Synthetic Asymptote Formulas of Square Coaxial Line

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Abstract ─ This paper derives simple computer-aided design (CAD) formulas of characteristic impedance and open-ended capacitance of square coaxial line by synthetic asymptote and moment method. These formulas can be used for the design of the 3-D frequency selective surface (FSS) with square coaxial line as unit cells. Both of the formulas can give good physical insights and have only one or two arbitrary constants to be matched with numerical results. A good agreement was found between the results by the formulas and numerical method.

Index Terms ─ CAD formulas, frequency selective surface (FSS), square coaxial line, synthetic asymptote.

I. INTRODUCTION

Frequency-selective surfaces (FSSs) have been studied over the past decades. A conventional FSS consists of two-dimensional (2-D) periodic unit cells, which are either printed on a dielectric layer or etched out of a conductive surface [1-2]. Recently, thanks to the significant improvements in computational electromagnetic methods and fabrication technology, three-dimensional (3-D) FSSs [3-5] have been proposed and studied.

Compared with 2-D FSSs, 3-D FSSs have one more design freedom and therefore more paths for electromagnetic wave propagating. The propagation paths may produce multiple transmission zeros and poles under proper design. For the 3-D FSS unit cell design, waveguide is good choice and studied in many references. In [3], 3-D FSS based on substrate integrated waveguide (SIW) structures is proposed and produce transmission zeros through the couplings between different resonant modes in these SIW cavities. In [4], Shen etc. proposed a 3-D FSS with the unit cell consists of vertical and horizontal double-sided parallel-strip lines and a thin metallic plate, which formed substrate and air paths to produce transmission poles and zeros. This unit cell in [4] can also be considered as a rectangular waveguide loaded with a planar circuit.

Another kind of waveguide, coaxial line, can also be used as the unit cell of 3-D FSS. To produce the propagation paths easily, square coaxial line is chosen as the unit cell of 3-D FSS [6] by the authors. Square coaxial line not only has one "coaxial path", but also introduces a "parallel plate path" between the adjacent unit cells to ease the analysis and design. And these two paths can provide multiple transmission poles and zeros under proper design.

In the FSS analysis and design, equivalent circuit model is a very useful tool and should be established. For the square coaxial line 3-D FSS, the equivalent circuit model has two important parameters, in particular, the characteristic impedance and the open-ended capacitance of the square coaxial line.

There are several references for calculating the characteristic impedance [7-9] and the open-ended capacitance [10-11] of square coaxial line, generally in analytical or numerical approach. All the closed form expressions in [7-11] have more than two constants to be curve-fitted. Synthetic asymptote technique is useful in deriving simple CAD formulas and such synthetic asymptote formulas have been obtained by the authors in a series papers [12-14]. Generally, synthetic asymptote is the sum of two regular asymptotes at the two limits of one parameter, with one or two arbitrary constants to be matched with numerical results. And the synthetic asymptote formula has good physical insights, and is useful in practical initial design.

In this paper, the formulas of the characteristic impedance and the open-ended capacitance of the square coaxial line are derived by synthetic asymptote. Unlike reference [6] by the authors, more detailed derivation will be given in this paper. Compared with numerical

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results by moment method [15] under quasi-static condition and full-wave EM commercial software HFSS [16], the average error of the formulas is less than 2% with the maximum error of 6.5%.

II. DERIVATION OF THE FORMULA OF CHARACTERISTIC IMPEDANCE

For the derivation of the formula of characteristic impedance, only TEM mode is considered in this paper. In the cross section view of the square coaxial line shown in Fig. 1, c is the side length of the inner conductor and a-b is the thickness of the outer conductor. The dielectric with dielectric constant ε_r is filled in between the inner and outer conductors. From the transmission line theory, the characteristic impedance Z_0 of the square coaxial line is:

$$Z_0 = \frac{1}{v_0\sqrt{C_0 C_{total}}}, \tag{1}$$

where v_0 is the light speed in free space. C_0 and C_{total} are the capacitances per unit length of the square coaxial line when the dielectric between the inner and outer conductors is free space and dielectric, respectively, and they can be derived by synthetic asymptote which is constructed by two regular asymptotes (near and far asymptotes) of one parameter. In this paper, this parameter is chosen as S=b/c.

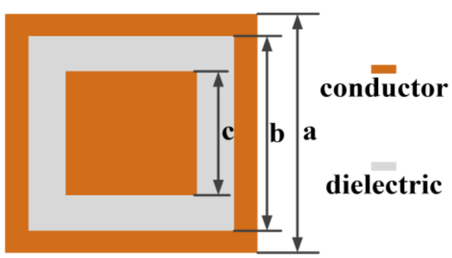


Fig. 1. Cross section of square coaxial line.

A. Near asymptote ($S\to 1$)

When $S\to 1$, the dielectric between the inner and outer conductors is very thin. This means that the near asymptote of the capacitance is that of the parallel plate with 4 sides of the inner conductor. Hence, we have:

$$C_{near} \overset{Asym}{\underset{S\to 1}{=}} C = 4\varepsilon_0\varepsilon_r \frac{c}{\frac{b-c}{2}} = 4\varepsilon_0\varepsilon_r \frac{2}{S-1}. \tag{2}$$

B. Far asymptote ($S\to\infty$)

When $S\to\infty$, the inner conductor is very far from the outer conductor and then the square coaxial line can be considered as a circular coaxial line. Therefore, the far asymptote is:

$$C_{far} \overset{Asym}{\underset{S\to\infty}{=}} C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(\frac{b}{c})} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(S)}. \tag{3}$$

Usually, synthetic asymptote is constructed by summing the regular asymptotes at the two limits of the parameter under the "asymptote consistency condition". The condition is that the far asymptote will approach zero or a pretty small number at the near parameter limit and vice versa for the near asymptote. From Eq. (3), one can see that when $S\to 1$, the far asymptote tends to infinity and will have effects on the near asymptote. Therefore, like the far asymptote formula in [12], Eq. (3) can be modified as following:

$$C_{far} \overset{Asym}{\underset{S\to\infty}{=}} C \approx \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(S)} - \frac{2\pi\varepsilon_0\varepsilon_r}{S-1}. \tag{4}$$

C. Synthetic asymptote

By adding the Eqs. (2) and (4) together, we obtain the synthetic asymptote:

$$C_{total} = \left\{ \begin{aligned} &(4\varepsilon_0\varepsilon_r \frac{2}{S-1})^m \\ &+\left[\frac{2\pi\varepsilon_0\varepsilon_r}{\ln(S)} - \frac{2\pi\varepsilon_0\varepsilon_r}{S-1}\right]^m \end{aligned} \right\}^{\frac{1}{m}}, \tag{5}$$

with the power of m. Matching with one data point with numerical computation at an arbitrary C_{total}, the power m is obtained as 1.08, the same as that of synthetic asymptote formula for microstrip in [12], which can make the average error of Eq. (5) less than 2%.

III. DERIVATION OF THE FORMULA OF OPEN-ENDED CAPACITANCE

To obtain the formula of the open-ended capacitance $C_{open\text{-}end}$ for the open-end of square coaxial line, Fig. 2 (a) gives the distribution of the fringe field at the open-end of square coaxial line. One can see that $C_{open\text{-}end}$ is half of the capacitance C_a of the metal plate and metal loop, a two conductors system in free space, as shown in Fig. 2 (b). We may use the synthetic asymptote and analytical moment method to derive the formula of C_a.

One should note that in this two conductors system, the voltages on metal plate and metal loop are different since they form a two-conductor transmission line. Assuming the voltage and charge on the metal plate of Fig. 2 (b) are V_1 and Q_1, while V_2 and Q_2 on metal loop, we then have:

$$V_1 = P_{11}Q_1 + P_{12}Q_2, \tag{6}$$

$$V_2 = P_{21}Q_1 + P_{22}Q_2, \tag{7}$$

with P_{12}=P_{21}, the mutual-potentials between the metal plate and the metal loop. P_{11} and P_{22} are, respectively, the self-potentials on metal plate and loop.

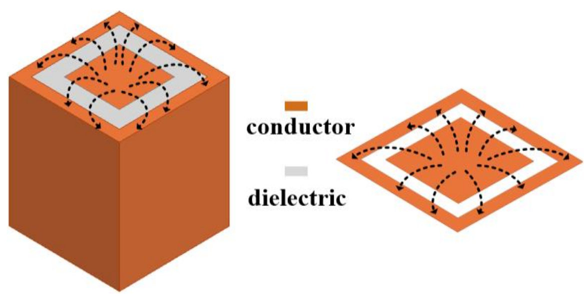
By setting V_1=1V and V_2=0V in Eqs. (6) and (7), the capacitance of the metal plate and loop C_a is then:

$$C_a = \frac{Q_1}{V_2} = \frac{P_{22}}{P_{11}P_{22} - P_{12}{}^2}. \tag{8}$$

And the open-ended capacitance ($C_{open\text{-}end}$) of square coaxial line in Fig. 2 (a) is:

$$C_{open-end} = \frac{1}{2}C_a = \frac{1}{2}\frac{P_{22}}{P_{11}P_{22} - P_{12}{}^2}. \tag{9}$$

To obtain the capacitance C_a, one can see that the self and mutual-potential in the above Eq. (8) should be obtained firstly.



(a) (b)

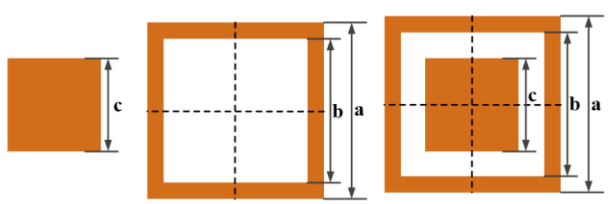
Fig. 2. Schematic diagram of electric field distribution. (a) The open end of square coaxial line. (b) Metal plate and loop in free space.

A. Self-potentials P_{11} and P_{22}

The self-potential P_{11} of the metal plate as depicted in Fig. 3 (a) can be calculated by the "root of area" formula in [13], that is:

$$P_{11} = \frac{1}{C_{11}} = \frac{1}{c_{f1}\varepsilon_0\sqrt{8\pi c^2}}, \tag{10}$$

with the "shape factor" c_{f1}= 0.9.



(a) (b) (c)

Fig. 3. Three kinds of structures: (a) metal plate, (b) metal loop, and (c) metal plate and metal loop, for the self and mutual potentials calculation in free space.

For the self-potential P_{22}, the metal loop can be divided into 4 segments, as shown in Fig. 3 (b). Using the analytical moment method and synthetic asymptote formulas in [13], P_{22} can be expressed as:

$$P_{22} = \frac{1}{c_{f1}\varepsilon_0\sqrt{8\pi a^2}} + \frac{1}{4}\frac{1}{c_{f2}\varepsilon_0\sqrt{8\pi}}\left(\frac{1}{\sqrt{\frac{a^2-b^2}{4}}} - \frac{1}{\sqrt{\frac{a^2}{4}}}\right), \tag{11}$$

where the $c_{f2} = \dfrac{3.5a}{b}$ is the "shape factor" of the metal loop obtained by matching with numerical results.

B. Mutual-potential P_{12}

For the mutual potential P_{12}, we may first obtain the capacitance C_b of the metal plate and metal loop in Fig. 3 (c) with the *same* potential on them, in other words, V_2=V_1=1V for example. In this case, the metal plate and loop can be considered as a large solid metal plate with a square slot etched on it. Therefore, the capacitance C_b has the same form as that of Eq. (11), i.e.,

$$\frac{1}{C_b} = \frac{1}{c_{f1}\varepsilon_0\sqrt{8\pi a^2}} + \frac{1}{4}\frac{1}{c_{f3}\varepsilon_0\sqrt{8\pi}}\left(\frac{1}{\sqrt{\frac{a^2}{4} - \frac{b^2-c^2}{4}}} - \frac{1}{\sqrt{\frac{a^2}{4}}}\right), \tag{12}$$

with $c_{f3} = \dfrac{3.5a}{\sqrt{b^2-c^2}}$ by substituting $\sqrt{b^2-c^2}$ for b of c_{f2}.

On the other hand, by setting V_2= V_1=1V in Eqs. (6) and (7), C_b can be expressed as:

$$C_b = \frac{Q_1 + Q_2}{V_1} = \frac{P_{11} + P_{22} - 2P_{12}}{P_{11}P_{22} - P_{12}{}^2}. \tag{13}$$

After some manipulations, the mutual-potential P_{12} can be derived from Eq. (13):

$$P_{12} = \frac{1}{C_b} \pm \sqrt{\frac{1}{C_b{}^2} - \frac{P_{11} + P_{22}}{C_b} + P_{11}P_{22}}. \tag{14}$$

In Eq. (14), the mutual-potential P_{12} has two possible results due to the sign of "±". However, the mutual-potential should be less than the self-potential from the physical insight. Figure 4 shows the P_{12} with sign of "+"and "-" for different c/b of the square coaxial line, where a=10mm and b=9mm. As can be seen, negative sign "-" in Eq. (14) is suitable for the calculation of P_{12}. Many computations have been done for other different values of the structure parameters a, b, and c, and we find that "-" in Eq. (14) should be taken for P_{12} calculation.

Substituting the potentials P_{11}, P_{22}, and P_{12} by Eqs. (10), (11), and (14) with negative sign of "-", respectively, into Eq. (9), the open-ended capacitance of the square coaxial line can be obtained.

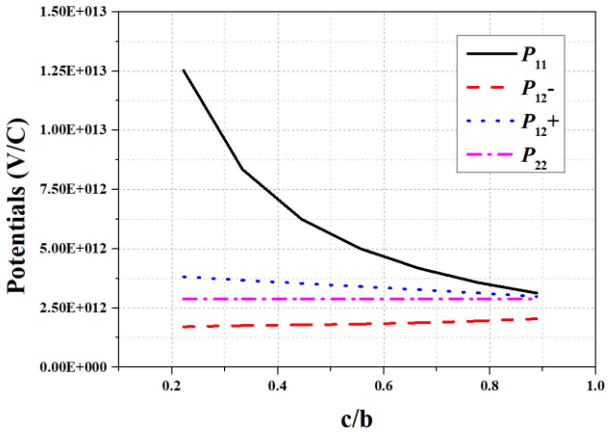


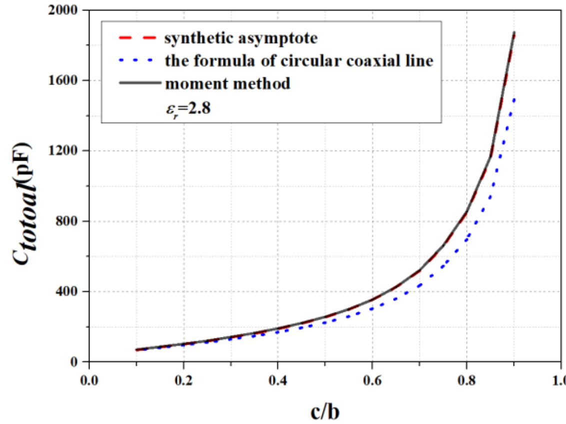
Fig. 4. The P_{12} with sign of "+"and "-" for different *c*/*b* of the square coaxial line, where *a*=10mm and *b*=9mm.

IV. RESULTS

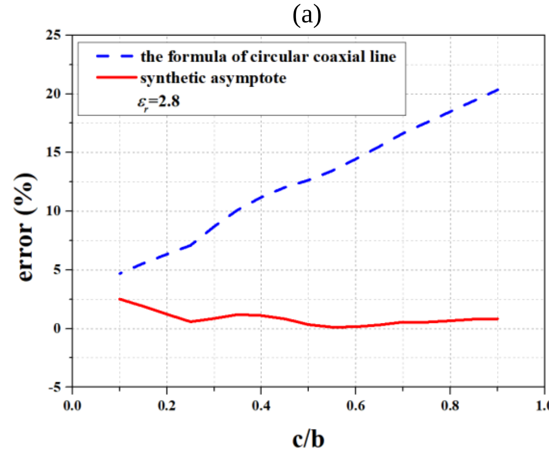
To verify the accuracy of the formulas derived in this paper, the moment method [15] are used for comparison. As shown in Fig. 5 (a), good agreement can be observed between the results by moment method and synthetic asymptote formula for the per unit length capacitance of square coaxial line. The average error is less than 2% with the maximum error of 2.5%. Moreover, the results by the formula of circular coaxial line are also added in Fig. 5. One can see that the formula of circular coaxial line can be used for the calculation of the per unit length capacitance of the square coaxial line when *c*/*b* is very small. However, when *c*/*b* increases, the discrepancy becomes larger, which means the formula of circular coaxial line will not be suitable. Figure 5 (b) shows the calculation error of Fig. 5 (a), and it's apparent that the formula derived by synthetic asymptote has a better performance.

As shown in Fig. 6 (*a*=10mm), the results of the open-ended capacitance of square coaxial line by synthetic asymptote formula and moment method [15] are given for different *b*/*a* and *c*/*b*. The results by synthetic asymptote formula are of good accuracy comparing with those by moment method. The average error is less than 2% with the maximum error of 6.5%.

In practice, the characteristic impedance and propagation constant are two important parameters for the transmission line, and the comparisons with HFSS are shown in Fig. 7 (a), where the dimensions of the square coaxial line are, respectively, *a*=10mm, *b*=8mm, *c*=6mm with the length *l*=50mm. One can see that good agreements are achieved with the average error of 2%. The verification of open-ended capacitance is realized by the S_{11} comparisons of the square coaxial line with an open end as illustrated in Figs. 7 (b) and (c) with average error less than 2%.



(a)



(b)

Fig. 5. (a) Comparison of the capacitance C_{total} obtained by the formulas and moment method with ε_r=2.8. (b) The errors of the formula of circular coaxial line and synthetic asymptote, compared with moment method.

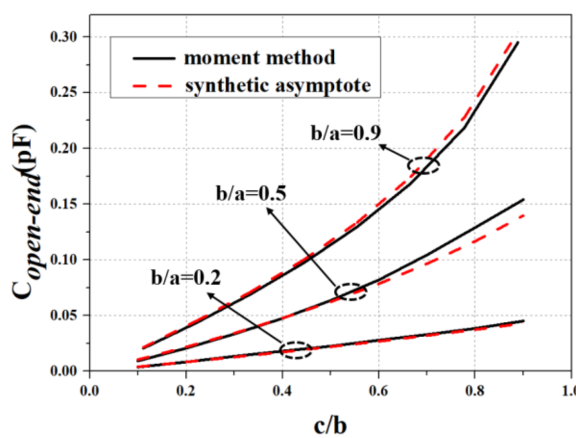


Fig. 6. Comparison of the square coaxial line open-ended capacitance by the synthetic asymptote formulas and moment method, where *a*=10mm.

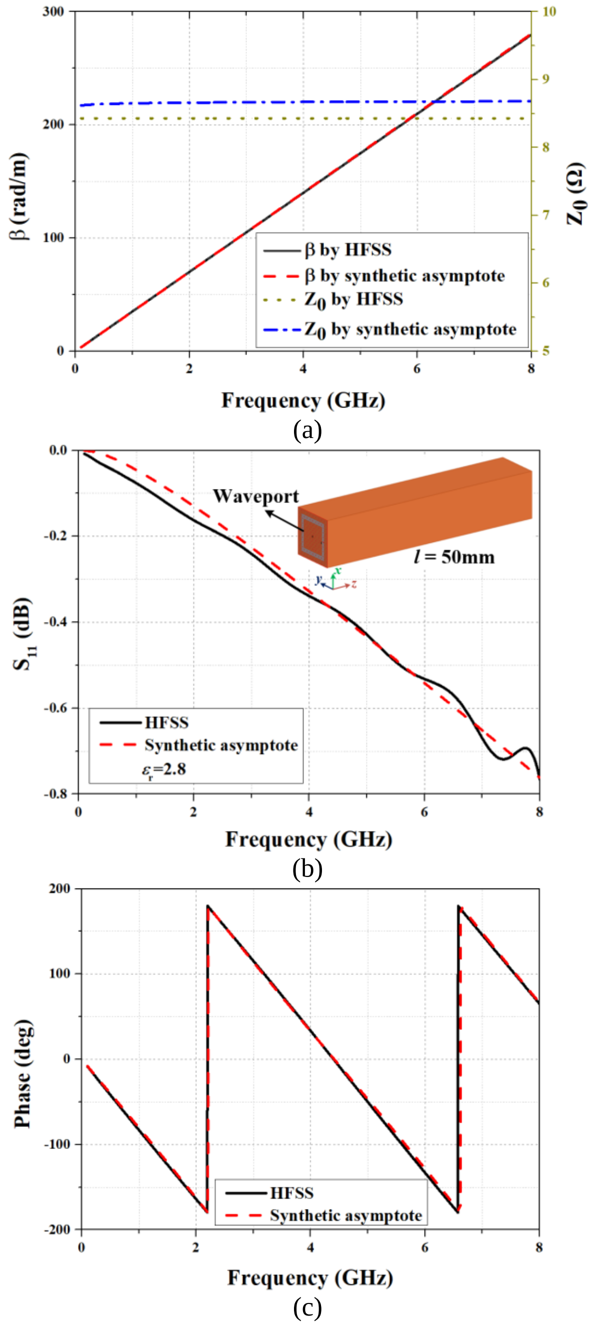


Fig. 7. Comparison of (a) the characteristic impedance and propagation constant, (b) magnitude, and (c) phase of S_{11} of the square coaxial line with an open end.

V. CONCLUSIONS

In this paper, the formulas of characteristic impedance and open-ended capacitance of square coaxial line are derived by synthetic asymptote. All of the formulas are depend on the structure parameters of square coaxial line and have good physical insights. The results of these formulas are compared with numerical methods and good agreement can be found between them.

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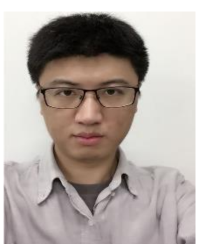

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