Electromagnetic Diffraction Modeling: High Frequency Asymptotics vs. Numerical Techniques

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Abstract **—** Electromagnetic diffraction modelling and recent numerical simulation approaches, on the canonical 2D non-penetrable wedge scattering problem, are reviewed in this introduction paper.

Index Terms **—** Diffraction, electromagnetics, finite element method (FEM), geometric optics (GO), geometric theory of diffraction (GTD), method of moments (MoM), physical optics (PO), physical theory of diffraction (PTD), simulation, time domain finite difference method (FDTD), wave scattering.

I. INTRODUCTION

We have witnessed transformation from engineering electromagnetics to electromagnetic engineering [1-4]. This is merely because of technological developments we have had for the last two-three decades. Understanding and using electromagnetic theory has become a must in many engineering disciplines. One important topic is electromagnetic scattering, and diffraction is the most critical phenomena that has been investigated analytically and numerically for a long time [5-36].

Electromagnetic (EM) scattering from wave – object interactions using analytical solutions is limited to structures whose surfaces can be described by orthogonal curvilinear coordinates. Most of these solutions are in the form of infinite series, which are poorly convergent when the dimensions of the object are greater than a few wavelengths. Many practical scattering problems have no closed-form solutions. Because of this, high frequency asymptotic (HFA) techniques have been used when the dimensions of the scattering object are many wavelengths. Both ray-type *Geometrical Theory of Diffraction* (GTD) [5-8] and the wave- (i.e., induced-source)-based *Physical Theory of Diffraction* (PTD) [9-11] have received considerable attention in the past several decades. A short summary on HFA is given in [13].

Diffraction from a two-dimensional (2D) non-penetrable wedge is a canonical structure for all these HFA methods (see, Fig. 1). The source locations 1 and 2 belong to single-side (SSI) and double-side (DSI) illuminations, respectively. Note that, there is a shadow region for SSI where only diffraction fields exist. The two critical angles reflection-shadow boundary (RSB) and incident-shadow boundary (ISB) separate three regions. In Region I, incident, reflected, and diffracted fields exist. In Region II, only incident and diffracted fields exist. In Region III (i.e., in the shadow region) only diffracted fields exist. In the DSI scenario, there are two RSBs which separate regions with and without reflected fields.

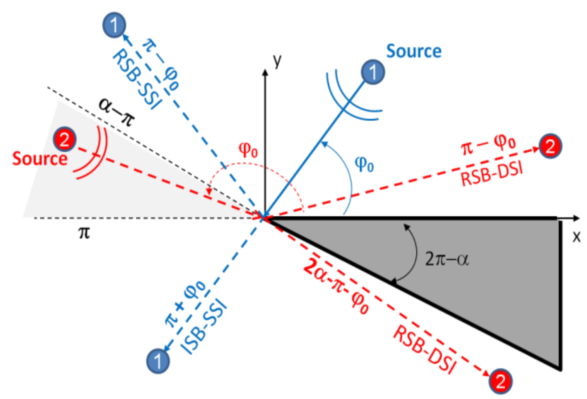

Fig. 1. Wedge scattering scenarios (1: single side illumination; 2: double side illumination).

In the case of acoustic waves, the two boundary conditions (BC) appropriate for the non-penetrable wedge are acoustically soft (SBC) and hard (HBC) wedges. In electromagnetics, they correspond to transverse magnetic (SBC $\rightarrow$ TM) and transverse electric (HBC $\rightarrow$ TE) cases, respectively. The field components used in these two cases, respectively, are the *z*-components of electric (E_z) and magnetic field (H_z) intensities. Mathematically, they are Dirichlet and Neumann BCs, respectively.

Wedge scattering has also been modeled with numerical models such as the Finite-Difference Time-Domain (FDTD) [16,17], Method of Moments (MoM)

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[18-23], and Finite Element Method [33,34]. The following sections summarize these models and techniques with characteristic applications.

II. NUMERICAL MODELING OF DIFFRACTED WAVES

A. FDTD approach

Wedge scattering can be modeled with the FDTD method [20-28] and scattered, reflected and diffracted fields can be separated in both time and frequency domains. Early approaches were based on the separation of incident, reflected, and diffracted pulses in the time domain using time-gating approach [20-22].

The methods discussed in [23-29] use multi-step techniques in separation of both diffracted and fringe fields as pictured in Fig. 2. Here, scenario (a) yields total fields; incident, reflected, and diffracted field components in Region I; incident and diffracted field components in Region II; and only the diffracted fields in Region III.

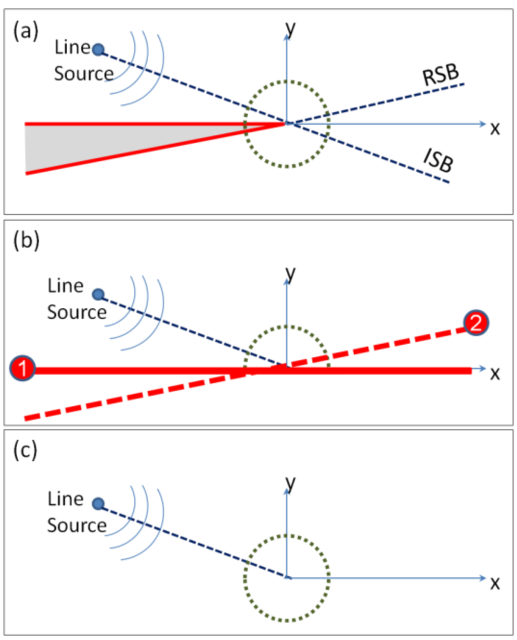

Fig. 2. Multi-step FDTD-based diffraction approach: (a) the wedge scenario, (b) infinite-plane problem, and (c) free-space scenario [25].

Scenario (b) in the figure is the infinite-plane scenario ($\alpha = 180^{\circ}$, Plane-1) which yields total fields on the upper half plane ($0 \leq \varphi < \pi$). Since there is no edge or tip, the total fields include only incident and reflected fields; and do not contain diffracted fields. Finally, scenario (c) yields only the incident fields in the free-space.

The FDTD simulation is run separately for each of these three scenarios and time-domain data are recorded. Subtracting the time data of the second scenario from the first scenario in Region I ($0 \leq \varphi < \pi - \varphi_0$); and the time data of the third scenario from the first scenario in Region II ($\pi - \varphi_0 \leq \varphi < \pi + \varphi_0$) will yield diffracted-only fields all around the wedge [23-25].

B. MoM approach

Diffracted fields can also be obtained with the two-step MoM approach as introduced in [30-32]. Figure 3 shows the two (i.e., the wedge and infinite-plane) scenarios used for this purpose.

MoM is a general procedure and frequency domain approach for solving linear equations. Many problems that cannot be solved exactly can be solved approximately by this method. It has been applied to a broad range of EM problems since the publication of the book by Harrington [37]. A useful tutorial has just been published [38]. MoM is a semi-analytical-numerical model which needs the Green's function solution of the problem at hand.

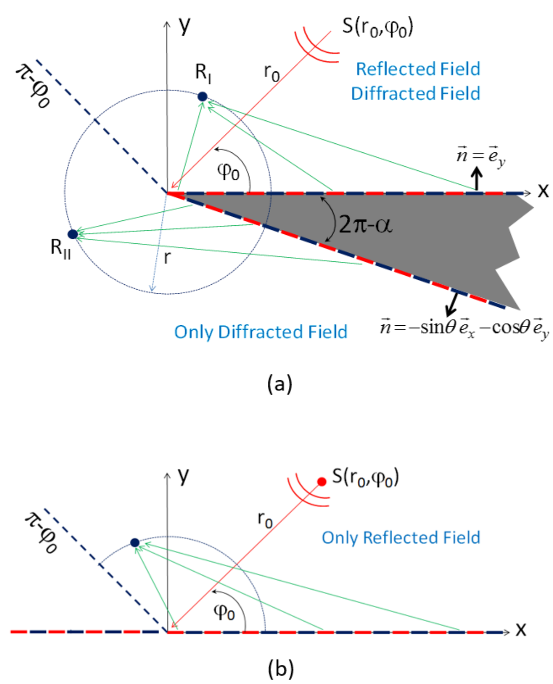

Fig. 3. The two-step MoM-based diffraction approach: (a) the wedge scenario, and (b) infinite-plane problem [30].

The two-step MoM approach [30] is applied as follows. In Fig. 3 (a), incident fields are injected analytically, therefore MoM solutions directly yield the scattered fields which contains reflected and diffracted fields. The MoM solution of the half-plane scenario in Fig. 3 (b) yields the reflected fields in the region up to the critical angle $\varphi=\pi-\varphi_0$. During the MoM implementation of the wedge scattering first, incident fields upon segments are calculated using the Green's function of the problem and the impedance matrix is formed. Then 2N by 2N matrix system is solved and source-induced segment currents are obtained. Finally, scattered fields on the chosen observation points are calculated from the superposition of segment radiations using the Green's function. Total fields are obtained by adding the direct wave from the source to the receiver. The diffracted-only fields can be obtained using the MoM procedure if reflected fields in ($0 \le \varphi < \pi - \varphi_0$) are subtracted. The reflected fields in this region) can be obtained with the scenario in Fig. 3 (b).

C. FEM approach

Field components around the 2D non-penetrable wedge can also be extracted via FEM [33-34]. FEM is a variational method that is developed for approximate solution of boundary value problems governed by partial differential equations. It has been widely used due to its flexibility in handling arbitrary geometries and material non-homogeneities.

Consider, the wedge problem in Fig. 4 (a). The open-region of the computational domain is terminated by PML blocks. The dotted observation circle represents the positions of receivers all of which will record the scattered fields. The three-step diffracted field extraction is as follows:

(i) FEM is run for the structure in Fig. 4 (a) and the scattered fields are recorded on the observation circle.

(ii) FEM is run for the problem in Fig. 4 (b), where the right edge of the object is extended over the vertical direction, and the scattered fields are recorded only on the blue-dotted part of the observation circle. These fields correspond to the reflected fields from the top face of the wedge.

(iii) The same is repeated for the problem in Fig. 4 (c) and fields reflected from the bottom face of the wedge are obtained.

Finally, the diffracted field is obtained by subtracting the fields in steps (ii) and (iii) from the scattered fields in step (i).

III. NUMERICAL MODELING OF FRINGE WAVES

Electromagnetic and/or acoustic waves interact with objects and induce surface currents. These surface currents contain both uniform (PO) and non-uniform (PTD) currents if there is an edge and/or tip. The non-uniform currents are called fringe currents and fields generated by these currents are called fringe fields. In order to calculate fringe waves, one needs to separate source-induced non-uniform and uniform currents.

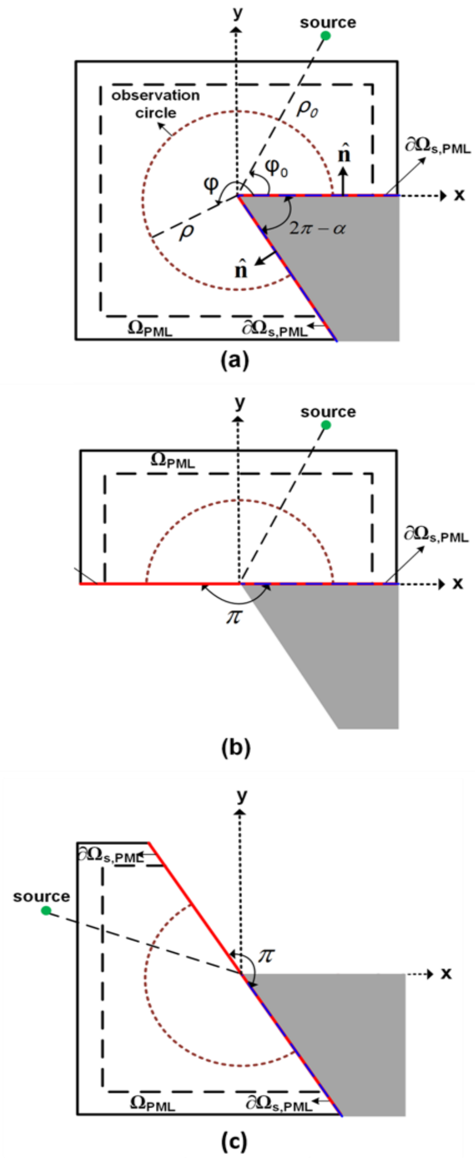

Fig. 4. FEM-based diffraction modeling: (a) original geometry, (b) modified geometry for obtaining the PO currents for SSI, and (c) modified geometry for obtaining the PO currents for SSI [34].

Fringe currents can be extracted with all these three methods (FDTD [29], MoM [31], and FEM [35]) by using similar multi-step procedures. First, standard procedures are applied to the wedge problem and surface currents are obtained. The currents on the illuminated face of the wedge contain both uniform and non-uniform currents; only non-uniform currents exist on the shadow face. Then, infinite-plane scenario is used and (since there is no edge or tip type discontinuity) only uniform currents are obtained. Subtracting (that part of corresponding) infinite-plane currents from the illuminated face wedge currents yields the non-uniform currents on the top face. The bottom face of the wedge already has non-uniform currents. The scattered waves superposed using non-uniform currents then yield the fringe waves.

Note that, the infinite-plane scenario must be repeated for the other face of the wedge for the DSI in both diffracted and fringe field simulations.

IV. NUMERICAL EXAMPLES

The tutorial in [15] summarizes HFA models and the MATLAB based virtual diffraction tool presented in [16] can be used to visualize total and diffracted fields around a 2D non-penetrable wedge. The front panel of this virtual tool is shown in Fig. 5.

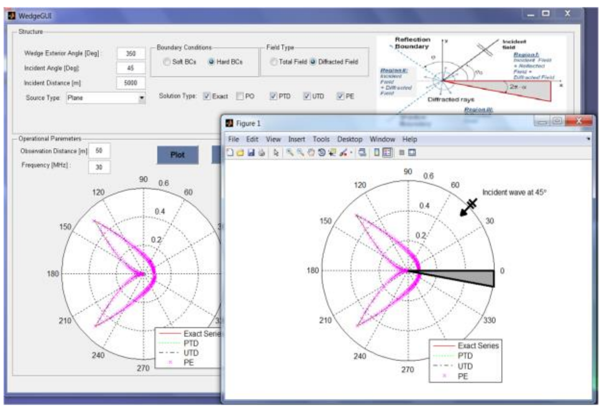

Fig. 5. The front panel of the WedgeGUI tool [16].

The top block of the panel is reserved for the structure. The wedge figure is shown on the top right. The wedge exterior angle, incident distance/angle are supplied on the top left. The user also selects either of the Soft and Hard BCs; total and diffracted fields in this block. A pop-up menu allows the user to choose a plane wave or a line source excitation. For each source type the methods used in simulations are given with tick boxes. Multiple selection is possible. An example generated with this tool is given in Fig. 6. Here, total and diffracted fields for both SBC and HBC cases are shown.

The next examples belong to numerical techniques. In Fig. 7, electromagnetic scattering around a 60°-wedge with non-penetrable boundaries is shown.

Here, MoM results are compared with HFA results. On the left, total fields around the wedge is presented. On the right, only diffracted fields are plotted. The angle of incidence is 60°; this corresponds to SSI.

As observed in the total fields plot, strong interference occurs between incident and reflected fields and lobes are formed. The total field on the shadow region only contains diffracted fields. As observed in the diffracted fields plot, maximum diffraction occurs along the two critical angles.

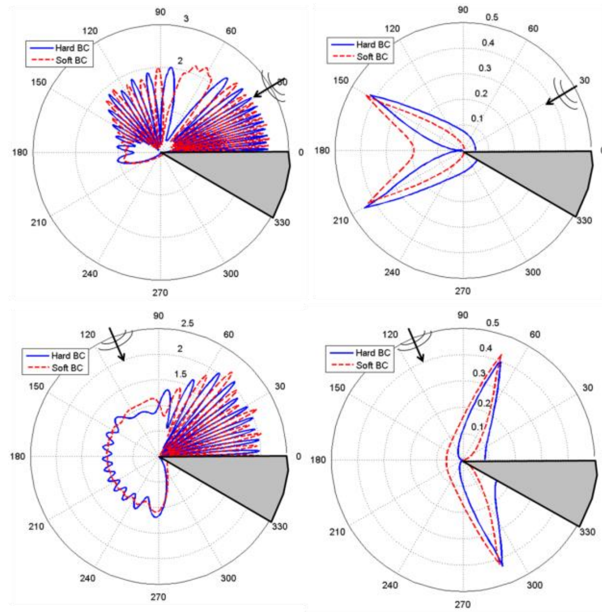

Fig. 6. EM scattering around a 30°-wedge, (Left) total fields, (Right) diffracted fields (r=5λ, kr=31.4).

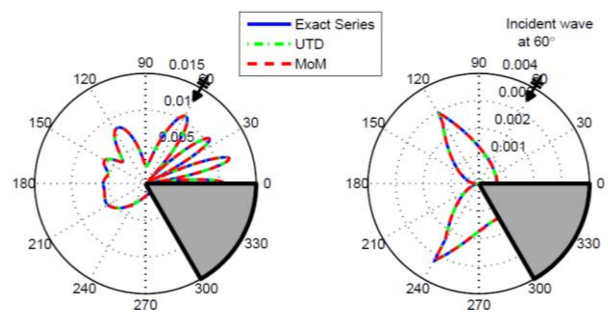

Fig. 7. EM scattering around a 60°-wedge, (Left) total fields, (Right) diffracted fields (TE/HBC case). The receivers are located on a circle around the wedge with radius r=2λ. Plane wave excitation is used [30].

Figure 8 belongs to the same wedge with similar comparisons but for DSI. Here, φ_0=150° and r=2λ. The results belong to HFA, MoM, and FDTD simulations. As observed, there is a perfect agreement among the results.

Fringe waves represent the part of the total edge-diffracted waves generated by source-induced fringe surface currents. These waves can be generated directly using fringe currents. Fringe fields around a 60°-wedge for both SBC and HBC cases are plotted in Fig. 9. Here, only PTD and MoM results are given for a clear visualization.

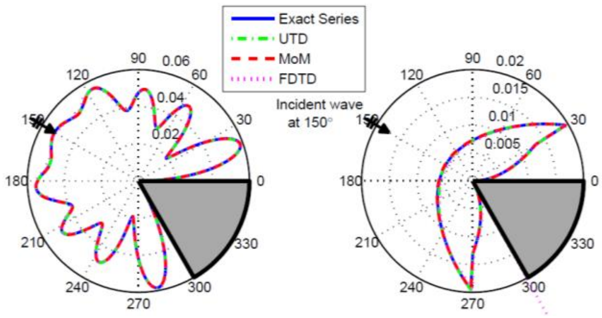

Fig. 8. EM scattering around a 60°-wedge, (Left) total fields, (Right) diffracted fields (TE/HBC case). Plane wave excitation is used [30].

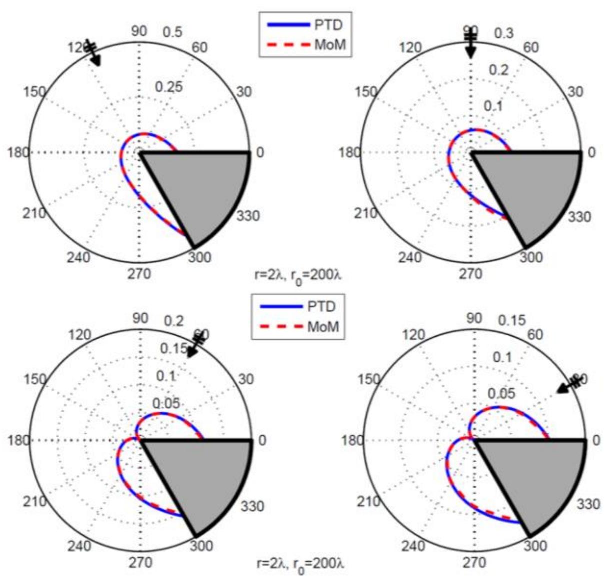

Fig. 9. Fringe fields vs. angle around a 60°-wedge. (Top) TM/SBC case, (Bottom) TE/HBC case. A line source excitation is used [31].

The last example in Fig. 10 belongs to fringe fields around a 30°-wedge with non-penetrable hard boundaries. All the methods are used here. As observed, PTD, MoM, FDTD, and FEM results agree very well; the incorrect result belongs to MTPO [36].

Note that, the free virtual tools presented in [26] (based on FDTD method), [30] and [32] (based on MoM) can also be used to visualize EM scattering around the 2D non-penetrable wedge comparatively.

V. CONCLUSION

Understanding electromagnetic wave scattering is critical in many applications, especially in designing reliable surveillance systems and low visible air and surface targets. This used to be done using approximate analytical models such as GO, GTD, UTD and PO, PTD, widely known as high frequency asymptotics.

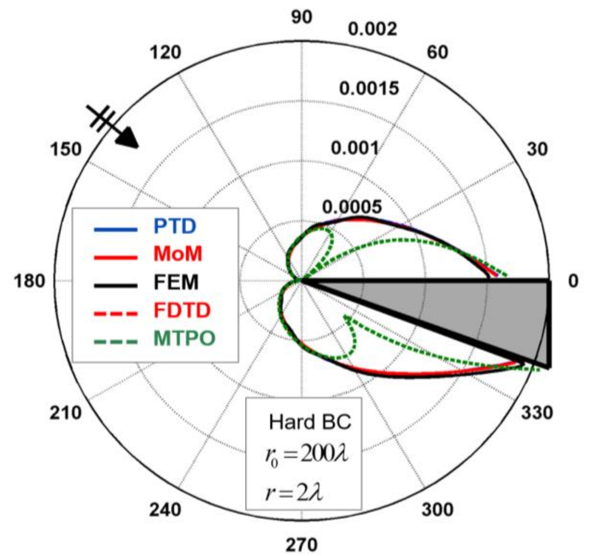

Fig. 10. Fringe fields vs. angle around a 30°-wedge (TE/HBC case).

The GO can model reflections and refractions but fails to account for the field intensity in shadow regions. GTD describes diffraction everywhere except at and near incidence and reflection shadow transitions; UTD removes the discontinuities along these shadow boundaries. However, GO/GTD/UTD fails near caustics. The PTD supplements PO to provide corrections that are due to diffractions at edges of conducting surfaces. Ufimtsev suggested the existence of nonuniform (*fringe*) edge currents in addition to the uniform physical optics surface currents.

Note that, GO/GTD/UTD is simple to apply, can be used to solve complicated problems that do not have exact solutions, provides physical insight into the radiation and scattering mechanisms from the various parts of the structure and can be combined with other techniques, such as MoM, to form a hybrid method. On the other hand, PO/PTD provides correctly only the first asymptotic terms for main components of the scattered field in 3D problems, allows constructing relatively simple solutions of various practical problems, provides uniform asymptotics for the scattered field which are valid both in the ray regions and in the vicinity of foci and caustics, clarifies the physical structure of the scattered field, establishes the diffraction limit of reduction of scattering by absorbing coatings, and can be utilized to develop efficient hybrid techniques.

Parallel to the use of high speed, huge memory computers, novel numerical models have also begun to be used in scattering modeling. Recent studies have focused on the identification and isolation of diffracted and fringe wave components using well-known numerical models such as FDTD, MoM, and FEM. The use and success of these numerical models are promising in modeling and simulation realistic objects in 3D.

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