# A New Multifractal Geometry for Design of Frequency Selective Surfaces with Dual Band Response

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*Abstract* — In this paper, we proposed a new multifractal geometry for design of frequency selective surfaces (FSS) with dual band response. The proposed new geometry is called Ericampos. The main advantage of the proposed geometry is to design dual band FSS and makes it flexible in terms of controlling resonance and bandwidth. In addition, the proposed structure is easy to implement. The validation of the proposed structure was initially verified through simulations in a commercial software and then the structure was built and experimental results were obtained. A good agreement between numerical and experimental results is observed. The structure shows angular stability and polarization independence.

*Index Terms* – Angular stability, FSS, multifractal Ericampos geometry, polarization independence.

# **I. INTRODUCTION**

The technological development in recent decades in the construction of planar structures has performed a key role in the implementation of devices with low weight, small volume and low cost, making these structures attractive for applications in aerospace systems and wireless communications, such as WLAN, Bluetooth, WMAN, among others.

In literature, there is a large number of works with frequency selective surfaces (FSS) and may be noted that the application of fractal geometry in FSS is increasing.

The study of fractal geometry is not something recent. In fact, it began in 1974, with the mathematician Benoît Mandelbrot using the word "fractal" to indicate objects whose complex geometry could not be characterized as an integral dimension. His examples included galaxies, coastal areas, snowflakes and the Cantor set.

Several researchers have focused on studies with multiband FSS response lately. This is because various applications such as mobile communications and wireless computer networks need multiband operation with resonance frequencies closed. Fractal geometry is a good solution for this problem. These structures are recognized by their properties of self-similarity and fractional dimension [1].

Fractal curves are based on a mathematical concept geometry [2]. The FSS fractal geometric shape has a large effective length, which may be designed in various ways. Fractal shapes has some interesting properties such as the ability to obtain an arbitrarily large electrical length confined to a finite volume.

The use of fractal elements in FSS design allows the development of compact spatial filters with better performance compared with conventional structures [3]. Several iterations in fractals can be used to design FSS with multiband frequency response associated with selfsimilarity structure contained in [4].

Many studies on the FSS with fractal elements were performed by analyzing only monofractal geometry structures, where the proportion of the adjacent resonance frequencies is approximately equal to the fractal dimension [5]-[7]. However, these structures cannot be used in multiband designs with different ratios of resonance frequencies.

In this work, an analysis of FSS with a new type of multifractal geometry, which is named Ericampos, is performed. This structure optimize the design of multiband FSS structures and allows construction of structures with different proportions of multiband frequency resonances.

## **II. MULTIFRACTAL GEOMETRY**

The proposed multifractal geometry is a new multifractal geometry, developed by the authors of this paper. Its geometric construction starts with a square as initiator element,  $Q_0$ , and the generator element,  $Q_1$ , is a combination of a square element with mass probability  $p_1$  and four square elements with mass probability  $p_2$ , each. This process is repeated *ad infinitum* obtained for each element. Therefore, after *m* iterations we have 5m elements, where  $m = 0, 1, 2, ..., \infty$  and  $Q_n$  is the *nth* fractal element. The ratio  $\rho$  is defined as:

$$\rho = \frac{p_1}{p_2},\tag{1}$$

Thus, be  $\mu_n$  the probability measure of the *nth* level fractal, then, if *I* is an interval of  $Q_n$  we have:

$$\mu_n(I) = p_1^k p_2^{n-k}, (2)$$

where, by construction of *I*, the element with probability  $p_1$  is taken *k* times and the elements with probability  $p_2$  are taken n - k times. For example, for  $Q_1$ , k = 1 and n = 4.

Now, generalizing the above definition for all elements, it has  $Q_n(y)$  as the set that contains the subintervals of length  $\delta_n$ , then the local dimension is defined as:

$$y = \frac{\log(\mu_n(I))}{\log(\delta_n)}.$$
(3)

Also,  $N_{\delta n}$  is the number of elements with length  $\delta_n$  contained in  $Q_n(y)$ , then:

$$N_{\delta_n} = 4^k \, \frac{n!}{k!(n-k)!}.\tag{4}$$

If we consider only the *y* values greater than zero, we define a function f(y), or multifractal dimension, as:

$$f_n(y) = \frac{\log(N_{\delta_n})}{\log(\delta_n)}.$$
(5)

The proposed geometry is shown in Fig. 1. Figure 1 (a) has a monofractal geometry. Figure 1 (b) has the multifractal geometry. So, as you can see, the fractal dimension of the geometry is changed independently. This provides greater design flexibility.

In Fig. 2 is shown a spectral diagram for the proposed structure. It can be seen that the multifractal spectrum has a maximum value of the fractal dimension of monofractal, or  $\rho = 1$  and the area occupied by the conducting patches is highly concentrated. It is observed that reducing  $\rho$  the multifractality of the system increases and favors the spreading of the area occupied by the conducting patches.



Fig. 1. Proposed geometry: (a) monofractal, and (b) multifractal.



Fig. 2. Multifractal spectrum for different ratios fractality.

To analyze the effect of multifractality, simulations of the transmittance were performed for different values of  $\rho$  and for vertical and horizontal polarizations. In Fig. 3, we can observe the coefficient of transmission for different simulation scenarios and vertical polarization. It appears that the increased multifractality enables a reduction in the resonance frequency and increased bandwidth, which does not occur for monofractal geometries. In monofractal case, there is a reduction in the frequency and bandwidth. In Fig. 4, we observe the same behavior for horizontal polarization. Thus, it can be seen that the structure has the independence of polarization.



Fig. 3. Transmission coefficient for various ratios of fractality and vertical polarization.



Fig. 4. Transmission coefficient for various ratios of fractality and horizontal polarization.

After previous simulations, was designed a FSS with multifractal geometry Ericampos level 2. The geometry was obtained from an initiator element (squared patch) with 15 mm of side. The individual cell has periodicity of 15.5 mm in the x and y directions. Probabilities  $p_1$  and  $p_2$  equal to 0.0178 and 0.1878, respectively, were used, giving a ratio  $\rho = 0.095$ .

In Fig. 5, we can see the frequency response of the FSS designed for different angles of incidence and vertical polarization. It may be noted that there is no degradation in resonance frequency or bandwidth. This shows that the geometry is stable in terms of incidence angles. In the simulations, the angles of incidence for an incident plane wave were changed from  $0^{\circ}$  to 45°. This range covers almost all applications of interesting for FSS.

In Fig. 6, we can see the frequency response of the FSS designed for different angles of incidence and horizontal polarization. It may be noted that, as in the case of vertical polarization, no degradation in

resonance frequency or bandwidth. This shows that the structure has the independence of polarization.

It can be seen that the designed structure has possible applications in the S- and X-band.



Fig. 5. Transmission coefficient of the FSS designed for different angles of incidence and vertical polarization.



Fig. 6. Transmission coefficient of the FSS designed for different angles of incidence and horizontal polarization.

# **III. EXPERIMENTAL RESULTS**

To validate the analysis performed, a prototype FSS was designed and built, and experimental characterizations were performed. Thus, we could compare the simulated results with measurements. The measurements were carried out using a vector network analyzer from Rohde & Schwarz (ZVB-14), which operates at 14 MHz to 10 GHz, and two horn antennas operating in the band from 700 MHz to 18 GHz with 16 dBi gain. A photograph of the measurement setup is shown in Fig. 7. Bulkhead on which is fixed the FSS is circulated by absorbers to avoid diffraction at the edges and lined with metal to ensure that the signal passes only through the FSS window.



Fig. 7. Photograph of the measurement setup.

In Fig. 8, can be seen a comparison between numerical and measured results for cascaded FSS for normal incidence. The measured and simulated results show good agreement. At the second resonance, a little difference at the resonance frequency can be noted. This may be occuring because the FSS was built with a milling machine table and this may have slightly reduced the thickness of the dielectric, which would be more critical at higher frequencies. This would explain the small difference between the simulated and measured results, at the second resonance.



Fig. 8. Comparison between simulated and measured results of the normal incidence transmission coefficient of the FSS designed.

In Figd. 9 and 10, we can see measured results of vertical and horizontal polarizations, respectively, with oblique incidence. The angles of incidence ranging from 0 to 30 degrees. The measured results show that the bandwidth does not suffer degradation in both

cases, for different incidence angles, confirming the stability angle and polarization independent. The angles of incidence were changed from  $0^{\circ}$  to  $30^{\circ}$  because we have limitations at the measurement setup.



Fig. 9. Transmittance measured FSS designed for different angles of incidence and vertical polarization.



Fig. 10. Transmittance measured FSS designed for different angles of incidence and horizontal polarization.

#### **IV. CONCLUSION**

In this paper, we presented a new multifractal geometry for design of dual band FSS. The new geometry was named Ericampos and the FSS presented potential for applications in S- and X-band. The main advantage of the proposed structure is to design multiband FSS with multiple frequency ratios between the adjacent bands and a facility to build the designed structures. In addition, the proposed structure increases the degree of freedom in design of multiband structure according to the number of fractal iterations. The validation of the proposed structure was initially verified through simulations in a commercial software and then with measurements. A good agreement between simulated and measured results was obtained. The structure shows angular stability and polarization independence.

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