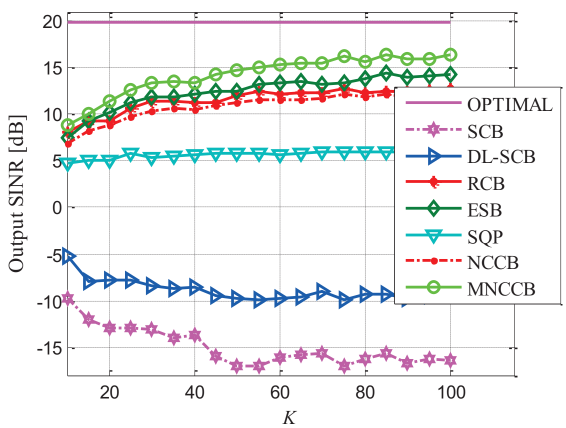

Fig. 5. Output SINR versus K at 5° pointing error.

Example 3: *Effect of the signal direction mismatch*

In this experiment, the steering direction error is preselected as [-8°-8°]. The performance of output SINR versus signal direction mismatch is given in Fig. 6. We can clearly see that the performance of the SCB, DL-SCB and RCB are severely deteriorated with an increase of the signal direction mismatch, while the output SINR of MNCCB is stable. It is worth noting that when the angle error is 8°, the output SINR of the NCCB and SQP are -7.5 dB and 2.2 dB, respectively, while the output SINR of the MNCCB is 7.9 dB, which exceeds the NCCB 15.4 dB.

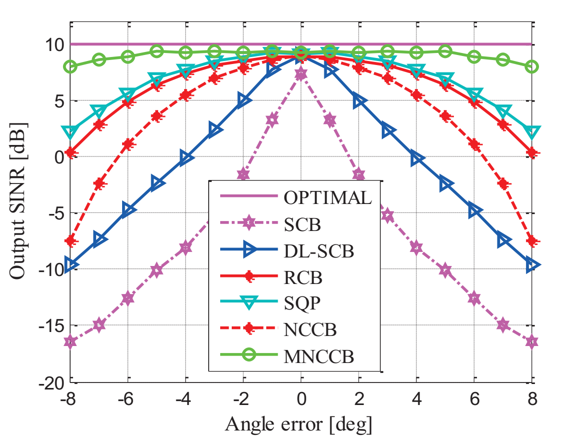

Fig. 6. Output SINR against pointing error.

Example 4: *Effects of the parameters m and ξ*

In this example, we will discuss the effects of the parameters m and ξ on the performance of the MNCCB algorithm with the presumed DOA is 5°. The output SINRs with different m and ξ are shown in Fig. 7. It can be seen that the MNCCB is not insensitive to parameter m and ξ. The output SINR curves are almost the same for different m and ξ .The proposed MNCCB beamformer can provide a good performance over a wide range of ξ making the proposed MNCCB operable and practical compared with previously proposed methods.

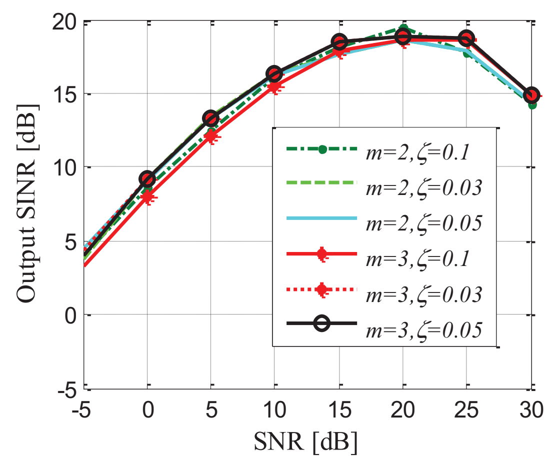

Fig. 7. Output SINR versus SNR.

V. CONCLUSION

In order to improve the degradation of adaptive beamformer with large ASV mismatch, a robust adaptive beamformer denoted as MNCCB was proposed and its performance was verified in detail. The proposed MNCCB was realized via the modification of the norm constraint, which was to add an orthogonal projection in the early reported NCCB to improve its robustness. As a result, the proposed MNCCB could give better performance than the NCCB and the diagonal loading algorithms. Theoretical analysis and numerical examples were presented to improve the performance of previous beamformers. Simulation results demonstrated that the proposed MNCCB can not only provide better interference suppression, but also achieve higher output SINR with steering vector mismatch in comparison with existing popular robust beamforming algorithms.

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