Uncertainty Analysis in EMC Simulation Based on Improved Method of Moments

Jinjun Bai, Gang Zhang, Lixin Wang, and Tianhao Wang

School of Electrical Engineering and Automation
Harbin Institute of Technology, Harbin, 150001, China
13B906024@hit.edu.cn, zhang_hit@hit.edu.cn, wlx@hit.edu.cn, hao3916@163.com

Abstract **─** Recently, many uncertainty analysis methods have been taken into consideration in electromagnetic compatibility (EMC) simulation. As a traditional method, the Method of Moments (MoM) owns many advantages compared with other methods, especially in calculating the high dimension problems. However, its main disadvantage is the poor accuracy. In this paper, the Richardson extrapolation has been used to improve the MoM in order to promote the accuracy. By using feature selective validation (FSV), the effectiveness of the improvement can be obviously shown compared with the standard results calculated by the Monte Carlo Method (MC).

Index Terms **─** EMC simulation, feature selective validation, Method of Moments, Richardson extrapolation, uncertainty analysis.

I. INTRODUCTION

Nowadays, the electromagnetic compatibility (EMC) community is facing a growing demand for taking uncertainty into consideration in EMC simulation. The MC [1-3] is a widely used uncertainty analysis method. Random input parameters are sampled according to their distributions, and a very large number of simulations are required to achieve accurate information. Therefore, low computational efficiency makes the MC uncompetitive, though the MC owns high accuracy. In theoretical research, the results calculated by the MC can be regarded as the standard answers in order to verify other uncertainty analysis methods.

Recently, another effective method, called the Stochastic Galerkin Method (SGM), has been taken into consideration in EMC uncertainty analysis by Canavero [4,5]. Many typical EMC problems with uncertainty parameters have been solved successfully by the SGM. The Stochastic Collocation Method (SCM) is another uncertainty analysis method which has also been applied in EMC [6,7]. These two methods are both rooted in the generalized polynomial chaos expansion theory, and they can reach a high accuracy with high computational efficiency. However, if the dimension of random variables is high enough, the realization of the SCM and the SGM will be impossible. It is well-known that such problem is also named ‘curse of dimensionality’.

The MoM [5,8] is another traditional uncertainty analysis method, and it would not be affected by ‘curse of dimensionality’. In addition, easy to realize and high computational efficiency are another two advantages of the MoM. However, the main disadvantage of the MoM is the low accuracy. In the paper, a novel method is presented in order to improve the accuracy of the MoM by using the Richardson extrapolation [9,10]. At last, a published example of one dimension wave propagation with uncertainty medium parameters is calculated by the MoM, the MC and the improved MoM. By using FSV, the improvement in accuracy of the novel method can be seen clearly.

The structure of the paper is as follows. In Section II, the uncertainty analysis in EMC simulation is introduced; Section III employs a brief description of the MoM; the improved MoM by using the Richardson extrapolation can be seen in Section IV; algorithm validation is shown in Section V; Section VI provides a summary of this paper.

II. THE UNCERTAINTY ANALYSIS IN EMC SIMULATION

As for the traditional EMC simulation, all the input parameters are supposed certain. However, in actual situation, some input parameters are uncertain as the uncertainty in measurement or limited knowledge. Therefore, such parameters should be modeled by the random variables. There is no doubt that the uncertainty analysis is to calculate the output parameters under the influence of uncertain input parameters.

Maxwell’s equations for a wave propagating in a linear isotropic homogeneous material along the z-axis in 1D are shown as (1) and (2):

$$-\frac{\partial E_x(z,t)}{\partial z} = \mu \frac{\partial H_y(z,t)}{\partial t}, \tag{1}$$

$$-\frac{\partial H_y(z,t)}{\partial z} = \varepsilon \frac{\partial E_x(z,t)}{\partial t} + \sigma E_x(z,t), \tag{2}$$

Submitted On: February 2, 2015
Accepted On: December 6, 2015

where $E_x(z,t)$ represents the electric field oriented in the *x* direction, at a position *z* and time *t*. Similarly, $H_y(z,t)$ stands for the magnetic field oriented in the *y* direction at a position *z* and time *t*. The symbols μ, ε and σ represent the permeability, permittivity and conductivity of the medium in which the electromagnetic fields propagate.

Suppose that the medium parameters μ and ε are uncertain as the limited knowledge. It is obviously that a random event θ should be introduced in the Maxwell's equations. Therefore, the Maxwell's equations are transformed into stochastic differential equations, shown in (3) and (4). Furthermore, the output parameters E_x and H_y which we care about are also affected by the random event θ:

$$-\frac{\partial E_x(z,t,\theta)}{\partial z} = \mu(\theta)\frac{\partial H_y(z,t,\theta)}{\partial t}, \tag{3}$$

$$-\frac{\partial H_y(z,t,\theta)}{\partial z} = \varepsilon(\theta)\frac{\partial E_x(z,t,\theta)}{\partial t} + \sigma E_x(z,t,\theta). \tag{4}$$

Finite random variables can be used to model the random event θ shown as (5). According to the Karhunen-Loeve theory [11], the random variables can be obtained which are independent with each other. The independence is the basis of the subsequent processing:

$$\xi(\theta) = \{\xi_1(\theta),\ \xi_2(\theta),\ \cdots,\ \xi_n(\theta)\}. \tag{5}$$

As for the MC, the independent random variables are sampled according to their distributions, and a mass of certain Maxwell's equations will be obtained. By solving every certain Maxwell's equations, the statistical property of the solutions will be the answer of the uncertainty analysis.

Furthermore, in the results of the uncertainty analysis, there is no denying that the expectation and the variance are two main standards.

III. THE METHOD OF MOMENTS

The MoM is a traditional uncertainty analysis method with high computational efficiency. And it relies on first order truncated Taylor series expansions to obtain the estimates of the expectation and variance. The introduction of MoM is shown at first.

Suppose that *y* is the output parameter which we are interested in, and ξ_1 is the random variable that is modeled by the uncertain input parameters. The Taylor series form in the point $\xi_1 = \overline{\xi_1}$ of one random variable is shown as (6):

$$\begin{aligned} y(\xi_1) &= y(\overline{\xi_1}) + \left.\frac{dy}{d\xi_1}\right|_{\xi_1=\overline{\xi_1}} (\xi_1 - \overline{\xi_1}) \\ &+ \left.\frac{d^2 y}{d\xi_1^{\,2}}\right|_{\xi_1=\overline{\xi_1}} \frac{(\xi_1 - \overline{\xi_1})^2}{2} + \ldots, \end{aligned} \tag{6}$$

where $\overline{\xi_1}$ means the mean value of ξ_1. By using (6), the expectation of y and y^2 will be estimated, shown as (7) and (8):

$$E(y) = y(\overline{\xi_1}) + \frac{1}{2}\left.\frac{d^2 y}{d\xi_1^{\,2}}\right|_{\xi_1=\overline{\xi_1}} \sigma_{\xi_1}^2 + \ldots, \tag{7}$$

$$\begin{aligned} E(y^2) &= y^2(\overline{\xi_1}) + \frac{1}{2}\left.\frac{d^2(y^2)}{d\xi_1^{\,2}}\right|_{\xi_1=\overline{\xi_1}} \sigma_{\xi_1}^2 \\ &= y^2(\overline{\xi_1}) + \left.\left[(\frac{dy}{d\xi_1})^2 + \frac{dy}{d\xi_1}\frac{d^2 y}{d\xi_1^{\,2}} \right]\right|_{\xi_1=\overline{\xi_1}} \sigma_{\xi_1}^2, \end{aligned} \tag{8}$$

where $\sigma_{\xi_1}^2$ is the variance value of the random variable ξ_1, and $E(y)$ is the expectation of the output parameter *y*.

Ignoring the higher order terms, the expectation and the variance of *y* can be obtained easily, shown as (9) and (10):

$$E(y) \approx y(\overline{\xi_1}), \tag{9}$$

$$\sigma(y)^2 = E(y^2) - E(y)^2 \approx (\frac{dy}{d\xi_1})^2 \sigma_{\xi_1}^2. \tag{10}$$

Now, the MoM is generalized into multi-dimensional random variables. As for the random space shown as (5), two random variables $\xi(\theta) = \{\xi_1(\theta),\ \xi_2(\theta)\}$ are supposed. The results of the expectation and the variance in output parameters are shown in (11) and (12):

$$E(y(\xi)) = y(\overline{\xi_1}, \overline{\xi_2}), \tag{11}$$

$$\sigma(y(\xi)) = \sqrt{(\frac{dy}{d\xi_1})^2 \sigma_{\xi_1}^2 + (\frac{dy}{d\xi_2})^2 \sigma_{\xi_2}^2}, \tag{12}$$

where $y(\overline{\xi_1}, \overline{\xi_2})$ stands for making certain simulation with the input parameters $\overline{\xi_1}$ and $\overline{\xi_2}$ by the use of certain EMC solver. The sensitivity from the input parameter to the output parameter is shown as (13):

$$\frac{dy(\xi)}{d\xi_1} = \frac{y(\overline{\xi_1} + \delta_1, \overline{\xi_2}) - y(\overline{\xi_1}, \overline{\xi_2})}{\delta_1}, \tag{13}$$

where δ_1 is a small perturbation chosen according to the variance of the input parameters [5].

From (11) and (12), it is clearly seen that only certain simulations in specific points are needed during the MoM. Thus, no changing in original solver makes the MoM easy to realize. Furthermore, just *N+1* times of the certain simulations will be enough in an uncertainty analysis problem with *N*-dimensional random variables, so the MoM is in a high computational efficiency. In another word, the MoM cannot be affected by 'curse of dimensionality'.

However, ignoring the higher order terms in the Taylor series will bring some errors; thus, next section presents an improved method which aims to decrease such errors.

IV. THE IMPROVED METHOD OF MOMENTS

As (12) and (13) shown, the derivative needs to be estimated in calculating the variance. The bad estimation of the derivative may lead to the poor accuracy of the MoM.

The Richardson extrapolation is a method of numerical calculation which can improve the accuracy of calculating the derivative [7,8]. Furthermore, it can improve the accuracy in the variance calculating in the MoM.

According to (13), the error of the derivative calculating is in (14) and (15):

$$y(\xi_1+\delta_1) = y(\xi_1) + \delta_1 \times y^{'}(\xi_1) + \frac{\delta_1^{\,2}}{2} \times y^{''}(\xi_1) + o(\delta_1^{\,2}), \qquad (14)$$

$$y^{'}(x) = \frac{y(\xi_1+\delta_1) - y(\xi_1)}{\delta_1} - \frac{\delta_1}{2} \times y^{''}(\xi_1) + o(\delta_1), \qquad (15)$$

where $o(\delta_1)$ stands for the infinitesimal of higher order of δ_1. Thus, the error of the estimation in (13) is $\frac{\delta_1}{2} \times y^{''}(\xi_1) - o(\delta_1) = o(1)$.

Suppose $N(\delta_1) = \frac{y(\xi_1+\delta_1) - y(\xi_1)}{\delta_1}$, and (16) can be obtained by using (15):

$$y^{'}(\xi_1) = N(\delta_1) - \frac{\delta_1}{2} \times y^{''}(\xi_1) + o(\delta_1). \qquad (16)$$

According to (16), (17) can be got easily by replacing δ_1 with $\frac{\delta_1}{2}$:

$$y^{'}(\xi_1) = N(\frac{\delta_1}{2}) - \frac{1}{2} \times \frac{\delta_1}{2} \times y^{''}(\xi_1) + o(\delta_1). \qquad (17)$$

Simple calculating process is done with (16) and (17), and the final result is shown in (19):

$$2 \times y^{'}(\xi_1) = 2 \times N(\frac{\delta_1}{2}) - \frac{\delta_1}{2} \times y^{''}(\xi_1) + o(\delta_1), \qquad (18)$$

$$y^{'}(\xi_1) = 2 \times N(\frac{\delta_1}{2}) - N(\delta_1) + o(\delta_1). \qquad (19)$$

Obviously, the result in (15) is the calculating process in the MoM, the error is $o(1)$. On the other hand, the result derivative calculating in (19) is $2 \times N(\frac{\delta_1}{2}) - N(\delta_1)$, and the error is $o(\delta_1)$.

In terms of (19), (13) can be rewritten as it is shown in (20):

$$\frac{dy(\xi)}{d\xi_1} = 2 \times \frac{y(\overline{\xi_1} + \frac{\delta_1}{2}, \overline{\xi_2}) - y(\overline{\xi_1}, \overline{\xi_2})}{\frac{\delta_1}{2}} - \frac{y(\overline{\xi_1} + \delta_1, \overline{\xi_2}) - y(\overline{\xi_1}, \overline{\xi_2})}{\delta_1}. \qquad (20)$$

By combining (11) and (12), the uncertainty analysis results of the improved MoM can be obtained.

In this section, the Richardson extrapolation has been introduced into the MoM in estimating the variance. Furthermore, the improvement from $o(1)$ to $o(\delta_1)$, is clearly seen in mathematics. As for computational efficiency, in N-dimensional random variables, *2N+1* times of the certain simulations are needed. The computational efficiency in the improved MoM is slightly lower than the MoM, but it is also much higher than other methods. Furthermore, the proposed method retains all the advantages of the MoM.

V. ALGORITHM VALIDATION

In this section, a simple example in EMC uncertainty analysis is shown in order to observe the improvement. The example is published in literature [7].

One dimension wave propagation example with uncertainty medium parameters is shown as Fig. 1. The space step is $1.5\times10^{-2}m$ and the time step is $5.0\times10^{-11}s$. The number of discrete points in the electric field intensity is 151, and it is 150 in the magnetic field intensity. The sine excitation source is in the first discrete point with the amplitude $2.7\times10^{-3}V/m$ and the frequency $1.0\times10^{9}Hz$.

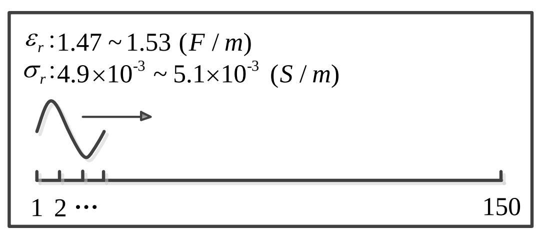


Fig. 1. The model of the example.

Suppose that the dielectric coefficient ε_r and the conductivity σ_r are uncertain parameters, and they are both in uniform distribution. The dielectric coefficient is $1.47 \sim 1.53(F/m)$, and the conductivity is $4.9\times10^{-3} \sim 5.1\times10^{-3}(S/m)$.

The random variables ξ_1 and ξ_2 can be used to model such uncertain input parameters, as (21) and (22) shown:

$$\varepsilon_r = \varepsilon_r^* \times (1 + 0.02 \times \xi_2), \quad (21)$$

$$\sigma_r = \sigma_r^* \times (1 + 0.02 \times \xi_1), \quad (22)$$

where, the mean values of the input parameters are $\varepsilon_r^* = 1.5\ (F/m)$ and $\sigma_r^* = 5 \times 10^{-3}\ (S/m)$. The distributions of the random variables are both uniform distribution in [-1, 1].

The small perturbations δ_1 and δ_2 are chosen as (23) and (24) shown:

$$\delta_1 = \varepsilon_r^* \times 0.02 = 0.03\ (F/m), \quad (23)$$

$$\delta_2 = \sigma_r^* \times 0.02 = 0.1 \times 10^{-3}\ (S/m). \quad (24)$$

Three uncertainty analysis methods, the MC, the MoM and the improved MoM are used in calculating this example. The certain simulation solver is the finite difference time domain (FDTD). The number of iteration times in time is 200.

Figure 2 shows the results in expectation, and it is calculated according to (11). The expectation of the magnetic field intensity and the electric field intensity in every discrete point is given, which is calculated by three different uncertainty analysis methods.

Table 1 is the evaluation result of the simulation results in Fig. 2 by using FSV. FSV can provide the credibility evaluation result in EMC simulation results. Both qualitative and quantitative description can be given. Total-GDM, a value to give a quantitative description in FSV, manifests the validity of simulation result. More details can be found in [12,13].

Taking the results calculated by the MC as the standard data, the value in Table 1 is the total-GDM value between the MC results and another method's results. According to the criterion in FSV [12,13], the calculated results in the MoM or the improved MoM are both 'Excellent'.

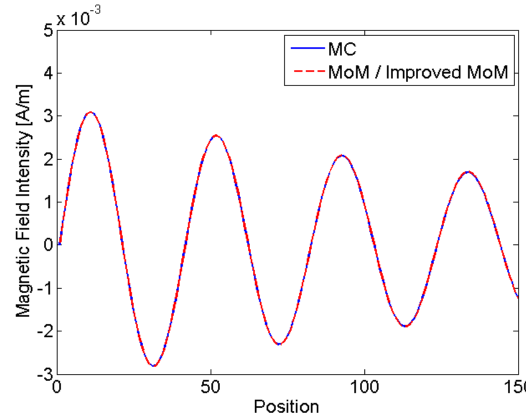
Figure 3 shows the results in variance in every discrete point, and Table 2 is the calculating results by using FSV. In Table 2, the result in the MoM is 'Very Good' and the improved MoM is 'Excellent'.

In other words, the improved MoM is as accurate as the MC, and much more accurate than the MoM. Thus, the improvement of the improved MoM is obvious.

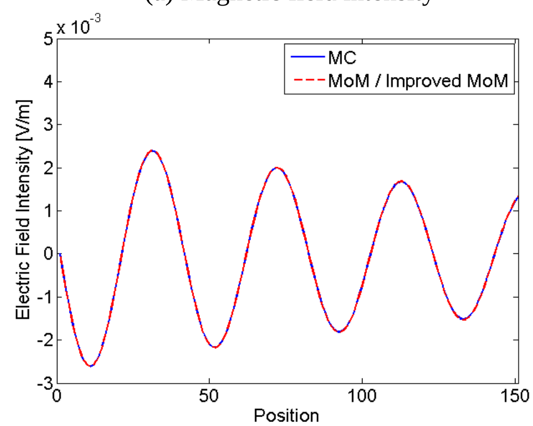
At last, the computational efficiency of three methods is compared in Table 3. The MoM and the improved MoM are in the same level, but the computational efficiency of the MC is much lower than them. The simulation times in MC are 4000, and the results would be proved convergence by using the judgment method mentioned in literature [5].

In conclusion, a simple example of EMC uncertainty analysis is given in the section, and it is proved that the improvement of the proposed method is obvious by the use of the FSV. And only a little additional calculating is needed in the improvement.

In a word, the improved MoM can reach a high accuracy with high computational efficiency.



(a) Magnetic field intensity

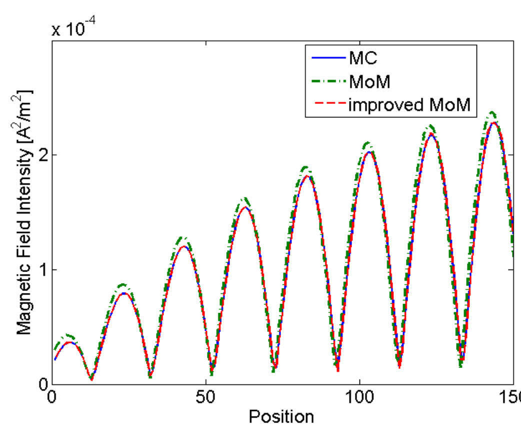


(b) Electric field intensity

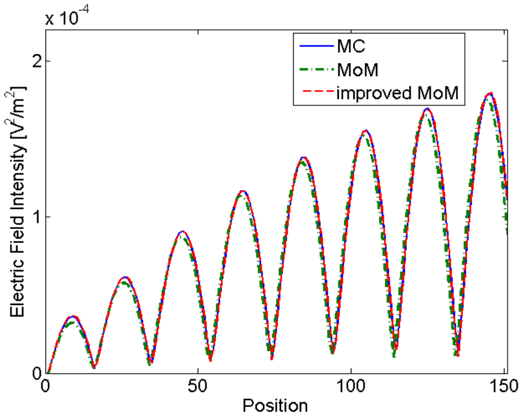
Fig. 2. The results in expectation.

Table 1: The total-GDM value of the results in expectation

Expectation	MoM/Improved MoM
Electric field intensity	0.0062
Magnetic field intensity	0.0054



(a) Magnetic field intensity



(b) Electric field intensity

Fig. 3. The results in variance.

Table 2: The total-GDM value of the results in variance

Variance	MoM	Improved MoM
Electric field intensity	0.1686	0.0192
Magnetic field intensity	0.1561	0.0109

Table 3: The comparison of computational efficiency

	MC	MoM	Improved MoM
Simulation time	86.5s	0.06s	0.1s
Times	4000	3	5

VI. CONCLUSION

In this paper, a novel method based on Richardson extrapolation is presented to make uncertainty analysis in EMC simulation, aiming at improving the accuracy of the MoM. By using a published example, the improvement of the proposed method is obviously shown. In conclusion, it is proved that the proposed method not only improves the accuracy greatly, but also remains all the advantages in the MoM.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundational of China under Grant 51507041.

REFERENCES

[1] M. Wu, et al., "Estimation of the statistical variation of crosstalk in wiring harnesses," *IEEE International Symposium on Electromagnetic Compatibility*, pp. 1-7, 2008.

[2] G. Spadacini and S. A. Pignari, "Numerical assessment of radiated susceptibility of twisted-wire pairs with random nonuniform twisting," *IEEE Transactions on Electromagnetic Compatibility*, vol. 55, no. 5, pp. 956-964, 2013.

[3] D. Srivastava, et al., "Computation of protection zone of a lightning rod using method of moments and Monte Carlo integration technique," *Journal of Electromagnetic Analysis and Applications*, vol. 3, no. 4, pp. 118-121, 2011.

[4] P. Manfredi and F. G. Canavero, "Polynomial chaos representation of transmission-line response to random plane waves," *2012 International Symposium on Electromagnetic Compatibility*, pp. 1-6, 2012.

[5] R. S. Edwards, A. C. Marvin, and S. J. Porter, "Uncertainty analyses in the finite-difference time-domain method," *IEEE Transactions on Electromagnetic Compatibility*, vol. 52, no. 1, pp. 155-163, 2010.

[6] Y. Bagci, et al., "A fast Stroud-based collocation method for statistically characterizing EMI/EMC phenomena on complex platforms," *IEEE Transactions on Electromagnetic Compatibility*, vol. 51, no. 2, pp. 301-311, 2009.

[7] B. Jinjun, Z. Gang, et al., "Uncertainty analysis in EMC simulation based on Stochastic collocation method," *2015 IEEE International Symposium on Electromagnetic Compatibility*, pp. 930-934, 2015.

[8] R. W. Walters and L. Huyse, *Uncertainty Analysis for Fluid Mechanics with Applications*, no. ICASE-2002-1, NASA/CR-2002-211449, REC Warangal, 2002.

[9] S. A. Richards, "Completed Richardson extrapolation in space and time," *Communications in Numerical Methods in Engineering*, vol. 13, no. 7, pp. 573-582, 1997.

[10] P. J. Roache, Patrick and P. M. Knupp, "Completed Richardson extrapolation," *Communications in Numerical Methods in Engineering*, vol. 9, no. 5, pp. 365-374, 1993.

[11] M. Loeve, *Probability Theory*, Springer-Verlag Berlin and Heidelberg GmbH & Co.K, 1978.

[12] IEEE Standard for Validation of Computational Electromagnetics Computer Modeling and Simulations, IEEE STD 15971-2008, pp. 1-41, 2008.

[13] IEEE Recommended Practice for Validation of Computational Electromagnetics Computer Modeling and Simulations, IEEE STD 15972-2010, pp. 1-124, 2011.

Jinjun Bai received the B.Eng. degree in Electrical Engineering and Automation from the Harbin Institute of Technology, Harbin, China, in 2013.

He is currently working toward the Ph.D. degree in Electrical Engineering at the Harbin Institute of Technology, Harbin, China. His research interests include uncertainty analysis methods in EMC simulation and the credibility evaluation of uncertainty analysis results in EMC simulation.

Gang Zhang received the B.Sc. in Electrical Engineering from China University of Petroleum, Dongying, China, in 2007, and the M.Sc. and Ph.D. degrees in Electrical Engineering from Harbin Institute of Technology (HIT), Harbin, China, in 2009 and 2014, respectively.

He is now with the Faults Online Monitoring and Diagnosis Laboratory at Harbin Institute of Technology. His research interests include analysis of electromagnetic compatibility, electromagnetic simulation, and the validation of CEM.

Lixin Wang received the B.S. degree in Electrical Engineering from Nankai University, Tianjin, China, in 1988, and the M.S. and D.Sc. degrees in Electrical Engineering from Harbin Institute of Technology (HIT), Harbin, China, in 1991 and 1999, respectively.

He is currently a Professor of Power Electronic and Electric Drives at the HIT. He conducts research with Faults Online Monitoring and Diagnosis Laboratory, HIT, on a wide variety of topics including electromagnetic compatibility at the electronic system level, aircraft electromechanical fault diagnosis expert system and prediction and health management (PHM) of Li-ion battery.

Tianhao Wang received the B.Eng. degree in Electrical Engineering and Automation from the Harbin Institute of Technology (HIT), Harbin, China, in 2014.

She is currently working toward the M.Sc. degree in HIT and involving in the research of uncertainty analysis method of random cables.