

# Integral Equation Methods for Near-Field Far-Field Transformation

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**Abstract**— Antenna measurements are often carried out in the radiating near-field of the antenna under test. Near-field transformation algorithms determine an equivalent sources representation of the antenna in an inverse process and field values in almost arbitrary distances can be computed. In this paper two integral equation methods for the near-field transformation are presented, which are especially suitable for electrically large antennas, irregular sample point distributions, higher order probes, and non-ideal measurement environments.

**Index Terms**— Integral Equation Methods, Near-Field Far-Field Transformation, Plane Wave Expansion, Equivalent Current Methods.

## I. INTRODUCTION

The radiation characteristic of an antenna under test (AUT) can be determined employing one of the various measurement techniques, e.g. far-field, compact range or near-field measurements [1]. For electrically large antennas, which achieve far-field conditions in a distance of several tens or even hundreds of meters, indoor far-field measurements are not applicable due to the limited size of the measurement facility. In open field test ranges the environmental conditions are difficult to control for precise measurements. In near-field measurement techniques the radiated field distribution of the AUT is measured in the radiating near-field and afterwards processed into the far-field or even other observation locations, typically outside the AUT minimum sphere as illustrated in Fig. 1. With the computational resources available nowadays, near-field transformation algorithms allow to compute the far-field pattern of the AUT with

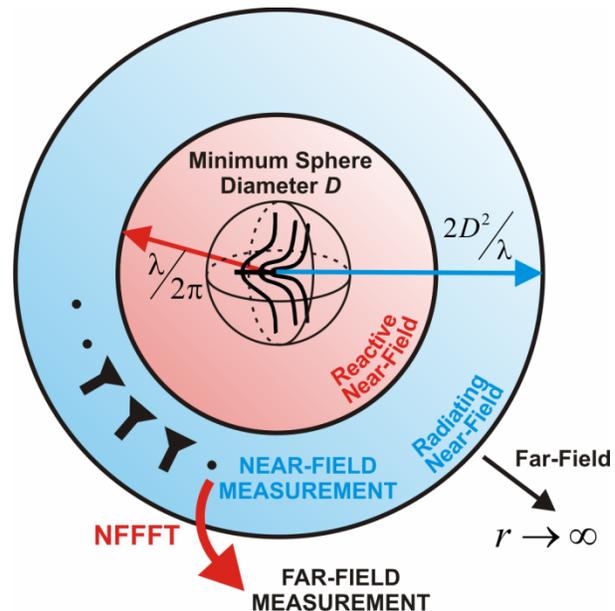


Fig. 1. Antenna field regions and measurement setup.

accuracies comparable to a direct far-field measurement.

In the transformation algorithm, the radiated field distribution of the AUT is represented by equivalent sources and their unknown amplitudes are determined in an inverse process from the measured near-field values.

In practice a near-field probe with finite geometrical extent and a corresponding receiving characteristic is used to probe the AUT field distribution. For a measurement of the electric field strength at a discrete sampling point, the probe kind of integrates the field over its volume resulting in an output signal proportional to the weighted field distribution around the sampling location (see Fig. 2). To compensate this effect, a probe correction is

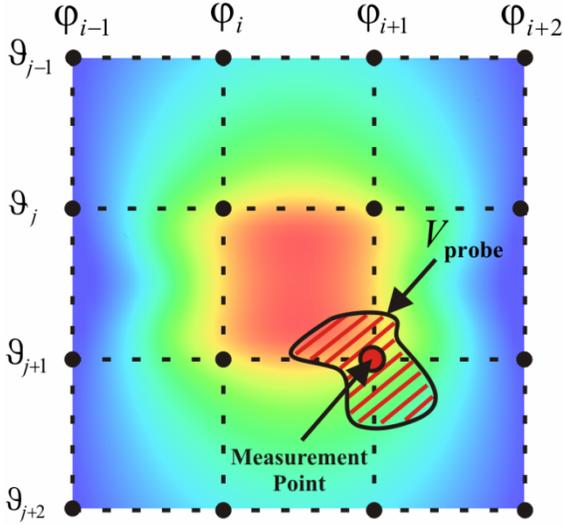


Fig. 2. Probe weighting effect in electric near-field measurement.

employed in most of the transformation algorithms [2].

Depending on the kind of measurement, various near-field transformation algorithms exist, all with their own benefits and drawbacks.

One of the major categories are algorithms working with eigenmode expansions of the AUT fields [3], for example spherical, cylindrical, and plane waves for spherical [4], cylindrical, and planar measurement surfaces [5], respectively.

To relate the amplitudes of the waves to the measured near-field values in an efficient manner, the orthogonality of the eigenmodes is utilized. This requires an often regular measurement grid

on the corresponding coordinate surfaces, even though some techniques have been proposed for spherical [6] and planar [7] near-field measurements with non-ideal probe locations.

The computational complexity of the probe correction strongly depends on the measurement geometry. While general probes can be corrected efficiently for planar near-field measurements [5], a full correction of higher order probes for spherical near-field measurements becomes time consuming since either the measurement or the transformation time [8] is increased. Nevertheless efficient formulas for so-called first order probes with an azimuthal mode spectrum restricted to the  $\mu=\pm 1$  modes are well-known [4].

A second category of near-field transformation algorithms works with integral equation evaluations. Equivalent current methods (ECM) [9-12] assume equivalent Huygens currents either on a fictitious surface (green sphere in Fig. 3 left) or the radiating structure itself (red arrows on horn antenna surface mesh Fig. 3 left). The currents are related to the field values employing a field integral equation. For an efficient solution, this equation can be evaluated using fast solver techniques like the multilevel fast multipole method (MLFMM) [13-14]. As such, a plane wave expansion is employed to convert the equivalent Huygens currents into propagating plane waves (Fig. 3 middle), which can be translated to the field probe position efficiently.

A full probe correction is achieved by weighting the incident plane waves with the probe's far-field pattern prior to superposition (Fig. 3 right)

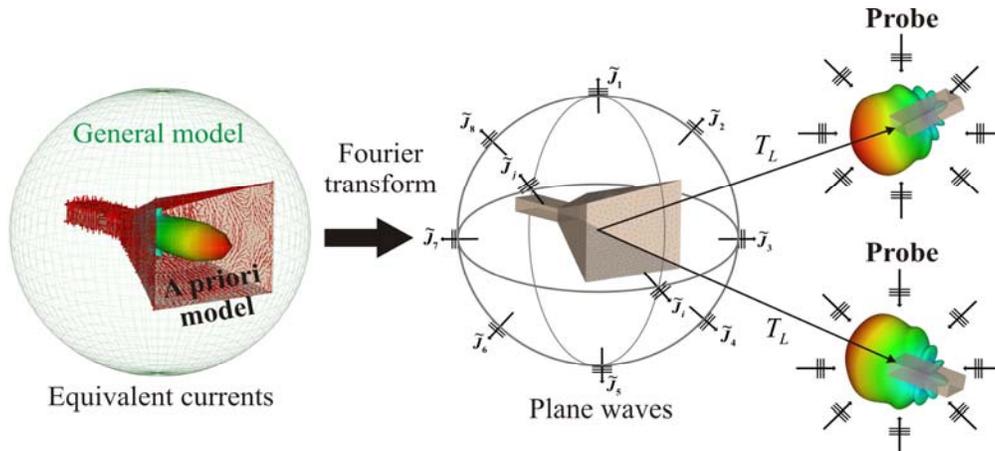


Fig. 3. Equivalent current and plane wave representation of AUT.

without increasing the algorithms complexity. A second approach, referred to as plane wave based near-field far-field transformation (PWNFFFT), utilizes the plane waves as equivalent sources directly [15]. Due to the integral equation formulation, both approaches are well suited for irregular measurement surfaces. The fast solver techniques with a low complexity also allow the transformation of electrically large antennas.

In the following sections, the ECM and PWNFFFT approaches are discussed and some results are shown. Further, some remarks on electrically large antennas and non-ideal measurement environments are given.

## II. EQUIVALENT CURRENT METHOD

According to Huygens' principal, the electric field strength

$$\mathbf{E}(\mathbf{r}_M) = \iint_A \left[ \bar{\mathbf{G}}_J^E(\mathbf{r}_M, \mathbf{r}') \cdot \mathbf{J}_A(\mathbf{r}') + \bar{\mathbf{G}}_M^E(\mathbf{r}_M, \mathbf{r}') \cdot \mathbf{M}_A(\mathbf{r}') \right] dA' + \mathbf{E}^{\text{inc}}(\mathbf{r}_M) \quad (1)$$

due to a radiating or scattering object at a measurement point  $\mathbf{r}_M$  can be computed from the electric and magnetic Huygens currents  $\mathbf{J}_A(\mathbf{r}')$  and  $\mathbf{M}_A(\mathbf{r}')$  assumed on the surface  $A$ , either a fictitious surface or the radiating/scattering structure itself.  $\bar{\mathbf{G}}_J^E(\mathbf{r}_M, \mathbf{r}')$  and  $\bar{\mathbf{G}}_M^E(\mathbf{r}_M, \mathbf{r}')$  are the dyadic Green's functions of free space and  $\mathbf{E}^{\text{inc}}(\mathbf{r}_M)$  is the incident field used as excitation for scattering investigations. In the following, the paper focuses on antenna measurements, where no incident electric field  $\mathbf{E}^{\text{inc}}(\mathbf{r}_M)$  is present. The ECM relating the equivalent Huygens currents to the measured probe signals is developed in the following. The formulation starts with the output signal

$$\mathbf{U}(\mathbf{r}_M) = \iiint_{V_{\text{probe}}} \mathbf{w}_{\text{probe}}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r}) dV \quad (2)$$

of the field probe measuring the radiated near-field distribution. It is obtained by weighting the electric field over the probe volume according to the spatial probe characteristic  $\mathbf{w}_{\text{probe}}(\mathbf{r})$  as seen in Fig. 2.

The electric and magnetic surface currents characterizing the AUT are discretized on a triangular surface mesh [12] utilizing Rao-Wilton-

Glisson (RWG) basis functions  $\boldsymbol{\beta}(\mathbf{r})$  [16] resulting in

$$\mathbf{U}(\mathbf{r}_M) = \iiint_{V_{\text{probe}}} \mathbf{w}_{\text{probe}}(\mathbf{r}) \cdot \left[ \sum_p J_p \iint_A \bar{\mathbf{G}}_J^E(\mathbf{r}_M, \mathbf{r}') \cdot \boldsymbol{\beta}_p(\mathbf{r}') dA' + \sum_q M_q \iint_A \bar{\mathbf{G}}_M^E(\mathbf{r}_M, \mathbf{r}') \cdot \boldsymbol{\beta}_q(\mathbf{r}') dA' \right] dV. \quad (3)$$

$J_p$  and  $M_q$  are the unknown current expansion coefficients. Applying Gegenbauer's addition theorem together with a plane wave expansion, the spatial integral for the probe signal can be cast in a spectral integral

$$\mathbf{U}(\mathbf{r}_M) = -j \frac{\omega \mu}{4\pi} \left[ \sum_p J_p \oint T_L(\hat{k}, \hat{r}_M) \mathbf{P}(\hat{k}, \mathbf{r}_M) \cdot (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \tilde{\boldsymbol{\beta}}_p(\hat{k}) d\hat{k}^2 + \sum_q M_q \iint T_L(\hat{k}, \hat{r}_M) \mathbf{P}(\hat{k}, \mathbf{r}_M) \cdot \frac{1}{Z} (\tilde{\boldsymbol{\beta}}_q(\hat{k}) \times \hat{k}) d\hat{k}^2 \right] \quad (4)$$

over the Ewald sphere analog to the fast multipole method (FMM) [13-14]. The spatial basis functions  $\boldsymbol{\beta}(\mathbf{r})$  are Fourier transformed into their spectral counterparts  $\tilde{\boldsymbol{\beta}}(\hat{k})$ , i.e. the corresponding plane wave representation. The spatial probe weighting function  $\mathbf{w}_{\text{probe}}(\mathbf{r})$  is Fourier transformed as well into the spectral probe correction coefficient  $\mathbf{P}(\hat{k}, \mathbf{r}_M)$ . This is simply the product of the probe's far-field pattern and the antenna factor, relating the electric field to the probe signal. The plane waves are translated from the AUT to the field probe position  $\mathbf{r}_M$  by multiplication with the diagonal translation operator  $T_L(\hat{k}, \hat{r}_M)$ . Then, they are weighted with the probe correction coefficient and superimposed to give the measured probe signal. The diagonal form of the translation operator is a key factor for the realization of a fast integral equation solver. The FMM acceleration is implemented in a multilevel fashion (MLFMM) similar to [17] and further described in section VI.

To determine the unknown current expansion coefficients in an inverse process, the probe output signal is measured at several points. Electrically large AUTs require a huge number of unknowns in

order to model the radiation behavior accurately and a large number of measurement points is also required for the inverse solution. Due to the high complexity of direct solvers, the resulting normal system of equations is solved by the iterative generalized minimum residual method (GMRES) [18].

In addition to far-field computations, ECMs are also suitable for antenna diagnostics, especially if a priori knowledge is given. Therefore, the equivalent currents on the radiating structure can be directly evaluated in order to inspect the antenna's functioning. It is further noted that ECMs are suitable for near-field measurements close to the AUT, when modal expansion methods might no longer be applicable.

Key features of the presented ECM include:

- Antenna diagnostics possible
- Near-field measurements close to AUT possible

### III. PLANE WAVE BASED NEAR-FIELD TRANSFORMATION

The second approach (PWNFFFT) utilizes directly plane waves as equivalent sources representing the AUT. The spectral plane wave representation of the AUT is obtained from the electric equivalent Huygens currents by Fourier transform according to

$$\tilde{\mathbf{J}}_A(\hat{k}) = \iiint_{V_{AUT}} \mathbf{J}_A(\mathbf{r}') e^{j\hat{k} \cdot \mathbf{r}'} dV' \quad (5)$$

without any prior discretization. The same is done for the magnetic currents. The output signal of the field probe is thus obtained from Eq. (4) as

$$\mathbf{U}(\mathbf{r}_M) = -j \frac{\omega\mu}{4\pi} \left[ \iint T_L(\hat{k}, \hat{r}_M) \mathbf{P}(\hat{k}, \mathbf{r}_M) \cdot (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \tilde{\mathbf{J}}(\hat{k}) d\hat{k}^2 \right] \quad (6)$$

Plane waves representing electric and magnetic currents are combined to the total plane wave spectrum

$$(\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \tilde{\mathbf{J}}(\hat{k}) = (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \tilde{\mathbf{J}}_A(\hat{k}) + \frac{1}{Z} (\tilde{\mathbf{M}}_A(\hat{k}) \times \hat{k}) \quad (7)$$

for convenience. The further steps, translation and

probe correction as well as the entire solution process are similar to the ECM. The plane waves used as equivalent sources are proportional to the desired far-field pattern of the AUT. Therefore, an additional far-field computation from the determined sources is no longer required.

Key features of the presented PWNFFFT include:

- Minimum number of unknowns possible
- No separate far-field computation

### IV. NEAR-FIELD TRANSFORMATION ALGORITHM

The utilization of the presented methods for near-field measurements is addressed in this section and shown in the flowchart in Fig. 4.

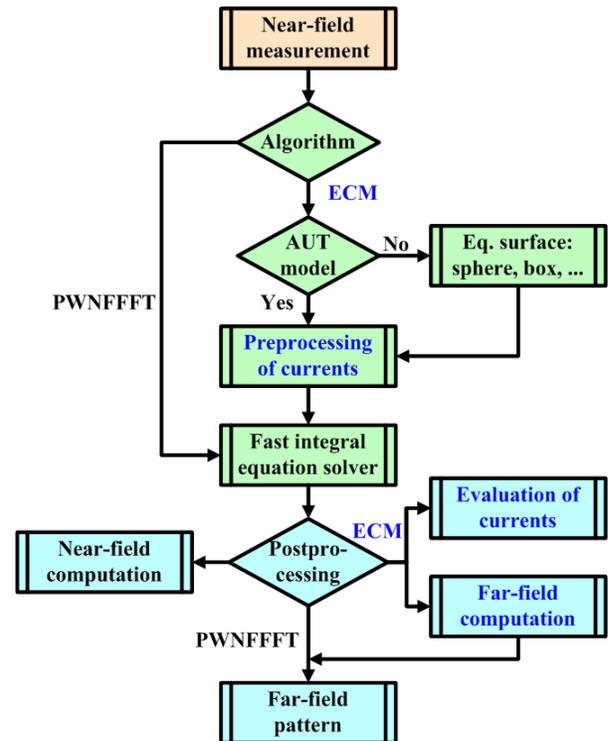


Fig. 4. Flowchart of near-field transformation.

First the near-field of the AUT is sampled in typically two polarizations. For the ECM it is possible to assume the equivalent currents on a model of the AUT. Alternatively they can be assumed on arbitrary surfaces, typically enclosing the AUT. The currents are converted to propagating plane

waves in a preprocessing step. That is where the PWNFFFT starts. The inverse problem is solved employing the FMM fast integral equation solver. Near-field values can be computed from both the equivalent currents as well as the plane waves. For the PWNFFFT no further computations are required to obtain the far-field pattern of the AUT, whereas further computations are required for the ECM. For the ECM, the equivalent currents can be evaluated for diagnostic purposes.

**V. RESULTS**

Both ECM and PWNFFFT algorithms have been applied to a near-field measurement scenario. A Kathrein base station antenna was measured at 1.92 GHz using a spherical NSI near-field scanner [19] and an open-ended waveguide probe. The antenna has a height of 1.3 m which equals  $8.3 \lambda$ . The parameters of the measurement setup are summarized in Table 1. Fig. 5 shows the equivalent currents determined by the ECM approach on a rectangular box surrounding the AUT. Some clues on the radiating elements inside the radome can be obtained. Nevertheless, a model of the base station antenna would deliver more detailed diagnostic information like the excitation levels of the single radiators.

Table 1. Parameters of measurement setup.

<b>AUT</b>	Kathrein base station antenna 742 445
<b>Measurement type</b>	Spherical
<b>Probe</b>	WR 430 OEWG
<b>Frequency</b>	1.92 GHz
<b>Antenna size</b>	1.3 m $\square$ $8.3 \lambda$
<b>Measurement distance</b>	2.715 m

The transformed far-field pattern is shown in Fig. 6 in E- and H-plane cuts and compared to the reference pattern obtained from the commercial NSI2000 software. With respect to the large dynamic range of 60 dB in the E-plane cut, a good agreement with the reference could be achieved.

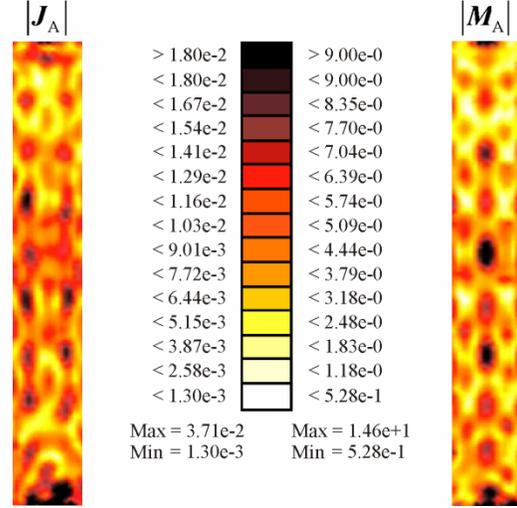


Fig. 5. Equivalent currents on rectangular box surrounding base station antenna.

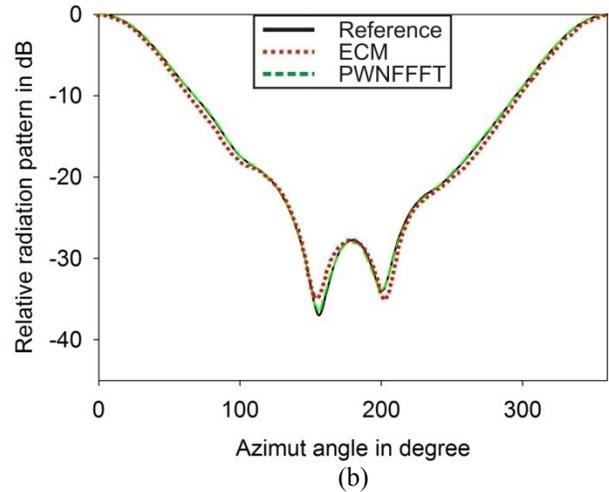
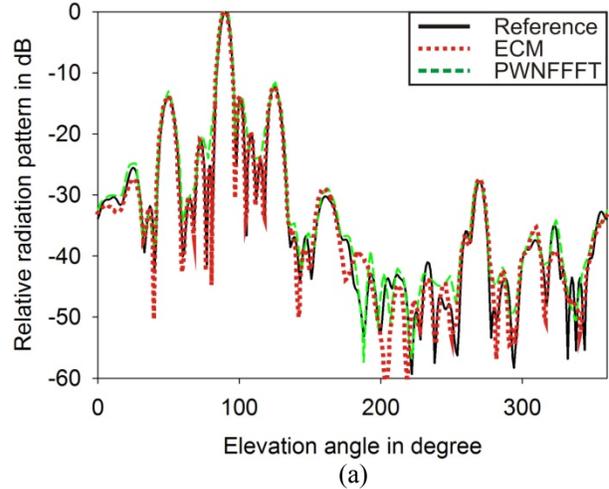


Fig. 6. Reference and transformed far-field patterns of base station antenna. (a) E-plane cut. (b) H-plane cut.

## VI. ELECTRICALLY LARGE ANTENNAS AND NON-IDEAL MEASUREMENT ENVIRONMENTS

The low complexity of the algorithms due to the diagonal translation operators can be further enhanced in a multilevel version [12,20] analog to the multilevel fast multipole method (MLFMM) [14]. Therefore the measurement points are grouped in a multilevel box structure and the plane waves are no longer translated to every measurement point explicitly. Instead the plane waves are translated to the box centers on the highest level and are further processed through the different levels towards the measurement points using disaggregation and antepolation. Disaggregation is a simple phase shift between the box centers on adjacent levels or the lowest level of the box structure and the measurement points respectively. Antepolation can be seen as counterpart to interpolation and it reduces the sampling rate of the plane wave spectrum according to its spectral content with decreasing box sizes on the various levels. The probe correction is performed on the lowest level of the box structure for a minimum number of plane wave samples. The hierarchical field representation is the principal point for reducing the computational complexity of the algorithm from  $O(N^2)$  to  $O(M\log N)$ ,  $N$  being the number of measurement points.

For measurement points fulfilling the far-field condition, efficient far-field translations, utilizing a single plane wave in the direction towards the measurement point, can be used. In order to relax the far-field criterion, the AUT can be recursively subdivided into smaller source boxes with a reduced far-field distance. The probe output voltage is obtained as superposition of the individual source boxes. Near- and far-field translations are combined in a hybrid approach in order to optimize the overall complexity [21].

The plane wave characteristic of the equivalent sources allows to utilize reflection and diffraction concepts also in near-field distance to the AUT. Subdividing the AUT in source boxes and utilizing far-field translations, infinite perfectly conducting ground planes and dielectric halfspaces, as approximation for real ground effects, can be considered in the transformation algorithm by superimposing ground reflected waves with the line-of-sight waves [20]. More complex obstacles and

scattering objects can be considered by an MLFMM-UTD hybrid approach [22], if sufficient a priori knowledge is given. Unknown scattering objects and non-ideal measurement environments are modeled as additional sources via scattering centers [23]. The plane waves representing the AUT as well as the additional scattering centers are determined in the inverse solution process. Only some oversampling of the measured fields is required to determine the additional unknowns.

Key features of the algorithms include:

- Low complexity of  $O(M\log N)$
- Arbitrary measurement grids possible
- Full probe correction
- Antenna diagnostics
- Integration of scattering contributions possible

## VII. CONCLUSION

An equivalent current method as well as a plane wave based near-field transformation have been discussed. Due to the integral equation formulation, these approaches are well suited for irregular measurement grids and a full probe correction is easily integrated without increasing the complexity. Fast solver techniques and a hybrid formulation utilizing combined near- and far-field translations allow an efficient transformation also for electrically large antennas with a low complexity. The plane wave based formulation allows for the compensation of ground reflections and also the effects of non-ideal measurement environments can be countered by introducing a scattering center approach.

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