

Biomedical Magnetic Induction Tomography: An Inhomogeneous Green's Function Approach

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Abstract — Magnetic induction tomography aims to reconstruct the passive electric properties of an object by measuring its scattered magnetic field. Current state-of-the-art numerical techniques are based on differential formulations such as the finite element method. A formulation based on volume integral equations has not yet been applied to its biomedical field and could improve the reconstruction speed by reducing the number of unknowns. This paper investigates salient characteristics of the approach and offers a solution based on inhomogeneous Green's functions.

Index Terms — Biomedical, magnetic induction, volume integral.

I. INTRODUCTION

Biomedical magnetic induction tomography (MIT) is a novel imaging technique with applications to stroke detection, inductive measurement of wound conductivity and lung imaging [1]. Its main benefits are a very low operating and construction cost, non-ionizing interactions with matter and contactless operation. A low frequency field (in the kHz to the MHz range) is used to induce eddy currents inside an object of interest. The eddy currents depend on the passive electric properties (PEP) of the object and these in turn induce a secondary magnetic field, which can be measured at sensors outside the volume of interest. Computation of the scattered magnetic field is done through the forward model. Current state-of-the-art forward models are based on approximate, differential or surface integral equations. Here, the main challenge is to obtain accurate results while maintaining a low computation cost. The forward model may be formulated as a large linear system, which needs to be solved iteratively. While a finer discretization may yield more accurate results, it also leads to a more computationally burdensome model. For imaging purposes, the PEP of the domain of interest must be reconstructed from the known incident and scattered fields, which is referred to as the inverse problem. The inverse problem is solved iteratively by minimizing the differences between the measured scattered magnetic

field and the forward model evaluation for estimated values of the PEP to which a penalty term may be added. For each iteration of the inverse solver, the forward problem must be solved, so an efficient forward model is critical. Biological tissues in the MIT frequency range may be considered as penetrable objects and volume integral equations (VIEs) can adequately model the problem. VIEs offer several advantages such as the need to discretize a much smaller domain than differential methods, a better accuracy than approximate methods and a wider range of applicability than surface integral methods. Although VIEs are commonly used in a higher frequency range (e.g., microwave tomography) and for non-destructive evaluation (NDE), specific issues plague their application to biomedical MIT. Since both permittivity and conductivity are being reconstructed (as opposed to only conductivity for NDE) and because at these frequencies biological tissues have very high dielectric properties [2], conditioning issues arise. For large domains requiring the implementation of iterative solvers, this yields a slow convergence of both the forward and the inverse solvers [1]. This paper presents and validates a model that tackles this conditioning issue arising from the high contrasts in biomedical MIT by using Green's function theory.

II. PROBLEM FORMULATION

A. Volume integral equation based model

The standard VIE is obtained by using Green's function and the volume equivalence principle to solve the wave equation [3]. Discretization is performed by the method of moments with Dirac basis functions. The scattered magnetic field at the sensors is obtained through:

$$\mathbf{h}^{scat}(\mathbf{x}) = -\tilde{\sigma}_b \mathbf{X} \mathbf{G}^R \mathcal{L}^{-1} \mathbf{e}^0, \quad (1)$$

with $\mathcal{L}^{-1} = (\mathbf{I} - k_b^2 \mathbf{G} \mathbf{X})^{-1}$ the scattering operator, \mathbf{X} a diagonal matrix containing the electric contrast values $X(\mathbf{r}) = \tilde{\sigma}(\mathbf{r})/\tilde{\sigma}_b - 1$, with $\tilde{\sigma}_b = \sigma - j\omega\epsilon_r\epsilon_0$ the background complex conductivity, $\mathbf{G} = \Delta V \left(\mathbf{I} + \frac{\nabla\nabla}{k_b^2} \right) g$ the dyadic Green's function, $g(\mathbf{r}, \mathbf{r}') = e^{jk_b|\mathbf{r}-\mathbf{r}'|}/(4\pi|\mathbf{r}-\mathbf{r}'|)$ denotes the scalar Green's function, ΔV the

discretization volume and \mathbf{e}^0 is the incident electric field. The matrix $\mathbf{G}^R = \xi \nabla g$ is defined through the Levi-Civita tensor and represents the cross product of the gradient and the scalar Green's function.

The selected criterion is the L_2 norm of the differences between the measurements and the forward model evaluation. To deal with the ill-posedness of the problem, an additive regularization term is added to include *a priori* information about the domain to reconstruct. An $L_2 L_1$ penalization term on the first differences of the contrast is chosen in order to preserve the object edges. The function $R(\mathbf{X}) = \lambda \sqrt{\delta^2 + t(\mathbf{X})^2}$ is well adapted, where t is the sum of contrast variations along each axis, δ is a hyperparameter that defines the transition between the quadratic and linear behaviors of the penalty function and λ is the regularization parameter. The cost functional to minimize is therefore: $F(\mathbf{X}) = \|y - f(\mathbf{X})\|_2^2 + \lambda R(\mathbf{X})$. To minimize this cost functional, the L-BFGS algorithm is chosen. This quasi-Newton method offers a lower computation cost per iteration than a Newton method, while converging faster than a gradient method. The gradient of the cost functional is obtained by computing its Fréchet derivative. Straightforward computations yield $\nabla F = \mathbf{J}^+(y - f) + \lambda R'_{HS}$ where Jacobian \mathbf{J} can be expressed as:

$$\mathbf{J} = \mathbf{G}^R (\mathbf{I} - \mathbf{G}\mathbf{X})^{-T} \text{diag}\{\mathbf{e}\}. \quad (2)$$

The issue with the VIE model is that the scattering operator \mathcal{L} is ill-conditioned due to the high contrast values of biological tissues in this low frequency range. This implies a very slow convergence of the forward model. The poor conditioning of the forward model also results in a slow convergence of the inverse solver. This yields a very high computation cost, which makes the model impractical for realistic applications.

B. Inhomogeneous Green's function based model

A solution to the problem's poor conditioning and slow convergence is to perform a change of variables to a new contrast value \mathbf{X}_2 , which is defined as a perturbation in an otherwise healthy tissue. Since in biomedical imaging the goal is to identify a small perturbation in a known background, this healthy tissue can be included into the background propagation medium. Two approaches may be considered. The first one is the linear embedding via Green's operator (LEGO), which makes use of the port theory to include the mutual scattering events between the background and the perturbation domain [4], but is computationally burdensome. The second one is based on the Green's function theory [5]. If one considers an inhomogeneous propagation medium including the healthy tissue, an inhomogeneous Green's function (IG) can be computed by solving $\mathbf{G}^{in} = \mathcal{L}_1^{-1} \mathbf{G}_{12}$. Domains 1 and 2 respectively denote the inhomogeneous propagation medium and the volume of interest, which contains the contrast values

\mathbf{X}_2 that are to be determined. After discretization, the forward model is given by:

$$\mathbf{h}^{scat}(\mathbf{X}_2) = \mathbf{G}^R \mathbf{X}_1 (\mathbf{e}_1 + \mathbf{G}^{in} \mathbf{X}_2 \mathbf{e}_2). \quad (3)$$

Here, $\mathbf{e}_2 = \mathcal{L}_2^{-1} \mathbf{e}_{1|2}$ and $\mathbf{e}_1 = \mathcal{L}_1^{-1} \mathbf{e}_1$ are the total fields in each domain and $\mathbf{e}_{1|2}$ is the total field in Domain 2 due to the inhomogeneous Domain 1. Matrices involving \mathbf{X}_1 may be computed only once so that only \mathbf{e}_2 needs to be computed at each iteration of the inverse solver. The new scattering operator \mathcal{L}_2^{-1} has a much better conditioning and convergence of both the forward and inverse problems are greatly improved. The new Jacobian is given by:

$$\mathbf{J} = \mathbf{G}^R \mathbf{X}_1 \mathbf{G}^{in} (\mathbf{I}_2 + \mathbf{X}_2 \mathcal{L}_2^{-1} \mathbf{G}_{22}) \text{diag}\{\mathcal{L}_2^{-1} \mathbf{e}_{1|2}\}. \quad (4)$$

III. RESULTS

The LEGO and IG models were compared to the standard VIE for a dielectric sphere emulating white matter with a layer of cerebrospinal fluid and a small blood perturbation as seen in Fig. 1 (a). The left pane of Fig. 1 (c) shows good agreement between the scattered magnetic field for the three models and the right pane shows improvement in the convergence of the inverse solver for the LEGO and IG models. In Fig. 1 (b) the reconstructed domain using the IG model is presented. The data were generated using the VIE model with 80 dB of added Gaussian noise in order to avoid the inverse crime. We notice that, although the boundaries of the perturbation are properly identified, the conductivity values are slightly underestimated, which seems to be an effect of the regularization.

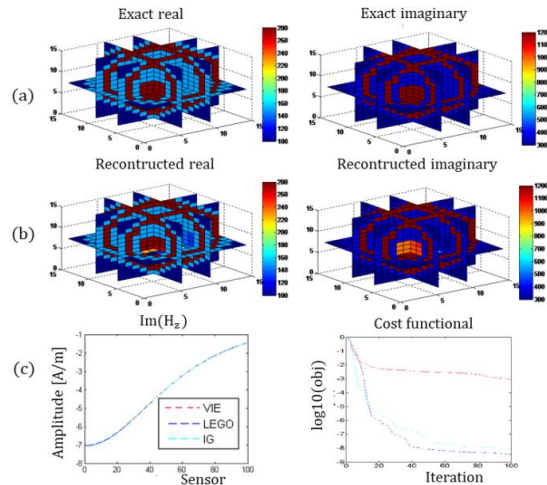


Fig. (a) Exact and (b) reconstructed domains, and (c) $\text{Im}\{H_z\}$ component of the scattered magnetic field and convergence of the cost functional for each model. The simulation domain is a cube with 20 cm edges, discretized onto a 15 X 15 X 15 Cartesian grid. It is illuminated by 15 sources (z oriented magnetic dipoles of unit amplitude) and 15 detectors are placed circularly ($r = 16$ cm) on three planes ($z = -2.5; 0; 2.5$ cm).

IV. DISCUSSION

These results indicate that, although the standard VIE model is not suitable for MIT, the alternate IG model offers considerable improvements regarding convergence of both the forward and inverse solvers. Although the proposed method shows some advantages over state-of-the-art numerical methods such as fewer unknowns and a structure well suited to parallelization, further work is required in order to reconstruct images with a lower SNR and to reduce computational cost through numerical methods such as Block algorithms and preconditioning.

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