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# Parallel Implementation and Branch Optimization of EBE-FEM Based on CUDA Platform 

Yan Zhang, Xiuke Yan, Xudong Ren, Sheng Wang, Dongyang Wu, and Baodong Bai

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#### Abstract

The finite element analysis of large complex structures makes higher demand on memory capacity and computation speed, which leads to the inefficiency of traditional serial finite element method (FEM) for such large-scale problems. In this paper, the element-byelement finite element method (EBE-FEM) has been implemented parallelly on CUDA (Compute Unified Device Architecture) platform, and been programmed using $C++$ language. The thread branches that exist in parallel reduction program have been researched and optimized to improve parallel efficiency. The correctness of algorithms and programs are verified by the analysis of an open slot of motor. The optimized parallel program is applied to analyze the main magnetic field of a singlephase transformer. The results show that the EBE-FEM implemented on CUDA platform is more effective than serial EBE-FEM, and branch optimization can improve the speedup further.


Index Terms - Branch optimization, CUDA, EBEFEM, parallel computation.

## I. INTRODUCTION

When FEM is used to analyze the electromagnetic field of large electrical equipment, huge amounts of meshes are needed to get more accurate results, which leads to a large scale of computation. Using traditional serial FEM to calculate large-scale numerical problems, there will be problems such as too long calculation time and large calculation error, sometimes even unable to calculate. Therefore, more and more parallel FEM (PFEM) are developed and applied to numerical calculation and EBE-FEM is one of them [1]. EBE-FEM avoids the formation and storage of the global coefficient matrix, and has no requirement or restriction on the geometric shape and element number of the structure in the region $[2,3]$. The parallel computing of EBE-FEM can be implemented on the elements level, it has high parallelism. Combining EBE-FEM with advanced computing platform can solve the bottleneck problem of large-scale numerical computation.

GPUs represent one of the newest types of parallel
processors. For its many-core nature, it is widely used in parallel calculation $[4,5]$. CUDA is a CPU+GPU heterogeneous parallel computing platform [6,7], which provides a reliable programming environment for GPU to perform data parallel processing. The EBE-FEM can be thought as a method which transforms a highly memory dependent problem to a massively computational dependent one, the latter can be parallelized efficiently. CUDA platform can make full use of the advantages of high parallelism of EBE-FEM [8,9]. GPUs are very good at computing, but weaker in logical judgments. For highparallel and intensive computing, whether it is a small problem or a large problem, running on GPU in parallel is better than it on CPU in serial. Recent years, some CUDA function libraries, such as cuBLAS, cuBLASXT, etc. have been developed to implement computation on GPUs. However, they sometimes show poor speedup performance due to their non-optimized operating process. Thread branch may appear in the process of EBE-FEM parallel implementing on CUDA platform. It is one of the main factors to reduce the parallel efficiency. This problem can be solved by thread-data remapping method [10].

In this paper, EBE-FEM combined with Jacobi preconditioned conjugate gradient (J-PCG) method has been researched to realize parallel computation on CUDA platform. The thread branches in reduction operation has been investigated and thread-data remapping method on instruction-level is proposed to optimize addressing process of reduction operation. All the corresponding programs have been developed in $\mathrm{C}++$. The correctness of the proposed algorithms and programs are verified by the analysis of an open slot of motor. The algorithms and programs have been applied to calculate the magnetic field in a single phase power transformer, the results have been analyzed and discussed.

## II. METHOD DESCRIPTION

## A. EBE technique

Using FEM, the Maxwell electromagnetic field equations can be discretized as linear equations:

$$
\begin{equation*}
\mathbf{A x}=\mathbf{b} \tag{1}
\end{equation*}
$$

where $\mathbf{A}$ is the whole stiffness matrix, $\mathbf{x}$ the vector of unknown variables and $\mathbf{b}$ the system vector.

According to EBE-FEM, (1) can be expressed as: $\left(\sum_{e=1}^{E}\left(\mathbf{Q}^{(e)}\right)^{T} \mathbf{A}^{(e)} \mathbf{Q}^{(e)}\right)\left(\sum_{e=1}^{E}\left(\mathbf{Q}^{(e)}\right)^{T} \mathbf{x}^{(e)}\right)=\left(\sum_{e=1}^{E}\left(\mathbf{Q}^{(e)}\right)^{T} \mathbf{b}^{(e)}\right)$,
where $\mathbf{A}^{(e)}$ is element stiffness matrix, $\mathbf{b}^{(e)}$ the element vector and matrix $\mathbf{Q}^{(e)}$ represents transition between local and global numbering of the unknown variables for the element.

The main operation of CG method is the inner product of vectors, which is appropriate to realize parallel computation. Therefore, CG method has been used to solve the equations of EBE-FEM. The iteration of CG method can be expressed by:

$$
\begin{gather*}
\mathbf{x}_{0}=(0,0, \ldots 0)^{T}, \mathbf{r}_{0}=\mathbf{P}_{0}=\mathbf{b}  \tag{3}\\
\alpha_{k}=\frac{\left(\mathbf{P}_{k}, \mathbf{b}\right)}{\left(\mathbf{P}_{k}, \mathbf{A} \mathbf{P}_{k}\right)}  \tag{4}\\
\mathbf{x}_{k+1}=\mathbf{x}_{k}+\alpha_{k} \mathbf{P}_{k}  \tag{5}\\
\mathbf{r}_{k+1}=\mathbf{b}-\mathbf{A} \mathbf{x}_{k+1} \tag{6}
\end{gather*}
$$

if $\mathbf{r}_{k}<\varepsilon$, stop iteration, $\mathbf{x}=\mathbf{x}_{k}$; else, update $\beta_{k}$ and $\mathbf{P}_{k}$ :

$$
\begin{align*}
\beta_{k} & =\frac{\left(\mathbf{r}_{k+1}, \mathbf{A} \mathbf{P}_{k}\right)}{\left(\mathbf{P}_{k}, \mathbf{A} \mathbf{P}_{k}\right)},  \tag{7}\\
\mathbf{P}_{k+1} & =\mathbf{r}_{k+1}+\beta_{k} \mathbf{P}_{k}, \tag{8}
\end{align*}
$$

return to the equation (4), and continue iteration process.
In EBE method, the inner product ( $\mathbf{r}, \mathbf{r}$ ) and ( $\mathbf{P}, \mathbf{A P}$ ) can be calculated on each element as follow:

$$
\begin{gather*}
(\mathbf{r}, \mathbf{r})=\mathbf{r}^{T} \mathbf{r}=\left(\mathbf{Q}^{T} \mathbf{r}^{e}\right)^{T} \mathbf{Q}^{T} \mathbf{r}^{e}=\sum_{e=1}^{E}\left(\mathbf{r}^{e}\right)^{T} \mathbf{s}^{(e)}  \tag{9}\\
\mathbf{s}^{(e)}=\mathbf{r}^{e} \oplus \sum_{j=\operatorname{adj}(e)} \mathbf{r}^{j}  \tag{10}\\
(\mathbf{P}, \mathbf{A P})=(\mathbf{Q P})^{T} \mathbf{A}^{e} \mathbf{Q} \mathbf{P}=\sum_{e=1}^{E}\left(\mathbf{P}^{(e)}\right)^{T} \mathbf{A}^{e} \mathbf{P}^{(e)},  \tag{11}\\
\mathbf{Q}=\left(\mathbf{Q}^{(1) T}, \mathbf{Q}^{(2) T}, \ldots, \mathbf{Q}^{(E) T}\right)^{T} \tag{12}
\end{gather*}
$$

where $\mathbf{r}$ is the global residual vector, $\mathbf{r}^{e}$ is the local element residual vector, $\oplus$ refers to accumulation of the contribution made by all elements to the nodes of the element and $\operatorname{adj}(e)$ represents the adjacent element which share the common node with element $e$.

In order to improve its convergence further, the Jacobi preconditioned (JP) technology is applied to EBE-CG. The mathematic model of EBE-J-PCG was presented in [9]. The multi-core nature of GPU can exploit the parallelism of EBE-CG method considerably on CUDA platform.

## B. Parallel realization on CUDA platform

CUDA is CPU+GPU heterogeneous computing platform, the programming model ensures that the GPU and CPU complement each other, which executes the complex logic control tasks on CPU and data-parallel computation-intensive tasks on GPU. Implementing EBE-FEM on CUDA, the CPU+GPU collaborative computing model is shown in Fig. 1.


Fig. 1. Flow of CPU+GPU heterogeneous computing model of EBE-FEM.

In GPU solution, the EBE-J-PCG iterating solution for all elements can be operated at the same time. The calculations are performed by kernels with different function on GPU. The parallel computation on CUDA platform is realized by executing the kernel functions in parallel through thousands of threads. The inner product of vectors is executed parallelly for all elements by two kernels are shown as Fig. 2.


Fig. 2. Kernel functions with reduction operations.
The iterative process of EBE-J-PCG method can be given as:
(1) Initialization
(a) Set initial value,

$$
\begin{equation*}
\mathbf{x}_{0}^{(e)}=0, \mathbf{r}^{(e)}=\mathbf{b}^{e}-\mathbf{A}^{e} \mathbf{x}_{0}^{(e)} \tag{13}
\end{equation*}
$$

(b) Jacobi precondition,

$$
\begin{equation*}
\mathbf{m}^{e}=\operatorname{diag}\left(\mathbf{A}^{e}\right) \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{m}^{(e)}=\mathbf{m}^{e} \oplus \sum_{j=\operatorname{adj}(e)} \mathbf{m}^{j} \tag{15}
\end{equation*}
$$

(c) Solve equations,

$$
\begin{equation*}
\mathbf{m}^{(e)} \mathbf{h}^{e}=\mathbf{r}^{e} \tag{16}
\end{equation*}
$$

(d) Calculate $\gamma_{0}=(\mathbf{r}, \mathbf{h})$.

For $\mathrm{e} \in(1,2, \ldots, \mathrm{E})$, use kernel 1 shown as Fig. 2.
(e) Calculate $\mathbf{p}^{(e)}$,

$$
\begin{equation*}
\mathbf{p}^{(e)}=\mathbf{h}^{(e)} \tag{17}
\end{equation*}
$$

(2) Calculate $\alpha$.

For $\mathrm{e} \in(1,2, \ldots, \mathrm{E})$, use kernel 1 and kernel 2 shown as Fig. 2,

$$
\begin{equation*}
\alpha=\frac{\left(\mathbf{P}_{k-1}, \mathbf{h}_{k-1}\right)}{\left(\mathbf{P}_{k}, \mathbf{A} \mathbf{P}_{k}\right)} \tag{18}
\end{equation*}
$$

(3) Update $\mathbf{x}^{(e)}$ and $\mathbf{r}^{e}$,

$$
\begin{align*}
& \mathbf{x}^{(e)}=\mathbf{x}^{(e)}+\alpha \mathbf{p}^{(e)}  \tag{19}\\
& \mathbf{r}^{e}=\mathbf{r}^{e}-\alpha \mathbf{A}^{e} \mathbf{p}^{(e)} \tag{20}
\end{align*}
$$

(4) Solve equations,

$$
\begin{equation*}
\mathbf{m}^{(e)} \mathbf{h}^{e}=\mathbf{r}^{e} \tag{21}
\end{equation*}
$$

(5) Calculate $\gamma_{\text {new }}=(\mathbf{r}, \mathbf{h})$.

For $\mathrm{e} \in(1,2, \ldots, \mathrm{E})$, use kernel 1 shown as Fig. 2.
(6) Judge convergence.

If $\gamma_{\text {new }}<\delta \gamma_{0}$, the calculation stops, else,
update $\mathbf{p}^{(e)}$,

$$
\begin{equation*}
\mathbf{p}^{(e)}=\mathbf{h}^{(e)}+\left(\gamma_{\text {new }} / \gamma_{0}\right) \mathbf{p}^{(e)}, \gamma_{0}=\gamma_{\text {new }} . \tag{22}
\end{equation*}
$$

Then, return to (2), where he is the element preprocessing vector, me is the vector of the main diagonal elements of each element.
Both of two kernels shown in Fig. 2 involve reduction operation. In EBE-J-PCG method, lots of reduction operations generate a large number of thread branches, which eventually lead to a deterioration in parallel efficiency. Although thousands of threads execute the same kernel functions, their operations and processed data may be different, which is due to the different allocation methods of threads.

## C. Branch optimization method of thread-data remapping

Generally, threads are executed in parallel, however, if different threads contain different control conditions, threads that execute different condition paths can only be executed serially. This is called thread branch, which is one of the main factors affecting GPU parallel efficiency.

Path vector is introduced to express thread branches in a piece of code. $\mathbf{V}$ represents all possible thread path sets, take a warp containing 4 threads as an example:

$$
\begin{equation*}
\mathbf{V}[\mathrm{tid}]=\left\{\mathrm{p}_{m}[\mathrm{tid}], \mathrm{p}_{n}[\mathrm{tid}], \mathrm{p}_{k}[\mathrm{tid}], \mathrm{p}_{l}[\mathrm{tid}]\right\}, \tag{23}
\end{equation*}
$$

where $\mathrm{p}[t i d]$ is the execution path, tid is the address of the current thread, and $m, n, k, l$ represent four possible different paths respectively. When there is only one element in $\mathbf{V}$, such as $\mathbf{V}[$ tid $]=\left\{\mathrm{p}_{m}[\right.$ tid $\left.]\right\}$, thread branches do not occur in a warp.


Fig. 3. Original thread-data mapping.
Suppose there are four threads in a warp shown as Fig. 3, squares in two different colors represent the data to be processed differently. The mapping relationship in warp1 is:

$$
\begin{align*}
& \text { Thread } \operatorname{ID}[j](j=0,1) \rightarrow \text { Data1 }[i](i=0,1),  \tag{24}\\
& \text { Thread } \operatorname{ID}[j](j=2,3) \rightarrow \operatorname{Data} 2[i](i=2,3) \tag{25}
\end{align*}
$$

The mapping relationship in warp2 is:
Thread $\operatorname{ID}[\mathrm{j}](\mathrm{j}=4,5) \rightarrow$ Data2 $[\mathrm{i}](\mathrm{i}=4,5)$,
A mapping relationship represents an execution path. The path vector is $\mathbf{V}[t i d]=\left\{\mathrm{p}_{1}[\right.$ tid $], \mathrm{p}_{2}[$ tid $\left.]\right\}$ for both warp1 and warp2. Two execution paths exist in both warp1 and warp2, which results in thread branches.

To eliminate the threads branch, the mapping between thread and data can be reset, switch threads that execute the same code in different warp to the same warp so that all threads in a warp will take the same path.


Fig. 4. Redirected thread-data mapping.
As shown in Fig. 4, change the direction of thread mapping, remap the threads in warp1 and warp2 so that the same type of data will be processed in the same warp. After remapping, the mapping relationship in warp1 is: Thread ID $[\mathrm{j}](\mathrm{j}=0,1,2,3) \rightarrow \operatorname{Data1}[\mathrm{i}](\mathrm{i}=0,1,6,7)$. (28)

The mapping relationship in warp 2 is:
Thread ID $[\mathrm{j}](\mathrm{j}=4,5,6,7) \rightarrow \operatorname{Data} 2[\mathrm{i}](\mathrm{i}=2,3,4,5),(29)$ therefore, path vector has been converted to $\mathbf{V}[$ tid $]=\left\{p_{1}[\right.$ tid $\left.]\right\}$ for warp1 and $\mathbf{V}[$ tid $]=\left\{p_{2}[\right.$ tid $\left.]\right\}$ for
warp2, no thread branches exists in them.

## D. Branch optimization of reduction operation in EBE-FEM

Generally, reduction operation adopts adjacent addressing mode, but this mode will generate many thread branches when EBE-FEM is executed on CUDA platform.


Fig. 5. Adjacent addressing mode.
Figure 5 shows the thread-data mapping of reduction program using adjacent addressing (Assume the reduction operation generates the sum of eight input data). Threads with even address execute addition operation for two adjacent data, threads with odd address do nothing except the parity checking of thread address. Even if only one thread involved in reduction operation, it needs to wait for the other seven threads to perform parity.

All threads in the warp need to be determined the parity, which will generate lots of thread branches. Thread-data remapping method is applied to addressing process to obtain better allocation of threads. Figure 6 shows the optimized addressing mode.


Fig. 6. Optimized addressing mode.
Change the thread data mapping relationship of adjacent addressing, the data involved in reduction operation are divided into two groups on average. The threads that execute addition operation are mapped sequentially to two data from the first group and the second group respectively. The threads in the second group are not involved in any operation, so they can
be stopped by hardware, which will reduce useless operations and waiting. After thread-data remapping, the judgments for parity have been eliminated, thread branches have been reduced, which would improve the computational efficiency.

Branch optimization is an optimization method to GPU parallel technology, which improves parallel efficiency by reducing the number of logical judgments. In a multi-GPU environment, the degree of parallelism is higher and branch optimization is more critical, which can improve parallel efficiency more effectively.

## III. APPLICATION AND ANALYSIS

The proposed method has been applied to analyze the magnetic field in an opening slot of motor and the field produced by a single-phase power transformer. All the programs have been developed in C++. The computations have been carried out on the Intel Xeon E5-2650 v2, 2.6 GHz server with dual GPU (Nvidia Quadro K2000) and 256 GB memory.

The main computation in this paper is the inner product of vectors, which is not complicated, and a single GPU can meet the computing requirements. In the subsequent research of multiphysics coupling problems, if the kernel function involves intensive complex parallel calculations, the multi-GPU method will be considered.

## A. Magnetic field in an opening slot of motor

The opening slot of motor is shown as Fig. 7, it is an example to illustrate the traditional FEM in a book [11]. Due to its very simple model, few meshes and very regular mesh shapes, the magnetic potential on each node can be determined accurately by traditional FEM, and the results are given by the book.


Fig. 7. Model and mesh of opening slot of motor.
In the model, AE and FG are the center lines of stator slot and tooth respectively, and AF is the center line of the air gap. Assuming BC is a magnetic line, rectangular ABDE can be taken as the solving region. Taking scalar magnetic potential $\varphi_{m}$ as variable, the first boundary condition exists on AB and CDE. Setting $\varphi_{m}=0$ on CDE , and $\varphi_{m}=700 \mathrm{~A}$ on AB , the magnetic potential on each node can be obtained.


Fig. 8. Comparison of calculation results.
Figure 8 shows the comparison of the results calculated by parallel program of EBE-J-PCG method and traditional FEM. The maximum local error is $0.83 \%$, which verifies the correctness of the EBE-J-PCG method and program.

## B. Magnetic field analysis of a single-phase transformer

This method is applied to calculate the 2D quasistatic magnetic field distribution of single-phase DSP$241000 \mathrm{kVA} / 500 \mathrm{kV}$ transformer which the secondary side is opened and the primary side is excited by rated current. Figure 9 shows the model and meshes of the transformer. The distribution of the magnetic lines and the magnetic flux density are shown in Figs. 10 and 11.


Fig. 9. The model and mesh of the transformer.


Fig. 10. Distribution of magnetic lines in transformer.


Fig. 11. Distribution of magnetic flux-density in transformer.

To investigate the effect of branch optimization, the transformer model has been divided into different mesh size. Furthermore, the magnetic field in the transformer is calculated by serial EBE-J-PCG, unoptimized parallel EBE-J-PCG and optimized parallel EBE-J-PCG respectively for comparison. The serial EBE-J-PCG program runs on CPU only, while the two parallel programs run on CUDA platform. The computation accuracy of the three methods are the same under the same mesh. Table 1 shows the calculation time of serial EBE-J-PCG program and speedups of parallel EBE-JPCG program in different mesh size. Speedup is obtained by dividing serial time by parallel time (All the time in the table is in minutes).

Table 1: Calculation results of transformer model

| Elements | Serial <br> Time | Parallel without <br> Optimization | Parallel with <br> Optimization |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Time | Speedup | Time | Speedup |
| 8732 | 1.5 | 0.7 | 2.1 | 0.4 | 3.8 |
| 11443 | 2.6 | 1.2 | 2.2 | 0.6 | 4.3 |
| 40816 | 45.2 | 18.5 | 2.4 | 10.2 | 4.4 |
| 171338 | 1541.3 | 593.7 | 2.6 | 174.5 | 8.8 |

It can be seen from Table 1, compared with serial EBE-J-PCG method running on CPU only, parallel implementation on CUDA platform can improve the computation efficiency. However, thread branches in parallel computing can degrade computational efficiency, which can be seen from the similarity of speedup ratios obtained by non-optimized programs in computing different number of mesh. The proposed thread-data remapping method can solve this problem, the speedup ratios of parallel computation are improved obviously after optimization. Moreover, the larger scale computation is involved, the better acceleration can be obtained.

## VI. CONCLUSION

In this paper, the EBE-J-PCG method has been implemented in parallel on CUDA platform, and thread branche in GPU kernel has been researched. The
proposed thread-data remapping method can solve the problem of deterioration in parallel efficiency caused by thread branches, and this optimization method can improve the speedup ratio more obviously. Except the reduction operations, any branching parts of parallel programs based on CUDA platform can adopt the branch optimization method to improve parallel efficiency.

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# High-Order Small Perturbation Method of Arbitrary Order for Conducting Rough Surface Scattering under TE Incidence 

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#### Abstract

A novel closed-form high-order small perturbation method (HOSPM) for the analysis of scattering from 1-D conducting random rough surfaces under TE incidence is developed. The main theoretical contributions of the HOSPM are as follows: (1) our method yields a general high-order SPM form for scattered fields of arbitrary orders, (2) Faà di Bruno's formula is introduced into computational electromagnetics (CEM) for the first time to expand a tapered incident wave and its partial derivatives in power series form, and (3) the form is simple and easy to program and does not require any mathematical pretreatment. Comparisons are made between the method of moments (MOM) and different-order HOSPMs in terms of several aspects, including accuracy and time efficiency. The order convergence of the HOSPM is discussed, the regions of validity with regard to correlation lengths and root mean square (RMS) heights are demonstrated for the 2nd-order HOSPM, and the robustness of the 2 nd-order HOSPM is proven over a broad range of frequencies.


Index Terms - Bistatic scattering coefficients, highorder small perturbation method, rough surface scattering, tapered wave, TE incidence.

## I. INTRODUCTION

The scattering of EM waves from a random rough surface has been an important subject of research in recent decades because of its important applications in many diverse fields, including electromagnetic scattering [1-3], remote sensing [4], oceanography [5], communications [6], materials science [7-8], medical imaging [9] and applied optics [10-11]. Methods of studying rough surface scattering can be categorized into three groups: (1) approximation and analytical methods, (2) numerical methods and (3) semi-analytical methods. Approximation methods are based on physical
approximations and are aimed at providing closed-form formulae for the scattered fields. The basic idea of numerical methods is to discretize the continuous variable and continuous functions, convert the differential equations into the difference equations, convert the integral equations to the form of finite sum, establish the convergent algebraic equations, and use the computer technology to solve the problem. Semi-analytical methods are a combination of numerical methods and analytical methods. The main principle of the semianalytical methods is to reduce the dimension of multidimensional problems by using the family of lowdimensional solutions so as to simplify the calculation.

The small perturbation method (SPM) is a classic approximation method for rough surface with smallscale roughness [12-14]. The SPM produces a series expansion in the surface heights for a scattered field. There are two classic approaches to the SPM. The first one is based on the extended boundary condition (EBC) method. The surface currents on the rough surface are first calculated by applying the extinction theorem. The scattered fields can then be calculated from the diffraction integrals of the surface fields. The second approach makes use of the Rayleigh hypothesis to express the reflected and transmitted fields as upwardand downward-going waves, respectively. The field amplitudes are then determined from the boundary conditions [15]. Both perturbation methods yield the same expansion for the scattered fields.

There have been many quantitative studies aimed at analysis and applications of different-order SPMs. The first-order SPM has been used to predict scattering from multilayer stacks, especially light scattering from multilayer optical coatings [16-17]. The Bornapproximation first-order SPM is frequently applied to high-frequency scattering from marine sediments [18]. In the original paper [19], scattered fields of up to second order in surface height were considered for scattering
from 2-D random rough surfaces. In [20], scattered fields were derived to third order in the surface height, and expressions for the scattered and transmitted powers were developed for deterministic and stochastic surfaces as well as periodic and nonperiodic surfaces. The fourthorder SPM was investigated with regard to scattering from two rough surfaces in a layered geometry in [21].

Most SPM studies can only supply solutions in the form of limited series. Usually, as series order increases, the integral dimensionality increases. The necessary calculations are complicated to code when they involve multiple integrals. Because high-order mathematical forms are always complex and difficult to program, there is no general form for an SPM of arbitrary orders. In order to solve these problems, this paper presents a semianalytical version of the SPM for arbitrary orders, such that the highest order of the method can even be infinite. Therefore, the proposed method is called the high-order SPM (HOSPM). In this method, a tapered incident wave is used to avoid "edge diffraction". Because the incident wave should also be expended in series form, another new consideration in the HOSPM is the introduction of Faà di Bruno's formula to expand a tapered incident wave and its partial derivatives in power series form. This is the first time that Faà di Bruno's formula has been used in CEM.

There are several differences between the classic SPM and our method. (1) The classic SPM makes use of the Rayleigh hypothesis. Our method is based on the Ewald-Oseen extinction theorem [22]. (2) Although both methods use Taylor series to expand the scattered and incident fields, the spectral amplitude inside the Fourier transform is expanded in series form in the classic SPM, whereas the total field is expanded into a Taylor series in our method. (3) In the classic SPM, the scattered fields are classified as either coherent or incoherent waves. In our method, this classification is not needed, and the scattered fields are calculated from the diffraction integrals of the total surface fields. (4) In the classic SPM, the Fourier coefficients are determined via multiintegrals, and the integral dimensionality increases as the order of the terms increases. Our method involves only 1-D integrals regardless of the order and thus is simpler to express in mathematical form and much easier to code. (5) Edge diffraction is not considered in the classic SPM, whereas our method considers tapered waves to eliminate edge diffraction.

The HOSPM is based on the assumption that the boundary conditions are perturbed around those of a smooth surface. To verify the validity of the HOSPM, its solution should be compared with a solution obtained without imposing any restriction on the rough-surface properties. Numerical methods are most suitable for this purpose. One of the most widely used numerical methods is the method of moments (MOM). Therefore,
the MOM is chosen as the comparison method in this study to enable a more conclusive analysis. Simulations performed on 1-D one-layer conducting random rough surfaces with a Gaussian height distribution are reported. Such surfaces are the simplest to describe statistically and can be fully characterized by the surface correlation function. The Thorsos tapered wave [23] is chosen here for its accuracy and broad applications in the truncation of rough surfaces.

The paper is organized as follows. Section II gives the preliminaries. Section III presents all functions involved in the derivation of the HOSPM. In Section IV, many aspects of the HOSPM are analyzed, including accuracy, convergence, time efficiency, memory consumption, the influences of different correlation lengths and root mean square (RMS) heights, and frequency robustness. The conclusion is presented in Section V.


Fig. 1. Geometry for wave scattering by a conducting random rough surface.

## II. PRELIMINARIES

The model under investigation is the 1-D conducting random rough surface shown in Fig. 1, and a twodimensional scattering problem is considered. The analysis is conducted in the frequency domain, with the time-dependence factor $e^{-i \omega t}$ omitted throughout. The case of interest is the TE case, so the electric field has the form:

$$
\begin{equation*}
\mathbf{E}=\mathbf{y} \psi, \tag{1}
\end{equation*}
$$

where $\psi$ is the quantity of the electric field. Throughout the remainder of the paper, the scalar wave equation $\psi$ is studied instead of $\mathbf{E}$ for simplicity.

Because the wave equation $\psi$ is scalar, the scalar Green's Theorem is used, which has the form:

$$
\begin{align*}
& \iint_{V} d v\left[\psi(\mathbf{r}) \nabla^{2} g\left(\mathbf{r}, \mathbf{r}^{\prime}\right)-g\left(\mathbf{r}, \mathbf{r}^{\prime}\right) \nabla^{2} \psi(\mathbf{r})\right] \\
& =\oint_{\mathbf{S}} d \mathbf{S} \cdot\left[\psi(\mathbf{r}) \nabla g\left(\mathbf{r}, \mathbf{r}^{\prime}\right)-g\left(\mathbf{r}, \mathbf{r}^{\prime}\right) \nabla \psi(\mathbf{r})\right], \tag{2}
\end{align*}
$$

where $g\left(\mathbf{r}, \mathbf{r}^{\prime}\right)$ is the Green's function; $\mathbf{S}$ is the closed surface composed of the rough surface $\mathbf{S}_{r}$ (shown in Fig. 1) and the half-circle $\mathbf{S}_{i}$, with a radius extending to
infinity; $\mathbf{r}^{\prime}$ is above the surface $\mathbf{S}_{r}$; and $\mathbf{r}$ is on the surface $\mathbf{S}_{r}$.

Upon the application of the extinction theorem [24], supposing that the surface is perfectly conducting and employing the Dirichlet boundary condition $\psi(\mathbf{r})=0$ for $\mathbf{r}$ on $\mathbf{S}_{r}$, the expression for the scattered field should be:

$$
\begin{align*}
& \psi_{s}\left(\mathbf{r}^{\prime}\right)=\int_{S_{r}} d s \mathbf{n}_{s} \cdot\left[-g\left(\mathbf{r}, \mathbf{r}^{\prime}\right)\right] \nabla \psi(\mathbf{r}) \\
& =-\int_{S_{r}} d s g\left(\mathbf{r}, \mathbf{r}^{\prime}\right)\left[\mathbf{n}_{\mathrm{s}} \cdot \nabla \psi(\mathbf{r})\right]  \tag{3}\\
& =-\int_{-L / 2}^{L / 2} d x g\left(\mathbf{r}, \mathbf{r}^{\prime}\right) \sqrt{1+\left(\frac{d f}{d x}\right)^{2}} \frac{\partial \psi(\mathbf{r})}{\partial n_{s}}
\end{align*}
$$

where $\mathbf{r}^{\prime}$ is located in the upper space; $z=f(x)$ is the surface height profile of the rough surface, with the spectral density function $W(k)$; and $\mathbf{n}_{\mathrm{s}}$ is the unit normal vector of the rough surface pointing toward the upper space.

To prevent current discontinuity at the end points [24], we can use either a tapered incident wave or periodic boundary conditions [25-27]. Because the rough interface modeled in the simulation is of finite size, a tapered incident wave is used. The Thorsos tapered wave has been extensively applied for this purpose because of its low computational expense. The form of the tapered incident wave is as follows:

$$
\begin{equation*}
\psi_{i}(\mathbf{r})=e^{i k\left(x \sin \theta_{i}-z \cos \theta_{i}\right)[1+w(\mathbf{r})]-\frac{\left(x+z \tan \theta_{i}\right)^{2}}{g^{2}}} \tag{4}
\end{equation*}
$$

with

$$
\begin{equation*}
w(\mathbf{r})=\frac{2\left(x+z \tan \theta_{i}\right)^{2} / g^{2}-1}{\left(k g \cos \theta_{i}\right)^{2}} \tag{5}
\end{equation*}
$$

where $k$ is the wave number of free space, $\theta_{i}$ is the incidence angle, and $g$ is the tapering parameter.

## III. FORMULATION

## A. Solutions for the total and scattered fields

Because its variable has the form $\mathbf{r}=x \mathbf{x}+f(x) \hat{\mathbf{z}}$, the field $\psi(\mathbf{r})$ on the random rough surface is related to the profile of the scattering surface $f(x)$. In the case of a slightly rough surface, the RMS height $h$ of the surface is far smaller than both the incident wavelength $\lambda$ and the absolute value of $f(x)$. According to the power series expansion theory, which is valid when the variable (in our case, the absolute value of the height profile, $|f(x)|)$ is small, the field on the rough surface can be expanded as a Taylor series about the field on the mean surface $(z=f(x)=0)$ :

$$
\begin{align*}
& \psi(\mathbf{r})=\left.\sum_{m=0}^{n} \frac{f^{m}}{m!} \frac{\partial^{m} \psi}{\partial z^{m}}\right|_{z=0} \\
& =\left.\psi\right|_{z=0}+\left.f \frac{\partial \psi}{\partial z}\right|_{z=0}+\left.\frac{f^{2}}{2!} \frac{\partial^{2} \psi}{\partial z^{2}}\right|_{z=0}+\left.\frac{f^{3}}{3!} \frac{\partial^{3} \psi}{\partial z^{3}}\right|_{z=0}  \tag{6}\\
& +\left.\frac{f^{n}}{n!} \frac{\partial^{n} \psi}{\partial z^{n}}\right|_{z=0}
\end{align*}
$$

where $\psi$ and $f$ on the right are used in place of $\psi(\mathbf{r})$ and $f(x)$ for brevity. In this equation, $n$ can theoretically approach infinity. $\psi(\mathbf{r})$ is composed of an incident field and a scattered field. The certain part of $\psi(\mathbf{r})$ is the incident field, and the uncertain part is determined by the scattered field. The scattered field on the rough space as well as the total field can be expressed as the sum of the zeroth-order field component, the first-order field component, and all other components up through the $n$ thorder field component, as follows:

$$
\begin{equation*}
\psi_{s}(\mathbf{r})=\sum_{m=0}^{n} \psi_{m}^{s}(\mathbf{r}) . \tag{7}
\end{equation*}
$$

Based on equations (6) and (7), the different-order series expressions for the total field on the surface should be as follows:

$$
\begin{gather*}
\psi_{0}(\mathbf{r})=\left.\psi_{i}\right|_{z=0}+\left.\psi_{0}^{s}\right|_{z=0}  \tag{8.0}\\
\psi_{1}(\mathbf{r}) \\
=\left.\psi_{i}\right|_{z=0}+\left.\psi_{0}^{s}\right|_{z=0}+\left.\psi_{1}^{s}\right|_{z=0}+\left.f \frac{\partial}{\partial z}\left(\psi_{i}+\psi_{0}^{s}\right)\right|_{z=0}  \tag{8.1}\\
\psi_{2}(\mathbf{r})=\left.\psi_{i}\right|_{z=0}+\left.\psi_{0}^{s}\right|_{z=0}+\left.\psi_{1}^{s}\right|_{z=0}+\left.\psi_{2}^{s}\right|_{z=0} \\
+\left.f \frac{\partial}{\partial z}\left(\psi_{i}+\psi_{0}^{s}+\psi_{1}^{s}\right)\right|_{z=0}+\left.\frac{f^{2}}{2} \frac{\partial^{2}}{\partial z^{2}}\left(\psi_{i}+\psi_{0}^{s}\right)\right|_{z=0},  \tag{8.2}\\
\vdots \\
\psi_{n}(\mathbf{r})=\left.\sum_{m=0}^{n} \frac{f^{m}}{m!} \cdot \frac{\partial^{m}}{\partial z^{m}}\left(\psi_{i}+\sum_{l=0}^{n-m} \psi_{l}^{s}\right)\right|_{z=0} \tag{8.n}
\end{gather*}
$$

where $\psi_{i}, \psi_{0}^{s}, \psi_{1}^{s}, \ldots$ and $\psi_{n}^{s}$ are used as abbreviations for $\psi_{i}(\mathbf{r}), \psi_{0}^{s}(\mathbf{r}), \psi_{1}^{s}(\mathbf{r}), \ldots$ and $\psi_{n}^{s}(\mathbf{r})$, respectively. Based on equation (8.n), the partial derivatives of $\psi_{n}(\mathbf{r})$ with respect to $x$ and $z$ should be:

$$
\begin{gather*}
\frac{\partial \psi_{n}(\mathbf{r})}{\partial x}=\left.\sum_{m=1}^{n+1} \frac{f^{m-1}}{(m-1)!} \cdot \frac{\partial^{m-1}}{\partial z^{m-1}}\left(\frac{\partial \psi_{i}}{\partial x}+\sum_{l=0}^{n-m+1} \frac{\partial \psi_{l}^{s}}{\partial x}\right)\right|_{z=0},  \tag{9}\\
\frac{\partial \psi_{n}(\mathbf{r})}{\partial z}=\left.\sum_{m=1}^{n} \frac{f^{m-1}}{(m-1)!} \cdot \frac{\partial^{m}}{\partial z^{m}}\left(\psi_{i}+\sum_{l=0}^{n-m} \psi_{l}^{s}\right)\right|_{z=0} \tag{10}
\end{gather*}
$$

Because equations (8.0) to (8.n) represent total field expressions of different orders, each of them should obey the same boundary condition at the interface. By
simultaneously applying the Dirichlet boundary condition to equations (8.0) ~ (8.n), the 0th-order through $n$ thorder expressions for the scattered field on the mean surface $z=0$ can be obtained as follows:

$$
\begin{gather*}
\left.\psi_{0}^{s}(\mathbf{r})\right|_{z=0}=-\left.\psi_{i}\right|_{z=0}  \tag{11.0}\\
\left.\psi_{1}^{s}(\mathbf{r})\right|_{z=0}=  \tag{11.1}\\
\left.\psi_{2}^{s}(\mathbf{r})\right|_{z=0}=-\left.f \frac{\partial\left(\psi_{i}+\psi_{0}^{s}\right)}{\partial z}\right|_{z=0} \\
 \tag{11.2}\\
-\left.\frac{f^{2}}{2!} \frac{\partial^{2}\left(\psi_{0}^{s}\right.}{\partial z}\right|_{z=0} \\
\left.\psi_{n}^{s}(\mathbf{r})\right|_{z=0}= \\
\vdots  \tag{11.n}\\
\\
\end{gather*}
$$

To obtain the $n$ th-order partial derivatives of the scattered field on the rough surface $\psi_{n}^{s}(\mathbf{r})$, the spectral domain integral is used as follows:

$$
\begin{equation*}
\psi_{n}^{s}(\mathbf{r})=\int_{-\infty}^{\infty} d k_{x} A_{n}\left(k_{x}\right) e^{i\left[k_{x} x+k_{z} f(x)\right]} \tag{12}
\end{equation*}
$$

where $k_{z}=\sqrt{k^{2}-k_{x}^{2}}$. In this way, the scattered field is expressed in the space domain as an accumulated spectrum of waves with different propagation directions and different amplitudes. $A_{n}\left(k_{x}\right)$ is the amplitude of each wave in the spectrum. The following relationships are obtained on the mean surface:

$$
\begin{gather*}
\left.\psi_{n}^{s}(\mathbf{r})\right|_{z=0}=\int_{-\infty}^{\infty} d k_{x} A_{n}\left(k_{x}\right) e^{i k_{x} x},  \tag{13}\\
A_{n}\left(k_{x}\right)=\frac{1}{2 \pi} \int_{-\infty}^{\infty} d x\left[\left.\psi_{n}^{s}(\mathbf{r})\right|_{z=0}\right] e^{-i k_{x} x},  \tag{14}\\
\left.\frac{\partial^{m} \psi_{n}^{s}(\mathbf{r})}{\partial z^{m}}\right|_{z=0}=i^{m} \int_{-\infty}^{\infty} d k_{x} k_{z}^{m} A_{n}\left(k_{x}\right) e^{i k_{x} x},  \tag{15}\\
\left.\frac{\partial \psi_{n}^{s}(\mathbf{r})}{\partial x}\right|_{z=0}=i \int_{-\infty}^{\infty} d k_{x} k_{x} A_{n}\left(k_{x}\right) e^{i k_{x} x},  \tag{16}\\
\left.\frac{\partial^{m}}{\partial z^{m}}\left(\frac{\partial \psi_{n}^{s}(\mathbf{r})}{\partial x}\right)\right|_{z=0}=i^{m+1} \int_{-\infty}^{\infty} d k_{x} k_{x} k_{z}^{m} A_{n}\left(k_{x}\right) e^{i k_{x} x} \tag{17}
\end{gather*}
$$

By calculating the Fourier transform of equation (14), $A_{n}\left(k_{x}\right)$ is determined. The amplitude is substituted into equations (15), (16) and (17), and the inverse Fourier transforms are applied subsequently. Accordingly, the partial derivative terms $\left.\left[\partial^{m} \psi_{n}^{s}(\mathbf{r}) / \partial z^{m}\right]\right|_{z=0}$, $\left.\left[\partial \psi_{n}^{s}(\mathbf{r}) / \partial x\right]\right|_{z=0}$ and $\left.\left\{\partial^{m} / \partial z^{m}\left[\partial \psi_{n}^{s}(\mathbf{r}) / \partial x\right]\right\}\right|_{z=0}$ can
be determined. These partial derivative terms are needed in the calculation of equations (8.0) ~ (8.n), (9) and (10). The partial derivatives of the tapered incident field are derived in Appendix.

## B. Bistatic scattering coefficients

As expressed in equation (3), the scattered field in the upper space can be obtained by integrating the Green's function with the directional derivative of the total field at the interface $\mathbf{S}_{r}$. The two-dimensional Green's function in equation (3) is:

$$
\begin{equation*}
g\left(\mathbf{r}, \mathbf{r}^{\prime}\right)=\frac{i}{4} H_{0}^{(1)}\left(k\left|\mathbf{r}-\mathbf{r}^{\prime}\right|\right) \tag{18}
\end{equation*}
$$

When this Green's function is expanded at infinity, the Hankel function $H_{0}^{(1)}\left(k\left|\mathbf{r}-\mathbf{r}^{\prime}\right|\right)$ can be approximated as follows when $\mathbf{r}^{\prime}$ is located at an infinitely far distance and the observation is in the direction of $\mathbf{k}_{\mathrm{s}}=\sin \theta_{\mathrm{s}} \mathbf{x}+\cos \theta_{\mathrm{s}} \mathbf{z}$ :

$$
\begin{align*}
& H_{0}^{(1)}\left(k\left|\mathbf{r}-\mathbf{r}^{\prime}\right|\right) \approx \sqrt{\frac{2}{\pi k\left|\mathbf{r}-\mathbf{r}^{\prime}\right|}} e^{i\left(k \left\lvert\, \mathbf{r}-\mathbf{r}^{\prime}-\frac{\pi}{4}\right.\right)}  \tag{19}\\
& \approx \sqrt{\frac{2}{\pi k r^{\prime}}} e^{i k r^{\prime}} e^{-i \frac{\pi}{4}} e^{-i k\left[x \sin \theta_{s}+f(x) \cos \theta_{s}\right]} .
\end{align*}
$$

Upon substituting equations (18) and (19) into equation (3) and setting the total field to be of $n$ th-order, the scattered fields can be written as:

$$
\begin{equation*}
\psi_{s}\left(\mathbf{r}^{\prime}\right)=-\frac{i}{4} \sqrt{\frac{2}{\pi k r^{\prime}}} e^{-i \frac{\pi}{4}} e^{i k r^{\prime}} \psi_{n}^{N_{s}}\left(\theta_{s}\right), \tag{20}
\end{equation*}
$$

with

$$
\begin{align*}
& \psi_{n}^{N_{s}}\left(\theta_{s}\right) \\
& =\int_{-L / 2}^{L / 2} d x e^{-i k\left[x \sin \theta_{s}+f(x) \cos \theta_{s}\right]} \sqrt{1+\left(\frac{d f}{d x}\right)^{2}} \frac{\partial \psi_{n}(\mathbf{r})}{\partial n_{s}}, \tag{21}
\end{align*}
$$

where

$$
\begin{equation*}
\frac{\partial \psi_{n}(\mathbf{r})}{\partial n_{s}}=\nabla \psi_{n}(\mathbf{r}) \cdot \mathbf{n}_{\mathrm{s}} \tag{22}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathbf{n}_{\mathrm{s}}=\frac{-\frac{d f}{d x} \hat{x}+\hat{z}}{\sqrt{1+\left(\frac{d f}{d x}\right)^{2}}} \tag{23}
\end{equation*}
$$

Consequently,

$$
\begin{align*}
& \psi_{n}^{N_{s}}\left(\theta_{s}\right) \\
& =\int_{-L / 2}^{L / 2} d x e^{-i k\left[x \sin \theta_{s}+f(x) \cos \theta_{s}\right]}\left[-\frac{d f}{d x} \frac{\partial \psi_{n}(\mathbf{r})}{\partial x}+\frac{\partial \psi_{n}(\mathbf{r})}{\partial z}\right] \tag{24}
\end{align*}
$$

where $\partial \psi_{n}(\mathbf{r}) / \partial x$ and $\partial \psi_{n}(\mathbf{r}) / \partial z$ can be obtained from equations (9) and (10).

The expression for the normalized far-field bistatic scattering coefficients (BSCs) is [24]:

$$
\begin{align*}
& \sigma_{n}\left(\theta_{i}, \theta_{s}\right)=\frac{r^{\prime} S_{s}\left(\mathbf{r}^{\prime}\right)}{-\int_{-\infty}^{\infty} d x\left(S_{\text {inc }} \cdot \hat{\mathbf{z}}\right)_{z=0}} \\
& =\frac{-\frac{1}{2 \eta k} \operatorname{Im}\left[\psi_{n}^{s}\left(\mathbf{r}^{\prime}\right) \nabla \psi_{n}^{s} *\left(\mathbf{r}^{\prime}\right)\right]}{P_{i n c}} . \tag{25}
\end{align*}
$$

For a plane wave, $\mathbf{S}_{i n c} \cdot \hat{\mathbf{z}}=-\cos \theta_{i} /(2 \eta)$, and $\eta$ is the intrinsic impedance of free space. Accordingly, the power $P_{i n c}$ received by the rough surface should be:

$$
\begin{equation*}
P_{i n c}=-\left.\int_{-\infty}^{\infty} d x\left(\mathbf{S}_{\text {incp }} \cdot \hat{\mathbf{z}}\right)\right|_{z=0}=-\int_{-\infty}^{\infty} d x \frac{1}{2 \eta} \cos \theta_{i} . \tag{26}
\end{equation*}
$$

The value of the quantity expressed in equation (26) approaches infinity. This makes the calculation of $\sigma_{n}\left(\theta_{i}, \theta_{s}\right)$ impossible. However, this dilemma can be resolved by using the Thorsos tapered wave as the incident wave[23]. In this case,

$$
\begin{align*}
& \sigma_{n(\text { tapered wave) }}\left(\theta_{i}, \theta_{s}\right) \\
& =\frac{\frac{1}{2 \eta} \frac{1}{8 \pi k}\left|\psi_{n}^{N_{s}}\left(\theta_{s}\right)\right|^{2}}{P_{\text {inc(tapered wave })}}  \tag{27}\\
& =\frac{\left|\psi_{n}^{N_{s}}\left(\theta_{s}\right)\right|^{2}}{8 \pi k g \sqrt{\frac{\pi}{2}} \cos \theta_{i}\left[1-\frac{1+2 \tan ^{2} \theta_{i}}{2 k^{2} g^{2} \cos ^{2} \theta_{i}}\right]},
\end{align*}
$$

where

$$
\begin{equation*}
P_{\text {inc }(\text { tapered wave })}=\frac{\cos \theta_{i}}{2 \eta} g \sqrt{\frac{\pi}{2}}\left(1-\frac{1+2 \tan ^{2} \theta_{i}}{2 k^{2} g^{2} \cos \theta_{i}}\right) . \tag{28}
\end{equation*}
$$

The BSCs can then be determined using equation (27) and the related equations above.

## C. Monte Carlo simulation

To obtain the statistical averages of the BSCs for random rough surfaces, Monte Carlo simulations are used. In the simulation process, independent samples of rough surfaces are first generated, and the BSCs for each sample are individually computed. Then, the statistical averages of the BSCs for $m$ independent computations are determined as follows:

$$
\begin{equation*}
\bar{\sigma}^{m}=\left[\sigma^{m}+\bar{\sigma}^{m-1} \cdot(m-1)\right] / m, \tag{29}
\end{equation*}
$$

where $m(1 \leq m \leq M)$ is the index representing the number of computations, $M$ is the total number of surface realizations, $\bar{\sigma}^{m}$ is the statistical average, and $\sigma^{m}$ is the value from the $m$ th simulation.

## IV. VALIDATION

In this section, several numerical examples are presented to evaluate the HOSPM in terms of many
aspects, including accuracy, convergency, time efficiency, the influences of correlation lengths and RMS heights, and frequency robustness. Because of its wide range of accuracy [28], the MOM is employed as the method for comparison. The general formulation of the MOM is described in [24], and all code for implementing the MOM is based on Dr. Tsang Leung's Electromagnetic Wave MATLAB Library. Because this paper considers a 1-D problem under TE incidence, all simulations default to HH polarization. To fully exploit the computational efficiency of FFT operations, the number of points on the rough surface is set to be a power of two.

## A. Accuracy and convergency

A table and a figure are presented in this subsection. Because subsection D addresses the influences of frequency, a fixed wavelength of $\lambda=1 \mathrm{~m}$ is considered in subsection A, B and C. All of the parameters used to generated the results shown in Fig. 2 are listed in the figure, including the rough surface length $L$, the RMS height $h$, the correlation length $l$, the tapering parameter $g$, the incidence angle $\theta_{i}$, the number of points $N$ on the surface and the number of iterations (samples) $M$; these are also the parameters used for the simulations reported in Table 1.

Table 1: The accuracy of HOSPM on backscattering and forward scattering direction and relative errors between different orders

| $\boldsymbol{n}$ | $\boldsymbol{\Delta}_{\mathbf{b}}(\mathbf{d B})$ | $\boldsymbol{\Delta}_{\mathbf{s}}(\mathbf{d B})$ | $\boldsymbol{\Sigma}_{\boldsymbol{n}}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.59 | 0.40 | $/$ |
| 2 | 0.23 | 0.41 | 0.027 |
| 3 | 0.24 | 0.39 | 0.049 |
| 4 | 0.24 | 0.39 | $1.90 \times 10^{-7}$ |
| 5 | 0.24 | 0.39 | $1.28 \times 10^{-5}$ |
| 6 | 0.24 | 0.39 | $1.13 \times 10^{-7}$ |
| 7 | 0.24 | 0.39 | $1.15 \times 10^{-8}$ |
| 8 | 0.24 | 0.39 | $1.75 \times 10^{-11}$ |

$n$ : order of HOSPM.
$\Delta_{\mathrm{b}}:\left|\sigma_{n(\mathrm{MOM})}-\sigma_{n(\mathrm{HOSPM})}\right|$ in the backscattering angle.
$\Delta_{\mathrm{s}}:\left|\sigma_{n(\mathrm{MOM})}-\sigma_{n(\mathrm{HOSPM})}\right|$ in the forward scattering angle.
$\Sigma_{n}$ : relative error for the $n$ th-order HOSPM.
Figure 2 shows the BSCs for 1-D perfectly electrically conducting (PEC) Gaussian random rough surfaces obtained using different-order HOSPMs and the MOM. In the figure, the curves obtained using the 1st-, 2nd-, 4th- and 8th-order HOSPMs do not show significant differences. The curve of MOM and the curves from HOSPMs match well to the left of the specular scattering point. The value of BSCs generally coincide between the HOSPMs and the MOM over
a wide range of scattering angles from precisely $-89^{\circ}$ to $41^{\circ}$. As can be seen from Fig. 2 in the manuscript, the BSCs generally coincide between the HOSPMs and the MOM over a wide range of scattering angles from approximately $-89^{\circ}$ to $41^{\circ}$, with good matching for many angles.


Fig. 2. Comparisons of the BSCs by 1st-order, 2nd-order, 4th-order and 8th-order HOSPM and MOM.

The biggest difference between the two methods lies in how to get the value of the term $\left[-\frac{d f}{d x} \frac{\partial \psi_{n}(\mathbf{r})}{\partial x}+\frac{\partial \psi_{n}(\mathbf{r})}{\partial z}\right]$ in the integrand in Eq. (21). MOM uses a numerical method which is solving the matrix, and HOSPM uses the method which is a series of formulas related to the series expansion. Therefore, the final values obtained by the two methods are difficult to be exactly the same, which is the main reason why the matching between HOSPM and MOM is not good for angle larger than $41^{\circ}$.

For more specific investigations, two representative angles, the backscattering angle and the forward scattering angle, are selected as objects of study. These two angles are also of the greatest interest compared with other angles.

Table 1 presents the results for the precision of the different-order HOSPMs in the backscattering and forward scattering directions and the convergence among the different-order HOSPMs. Comparisons of precision between $n$ th-order HOSPMs and the MOM in the backscattering and forward scattering directions are given in the second and third rows of Table 1. At 1st order and above, the HOSPM shows high precision, with differences of less than 0.6 dB between the two methods.

Since HOSPM are expressed in the form of series
of arbitrary orders, the order convergence should be discussed. The fourth row in Table 1 shows the iteration errors from the 2nd-order to the 8th-order HOSPM.

The relative errors for BSCs of different orders that are reported in the fourth row of Table 1 are denoted by $\Sigma_{n}$ and are defined as:

$$
\begin{equation*}
\Sigma_{n}=\max \left|\frac{\sigma_{n}\left(\theta_{i}\right)-\sigma_{n-1}\left(\theta_{i}\right)}{\sigma_{n-1}\left(\theta_{i}\right)}\right|,(n \gg 2) . \tag{30}
\end{equation*}
$$

As seen from Table 1, the relative error $\Sigma_{n}$ decreases as the order $n$ increases. If we set $5 \%$ as the threshold for convergence, the values obtained for the 1st-order HOSPM and above are acceptable. Because the relative errors of the 2rd-order HOSPM and above are negligible, the terms of 2 rd-order and above in the series can be ignored, consistent with the error property of power series.

These findings prove that our method is accurate and stable. Considering both the accuracy and convergence results, the BSCs obtained using the 1st-order HOSPM and above can be considered acceptable.

## B. Time and memory consumption

Without the application of any acceleration algorithms, the MOM has a memory requirement of $O\left(N^{2}\right)$ and a computational complexity of $O\left(N^{3}\right)$, where $N$ is the total number of sample points per interface. A precise analysis reveals that, the HOSPM requires $O[(n+1) N]$ memory and $O\left[(n+1)!n^{3} N\right]$ operations. Therefore, the HOSPM is theoretically superior to the MOM in both memory and complexity because of $n \ll N$. The HOSPM also exhibits computational advantages in practice, as shown in Table 2 and Fig. 3. Table 2 presents the time consumption data for the MOM and for the 2nd-, 3rd- and 4th-order HOSPMs, which vary with the number of points on the surface.

Table 2: The time cost by MOM and HOSPM of different orders (s)

| $\boldsymbol{N}$ | $\mathbf{2}^{\mathbf{8}}$ | $\mathbf{2}^{\mathbf{9}}$ | $\mathbf{2}^{\mathbf{1 0}}$ | $\mathbf{2}^{\mathbf{1 1}}$ | $\mathbf{2}^{\mathbf{1 2}}$ | $\mathbf{2}^{\mathbf{1 3}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t$ | 2.13 | 6.51 | 25.13 | 103.70 | 491.51 | 2532.0 |
| $t_{1}$ | 1.69 | 2.97 | 5.75 | 11.50 | 27.57 | 74.48 |
| $t_{2}$ | 4.61 | 8.44 | 16.61 | 35.83 | 87.56 | 238.2 |
| $t_{3}$ | 10.56 | 19.55 | 39.10 | 85.33 | 208.91 | 579.47 |
| $t_{4}$ | 22.68 | 41.81 | 85.62 | 186.47 | 458.19 | 1239.8 |

$t$ : the time cost by MOM.
$t_{1}$ : the time cost by HOSPM1.
$t_{2}$ : the time cost by HOSPM2.
$t_{3}$ : the time cost by HOSPM3.
$t_{4}$ : the time cost by HOSPM4.


Fig. 3. Time cost by MOM and HOSPMs with 1st-, 2nd-, 3rd- and 4th-order (in the unit of second).

Figure 3 illustrates the data summarized in Table 2. The parameters used to generate the results presented in Fig. 3 and Table 2 are listed in the figure. Among the different-order HOSPMs, the 1st-order method exhibits the best time performance. Its time curve is nearly linear, whereas the MOM has a quadratic cubic time curve. As the order of the HOSPM increases, the curve becomes steeper, indicating worse efficiency. However, even the 4th-order HOSPM still shows better efficiency compared with the MOM when the number of points is greater than $2^{12}$. Because of the use of FFT operations, the value of $N$ must have the form $2^{N_{1}}$, where $N_{1}$ is an integer. When $N_{1}=10$, the 1 st- and 2 nd-order HOSPM shows better time performance than the MOM; when $N_{1}=11$, the 1st-, 2nd- and 3rd-order HOSPMs perform better; and when $N_{1}=13$, the HOSPMs all perform better. Thus, we can conclude that our method shows good potential in terms of time efficiency for electrically large surfaces.

Considering the time efficiency, the 1st-order HOSPM is the best choice. Considering all the factor discussed above (the time efficiency, accuracy and convergency), the 2nd-order HOSPM is a better choice. Therefore, the following analysis will focus on the 2 ndorder HOSPM.

## C. Correlation length and RMS height

The correlation length is a fundamental quantity that describes a random rough surface. It provides a benchmark for estimating the level of independence between any two points on a random rough surface. If the distance between two points is larger than the correlation length $l$, the heights of these two points can be approximately regarded as independent. The RMS height $h$ is a fundamental quantity that describes the roughness of a rough surface. The larger the RMS height is, the rougher the surface is. Therefore, the appropriate ranges of RMS heights and correlation lengths in which our method is applicable should be discussed.

Considering the analysis presented in subsection A, the HOSPM will be studied only at 2 nd-order in this subsection and next for brevity. The regions of validity with regard to the backscattering and forward scattering angles for the 2nd-order HOSPM are examined based on two sets of simulations, as shown in Fig. 4. Each set consists of 325 different combinations of values of the varies from 0.1 to 2.5 in increments of 0.1 correlation lengths and RMS heights. The value of $k l$ $(0.16 \mathrm{~m} \leq l \leq 0.40 \mathrm{~m})$, whereas $k h$ varies from 0.05 to 0.65 in increments of $0.05(0.008 \mathrm{~m} \leq h \leq 0.104 \mathrm{~m})$.


Fig. 4. Contour plot of $\left|\sigma_{\text {MOM }}-\sigma_{\text {HOSPM } 2}\right|$ for backscattering and forward scattering direction when $\lambda=1 \mathrm{~m}, \theta_{\mathrm{i}}=45^{\circ}$, $L=25.6 \lambda, g=\lambda / 4, N=256$. Both $\sigma_{\text {MOM }}$ and $\sigma_{\text {HOSPM } 2}$ are in decibels. The area enclosed by pink lines in (a) is the overlapping area for (a) and (b) under the condition $\left|\sigma_{\text {MOM }}-\sigma_{\text {HOSPM } 2}\right| \leq 1 \mathrm{~dB}$.

For the backscattering case, as illustrated in Fig. 4 (a), the 2nd-order HOSPM and the MOM show very good agreement in a broad region where $k l$ spans the domain of the horizontal axis, and $k h$ covers most the area of the figure, leaving only a little space in the upper right corner, where both $k l$ and $k h$ approach maximums.

In the small region near $k l=2.5$ and $k h=0.65(l=0.40 \mathrm{~m}$ and $h=0.104 \mathrm{~m}$ ), the difference between the two methods eventually grows to 3 dB , indicating that the 2 nd-order HOSPM is not suitable for the backscattering case when the RMS height and correlation length are both large.

For the forward scattering case, as illustrated in Fig. 4 (b), there is a large region in which the difference between the two methods is below 1 dB . In this region, the value of $k l$ ranges from 0 to $2.5(0 \leq l \leq 0.40 \mathrm{~m})$, and $k h$ ranges from 0 to $0.65(0 \leq l \leq 0.103 \mathrm{~m})$. The error is approximately 5 dB in a small region near $k l=0.6$ and $k h=0.65$ ( $l=0.10 \mathrm{~m}$ and $h=0.103 \mathrm{~m}$ ). This small region
should be avoided when the HOSPM is used to estimate forward scattering.

The region enclosed by pink lines in Fig. 4 (a) represents the overlapping region for backscattering and forward scattering, where the condition $\mid \sigma_{\mathrm{MOM}^{-}}$ $\sigma_{\text {HOSPM } 2} \mid \leq 1 \mathrm{~dB}$ is satisfied for both cases. In this region, $k l$ ranges from 0 to $2.5(0 \leq l \leq 0.40 \mathrm{~m})$, and $k h$ ranges from 0.15 to $0.5(0.024 \mathrm{~m} \leq l \leq 0.80 \mathrm{~m})$. The overlapping area takes up most of the area under discussion. This provides a broad choice of suitable values when scattering data from both the backscattering and forward scattering directions are needed.

Table 3: The backscattering coefficients and forward scattering coefficients of different frequencies

| $\boldsymbol{f}(\boldsymbol{H z})$ | $\mathbf{B C}(\mathbf{d B})$ | $\mathbf{F C}(\mathbf{d B})$ | $\boldsymbol{f}(\boldsymbol{H z})$ | $\mathbf{B C}(\mathbf{d B})$ | $\mathbf{F C}(\mathbf{d B})$ | $\boldsymbol{f}(\boldsymbol{H z})$ | $\mathbf{B C}(\mathbf{d B})$ | $\mathbf{F C}(\mathbf{d B})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 . 0 G}$ | -20.27 | 11.08 | $\mathbf{1 5 . 0 G}$ | -19.78 | 11.08 | $\mathbf{3 0 0 G}$ | -20.06 | 11.1 |
| 1.5G | -21.14 | 11.07 | $\mathbf{3 0 . 0 G}$ | -19.89 | 11.08 | $\mathbf{3 3 3 G}$ | -20.37 | 11.08 |
| 3.0G | -20.35 | 11.06 | $\mathbf{3 3 . 3 G}$ | -19.37 | 11.06 | $\mathbf{3 7 5 G}$ | -20.20 | 11.09 |
| 3.3G | -19.95 | 11.04 | $\mathbf{3 7 . 5 G}$ | -19.35 | 11.12 | $\mathbf{4 2 9 G}$ | -19.61 | 11.10 |
| 3.7G | -20.20 | 11.13 | $\mathbf{4 2 . 9 G}$ | -20.37 | 11.09 | $\mathbf{5 0 0 G}$ | -19.83 | 11.07 |
| 4.3G | -20.82 | 11.13 | $\mathbf{5 0 . 0 G}$ | -19.81 | 11.10 | $\mathbf{6 0 0 G}$ | -19.90 | 11.08 |
| 5.0G | -21.32 | 11.08 | $\mathbf{6 0 . 0 G}$ | -20.12 | 11.09 | $\mathbf{7 5 0 G}$ | -20.83 | 11.11 |
| $\mathbf{6 . 0 G}$ | -19.68 | 11.12 | $\mathbf{7 5 . 0 G}$ | -20.13 | 11.08 | $\mathbf{1 . 0 T}$ | -19.73 | 11.04 |
| 7.5G | -20.21 | 11.09 | $\mathbf{1 0 0 . 0 G}$ | -19.51 | 11.05 | $\mathbf{1 . 5 T}$ | -19.62 | 11.09 |
| $\mathbf{1 0 . 0 G}$ | -20.04 | 11.07 | $\mathbf{1 5 0 . 0 G}$ | -19.71 | 11.12 | $\mathbf{3 . 0 T}$ | -19.86 | 11.07 |

BC : the abbreviation of backscattering coefficient.
FC: the abbreviation of forward scattering coefficient.

## D. Frequency robustness

As discussed above, the 2nd-order HOSPM has been proven to be good for both the backscattering and forward scattering angles. This section will focus on the frequency stability of the BSCs for these two angles. The analysis considers a range of frequencies from 1.0 GHz to 3.0 THz . A table and a figure are presented to illustrate the analysis. The univariate boxplot introduced by [29] is used to analyze the frequency robustness of the HOSPM, as shown in Fig. 5.

- Table of backscattering and forward scattering coefficients (Table 3). Both the backscattering and forward scattering coefficients are stable for frequencies from 1.0 GHz to 3 THz . The values of the backscattering coefficients vary only slightly. The difference between the largest and smallest values is less than 2 dB . The values vary even less for the forward scattering coefficients, with a difference between the largest and smallest values of no more than 0.1 dB .
- Box plot (Fig. 5). This plot visualizes the backscattering and forward scattering coefficients shown in Table 3 for different frequencies. The box plot displays the distributions of the sorted backscattering and forward scattering coefficients, where the observations along the x axis represent the different observation sets and the observations on the $y$ axis represent the values obtained. For both sets, most values are very closely clustered, with no outliers. This plot also yields various statistical summary measures for the data in our sets. For the backscattering coefficients, the: minimum value is -21.14 dB , the median is -20.26 dB , and the maximum value is -19.65 dB . For the forward scattering coefficients, the: minimum value is 11.03 dB , the median is 11.09 dB , and the maximum value is 11.16 dB .
In brief, our method can be applied over a wide range of frequencies with high numerical stability.


Fig. 5. Boxplot: backscattering and forward scattering coefficients by 2nd-order HOSPM among different frequencies.

## V. DISCUSSION

The proposed method provides general explicit closed-form HOSPM expressions of arbitrary orders for solving the problem of the EM fields scattered from 1-D conducting random rough surfaces under TE incidence. The main contributions of this new method are four-fold: first, the surface field is treated as a function of the surface profile, allowing the total field to be expanded in power series form; second, the classic SPM is restricted to series of order two, whereas the applicability of our method extends to arbitrary orders; third, Faà di Bruno's formula is introduced into CEM for the first time to expand a tapered incident wave and its partial derivatives in power series form; and fourth, the obtained mathematical form is simple and easy to program and does not require any mathematical pretreatment.

In the context considered here, our method has been validated from many perspectives.

- In simulation A, the 1st-, 2nd-, 4th- and 8th-order HOSPMs were compared with the numerically exact MOM. The accuracy of the HOSPM at 1storder and above was verified to be high. The convergency of the HOSPM was discussed with regard to the backscattering and forward scattering angles. The 1st-order form and above are considered to be satisfactory.
- In simulation B, the computational complexities of the 1st-, 2nd-, 3rd-, and 4th-order HOSPMs and the MOM were compared to investigate the efficiency of the proposed method. The results show that the larger the scale of the problem is, the better the HOSPM performs. Considering all factors mentioned above (accuracy, convergence, and time efficiency), the 2nd-order HOSPM applied to the
backscattering and forward scattering angles was selected as a representative pair of cases for further study.
- In simulation C, the regions of validity of the 2 ndorder HOSPM with respect to correlation lengths and RMS heights were investigated. It was shown that the HOSPM exhibits high precision and stability over a broad range of correlation lengths and RMS heights.
- In simulation D , the values of the backscattering and forward scattering coefficients were determined for 30 different frequencies. The method was confirmed to be stable over a broad frequency spectrum ranging from 1.0 GHz to 3.0 THz .


## VI. CONCLUSION

We can conclude that HOSPM is a method with a rigorous mathematical form, high accuracy, high efficiency and greater suitability for application to a wide variety of different rough surfaces and different frequencies. Moreover, this model can further be used to study radar echoes from dynamic ocean surfaces and various inclusions in studies of composite EM scattering from targets and sea backgrounds. A fast and accurate forward model is necessary to ensure a successful inversion process. Therefore, our method is necessary and important for many inversion scenarios. It can be used in the retrievals of subsurface soil moisture measurements, in planetary exploration, and in the analysis of other natural scenes and can also serve as an important tool for radar system design. Hence, our future work will focus on higher-order solutions for dielectric or multilayer rough surfaces with an arbitrary number of rough interfaces under both TE and TM incidence.

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## APPENDIX: EXPRESSIONS FOR THE PARTIAL DERIVATIVES OF A TAPERED INCIDENT WAVE

The computation of the derivatives of $e^{f(x)}$ is both useful and ubiquitous in analysis. Fà $\{\mathrm{a}\}$ di Bruno's formula, named after Francesco Faà di Bruno, is a mathematical identity for generalizing this problem.

Because a Thorsos tapered incident wave can be expressed as,

$$
\begin{equation*}
\psi_{i}(\mathbf{r})=e^{\chi(\mathbf{r})} \tag{31}
\end{equation*}
$$

where

$$
\begin{gather*}
\chi(\mathbf{r})=i k\left(x \sin \theta_{i}-z \cos \theta_{i}\right)(1+w(\mathbf{r})) \\
-\frac{\left(x+z \tan \theta_{i}\right)^{2}}{g^{2}} \tag{32}
\end{gather*}
$$

the series expansion of this tapered incident wave can be derived using Faà di Bruno's formula[30]. This expansion has the form:

$$
\begin{align*}
& \psi_{i}^{(m)}(\mathbf{r}) \\
& =\sum_{k=1}^{m} \psi_{i}^{(k)}(\mathbf{r}) B_{m, k}\left(\chi^{\prime}(\mathbf{r}), \chi^{\prime \prime}(\mathbf{r}), \cdots, \chi^{(m)}(\mathbf{r})\right) . \tag{33}
\end{align*}
$$

Here, the $B_{m, k}\left(\chi^{\prime}(\mathbf{r}), \chi^{\prime \prime}(\mathbf{r}), \cdots, \chi^{(m)}(\mathbf{r})\right)$ are the Bell polynomials, which have the form,

$$
\begin{align*}
& B_{m, k}\left(\chi^{\prime}(\mathbf{r}), \chi^{\prime \prime}(\mathbf{r}), \cdots, \chi^{(m)}(\mathbf{r})\right) \\
& =\sum_{\substack{a_{1}+2 a_{2}+\cdots+m_{m} m \\
a_{1}+a_{2}+\cdots+a_{m}-k}} \frac{n!\left[\chi^{\prime}\right]^{a_{1}}\left[\chi^{\prime \prime}\right]^{a_{2}} \cdots\left[\chi^{(m)}\right]^{a_{m}}}{a_{1}!(1!)^{a_{1}} a_{2}!(2!)^{a_{2}} \cdots a_{m}!(m!)^{a_{m}}} \tag{34}
\end{align*}
$$

where $\chi^{\prime}, \chi^{\prime \prime}$ and $\chi^{(m)}$ are used as abbreviations for $\chi^{\prime}(\mathbf{r}), \chi^{\prime \prime}(\mathbf{r})$ and $\chi^{(m)}(\mathbf{r})$, respectively, and $a_{1}, a_{2}$, $\ldots$ and $a_{m}$ are all integers greater than or equal to zero. The Bell polynomials also satisfy the following relation ( $m \geqslant 1$ ):

$$
\begin{equation*}
k B_{m, k}=\sum_{l=k-1}^{m-1}\binom{m}{l} \chi^{(m-l)}(\mathbf{r}) B_{m, k-1}, \tag{35}
\end{equation*}
$$

where $B_{\mathrm{m}, k}$ and $B_{\mathrm{m}, k-1}$ are used as abbreviations for $\quad B_{m, k}\left(\chi^{\prime}(\mathbf{r}), \chi^{\prime \prime}(\mathbf{r}), \cdots, \chi^{(m)}(\mathbf{r})\right) \quad$ and $B_{m, k-1}\left(\chi^{\prime}(\mathbf{r}), \chi^{\prime \prime}(\mathbf{r}), \cdots, \chi^{(m)}(\mathbf{r})\right)$, respectively. This relation is useful for programming the necessary calculations.

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# Electromagnetic Waves Interaction with a Human Head Model for Frequencies up to $100 \mathbf{~ G H z}$ 

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#### Abstract

Specific absorption rate (SAR), penetration depth, and temperature rise in a one-dimensional (1D) dispersive human head model due to electromagnetic fields radiated by wireless communication systems operated up to 100 GHz are evaluated with the use of a Multiphysics model. In this model, the Debye model of human head tissue parameters is integrated into the finite-difference time-domain method with the use of the auxiliary differential equation to obtain solutions at multiple frequencies of interest using a single simulation. Then, the SAR, peneration depth, and temperature rise in the 1D head model are calculated for each frequency of interest. The effects of frequency on the SAR, penetration depth, and temperature rise in the head are investigated.


Index Terms - Dispersive tissues, FDTD method, human safety standard, penetration depth, millimeterwave radiation, SAR, temperature rise.

## I. INTRODUCTION

Due to the improvement in wireless communication applications such as fifth generation (5G) mobile systems [1], radar systems [2] for military and automotive industries, and medical treatment [3], the use of electromagnetic (EM) fields in centimeter and millimeter wave ranges is increasing day by day. For 3G mobile communication system, the frequency band was less than 6 GHz and for 5 G mobile communication system the frequency band is between 24 GHz and 52 GHz . In the next few years, we start to enter into the 6 G mobile communication system. Therefore, we should consider the effect of EM wave containing 100 GHz or higher frequency band on human tissues. It is important to investigate the absorption of EM energy and resulting temperature rise in human tissues due to EM fields in these frequency ranges.

In order to limit the temperature rise in the tissues resulting from the absorption of EM energy due to EM field exposure, international EM safety guidelines/ standards published by the Federal Communication Commission (FCC) [4], the International Commission on Non-Ionizing Radiation Protection (ICNIRP) [5], IEEE (C95.1-2005) [6], and Safety Code 6 (SC-6) [7] provide basic restrictions for the amount of absorbed EM energy in tissues and limits of incident power density (IPD). Table 1 gives maximum permissible exposure (MPE) limits of IPD, dependent on frequency range and exposure type (general public exposure (GPE) or occupational exposure (OE)).

The amount of EM energy absorbed by biological tissues is defined as specific absorption rate (SAR). The SAR presents an EM heat source for the tissues. The peak-spatial SAR averaged over 1 g of tissue $\left(\mathrm{SAR}_{1 \mathrm{~g}}\right)$ has been used as a restriction for frequencies from 100 kHz to 6 GHz in FCC, SC-6, and old versions of IEEE standards. The specified limits of $\mathrm{SAR}_{1 \mathrm{~g}}$ in head, neck, and trunk are $1.6 \mathrm{~W} / \mathrm{kg}$ for GPE and $8 \mathrm{~W} / \mathrm{kg}$ for OE, respectively. In the updated version of IEEE standards, the peak-spatial SAR is calculated over 10 g of tissues $\left(S_{10 \mathrm{~g}}\right)$ instead of 1 g and the upper frequency limit for evaluating SAR values has been changed from 6 GHz to 3 GHz . In ICNIRP, the $\mathrm{SAR}_{10 \mathrm{~g}}$ value is a good measure for assessing absorbed energy up to 10 GHz . The specified limits of $S A R_{10 g}$ are $2 \mathrm{~W} / \mathrm{kg}$ for GPE and 10 $\mathrm{W} / \mathrm{kg}$ for OE, respectively. In the IEEE standard, the frequency range from 3 GHz to 6 GHz is considered as a transition region for SAR and IPD. At frequencies above 6 GHz for FCC and SC-6, above 3 GHz for IEEE standard, and above 10 GHz for ICNIRP, SAR is not considered appropriate for evaluating exposure, and thus IPD is considered as a restriction.

At frequencies below 6 GHz , effects of EM fields
due to near-field or far-field sources on the threedimensional (3D) human head [8-15] and human eyes [16-19] have been investigated extensively using the finite-difference time-domain (FDTD) method. Furthermore, a one-dimensional (1D) multi-layered human head model [20-22] and body model [23-24] exposed to far-field sources at frequencies below 6 GHz have been studied using the FDTD method.

Table 1: MPE limits of safety standards/guidelines for GPE and OE

| Safety <br> Standards | Frequency <br> $(\boldsymbol{f}: \mathbf{G H z})$ <br> $\mathbf{G P E} / \mathbf{O E}$ | MPE Limit <br> for GPE <br> $\left(\mathbf{w} / \mathbf{m}^{2}\right)$ | MPE Limit <br> for OE <br> $\left(\mathbf{w} / \mathbf{m}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| IEEE [6] | $0.4-2 / 0.3-3$ | $f / 0.2$ | $f / 0.03$ |
|  | $2-100 / 3-300$ | 10 | 100 |
| FCC [4] | $0.3-1.5$ | $f / 0.15$ | $f / 0.03$ |
|  | $1.5-100$ | 10 | 50 |
|  | $0.4-2$ | $f / 0.2$ | $f / 0.04$ |
| SC- 6 [7] | $0.3-6 / 0.1-6$ | $0.02619 f^{0.6834}$ | $0.6455 f^{0.5}$ |
|  | $6-150$ | 10 | 50 |

At frequencies above 6 GHz , effects of EM fields on the 3D human head and human eye models have not been well investigated using the FDTD method, except for a few studies [25-29]. A 3D anatomical eye model exposed to EM fields was investigated at frequencies of 6,18 , and 30 GHz in [26] and 77 GHz in [25]. In [27], a 3D human brain model and eye model exposed to EM fields were investigated at frequencies between 1 GHz to 10 GHz . In [28-29], a 3D human head model with a dipole antenna was analyzed at frequencies up to 30 GHz . For frequencies above 30 GHz , the minimum wavelength in head tissues are very small and thus the 3D human head model has not been studied using the FDTD method, due to excessively long computation times and large memory requirements when the FDTD cell size is in the order of 0.05 of the wavelength in the tissue. Therefore, the FDTD method has been used to analyze a 1D multi-layered human model in [29-30] for frequencies from 1 to 30 GHz and from 3 to 300 GHz , respectively, and a part of the 3D human face model included eye tissues in [31] for frequencies up to 100 GHz due to far-field sources. Furthermore, analysis of a 1D multi-layer human tissue model for frequencies up to 100 GHz in [25] and up to 300 GHz in [32] has been carried out using an analytical method.

All biological tissues are dispersive, thus their EM parameters such as relative permittivity and conductivity change with frequency. Therefore, the EM simulation of the human head must be repeated for each frequency of interest, which leads to a large increase in computation time. In all previous work, except for the studies in [13-

14], each EM simulation has been performed for only one frequency of interest. In order to reduce the required computation time and get solutions for multiple frequencies of interest in a single simulation, an algorithm called a Multiphysics model was proposed in [13-14]. This model can be used to analyze the SAR, temperature rise, and radiation penetration depth in the human head at multiple frequencies in a single simulation. This model is based on the Debye representation of human head tissues which was conducted here for frequencies up to 100 GHz and is utilized in the FDTD formulation [33] for the dispersive tissues based on the auxiliary differential equation. Then, calculations of SAR and temperature rise using the Pennes bioheat equation [34] are performed. The Debye model of the tissues is constructed with three-term coefficients for three different frequency ranges: 100 MHz to $2 \mathrm{GHz}, 2 \mathrm{GHz}$ to 20 GHz , and 20 GHz to 100 GHz . These coefficients are determined by following the analysis in [35], based on data obtained from [36]. Therefore, the penetration depth, SAR and resulting temperature rise distributions in the human head model due to EM radiated fields can be calculated for a wide range of frequencies up to 100 GHz using the Multiphysics model.

In this paper, two 1D sections of the multi-layer head models based with and without eye tissues obtained from a 3D MRI images of the human head model are investigated using the Multiphysics model to show the effect of tissue types in the head model and to obtain the penetration depths, SAR and temperature rise distributions due to a far-field source at the frequencies up to 100 GHz using a single FDTD simulation. Eye tissues are chosen for investigation because, due to a lack of blood flow, they are most sensitive to EM heat sources. Numerical results obtained in this work are compared with published results for selective frequencies to assess the accuracy of the Multiphysics model and are useful for the development of EM safety guidelines/ standards at frequencies up to 100 GHz with faster simulation tool.

## II. NUMERICAL METHOD AND MODELS

## A. 1D multi-layer human head models

1D multi-layer head models analyzed in this work are obtained from a 3D realistic human head model [37]. The 3D head model consists of 21 biological tissues: skin, fat, bone, brain (grey and white matter), blood vessel, cartilage, cerebellum, cerebral fluid, cornea, lens, dura, eye sclera, gland, mucous membrane, muscle, nerve, tongue, tooth, trachea, and vitreous humor. Figure 1 shows a horizontal cross-section of the 3D human head model which consists of 2324(width) $\times 3120$ (depth) cubic cells.

Two cuts (Layer-A and Layer-B) in Fig. 1 are
leading to a planar 1D models for the investigation in this work. The Layer-A represented contains human head tissues without eye tissues, whereas the Layer-B contains human head tissues with eye tissues such as lens, cornea, eye sclera, and vitreous humor.


Fig. 1. Horizontal cross-section of a 3D human head model. (Layer A: solid line and Layer B: dashed line)

Table 2: Debye parameters of tissues for frequencies of 20 GHz to 100 GHz

| Tissue | $\boldsymbol{\varepsilon}_{\infty}$ | $\Delta \boldsymbol{\varepsilon}_{\boldsymbol{s} 1}$ | $\Delta \boldsymbol{\varepsilon}_{\boldsymbol{s} 2}$ | $\Delta \boldsymbol{\varepsilon}_{\boldsymbol{s} 3}$ | $\boldsymbol{\tau}_{\mathbf{1}}[\mathbf{p s}]$ | $\boldsymbol{\tau}_{\mathbf{2}}[\mathbf{p s}]$ | $\boldsymbol{\tau}_{\mathbf{3}}[\mathbf{p s}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Skin | 4.030 | 0.125 | 32.419 | 22.833 | 1.449 | 7.233 | 160.7 |
| Fat | 2.566 | 0.339 | 1.182 | 1.424 | 1.321 | 4.985 | 16.95 |
| B. Cortical | 2.647 | 0.695 | 2.683 | 6.163 | 1.354 | 5.566 | 19.22 |
| B. Marrow | 2.565 | 0.335 | 1.158 | 1.407 | 1.305 | 4.911 | 16.16 |
| Cartilage | 4.371 | 2.030 | 11.251 | 25.934 | 1.489 | 6.305 | 20.23 |
| Blood | 4.498 | 3.684 | 32.111 | 26.674 | 1.668 | 6.403 | 20.63 |
| Muscle | 4.490 | 3.793 | 29.742 | 19.355 | 1.606 | 5.797 | 17.37 |
| Tongue | 4.451 | 3.348 | 27.925 | 21.646 | 1.609 | 6.059 | 17.37 |
| Tooth | 2.647 | 0.695 | 2.683 | 6.163 | 1.354 | 5.566 | 19.22 |
| Trachea | 2.851 | 2.590 | 21.581 | 16.693 | 1.619 | 6.103 | 18.07 |
| Eye Sclera | 4.460 | 3.447 | 28.933 | 22.031 | 1.641 | 6.169 | 19.10 |
| Cornea | 4.465 | 3.452 | 28.828 | 22.807 | 1.667 | 6.276 | 21.60 |
| Lens | 4.376 | 2.803 | 23.426 | 18.052 | 1.610 | 6.059 | 69.76 |
| V. Humor | 6.552 | 34.203 | 11.316 | 139.74 | 6.701 | 19.16 | 873.5 |
| Nerve | 4.251 | 1.827 | 15.117 | 11.972 | 1.628 | 6.155 | 19.21 |
| Cerebellum | 4.403 | 2.954 | 24.443 | 20.383 | 1.674 | 6.315 | 22.97 |
| Dura | 4.602 | 3.754 | 19.037 | 19.513 | 1.479 | 5.559 | 18.61 |
| CSF | 4.614 | 4.717 | 39.895 | 31.694 | 1.711 | 6.386 | 24.34 |
| Gland | 4.494 | 3.673 | 30.674 | 23.645 | 1.609 | 6.058 | 17.31 |
| M. | 4.368 | 2.711 | 22.538 | 17.662 | 1.629 | 6.146 | 18.93 |
| Membrane |  |  |  |  |  |  |  |
| W. Matter | 4.315 | 2.212 | 18.093 | 14.502 | 1.594 | 6.058 | 17.77 |
| G. Matter | 4.428 | 3.123 | 25.887 | 20.355 | 1.622 | 6.128 | 18.66 |

## B. Debye coefficients of human head tissues

The Debye coefficients of the tissues are needed to obtain solutions at multiple frequencies of interest in a single EM simulation. The three-term Debye coefficients obtained by using the numerical technique proposed in [35] are used to accurately fit the experimental data provided in [36] for the biological tissues in the frequency ranges of 100 MHz to $2 \mathrm{GHz}, 2 \mathrm{GHz}$ to 20 GHz , and 20 GHz to 100 GHz . The three-term Debye coefficients for the frequency range between 20 GHz and 100 GHz are tabulated in Table 2.

## C. Incident plane wave and FDTD parameters

An incident plane wave with a Gaussian waveform containing all frequencies of interest up to 100 GHz is considered as the EM fields radiated by wireless communication systems. In this paper, the IPD of the incident plane wave are set to $100 \mathrm{~W} / \mathrm{m}^{2}$ and $10 \mathrm{~W} / \mathrm{m}^{2}$
which are maximum permissible exposure limits for occupational and public exposures [6], respectively.

The linearly polarized plane wave in the FDTD problem domain is generated on the total-field/scatteredfield (TF/SF) boundary [33]. The convolution perfect matching layer (CPML) [33] is used as an absorbing boundary to truncate the FDTD problem domain. The Courant-Friedrichs-Lewy (CFL) condition is used to determine the numerical stability in the FDTD method. This condition depends on the cell size of the FDTD problem domain. Thus, the cell size should be less than $\lambda_{\text {min }} / 20$, where $\lambda_{\text {min }}$ is the wavelength of the highest frequency in the head model. In order to satisfy this criterion, the cell size of the 1D head model is set to 0.0625 mm .

## D. SAR and temperature rise calculation for 1D multi-layered model

The electric field in time-domain is transformed to frequency domain by using the discrete Fourier transform (DFT) in each time-step of the FDTD simulation. After the FDTD simulation is completed, electric field ( $E$ ) in the frequency domain is used to calculate the steady-state SAR distribution in the 1D head model at each frequency of interest. The SAR equation for the 1D multi-layered model is written at a specific frequency and location as follows:

$$
\begin{equation*}
S A R(i)=\frac{\sigma(i)}{2 \rho(i)}\left(|E(i)|^{2}\right) \tag{1}
\end{equation*}
$$

where $\sigma$ and $\rho$ are conductivity $[\mathrm{S} / \mathrm{m}]$ and mass density $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ of tissue, respectively, and $i$ denotes the indexed cell. The algorithm specified in the IEEE C95.3 standard [38] is applied to calculate the $S A R_{1 g}$ and $S A R_{10 g}$ in the 1D head model.

The temperature rise in the 1D head model is calculated by using the Pennes bioheat equation in [34] and as implemented in [13-14]. The $\mathrm{SAR}_{1 \mathrm{~g}}$ distribution are used as EM heat source into the bioheat equation. All required thermal parameters for temperature rise calculations in the tissues are provided in [24].

## III. NUMERICAL RESULTS AND DISCUSSIONS

First, the results obtained in this investigation are compared to the limited results available in the literature to confirm the validity of our 1D multi-layered head models with the use of the Multiphysics model. Then, the penetration depth, SAR, and temperature rise in LayerA and Layer-B due to an EM plane wave are calculated using the Multiphysics model for three frequency ranges ( 100 MHz to 2 GHz , 2 to 20 GHz , and 20 to 100 GHz ).

## A. Comparison of results

In order to prove the validity of the Multiphysics
model, the maximum local SAR, $\mathrm{SAR}_{1 \mathrm{~g}}, \mathrm{SAR}_{10 \mathrm{~g}}$, and temperature rise values in the Layer-A model obtained by the Multiphysics model at the frequencies of $3,6,24$, 77 , and 100 GHz are compared to those values obtained analytically for a 1D multi-layer model consisting of only skin, fat, and muscle in [25]. The results listed in Table 3 are obtained when the IPD is $10 \mathrm{~W} / \mathrm{m}^{2}$. Although the 1D multi-layered models in [25] and in this work have different tissue layer thickness, and the EM and thermal parameters of tissues used are different, the results in Table 3 are in a good agreement with acceptable differences. For IPD is $100 \mathrm{~W} / \mathrm{m}^{2}$, the measured and simulated temperature rise calculated by using the finiteelement method (FEM) in [25] are 0.7 and $0.84{ }^{\circ} \mathrm{C}$ at 77 GHz , respectively, whereas the temperature rise obtained using our Multiphysics model is $0.64{ }^{\circ} \mathrm{C}$.

Furthermore, a 3D eye model has been analyzed at 77 GHz using the traditional FDTD method with the IPD of $10 \mathrm{~W} / \mathrm{m}^{2}$ in [25]. The reported $\mathrm{SAR}_{1 \mathrm{~g}}$ is $0.66 \mathrm{~W} / \mathrm{kg}$, whereas in this work, the $\mathrm{SAR}_{1 \mathrm{~g}}$ for Layer-B model obtained by the Multiphysics model at 77 GHz is 0.55 $\mathrm{W} / \mathrm{kg}$. The $\mathrm{SAR}_{1 \mathrm{~g}}$ values obtained in [25] and here are close to each other, even with the use of different dimensional models.

Table 3: Layer-A results compared with those from [25] when IPD $=10 \mathrm{~W} / \mathrm{m}^{2}$

| Freq. <br> $(\mathrm{GHz})$ | Methods | $\mathrm{SAR}_{\max }$ <br> $(\mathrm{W} / \mathrm{kg})$ | $\mathrm{SAR}_{1 \mathrm{~g}, \max }$ <br> $(\mathrm{~W} / \mathrm{kg})$ | $\mathrm{SAR}_{10 \mathrm{~g}, \max }$ <br> $(\mathrm{~W} / \mathrm{kg})$ | Temp. <br> Rise $\left({ }^{\circ} \mathrm{C}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | Result in [25] | 0.098 | 0.200 | 0.110 | $<0.1$ |
|  | Multiphysics | 0.548 | 0.154 | 0.101 | 0.074 |
| 6 | Result in [25] | 0.800 | 0.240 | 0.140 | $<0.1$ |
|  | Multiphysics | 0.888 | 0.260 | 0.144 | 0.038 |
| 24 | Result in $[25]$ | 7.740 | 0.420 | 0.200 | $<0.1$ |
|  | Multiphysics | 8.022 | 0.465 | 0.215 | 0.057 |
| 77 | Result in $[25]$ | 27.200 | 0.580 | 0.270 | $<0.1$ |
|  | Multiphysics | 24.511 | 0.581 | 0.270 | 0.064 |
| 100 | Result in $[25]$ | 33.900 | 0.620 | 0.290 | $<0.1$ |
|  | Multiphysics | 28.982 | 0.616 | 0.286 | 0.067 |

## B. SAR and temperature rise distribution on layer-A

The penetration depths, SAR, and temperature rise distributions on the Layer-A model due to the incident plane wave with an IPD of $100 \mathrm{~W} / \mathrm{m}^{2}$ are calculated up to 100 GHz using the Multiphysics model. However, one should point out that the data in [6] assumes $10 \mathrm{~W} / \mathrm{m}^{2}$, which is the maximum permissible limit for general public exposure. While, in this work, we assume that the incident power density is equal to $100 \mathrm{~W} / \mathrm{m}^{2}$ which is the maximum permissible limit for occupational exposure. That's why the results in Table-3 and those presented in the figures of this and the next section are having an approximately factor of 10 differences.

For Layer-A model, the maximum local SAR values and radiation penetration depths as a function of frequency are shown in Fig. 2. The penetration depths of an EM plane wave incident on the head model provide the distance where the local SAR values fall to $1 \%$ of their maximum. It has been realized that penetration
depths decrease exponentially with the increase of frequency, whereas maximum local SAR values increase with frequency, because the permittivity of tissues decreases and the conductivity of tissues increases at higher frequencies. Decreased permittivity causes more incident power to reach deeper tissues, while increased conductivity works to prevent this power from entering deeper tissues. The penetration depth becomes gradually less than 1 mm when the frequency gets closer to 100 GHz . The maximum $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ values up to 100 GHz are shown in Fig. 3.


Fig. 2. Maximum local SAR and penetration depth as a function of frequency.


Fig. 3. Max. $S A R_{1 g}$ and $S A R_{10 g}$ as a function of frequency for Layer-A.


Fig. 4. Maximum temperature rise as a function of frequency for Layer-A.

These values are less than the specified limits of $8 \mathrm{~W} / \mathrm{kg}$ for $\mathrm{SAR}_{1 \mathrm{~g}}$ and $10 \mathrm{~W} / \mathrm{kg}$ for $\mathrm{SAR}_{10 \mathrm{~g}}$. The calculated maximum temperature rise shown in Fig. 4 is less than $0.73{ }^{\circ} \mathrm{C}$ at all frequencies of interest. The maximum SAR value occurs at 100 GHz , whereas the maximum
temperature rise occurs at 3 GHz . This is because small penetration depths at high frequencies prevent incident power from entering deeper tissues and causing an increase in temperature. It has been realized that the maximum temperature rise at all frequencies of interest in the head model are linearly proportional to the maximum $S_{1 g}$ and $\operatorname{SAR}_{10 g}$ values, whereas they are not directly proportional to the maximum local SAR shown in Fig. 2. The local SAR and temperature rise distributions on the Layer-A model at specified frequencies up to 100 GHz are shown in Fig. 5 and Fig. 6 , respectively. It can be seen that the maximum values of local SAR generally occur at the skin surface of the head model, whereas the maximum temperature rise occurs at about 2.5 mm under the skin surface of the head model. Furthermore, the values of local SAR distributions exhibit faster decrease with the increase of the frequency, whereas the values of temperature rise distributions decrease gradually. Figure 7 shows the maximum temperature rise in Layer-A model at specified frequencies as a function of time. It can be seen that the maximum temperature rise is reached after 30 minutes of exposure.


Fig. 5. Local SAR distributions on Layer-A at specified frequencies.


Fig. 6. Temperature rise distributions on Layer-A at specified frequencies.


Fig. 7. Max. temperature rise as a function of time.
C. SAR and temperature rise distribution on layer-B

In order to show the effect of eye tissues on penetration depth, SAR, and temperature rise distributions, the Layer-B head model with eye tissues such as cornea, lens, sclera, and vitreous humor is analyzed using the Multiphysics model up to 100 GHz . The maximum local SAR values and radiation penetration depths as a function of frequency on the Layer-B model are shown in Fig. 8. The maximum $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ values up to 100 GHz shown in Fig. 9 are less than the specified limits of $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$. The calculated maximum temperature rise versus frequency are shown in Fig. 10. Numerical results show that the maximum local SAR values in Layer-B are slightly higher than those values in Layer-A, especially
for higher frequencies, whereas the maximum $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ values in Layer-B are slightly smaller than those values in Layer-A. However, the maximum temperature rise values in Layer-B are much higher than those values in the Layer-A, because Layer-B contains the eye tissues. The maximum temperature rise in the 1D human eye model is not linearly proportional in everywhere to the maximum $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ values. This is the reason that the temperature rise distribution is affected not only by the SAR distribution, but also by thermal parameters of eye tissues and penetration depth of the EM radiation. The local SAR and temperature rise distributions on the Layer-B model at specified frequencies up to 100 GHz are shown in Fig. 11 and Fig. 12, respectively. It can be seen from Fig. 6 and Fig. 12 that Layer-B allows higher temperature into deeper tissues than Layer-A. Figure 13 shows the maximum temperature rise in Layer-B at specified frequencies as a function of time.


Fig. 8. Maximum local SAR and penetration depth as a function of frequency.


Fig. 9. Max. $\mathrm{SAR}_{1 g}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ as a function of frequency for Layer-B.


Fig. 10. Maximum temperature rise as a function of frequency for Layer-B.


Fig. 11. Local SAR distributions on Layer-B at specified frequencies.


Fig. 12. Temperature rise distributions on Layer-B at specified frequencies.


Fig. 13. Max. temperature rise as a function of time.

## IV. CONCLUSION

The interaction between one-dimensional human head model and electromagnetic fields radiated by wireless communication systems up to 100 GHz has been investigated by using the Multiphysics model in a single simulation. Numerical results show that the maximum SAR values increase when the frequency gets closer to 100 GHz , whereas the penetration depths in the head model decrease exponentially. In order to show the effect of tissue types on the penetration depth, SAR and temperature rise distributions in the head model, two head models with and without eye tissues are investigated. For the layer-A model, the $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ values at 100 GHz are 6.16 and $2.86 \mathrm{~W} / \mathrm{kg}$, respectively. For the layer-B model, the $\mathrm{SAR}_{1 \mathrm{~g}}$ and $\mathrm{SAR}_{10 \mathrm{~g}}$ values at 100 GHz are 5.78 and $2.68 \mathrm{~W} / \mathrm{kg}$, respectively. These values are less than the specified limits of $8 \mathrm{~W} / \mathrm{kg}$ for $\mathrm{SAR}_{1 \mathrm{~g}}$ and $10 \mathrm{~W} / \mathrm{kg}$ for $\mathrm{SAR}_{10 \mathrm{~g}}$. The resulting maximum temperature rise at 100 GHz is $0.67{ }^{\circ} \mathrm{C}$ for the layer-A model and $0.9^{\circ} \mathrm{C}$ for the layer-B model. These values are less than the threshold temperature rise of $3-5^{\circ} \mathrm{C}$ for cataract formation and physiological damage [39] in the tissues. Numerical results obtained in this work are useful to evaluate the effect of EM fields radiated from wireless communications systems operated up to 100 GHz on the human head.

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# Acceleration of Dual Reflector Antenna Radiation Analysis using Double Bounce Physical Optics Accelerated using Multipole Method 

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#### Abstract

The multipole method is firstly used to accelerate the radiation analysis of the dual reflector antenna with the double-bounce physical optics. The algorithm starts with physical optics to calculate the equivalent electric current on the subreflector. Then the equivalent electric current on the main reflector can be obtained through the current on the subreflector. It should be noted that the modified multilevel fast multipole method (MLFMM) is applied to accelerate the calculation between the main reflector and the subreflector. In this way, the computation complexity is reduced greatly, thus the computational time can be significantly saved. At last, numerical results are given to demonstrate the superior efficiency and accuracy of the proposed method.


Index Terms - Double-bounce physical optics, dual reflector antenna, MLFMM.

## I. INTRODUCTION

Nowadays, the satellite communication has a wide application in both the military and civilian domains. The antenna technology plays a very important role in satellite communication. There are three kinds of satellite antennas, namely lens [1], phased array [2]-[3] and reflector [4]-[6] antennas. However, the weight of lens antenna is generally heavier than reflector antenna. Moreover, the phased array antenna generally need to hundreds of units in order to achieve the same gain of reflector antenna, which leads to exceed to the limit of the satellite. Therefore, the reflector antenna has a great advantage in volume and weight.

Full-wave simulation of electrically large size problems is impossible to be taken on conventional workstations due to their hardware limitations. Parallel technique provides a possibility through thousands of Giga Bytes (GB) of memory and processors connected by high-speed switches [7]. Moreover, it is an arduous task for the method of moments (MoM) for electrically large problems. In reference [8], a multi-level method based on near-field and far-field transformation of plane wave is proposed. This method uses diagonal
translation operator and is available for evaluating electrically large antennas. In [9], it presents the multilevel fast multipole method has been adapted to speed-up the inverse solution process. To rapidly analyze the radiation pattern of electrically large parabolic reflector antenna, the MoM accelerated with MLFMM is proposed [10]-[12]. In [13], it shows excellent agreement of physical optics results in the main lobe and out to several lobes has been found with those of MLFMM accelerated MoM technique. In order to further improve the speed of calculation, it is preferred to combine MoM with physical optics (PO) to analyze reflector antenna problems [14]. For the antenna radiation problems, it is a key to get the gain of main lobe. PO method can provide the main lobe gain with high efficiency. Therefore, it can be used as a promising tool to simulate the radiation pattern of antenna. In [15], PO is used to analyze the radiation pattern of a monopole antenna mounted on an aircraft. In order to analyze the reflector antenna efficiently, [16] applied graphics processing units to PO in computing the radiated fields. In [17], it proposes a new algorithm about multilevel physical optics for the effective evaluation of the wide angle radiation pattern. Then an acceleration technique was presented in [18] to fast evaluate the monostatic radar cross section. In order to get higher gain with limited size, single reflected antenna is replaced by dual reflector antenna. Reference [19] demonstrates MLFMM is applied to accelerate multiple reflection Physical Optics significantly. Recently, multilevel sampling and interpolation of phase-compensated and amplitudecompensated is applied to the double bounce through physical optics [20]. In [21], it is first to use the MLFMM algorithm for internal field calculations, and consider effect of internal shadowing on the factorized version of the field evaluation matrix, without compromising speed and accuracy. Although the double-bounce physical optics is accelerated by the above technology, it is still time-consuming to optimize large aperture antennas, especially in high frequency. It is significant to develop more efficient approach for
dual reflector antenna.
A new DBPO (double bouncing physical optics) technique is proposed to evaluate the radiation pattern of the dual reflector parabolic antenna. It is a novel combination between DBPO and MLFMM. In order to obtain the equivalent current of main reflector high efficiently, the effect of the subreflector's equivalent electric current on the main reflector's equivalent electric current is accelerated by MLFMM. Because of the distance between the main reflector and subreflector is long, we use window function to further accelerate the operation of transfer. Suppose $N_{1}$ and $N_{2}$ are the number of unknowns on the sub and main reflectors respectively. The computational complexity of computing the bounce between reflectors, that can be reduced from $O\left(N_{1} * N_{2}\right)$ to $O\left(N_{1}+N_{2}\right)$ by using the new approach.

With the increase of $N_{1}$ and $N_{2}$, the efficiency of new method is more evident. Several examples are presented to validate the superior efficiency and accuracy of the proposed method.

## II. THEORY AND FORMULATION

## A. Physical Optics

The radiation is obtained by the equivalent electric current on the surface of the metallic reflector. The equivalent currents are zero when the surface is not illuminated by incident plane wave, and it has the following expressions in the illuminated area:

$$
\begin{equation*}
\vec{J}\left(\vec{r}^{\prime}\right)=2 \vec{n} \times \vec{H}^{i n c}\left(\vec{r}^{\prime}\right) \tag{1}
\end{equation*}
$$

where $\hat{n}$ is the unit normal vector of the surface at point $\vec{r}^{\prime}, \vec{H}^{\text {inc }}(\vec{r})$ is the incident magnetic field. The radiation diagram of metallic surface as follows:

The radiation pattern is:

$$
\begin{equation*}
\vec{E}(\vec{r})=-j k_{0} \eta_{0}\left(\overline{\bar{I}}+\frac{1}{k_{0}^{2}} \nabla \nabla\right) \iint_{S} \vec{J}\left(\vec{r}^{\prime}\right) G\left(\vec{r} \mid \vec{r}^{\prime}\right) d s \tag{2}
\end{equation*}
$$

where $\eta_{0}=\sqrt{\frac{\mu_{0}}{\varepsilon_{0}}}$ presents the free space wave impedance, $k_{0}=\omega \sqrt{\varepsilon_{0} \mu_{0}}$ presents the free space wave number.

According to the far field approximation, the Green's function is simplified as follows:

$$
\begin{equation*}
G\left(\stackrel{\rightharpoonup}{r} \mid \vec{r}^{\prime}\right)=\frac{e^{-j k\left|\vec{r}-\vec{r}^{\prime}\right|}}{4 \pi\left|\vec{r}-\vec{r}^{\prime}\right|} \approx \frac{e^{-j k r}}{4 \pi r} e^{j k \vec{r}^{\prime} \cdot \hat{R}}, \tag{3}
\end{equation*}
$$

where $\hat{R}$ is the unit vector of the observed point in spherical coordinates in terms of the rectangular coordinates system, the expression is:

$$
\begin{equation*}
\hat{R}=\sin \theta \cos \phi \hat{x}+\sin \theta \sin \phi \hat{y}+\cos \theta \hat{z} . \tag{4}
\end{equation*}
$$

The radiation field of the reflector antenna is expressed as:

$$
\begin{equation*}
\stackrel{\rightharpoonup}{E}(\vec{r})=-j k_{0} \eta_{0}\left(\overline{\bar{I}}+\frac{1}{k_{0}^{2}} \nabla \nabla\right) \iint_{S} \bar{J}\left(\vec{r}^{\prime}\right) \frac{e^{-j k r}}{4 \pi r} e^{j k \vec{r}^{\prime} \cdot \hat{R}^{\prime}} d s^{\prime} \tag{5}
\end{equation*}
$$

Only the item of $1 / r$ is retained in the far field of radiation, and electric filed intensity is:
$\vec{E}(\vec{r})=-j k_{0} \eta_{0} \frac{e^{-j k r}}{4 \pi r}(\overline{\bar{I}}-\hat{R} \hat{R}) \iint_{S} J\left(\vec{r}^{\prime}\right) \frac{e^{-j k r}}{4 \pi r} e^{j k \vec{r}^{\prime} \cdot \hat{R}^{\prime}} d s^{\prime}$
where $\overline{\bar{I}}$ represents the dyadic unit vector, and $\hat{R} \hat{R}$ represents the dyadic unit vector of $\hat{R}$.


Fig. 1. Radiation of the single reflector antenna in PO.
In Fig. 2, for the dual reflector antenna, it is suitable to apply double bounce physical optics rather than PO. The radiation field of the main reflector is obtained through the equivalent electric current of the subreflector:

$$
\begin{align*}
& \vec{H}_{s c}=\iint \vec{J}_{s} \times \nabla^{\prime} g d S \\
& =\frac{1}{4 \pi} \iint \frac{e^{-j k r_{m}}}{r_{m}}\left(\vec{J}_{s} \times \vec{r}_{m}\right)\left(j k+\frac{1}{r_{m}}\right) d S \tag{7}
\end{align*},
$$

where $d S$ represents the area of each triangle in subreflector, and $\vec{J}_{s}$ represents the tangential equivalent electric current of each triangle in subreflector.


Fig. 2. Dual offset reflector antenna.

## B. A novel method based on MLFMM

Triangles are usually used to fit the surface of the object, and $\lambda / 10$ is taken as the mesh size. Where $\lambda$ is the wavelength of the incident wave. It is timeconsuming to get the incident field of the main reflector. Due to the fact that the distance between of the main reflector and the subreflector is farther than $10 \lambda$, where the main reflector regards as the far field for the subreflector. Therefore we speed up the process of (7) through a novel method, which modifies from MLFMM [22]-[24]. All the aggregations happen in subreflector, and all configurations appear in the main reflector. This is different from the MLFMM. In MLFMM, aggregation and configuration are determined by the distance between the groups. If the distance between two groups are close, the contribution of group is obtained by direct numerical calculation. Only the distance between two groups is long, it uses MLFMM to accelerate the contribution. In proposed algorithm, because the group of main reflector and the group of subreflector are far apart, all the contribution are evaluated by MLFMM. Through three processes of aggregation, transfer and configuration in Fig. 3, the acceleration of the computation is achieved. Firstly, the contribution of every source point is aggregated to the center of the corresponding group. In order to simplify the computation, the source point is defined as the center of the meshed triangle in the subreflector. The equivalent electric current of every point is considered as the basic function of corresponding triangle. Secondly, after finishing aggregation, the contribution of group of source point transfers to the group center of observed point. Thirdly, the contribution of observed point's group center is configured to the observed point, and the configuration is the inverse operation of aggregation.


Fig. 3. Principle of accelerating the effect of the subreflector to the main reflector in single level.

The main reflector and the subreflector are surrounded by box through octree grouping. Then the two reflectors are divided into many groups, and (7) is the contribution of the current. The entire solution region is first enclosed in a large group (the red cube at $\mathrm{n}+1$ level in Fig. 4), which is divided into eight smaller groups (the red cubs at $n$ level in Fig. 4). Each sub-
group is then recursively subdivided into smaller subgroups until the finest group (the yellow cube in Fig. 4) contains a few current elements. The level of groups is determined by the distance between the two elements in MLFMM. In Fig. 4, every group of n-1 level transfers at the $\mathrm{n}-1$ level. This is the operation of transfer. Meanwhile, every group of $n-1$ level gathers into the $n$ level, which is the father group of $n-1$ level. This is the operation of aggregation. Then every group of $n$ level is transferred at the n level. Every group of n level gathers into the $n+1$ level, which is the father group of $n$ level. Above operation is aggregation and transfer, and configuration is reverse of aggregation. Every group of $\mathrm{n}+1$ level is configurated to the n level. In MLFMM, the aggregation process stars from the bottom level, and the configuration process stars from the top level.


Fig. 4. Multi-level of aggregation, transfer and configuration.

If the distance between the main reflector and the subreflector, satisfying the following condition:

$$
\begin{equation*}
\left|\stackrel{V}{V}_{p q}\right|>\left|\stackrel{V}{r}_{m p}\right|+\left|\left.\right|_{n q}\right| \tag{8}
\end{equation*}
$$

Where $\stackrel{V}{p q}$ is orientation that from source group center of subreflector to observed group center of main reflector, and $\stackrel{V}{r}_{m p}$ is orientation that from source group center of main reflector to observed group center of main reflector. $\stackrel{V}{r}_{p q}$ is orientation that from group center of subreflector to group center of main reflector.

Because the distance between the main reflector and subreflector is far, the main reflector is regarded as the far-field of the subreflector. In this case, the operation of aggregation-transfer-configuration is to accelerate this progress, which solving the equivalent
electric current of main reflector from the equivalent current of subreflector.

The Green's function can be unfolded as:

$$
\begin{align*}
& g=\frac{e^{-j k \stackrel{v}{r}_{m n}}}{r_{m n}}  \tag{9}\\
& =-\frac{j k}{4 \pi} \oint e^{-j k \cdot\left(\stackrel{\mathrm{v}}{r_{m p}}+\stackrel{\mathrm{V}}{r_{q n}}\right)} T_{L}(\stackrel{\mathrm{v}}{k} \cdot \stackrel{\mathrm{v}}{r}) d^{2} k \\
& \nabla^{\prime} g=\frac{k^{2}}{4 \pi} \vec{k} f \int e^{-j k \cdot\left(\bar{r}_{m p}+\bar{r}_{q n}\right)} T_{L}(\vec{k} \cdot \vec{r}) d^{2} k,  \tag{10}\\
& T_{L}(\vec{k} \cdot \vec{r}) \approx \sum_{l=0}^{L}(-j)^{l}(2 l+1) h_{l}^{(2)}\left(k_{o} r_{p q}\right) P_{l}(\vec{r} \cdot \vec{k}), \tag{11}
\end{align*}
$$

where multipole mode number $L$ is infinite. In order to calculate the $T_{L}(\vec{k} \cdot \vec{r})$, $L$ is cut off. $L$ has an empirical formula:

$$
\begin{gather*}
L=k d+\alpha \log (\pi+k d),  \tag{12}\\
\alpha=-\log (\varepsilon),  \tag{13}\\
\beta=1.8(-\log (\varepsilon))^{2 / 3}, \tag{14}
\end{gather*}
$$

where $d$ represents the diagonal length of the group and $\varepsilon$ represents the precision.

Through transforming the Green's function, (7) is expressed as:

$$
\begin{align*}
& \vec{H}_{s c}=\iint \vec{J}_{s} \times \nabla^{\prime} g d S^{\prime} \\
& =\frac{k^{2}}{4 \pi} \iint \vec{J}_{s} \times \vec{k} \iint e^{-j k \cdot\left(\vec{r}_{m p}+\vec{r}_{q n}\right)} T_{L}(\vec{k} \cdot \vec{r}) d^{2} k d S^{\prime} \tag{15}
\end{align*}
$$

$\vec{J}_{s} \times \vec{k}^{-j k \vec{r}_{m p}}$ represents the factor of aggregation, $T_{L}(\vec{k} \cdot \vec{r})$ represents the factor of transfer and $e^{-j k \vec{r}_{q m}}$ represents the factor of configuration.


Fig. 5. Sketch map of window function.
In order to accelerate the operation of transfer, we add a window function to the transfer matrix:

$$
w_{l}=\left\{\begin{array}{cc}
1 & l \leq J  \tag{16}\\
\frac{1}{2}\left[1+\cos \left(\frac{l-J}{L-J} \pi\right)\right] & l>J
\end{array}\right.
$$

In Fig. 5, after the transfer factor multiplied by upper window function, the window function, like a bandpass filter, filters out the area that is not concerned. The transfer component at a certain distance form the center of the group becomes smaller. Because of the long distance between source and observation. Through the window function, we omit many of the angular spectral components that have less influence on the transfer process. Thus this function makes the calculation more efficient.

## C. NURBS modeling

The NURBS [25] surface is a bivariate piecewise rational function, which is P order in the direction of u and Q order in the direction of v . The basic expression is as follows:

$$
\begin{equation*}
\mathbf{S}(u, v)=\frac{\sum_{i=0}^{n} \sum_{j=0}^{m} w_{i j} N_{i, p}(u) N_{j, q}(v) P_{i j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} w_{i j} N_{i, p}(u) N_{j, q}(v)}, \tag{17}
\end{equation*}
$$

in which P and Q are the order, m and n are the number of control points in the direction of u and $\mathrm{v} . N_{i, p}(u)$ is P cubic B spline basis function, which is obtained by the node vector $U=\left[u_{0}, u_{1}, \ldots, u_{n+k+1}\right]$ according to the de-Boor-Cox recurrence formula. $P_{i j}$ represents the control points of surface, $w_{i j}$ is the weight of every control point:

$$
\begin{gather*}
N_{i, 0}(u)=\left\{\begin{array}{l}
1 \text { if } \mathrm{u}_{i} \leq u \leq \mathrm{u}_{i+1}, \\
0 \text { otherwise }
\end{array}\right.  \tag{18}\\
N_{i, p}(u)=\frac{u-u_{i}}{u_{i+p}-u_{i}} N_{i, p-1}(u)  \tag{19}\\
+\frac{u_{i+p+1}-u_{i}}{u_{i+p+1}-u_{i+1}} N_{i+1, p-1}(u)
\end{gather*}
$$

The piecewise rational base function is:

$$
\begin{equation*}
R_{i, j}(u, v)=\frac{w_{i j} N_{i, p}(u) N_{j, q}(v)}{\sum_{k=0}^{n} \sum_{l=0}^{m} w_{i j} N_{k, p}(u) N_{l, q}(v)} . \tag{20}
\end{equation*}
$$

The NURBS expression can be abbreviated as:

$$
\begin{equation*}
\mathbf{S}(u, v)=\sum_{i=0}^{n} \sum_{j=0}^{m} R_{i, j}(u, v) P_{i j} \tag{21}
\end{equation*}
$$

As is shown in Fig. 6, we change the model through changing the control points (red points), which are selected by Rhinoceros software. The weight of control points is set to 1 .


Fig. 6. NURBS fitting plane.


Fig. 7. Change value of two control points in $Z$ coordinate to get the new surface.

In Fig. 6, the x axis represents the direction of V , and the $y$ axis represents the direction of $U$. The surface around the control point varies with the control point and has no effect on the far curved surface. Through changing $7 * 9$ control points, we can achieve different surface what we want. In Fig. 7, changing the value of two control points in Z coordinate constantly, we will get many surface with different shapes. In the optimization process of the reflector antenna, it has a fixed contour. Only the value of the control point is changed in Z coordinate, the projection shape of the surface in the XOY plane is not affected. Therefore, the outline of the reflector antenna does not have any change. This is why the NURBS modeling is chosen to apply in the shape optimization of the reflector antenna.

## III. NUMERICAL RESULTS

The CPU has Intel core Q9500 and 8GB memory. The environment is based on the Fortran MPI.

In Fig. 8, it is the dual offset Cassegrain antenna with primary reflector diameter of 2.5 m and rectangular
secondary reflector diameter of $1.08 \mathrm{~m} * 1.06 \mathrm{~m}$, and the feed adopts the ideal horn feed with the right circular polarization. Since the antenna is a shared antenna, and it should be optimized simultaneously both at the receiving and transmitting frequency. The mesh size is 0.4 times the wavelength of the receiving frequency at 28 GHz , and the triangle of the primary reflector with 927395 , the secondary reflector with 150785 . The phi of simulation is 0 degree and the theta ranges from -5 degree to 5 degree. The tangential plane pattern of DBPO, DBPO with MLFMM at 18 GHz is shown in Fig.9, and DBPO and the proposed method is in good agreement in the main lobe, so is the near-sidelobe. The result of FEKO and the proposed method has difference in sidelobe. We pay attention to the main lobe and near-sidelobe in this problem. Therefore the proposed method meet the application requirements.


Fig. 8. Model of the dual offset Cassegrain antenna.


Fig. 9. Radiation pattern of the dual reflector antenna in different methods.

Table 1: Efficiency of different methods

|  | PO | Proposed <br> Method <br> (Stand-alone) | Proposed <br> Method (MPI <br> with 15 Core) |
| :---: | :---: | :---: | :---: |
| Time (s) | 16214.32 | 156.84 | 15.49 |

The enormous advantage of the proposed method is shown in Table 1. The proposed method is 103.38 times faster than traditional method (DBPO). The presented method combined optimization algorithm applies to optimize the dual reflection antenna, especially for the large aperture antenna.


Fig. 10. Model of the dual offset Cassegrain antenna.
In Fig. 10, the diameter of the main reflector is 2.5 m , the focal length is 3.74 m , and the center offset height is 3.12 m . The focal length of the subreflector is 1.456 m , and the distance of the hyperbolic vertex is 1.248 m . We model the main reflector and subreflector through NURBS, and select $9 * 9$ control points to change the shape of the main reflector and subreflector. The shape of the initial reflector and the coordinates of the control points are shown in Fig. 11.


Fig. 11. Initial main reflector and control points.
In Fig. 12 and Fig. 13, the center frequency point of transceiver is 28 GHz in Ka band. Since the performance of the 70 beams is approximately similar, it can only be optimized by taking several representative beams at the center and edge. For a high gain narrow beam antenna, we properly take less sampling point to the optimized target area. Here we take 8 sampling points form each edge of the beam and the symmetrical sampling points of equal distance, which guarantee the symmetry of the optimized beam. At the same time, a point is picked up at the center, which reduces the number of the sampling points and prevents the center of the radiation pattern from sinking. The optimization goal is that the directional coefficient of 0.8 degree beam is greater than 45 dB .

By optimizing the shape of the reflector, we achieve the radiation pattern which meets 45 dB as shown in Fig. 13.


Fig. 12. Isoline pattern of the initial beam at the frequency of 28 GHz .


Fig. 13. Isoline pattern of the optimized beam at the frequency of 28 GHz .

## VI. CONCLUSION

An innovative MLFMM-DBPO approach for calculating the radiation pattern of the dual reflector antenna is proposed in this work. The new method reduces the computational complexity and expedites the simulation of antenna with DBPO fundamentally. It is significant for optimizing the shape of large aperture antenna and analyzing the radiation of large electrical objects, such as the satellite antenna. According to the numerical results, the new algorithm is more efficient than traditional DBPO with encouraging accuracy.

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# Sparse Representation of Targets with Mixed Scattering Primitives 

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#### Abstract

A combination of two scattering primitives wedge diffraction primitives and isotropic point scatterers - is used to reconstruct far-field monostatic scattering patterns of several target geometries and addresses shortcomings of traditional approaches that only use a single type of primitive (e.g., approximations in analytic solutions, slow convergence). An $l_{1}$-norm minimization technique is applied to determine a set of weights for the point scatterers. We show that combining these two types of primitives yields better reconstruction performance than when each primitive type is used individually.


Index Terms - Geometric Theory of Diffraction (GTD), $l_{1}$-norm minimization, Radar Cross Section (RCS), scattering primitives, sparse representation, Uniform Theory of Diffraction (UTD).

## I. INTRODUCTION

It is well known that far-field scattering from a complex geometry can be estimated by decomposing the target into simple scattering primitives and summing their individual responses. Reconstruction of electromagnetic field quantities with isotropic point scatterers (IPSs) is a fundamental principle in SAR and ISAR processing [1]. The use of non-isotropic scattering primitives has also been investigated in [2,3]. Moreover, in [4], a dense array of IPSs is used as a part of an overcomplete dictionary. Sparse representations enable discrimination of target returns from nuisance returns that can arise from the measurement process.

In this work, we also seek to reduce the number of scattering centers required by introducing a wedge diffraction primitive (WDP) derived from GTD/UTD theory. We use the WDP to capture known scattering mechanisms based on the target's far-field pattern and allow the IPSs to recover the remaining coherent differences. We limit the analysis of this approach to planar cuts of 2D geometries and compare the results to reference data generated by a 3D Method of Moments (MoM) code. A normalization factor is applied to translate 2D echo width predictions to 3D RCS.

The use of multiple primitive types is also considered under the context of compressive sensing (CS). The theory states that the number of measurements required to successfully recover the sparse representation of a far-field pattern (via Basis Pursuit or other $l_{1}$-norm minimization algorithms) decreases substantially (even below the Nyquist sampling rate) as signal sparsity increases [5]. We posit that the sparsity of the far-field pattern representation can be improved by using WDPs to capture diffraction behavior and IPSs to capture the remaining difference between the diffraction contributions and the original far-field pattern. For example, a large portion of the monostatic response from a 2D flat plate can be accurately represented with two WDPs located at the ends of the plate. In contrast, a solution with two IPSs can only capture the specular response and would require significantly more points to accurately reconstruct the sidelobes of the monostatic return.

While sparsi ty can be satisfied, the efficacy of this technique as an application for CS is also dependent on the bounds of the Restricted Isometry Property (RIP) of the measurement matrix [6]. Determining these bounds explicitly is an NP-hard problem, but numerical experiments can provide a cursory and empirical assessment of its performance.

In the following sections, we describe the theory behind reconstruction via scattering centers and describe the framework that was developed to perform the reconstruction with a mixed set of primitives. We then present two simple cases to validate our approach, discuss the implications of their results and describe additional areas of investigation.

## II. SCATTERING CENTER THEORY

Consider a collection of spatially distributed scattering centers, each associated with a complex coefficient that modulates its magnitude and phase [7]. By adjusting the location and the complex value of each scattering center, the superposition of every scattering center's far-field response may yield a pattern that matches the far-field response of an arbitrary target
geometry. This concept is illustrated in Fig. 1 (a) and concisely summarized as:

$$
\begin{equation*}
S_{I P S}^{p r e d}(k, \hat{\mathbf{r}})=\sum_{n=1}^{N} \gamma_{n} e^{-j 2 k \hat{r} \cdot \mathbf{r}_{n}^{\prime}} \tag{1}
\end{equation*}
$$

where $S_{I P S}^{p r e d}$ is the far-field value synthesized with the wavenumber $k$ and observed from direction $\hat{\mathbf{r}}$. $S_{I P S}^{\text {pred }}$ is determined by the summation of $N$ IPSs that are located at $\mathbf{r}_{n}^{\prime}$ and modulated by the complex coefficient $\gamma_{n}$. The components of $\gamma_{n}$ control the magnitude and phase delay applied to the IPSs, while the operating frequency and the locations of the scattering centers with respect to the phase origin affect the exponential term in (1) and alters the phase oscillation rate of the far-field pattern.


Fig. 1. Far-Field approximation of a (a) IPS and (b) WDP.

WDPs, shown in Fig. 1 (b), are coherently summed together in a similar fashion as IPSs to generate far-field patterns. From [8], UTD diffraction coefficients reduce to GTD coefficients when the surfaces of the wedge are flat and the observation angles are not in the transition regions near the shadow boundaries. We implement 2D WDPs as:

$$
\begin{equation*}
D^{s, h}(\phi, m)=\frac{-e^{j \pi / 4} \sin (\pi / m)}{2 m \sqrt{2 \pi k}}\left[\left(\frac{1}{\cos \frac{\pi}{m}}\right) \mp\left(\frac{1}{\cos \frac{\pi}{m}-\cos \frac{2 \phi}{m}}\right)\right] \tag{2}
\end{equation*}
$$

where $m=(2 \pi-\alpha) / \pi, \alpha$ is the wedge angle and $\phi$ is the monostatic observation angle. In the second term between the brackets, the negative term corresponds to the soft polarization, while the positive term corresponds to the hard polarization (in our case, $\theta$ and $\phi$ polarization, respectively). This 2 D analytic solution assumes that the diffraction edge is always aligned along the $z$-axis and extends towards infinity. The RCS of a finite wedge of width $w$ can be determined by multiplying the 2D echo width by $2 \pi w^{2} / \lambda$, where wavelength $\lambda=2 \pi / k$. The diffraction coefficient $D^{s, h}$ replaces the IPS coefficient $\gamma_{n}$ in (1) and yields:

$$
\begin{equation*}
S_{W D S}^{p r e d}(k, \hat{\mathbf{r}})=\sum_{l=1}^{L} D_{l}^{s, h} e^{-j 2 k \hat{\mathbf{r}} \cdot \mathbf{r}_{l}^{\prime}}, \tag{3}
\end{equation*}
$$

to generate the far-field monostatic backscatter of the $L$ diffracting wedges.

We note that the formulation in (2) is valid for wedge angles of up to $180^{\circ}$ and does not address dihedral effects. Moreover, the scattering pattern arising from WDPs exhibit asymptotic behavior for monostatic angles that are normal to the faces of the wedge due to their vicinity to the shadow boundaries. These singularities occur in pairs for finite length wedges and additional considerations need to be made when geometries contain dihedral or concave regions.

## III. OPTIMIZATION FRAMEWORK

We use the IPS and WDP formulations to estimate a solution $S_{F F}^{e s t}$ for $S_{F F}^{r e f}$ by considering,

$$
\begin{align*}
& S_{F F}^{\text {ref }} \approx S_{F F}^{\text {est }}=S_{W D S}^{\text {pred }}+\Delta p \\
& =S_{W D S}^{\text {pred }}+\left(S_{F F}^{\text {ref }}-S_{W D S}^{\text {pred }}\right)  \tag{4}\\
& =S_{W D S}^{\text {pred }}+S_{I P S}^{\text {pred },}
\end{align*}
$$

where $S_{W D S}^{p r e d}$ utilizes a priori information about the geometry to generate a coarse estimate of $S_{F F}^{r e f}$ and the coherent difference $\Delta p$ is estimated with $S_{I P S}^{\text {pred }}$. An optimization framework, depicted in Fig. 2, was designed to determine $S_{W D S}^{p r e d}$ in a preprocessing stage and $S_{I P S}^{\text {pred }}$ with a sparse optimization stage.


Fig. 2. Optimization framework.
The framework was generalized to use a constrained minimization routine to estimate appropriate parameter values for multiple types of non-isotropic scattering primitives (including the WDP utilized in this study). We note that the determination of $\mathbf{r}_{l}^{\prime}$ and the dependent variables of $D_{l}^{s, h}$ in (3) can be a non-trivial problem with many local minima, especially when a priori information is limited. Because our investigation is focused on the feasibility of reducing the number of IPSs, we bypass the constrained minimization in these experiments and provide parameter values for the WDPs based on a priori knowledge of the scattering geometry as inputs instead.

The preprocessing stage synthesizes a far-field pattern from the WDPs and applies a global phase shift to the prediction that best matches the reference data. This is a necessary step in the event that the reference
data and WDP prediction from the analytic geometry have different phase origins. The phase-shifted WDP solution is then coherently subtracted from the $S_{F F}^{r e f}$ to yield $\Delta p$.

Shadowed regions are also determined in the preprocessing stage of the framework to prevent WDPs and IPSs from radiating through the boundaries of the target geometry. This is performed by tracing a vector from each primitive and scattering center to all far-field observation points and determining whether the ray intersects a facet of the geometry [11].

Modifications to support shadowing and the phase shift of the WDP solution augment the model in (4) with additional modulation terms such that,

$$
\begin{align*}
& \widehat{S}_{F F}^{e s t}(k, \widehat{\mathbf{r}})=\widehat{S}_{W D S}^{\text {pred }}+\widehat{S}_{I P S}^{\text {pred }} \\
& \widehat{S}_{F F}^{e s t}(k, \hat{\mathbf{r}})=\sum_{l=1}^{L} \beta_{l} e^{j \psi} S_{W D S}^{\text {pred }}+\sum_{n=1}^{N} \beta_{n} S_{I P S}^{\text {pred }}  \tag{5}\\
& =\sum_{l=1}^{L} \beta_{l} D_{l}^{s, h} e^{-j\left(2 k \hat{\mathbf{r}} \cdot \mathbf{r}_{l}^{\prime}-\psi\right)^{\prime}}+\sum_{n=1}^{N} \beta_{n} \gamma_{n} e^{-j 2 k \hat{\mathbf{r}} \cdot \mathbf{r}_{n}^{\prime}}
\end{align*}
$$

where $\beta_{l, n}$ represent the shadowing and angle constraints applied to the WDSs and IPSs respectively and $e^{j \boldsymbol{\psi}}$ represents the phase shift applied to the WDP solution.

In the sparse optimization stage, determining appropriate values of $\mathrm{r}_{\mathrm{n}}^{\prime}$ and $\gamma_{\mathrm{n}}$ in (1) often relies on $l_{1-}$ norm minimization techniques such as Basis Pursuit DeNoising (BPDN) in [4]. We note that BPDN provides a solution that minimizes the sum of the magnitudes of the complex coefficients, whereas an $l_{0}$-norm minimized solution minimizes number of complex coefficients contributing to the solution (true sparsity). A solution arising from an $l_{1}$-norm minimization routine is a good approximation to the $l_{0}$-norm minimized solution when compressive sensing characteristics are met, namely that the basis set that is used to represent the signal satisfies the RIP. Again, determining adherence to the RIP can be computationally intractable for non-random matrices, therefore we proceed to apply this technique with the understanding that $l_{1}$-norm minimized solutions may not strictly be the sparsest solution. In our framework, the SPGL1 library was leveraged to perform the BPDN optimization [9].

We note that primitive-based approaches are popular because generating far-field scattering from the primitives is straightforward. This is a key benefit and allows the optimization routine to iterate more quickly than in alternative approaches [10].

## IV. NUMERICAL EXPERIMENTS

We first apply our framework on a single flat plate, to demonstrate that the WDPs are implemented correctly and that BPDN can recover an adequate solution to $\Delta p$. Next, the framework is applied to an angled plate, which includes a dihedral response that cannot be captured with
the WDPs and shall be recovered with the IPSs. Figure 3 illustrates the two test geometries that are used to validate our approach.

The flat plate geometry is a $1 \mathrm{~m} \times 0.1 \mathrm{~m}$ plate with zero thickness, while the angled plate geometry consists of a $1 \mathrm{~m} \times 0.125 \mathrm{~m}$ and a $0.5 \mathrm{~m} \times 0.125 \mathrm{~m}$ plate joined at one end to form a $90^{\circ}$ angle (the latter dimension of each geometry is used to translate 2D echo width to 3D RCS). While the flat plate has no thickness, two variations of the angled plate were generated: one with zero thickness and one with 0.01 m thickness. The significance of the angled plate variations is discussed in Section IV-B.


Fig. 3. Flat plate and angled plate test geometries.
For all cases, only the points on the $z=0$ plane were used since the far-field patterns were limited to the $x y$ plane (elevation $\theta=90^{\circ}$, azimuth $\phi=\left[0,360^{\circ}\right]$ ) and there is no variation in either geometry in the $z$-direction. In all cases, we calculated the TM-polarized far-field backscatter response at $6 \mathrm{GHz}(\lambda=0.05 \mathrm{~m})$, ensuring that both targets are electrically large and amenable to high frequency approximations. Several mesh discretizations were generated to assess sparsity requirements for a given BPDN solution. Lastly, the BPDN parameters for error tolerance and maximum iterations were set to $10^{-3}$ and $10^{3}$, respectively, and were held constant over all experiments.

The reference data $S_{F F}^{\text {ref }}$ in our comparisons was generated with a MoM-based code to mitigate any contribution from measurement artifacts. We utilize a relative error norm as our metric for comparison, calculated as $\sum_{k, \hat{\mathbf{r}}}\left\|S_{F F}^{r e f}-\widehat{S}_{F F}^{e s t}\right\|_{2} /\left\|S_{F F}^{r e f}\right\|_{2}$, where $\|\cdot\|_{2}$ is the $l_{2}$-norm.

## A. Flat plate

Figure 4 illustrates our results from the flat plate geometry and compares the reference data against our method: the top plot compares the reference data $S_{F F}^{r e f}$ against the diffraction solution from the preprocessing stage $\hat{S}_{W D S}^{\text {pred }}$ as well as the combined solution $\hat{S}_{F F}^{\text {est }}=\widehat{S}_{W D S}^{\text {pred }}+\widehat{S}_{I P S}^{\text {pred }}$; the middle plot illustrates the performance of the sparse optimization stage by
comparing the IPS solution $\widehat{S}_{I P S}^{\text {pred }}$ with the difference pattern $\Delta p$ that the optimization attempts to recover; finally, the bottom plot depicts the coherent difference between $S_{F F}^{r e f}$ and $\widehat{S}_{F F}^{e s t}$.


Fig. 4. Reference, WDP and IPS solutions for a flat plate (magnitude). Discrepancies between the reference and WDP solution (top) are corrected with an IPS solution (middle) to yield low reconstruction error (bottom).
$\hat{S}_{W D S}^{\text {pred }}$ was generated by defining two WDPs at the ends of the plate with $\alpha=0^{\circ}$. The singularities from each primitive sum to generate the specular lobe at $\phi=90^{\circ}$. We observe that the diffraction solution compares well with the reference data until the monostatic angle approaches the grazing angle of the flat plate (a known deficiency in GTD analytic solutions). The coherent difference from the preprocessing stage stays largely within the -20 and -40 dB range and yields a relative error of 0.0731 .

After $\widehat{S}_{W D S}^{p r e d}$ is generated, $\Delta p$ is supplied to BPDN to synthesize $\widehat{S}_{I P S}^{\text {pred }}$. The result of the sparse optimization stage shows a well-converged solution and has a coherent difference that is largely below -60 dB. When the WDP and IPS solutions are combined to yield $\widehat{S}_{F F}^{e s t}$, we see overlay agreement with $S_{F F}^{r e f}$. The combined solution achieves a relative error of 0.0011 .

In contrast to our combined method, traditional scattering center reconstruction of far-field data utilizes IPSs exclusively to reconstruct the reference data rather than the delta pattern. We can assess the efficacy of the traditional approach by calculating the relative error as a function of the number of IPSs used to perform the reconstruction, as shown in Fig. 5. Using a $\lambda / 3$ sampling
to generate the IPSs candidates provides $N=61$ points. We see that both methods require all points to achieve the lowest errors, and the traditional method achieves a relative error of 0.0019 , compared to 0.0011 when the combined method is used (the WDP solution does not vary as a function of the number of IPSs).

We also observe that when scattering centers with the smallest magnitudes are removed from contributing to the far-field pattern, the error of the traditional approach increases more quickly than the combined approach. In this example, the traditional approach exceeds the error of the WDP solution when fewer than $N=60$ points are used for the reconstruction.


Fig. 5. Relative error norm of the flat plate.
These results indicate that a solution generated from a combination of WDPs and IPSs can achieve a more accurate reconstruction than either of the two separately. Moreover, for any desired level of error, the combined solution is sparser than the traditional method.

## B. Angled plate

The angled plate geometry provides a more challenging far-field pattern to reconstruct than the flat plate. In addition to the flat plate responses, a strong dihedral response will occur in the far-field azimuth sector $\phi=\left(180^{\circ}, 270^{\circ}\right)$. Knowing that the current implementation of WDPs cannot reconstruct the dihedral response, we limit their contributions to angles exclusive of the dihedral sector via $\beta_{l}$ in (5).

We note that even with the applied angle constraints, the WDPs may be inaccurate outside of the dihedral region, as well. According to [12], UTD WDPs fail near the shadow boundaries on concave shapes due to the fact that one of the WDPs is shadowed by obstructing geometry. The authors propose a separate type of diffraction coefficient to address dihedral effects by tracking rays that have multiple diffraction and reflection interactions on the target. Without introducing a third type of scattering center into the framework, we apply two additional WDPs located on the shared edge of the two plates (both with $\alpha=0^{\circ}$ ). This is analogous to two independent flat plates, where the additional WDPs complement the primitives on the open edges of the angled plate and compensate for the singularities that
arise from those primitives. In total, five WDPs are used: two for each flat plate and one for exterior corner of the angled plate and with $\alpha=90^{\circ}$. This arrangement yields a good approximation when compared to the reference data. The relative error norm over the far-field sector where the WDP solution is valid was calculated to be 0.0992 and is similar to the relative error norm achieved by the WDP solution for the flat plate geometry.

We note that IPSs will also encounter issues in pattern reconstruction of the angled plate due to the dihedral sector. We observed that the IPSs on a zerothickness angled plate failed to generate an adequate reconstruction since there is a large contrast in the farfield response of the dihedral and non-dihedral regions. Implementing a finite thickness model, shown in Fig. 6, and enforcing shadow boundaries mitigated these effects: an optimization of the zero thickness geometry resulted in an error of 0.7210 , while the finite thickness geometry achieved an error of 0.0477 .

However, even with the finite thickness geometry, additional non-physical aberrations are evident in the solution. We can observe the source of these errors by considering the angle sectors where each IPS contributes to the far-field. These sectors are discretized and plotted as vectors in Fig. 6.


Fig. 6. Active IPSs for $\phi=\left[0^{\circ}, 270^{\circ}\right]$ (red vectors) and $\phi=\left[270^{\circ}, 360^{\circ}\right]$ (green vectors).

The figure indicates that there are IPSs located in the interior region of the angled plate that contribute to both the dihedral and non-dihedral sectors of the far-field response. Moreover, the number of interior points contributing to the non-dihedral sector varies as the shadow boundary sweeps across the interior sector of the angled plate from $\phi=\left[116^{\circ}, 180^{\circ}\right]$ and $\left[270^{\circ}, 296^{\circ}\right]$. This variation causes the discontinuities in the far-field pattern shown in Fig. 7 and we see that the severity of the discontinuities decrease when the rate of variation decreases, namely when angle approaches either of the normal incident angles ( $\phi=180^{\circ}$ and $270^{\circ}$ ).

If the IPSs from the finite thickness model are used in the proposed method to reconstruct $\Delta p$ for the entire azimuth range $\phi=\left[0^{\circ}, 360^{\circ}\right]$, these discontinuities significantly degrade the reconstruction in the regions where the $\hat{S}_{W D S}^{\text {pred }}$ is already very good: under this arrangement, the method achieves a relative error norm of $0.0964(0.0905$ for $\lambda / 4,0.0830$ for $\lambda / 5)$. While it is
a slight improvement over the solution generated by WDPs alone, it does not provide a better solution than the traditional method. We speculate that, in addition to the discontinuities, the dynamic range of the delta pattern increases because the WDPs are restricted from contributing to the dihedral sector of $S_{F F}^{r e f}$. These effects ultimately make $\Delta p$ more difficult to reconstruct with IPSs.


Fig. 7. Discontinuities in non-dihedral sector of IPS solution due to interior IPS contributions.

As an alternative, we enforce additional constraints on the finite thickness model via $\beta_{n}$ such that the interior and exterior IPSs only contribute to the non-dihedral and dihedral sectors, respectively. Using this strict separation, $\hat{S}_{F F}^{e s t}$ from the combined method yields an improved relative error norm of 0.0238 and exceeds the performance of the traditional method. The results of this experiment are shown in Fig. 8.


Fig. 8. Reference, WDP and IPS solutions for an angled plate (magnitude only). Discrepancies between the reference data and WDP solution (top) are corrected with an IPS solution (middle) to yield low reconstruction error (bottom).

The figure also clearly shows the large dynamic range of $\Delta p$ where much of the dihedral sector stays above 0 dB and non-dihedral sector stays largely below -20 dB . With our proposed method of synthesizing $\widehat{S}_{F F}^{e s t}$, the errors achieved mostly fall below -20 dB .

Mirroring the analysis performed on the flat plate, we assess the trade between relative error norm and the number of IPSs used in the reconstruction of $S_{F F}^{r e f}$ and $\Delta p$, shown in Fig. 9. We note that the figure includes an additional dataset to show that, while the strict separation of the contributions of the inner IPSs to the dihedral sector and the outer IPSs to the non-dihedral sector was an effective strategy for synthesizing $\Delta p$, it was not effective when the IPSs were used to reconstruct $S_{F F}^{r e f}$. We speculate that the configuration that enforces strict separation does not provide an adequate number of IPSs to generate the narrow lobes that are present near the edges of the dihedral region in the far-field reference pattern. Conversely, the more permissive shadowing scheme provides enough of these point scatterers to generate narrow (but discontinuous) peaks to match the far-field reference pattern well, but detrimentally impacts the solution when they are used to match $\Delta p$ (which has lower and wider lobes).

We also observe a discontinuity in the solutions that rely exclusively on IPSs whereas the flat plate tests exhibited a monotonically decreasing error. This is because the magnitudes of the coefficients supporting the dihedral sector are significantly higher than those supporting the non-dihedral sector. For example, in the test case where $\Delta p$ was recovered via IPSs only, the removal of the lowest magnitude coefficients from reconstruction will incrementally degrade the nondihedral sector and only after the $39^{\text {th }}$ largest coefficient is removed will the dihedral reconstruction degrade.


Fig. 9. Relative error norm of the angled plate.
Overall, the results are consistent with those in the flat plate experiment. That is, the traditional approach that utilizes only IPSs to reconstruct far-field reference data is unable to reach the error levels that are achieved with the proposed approach. Moreover, if the smallest (in magnitude) non-zero coefficients are discarded from the reconstruction, the degradation of the solution from the
proposed method is more gradual than the traditional method.

Our numerical experiments are summarized in Table 1. We see with both geometries that a lower error is achieved when combining a WDP solution with an IPS solution to the delta pattern instead of the relying exclusively on WDPs or IPSs to reconstruct the far-field data. In the case of the angled plate, the result required manually setting boundaries on the range of angles where each primitive type contributes to the far-field pattern. Nonetheless, this is a valuable insight-if the primary goal is to find a compact representation of farfield data, this approach would prove to be very useful. With the proposed method, we can achieve a lower error with approximately the same number of point scatterers (WDPs and IPSs). Likewise, we have solutions that degrade more slowly with respect to how many IPSs are used to reconstruct the pattern when the IPSs are applied to a delta pattern rather than the far-field data.

Table 1: Relative error and (total point scatterer count) of $\lambda / 3$ discretized geometries

|  | Flat Plate | Angled Plate* |
| :--- | :--- | :--- |
| WDS Only | $0.0731(2)$ | $0.0992(5)$ |
| IPS Only | $0.0019(61)$ | $0.0477(184)$ |
| Combined | $0.0011(63)$ | $0.0238(189)$ |

* WDS case evaluated for non-dihedral sector only.


## V.SPARSE RECONSTRUCTION CONSIDERATIONS FOR COMPRESSIVE SENSING

In addition to investigating the reconstruction accuracy of the proposed method, we seek to understand how well IPSs perform as a sparse basis in the context of CS. With both the flat plate and angled plate geometries, we tested for solution convergence and robustness.

Figure 10 depicts the results from multiple discretizations of the flat plate geometry and how their solutions degrade as the weakest scattering centers are incrementally removed from the solution.

CS literature states that the recovery of signal is robust to noise and reconstruction accuracy should degrade gracefully with a given basis set due to the RIP (more specifically, the Null Space Property) [13]. We can see that the IPS basis can used to reconstruct $\Delta p$ in the proposed method and reconstruct $S_{F F}^{r e f}$ in the standard method. We noted previously that the degradation of $\widehat{S}_{F F}^{e s t}$ in the proposed method is more gradual than the far-field reconstruction in the standard method and we see that this remains true for other discretizations as well. However, the data also indicates that the solutions generated by the BPDN are not optimally sparse. The delta pattern and far-field pattern do not vary with respect to discretization, yet the number of IPSs required
to reconstruct those patterns does vary with respect to discretization.

We also observe that the numbers of candidate IPSs are $N=61,81$ and 101 for $\lambda / 3, \lambda / 4$ and $\lambda / 5$, respectively. At 6 GHz , the plate is $20 \lambda$ long and the solutions are effectively using all of the available scattering centers to determine a solution, even though it is known that a sparser solution exists (because the coarse discretizations are able to recover an equally accurate solution with fewer IPSs). While BPDN determines solutions with the smallest $\sum_{n}\left|\gamma_{n}\right|$, it does not guarantee a solution that minimizes the cardinality of $\gamma$ unless other CS criteria are met. These findings suggest that a basis set from IPSs does not satisfy the RIP.


Fig. 10. Robustness of proposed and traditional method solutions for $\lambda / 3$ to $\lambda / 5$ discretizations of the flat plate.

A similar analysis is performed on the data for the angled plate dihedral region, depicted in Fig. 11. Again, the number of IPSs required to reach a given level of error depends on the number of available IPSs. The rate of degradation is different from the flat plate case, however: the presence of longer tails on the reconstructions with the proposed method suggest that they have converged and while they are not ideal and optimally sparse solutions, they seem to be sparser and more robust than the reconstructions with the traditional method.


Fig. 11. Robustness of proposed and traditional method solutions for $\lambda / 3$ to $\lambda / 5$ discretizations of the angled plate.

These numerical experiments show that, while the technique is successful in generating point scatterer based (WDP and IPS) representations of the targets,
there may be limited utility as a basis for compressive sensing applications. The results show empirically that when a sparse representation of the target is used to generate far-field patterns (the traditional approach), perturbations in the sparse representation will introduce excessively large errors for the purpose of interpolation and extrapolation. The sparsity is slightly improved when IPS are employed to reconstruct delta patterns (the proposed approach), but their efficacy seems to be geometry dependent.

These initial results reveal areas that merit additional investigation. It would be prudent to integrate solutions for dihedral scattering mechanisms [14] into the framework which would allow the IPSs to recover a more simplified delta pattern. Additionally, we observed that the number of shadowed IPSs can vary rapidly and would introduce unwanted discontinuities in synthesized solution. Tapering or adjusting the angles that an IPS contributes to may address this issue and would improve how the IPSs perform on concave targets. Lastly, the optimization framework can be expanded to support multiple frequencies, multiple polarizations, non-planar observation geometries and bistatic quantities to possibly aid the convergence of the optimization routines and expand its applicability to a wider variety of test cases.

## VII. CONCLUSION

Using WDPs in conjunction with IPSs to reconstruct far-field patterns shows merit in simple cases and when they are applied judiciously. In our numerical experiments, we show that this approach can reduce the overall number of scattering centers required to replicate the scattering response of a flat and a right-angled plate. We also observed that $l_{1}$-norm minimization techniques may have difficulty finding maximally sparse solutions when IPSs are used as a basis set. Despite this, synthesized solutions are more robust when they are used to reconstruct a coherent difference pattern rather than the far-field data.

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# A Novel Omnidirectional Circularly Polarized Pagoda Antenna with Four Shorting Pins for UAV Applications 

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#### Abstract

A novel omnidirectional circularly polarized $(\mathrm{CP})$ pagoda antenna with four shorting pins is presented. In this structure, four curved branches are utilized to generate horizontal polarization, while coaxial cable and four shorting pins produce vertical polarization. Omnidirectional CP radiation is achieved by combining the radiation from the branches and the shorting pins. The curved branches loaded with shorting pins reduce the size of the antenna and the antenna is loaded with the lower substrate with two coupling patches to improve the bandwidth. The novel pagoda antenna is fabricated and measured. Its final dimensions are $0.42 \lambda_{0} * 0.42 \lambda_{0} * 0.33 \lambda_{0}$ ( $\lambda_{0}$ is the free-space wavelength at operating frequency). The impedance bandwidth ( $\mathrm{S}_{11} \leq-10 \mathrm{~dB}$ ) is 0.53 GHz and the axial ratio ( AR ) bandwidth ( $\mathrm{AR} \leq 3 \mathrm{~dB}$ ) is 0.95 GHz , which can be used for unmanned aerial vehicles' diagram transmission in China.


Index Terms - Circularly polarized (CP), coupling patch, curved branch, omnidirectional antenna, shorting pin.

## I. INTRODUCTION

Omnidirectional circularly polarized ( CP ) antennas have attracted more and more attention from experts and scholars due to they widely used in wireless communication, remote sensing and telemetry and satellite communication systems. Omnidirectional antenna can radiate electromagnetic wave in any direction in a plane, which is suitable for multipoint simultaneous communication and communication in uncertain position during movement. The CP antennas can accept arbitrary polarized wave, reduce multipath interference, and have high polarization isolation. Having both advantages of omnidirectional antenna and CP antenna, omnidirectional CP antenna can meet the requirements of accurate signal transmission of unmanned aerial vehicle (UAV) systems and be used as its image transmission antenna, which can effectively reduce the signal blind area and polarization interference.

In recent years, many methods of realizing omnidirectional CP antennas have been proposed, which
can be roughly classified into three categories. The first method is circular polarized method of omnidirectional antenna that a circular polarizer is loaded outside the omnidirectional antenna. In [1], the omnidirectional CP antenna is realized by using the parallelepiped medium element with high dielectric constant to embrace around the coaxial probe. In [2], the inner conductor is fed, while the spiral groove is opened on the outer cylinder conductor. The second method is element combination [3-6], which uses the combination of CP antenna elements to achieve omnidirectivity. In [3], circular open rings loaded with parasitic rings are used as CP antenna elements, and they are combined around cylinder to attain omnidirectivity. In [4], a rectangular ring is used as a CP antenna element.

However, omnidirectional CP antennas realized by these two methods are generally large in size and have high profile, which is difficult to install on small and lightweight UAVs. In order to reduce the size and profile, a third method is proposed, which combines the antenna that generates the vertical polarized wave with the antenna that generates the horizontal polarized wave [715]. In [7], a broadband omnidirectional CP antenna was proposed, in which two sets of dipoles were used to generate omnidirectional CP waves. In [8], capacitive feeding is used to widen the bandwidth of the antenna. The antenna's profile is greatly reduced, but the radius is still large. In [9], planar sector-shaped endfire elements are used to realize omnidirectional circular polarization of the antenna, which further reduces the antenna profile. V-shaped slot on the same principle was used in omnidirectional CP antenna [10], which also makes the size of the antenna even smaller. The profile and radius of this kind of antenna are small, but the bandwidth is too narrow to meet the actual needs.

Based on the comparison with the antenna mentioned above, this paper proposes an omnidirectional CP antenna suitable for UAV image transmission system. The antenna's size is relatively small and has an adequate bandwidth to ensure the transmission of information. Coaxial cable and four shorting pins
are used to generate vertical polarized waves, while four curved branches are used to generate horizontal polarized waves. The structure of curved branches loaded with shorting pins are used to reduce the size of the antenna, and the dielectric substrates at the bottom with two ring patches are coupled to extend the bandwidth. Finally, the antenna's size is $0.42 \lambda_{0} * 0.42 \lambda_{0} * 0.33 \lambda_{0}$. The impedance bandwidth of the antenna ( $\mathrm{S}_{11} \leq-10 \mathrm{~dB}$ ) ranges from 5.56 GHz to 6.09 GHz , and the axial ratio (AR) bandwidth ( $\mathrm{AR} \leq 3 \mathrm{~dB}$ ) is from 5.07 GHz to 6.02 GHz , effectively covering UAVs' diagram transmission frequency in China at 5.8 GHz . The configuration, principles, simulation used by Ansoft high-frequency structure simulator (HFSS) and results of the antenna are described detailedly in the following sections.

## II. ANTENNA CONFIGURATION

Figure 1 shows the configuration of omnidirectional CP pagoda antenna. It consists of three dielectric substrates with a thickness of 1 mm , a radiant patch, a reference ground, four shorting pins and four ring patches. The radiation patch and ground plane are located on the upper surface of the upper substrates and the middle substrates respectively, the material of which is FR-4 with the permittivity of 4.4 and the loss tangent of 0.002 . The radiation patch and ground plane each contains four curved branches in opposite direction connected by shorting pins at the end, which are commonly used to reduce the size of antenna. The coaxial cable is in direct contact with the radiation patch for feeding. The inner conductor of coaxial cable and shorting pins are both made of copper.



Fig. 1. Geometrical configuration of the proposed antenna. (a) Perspective view. (b) Side view. (c) Top view of the upper substrates. (d) Top view of the middle substrates. (e) Top view of the lower substrates.

The curved branches loaded with shorting pin reduce the size of the antenna, but the bandwidth is relatively narrow. In order to improve the bandwidth for better practicability, the antenna is loaded with the lower substrate with two ring patches on the upper surface and the lower surface respectively. The lower substrate with two ring patches acts as a choking coil, similar to a metal sleeve. Because it can be coupled with the radiation patch to enhance the radiation intensity, the lower substrate broadens both impedance bandwidth and AR bandwidth. The parameters of the antenna are listed as follows: $R_{l}=11 \mathrm{~mm}, R_{2}=5.1 \mathrm{~mm}, R_{3}=10.9 \mathrm{~mm}, R_{4}=9 \mathrm{~mm}$, $R_{5}=8 \mathrm{~mm}, R_{6}=6.9 \mathrm{~mm}, R_{7}=2.7 \mathrm{~mm}, R_{8}=4.5 \mathrm{~mm}, W_{l}=1 \mathrm{~mm}$, $W_{2}=2.75 \mathrm{~mm}, W_{3}=0.82 \mathrm{~mm}, W_{4}=0.9 \mathrm{~mm}, L_{I}=8.74 \mathrm{~mm}$, $L_{2}=5 \mathrm{~mm}, L_{3}=3 \mathrm{~mm}, H_{l}=4.65 \mathrm{~mm}, H_{2}=17.09 \mathrm{~mm}, H_{3}=24 \mathrm{~mm}$.

## III. PRINCIPLE AND METHODOLOGY

For omnidirectional CP pagoda antenna is understand clearly, working principle and parameters analysis of the proposed antenna are discussed in this section.

## A. Omnidirectional CP property

CP waves are composed of two orthogonal linearly polarized waves with a phase difference of $90^{\circ}$. According to the duality principle, the electric field $(\vec{E})$ in the far field generated by the magnetic currents on the pagoda antenna can be expressed as:

$$
\begin{equation*}
\vec{E}=\hat{\theta} E_{\theta}+\hat{\phi} E_{\phi} \tag{1}
\end{equation*}
$$

It is found that when there are more branches, omnidirectivity of the antenna is better, as shown in Fig. 2. And the operating frequency is reduced as the increase in the number of branches, which has the function of miniaturization, as shown in Fig. 3. However, considering the size of the antenna, four branches are finally chosen. The four branch elements are arranged in sequence and fed by coaxial cables, forming a clockwise current loop. The end of each branch element is connected to the ground plane by a shorting pin, that form a current path. According to the boundary condition, the clockwise current loop generates a vertical down magnetic pole ( $\vec{J}=\vec{n} \times \vec{H}$ ), which forms a horizontal polarization component, the $E_{\varphi}$ field. The radius of the current loop is $R_{1}$. The far field of the current loop $E_{\varphi}$ can be expressed as [16]:

$$
\begin{equation*}
E_{\varphi}=\frac{[I] R_{1} \omega \mu_{0}}{2 r} J_{1}\left(\beta R_{1} \sin \theta\right) \stackrel{\wedge}{\varphi} \tag{2}
\end{equation*}
$$

Where [I] represents the magnitude of current on the loop, $\omega$ is the operating frequency, $\mu_{0}$ is the freespace permeability, $J_{1}$ is Bessel's first order function, $\beta$ is the propagation parameter of space and $r$ is the distance between the antenna and the measuring point.

Vertical polarized waves are generated by coaxial cable and four shorting pins, which act as electrodes. What is more, the vertical polarized waves produced by the electrodes are omnidirectional. The height of the shorting pin can be seen as $H_{1}$. Its far field $E_{\theta}$ can be expressed as [16]:

$$
\begin{equation*}
E_{\theta}=j \frac{[I] H \omega \mu_{0}}{4 \pi r} \sin \theta \hat{\theta} \tag{3}
\end{equation*}
$$

Based on (2) and (3), it is noted that as long as the current [ $I$ ] through coaxial cable, curved branch and shorting pin is uniform, there will be a $90^{\circ}$ phase difference between the vertical polarized component and the horizontal polarized component. The CP radiation pattern will be generated when the magnitude of $E_{\theta}$ and $E_{\phi}$ are adjusted to be equal. As shown in the Fig. 4, it can be found that the current passing through does not change its direction in one time. Therefore, two polarization components exist $90^{\circ}$ phase difference to each other in the far field. Due to the compact structure of the antenna, two polarization components have the same amplitude and orthogonal to each other in space at the same time. Finally, the omnidirectional CP pagoda antenna is realized in theory.


Fig. 2. Three improved prototypes of the proposed antenna.


Fig. 3. Simulated $\left|\mathbf{S}_{11}\right|$ of Ant 1, Ant 2 and Ant 3.


Fig. 4. Current distribution of the proposed antenna at 5.8 GHz .

## B. Parametric study and analysis

Curved branch can change the path of current, effectively reducing the size of the antenna. The length of the branches is approximately determined by:

$$
\begin{equation*}
\frac{R_{6} \pi}{4}+L_{2}+\frac{W_{4}}{2}+R_{5}=\lambda \tag{4}
\end{equation*}
$$

Where $\lambda$ is the operating wavelength. As shown in Fig. 5 , the radius of the curved branch is much smaller than that of the straight branch by adjusting the width of the branches when the current passes through the same length.

The shorting pin at the end of the branch not only supports the upper and middle dielectric substrates, but also miniaturizes the antenna. The two ends of the radiation patch are open, and there must be zero potential points in the standing wave. Loading shorting pin at zero potential points, the original state of standing wave will be made from open circuit to short circuit state, equivalent to make $\lambda / 2$ to $\lambda / 4$ harmonic resonance. The antenna size ( $R$ ) can be approximately calculated by [17]:

$$
\begin{gather*}
R=\frac{\lambda}{4}=\frac{c}{4 \times \sqrt{e_{e q}} \times f},  \tag{5}\\
\varepsilon_{e q}=\frac{H_{1}+H_{2}}{\frac{H_{1}}{\varepsilon_{r}}+\frac{H_{2}}{\varepsilon_{\text {air }}}} . \tag{6}
\end{gather*}
$$

Where $c$ is the speed of light in vacuum, and $\varepsilon_{e q}$ is the
equivalent dielectric constant of multilayered substrate. Figure 6 compares the $S$ parameters of antenna with shorting pin and without shorting pin. As can be seen from Fig. 6, the impedance characteristic of the antenna with shorting pin is obviously better than that without shorting pin, and the resonant frequency of antenna reduces, thus achieving the miniaturization. Therefore, the antenna size is reduced due to the curved branch loaded with shorting pin.


Fig. 5. Radius of (a) curved branch and (b) straight branch.


Fig. 6. Simulated $\left|S_{11}\right|$ of the proposed antenna with and without shorting pins.

The lower substrate with two coupling patches can adjust the resonance point and enlarge the bandwidth of the antenna. Figure 7 shows the comparison of $S$ parameters and AR bandwidth of antennas with and without the coupling patches. It can be seen that the impedance bandwidth and AR bandwidth of the antenna both increase after the coupling patches loaded.


Fig. 7. Simulated (a) $\left|S_{11}\right|$ and (b) AR of the proposed antenna with and without coupling patches.

The coupling strength is affected by two factors [18], one is the distance between the upper substrate and the middle substrate $H_{1}$, the other is curved branch arc width of the radiation patch $W_{4}$. Figure 8 shows the influence of the distance $H_{1}$ on the operating frequency of the antenna. It can be seen from Fig. 8(a) that when the distance $H_{1}$ decreases, the resonant frequency of the antenna increases, while Fig. 8 (b) shows that AR points shift towards higher frequency. Figure 9 shows the effect of the width $W_{4}$. Figure 9 (a) shows that when $W_{4}$ decreases, the S-parameter curves of the antenna decrease while Fig. 9 (b) shows that the AR values have no obvious changes.


Fig. 8. Simulated (a) $\left|\mathrm{S}_{11}\right|$ and (b) AR of the proposed antenna with different $H_{1}$.


Fig. 9. Simulated (a) $\left|S_{11}\right|$ and (b) AR of the proposed antenna with different $W_{4}$.

## IV. EXPERIMENTAL RESULTS

In order to validate the proposed method, the antenna is fabricated, as shown in the Fig. 10. The antenna weighs only 4 grams, which makes it more suitable for UAVs diagram transmission because it's light enough. The measurements were implemented by an Agilent E8363B network analyzer and a far-field system in anechoic chamber.

Simulated and measured S parameters of antenna are shown in Fig. 11 (a). It can be seen that the measured S parameters agree with the simulated values well. The measured bandwidth of the antenna ( $\mathrm{S}_{11} \leq-10 \mathrm{~dB}$ ) is 0.64 GHz , from 5.56 GHz to 6.21 GHz , equivalent to $11 \%$ at 5.8 GHz . Figure 11 (b) shows simulated and measured AR bandwidth of antenna. The AR bandwidth ( $\mathrm{AR} \leq 3 \mathrm{~dB}$ ) is 0.9 GHz , from 5.10 GHz to 6.00 GHz , equivalent to $15.5 \%$. It can be found that the overlapped bandwidth of

Impedance and AR from 5.56 GHz to 6.00 GHz covers the UAVs diagram transmission frequency of 5.8 GHz in China.


Fig. 10. Photograph of the fabricated antenna.


Fig. 11. Simulated and measured (a) $\left|S_{11}\right|$ and (b) AR of the fabricated prototype.

Figure 12 shows simulated and measured AR of the antenna in the azimuth plane $\left(\theta=90^{\circ}\right)$ at 5.8 GHz . It can be seen that there are some small differences between the simulated and measured values due to fabrication errors. From the figure, when it is simulated, the ARs fluctuate within 1 dB . The measured ARs are slightly higher than the calculated results in all directions, but
they are all less than 3 dB in the plane, the ripple of which is 0.7 dB . Both the simulated and measured values meet the requirement of circular polarization, which indicates that the antenna has good CP characteristics at 5.8 GHz .


Fig. 12. Simulated and measured AR of the CP antenna in the azimuth plane $\left(\theta=90^{\circ}\right)$.

The simulated LHCP gain at 5.8 GHz is 1.12 dBic . Figure 13 shows the measured and calculated radiation patterns of the proposed antenna at 5.8 GHz in the azimuth (xy plane) and elevation (xz plane) planes, respectively. In $x y$ plane, both the simulated and measured values of left-hand circularly polarized (LHCP) fluctuate around 1 dB , while the simulated and measured values of right-hand circularly polarized (RHCP) are less than -15 dB . In xz plane, it also can be seen that the results of the simulated and measured agree well with each other, and the level differences between the RHCP and LHCP are more than 15 dB , that indicates the antenna is LHCP and has good omnidirectivity.

In order to show the advantage of the antenna, Table 1 compares the omnidirectional CP pagoda antenna with existing antennas in bandwidth and dimensions, from which the overall advantage of the novel pagoda antenna can be seen. Compared with the existing antenna, the overall size of novel pagoda antenna proposed is more advantageous. For the same size, the bandwidth is wider than that of the compared antenna.


Fig. 13. Simulated and measured radiation patterns in (a) azimuth (xy plane) and (b) elevation (xz plane) planes at 5.8 GHz .

Table 1: Performance comparison of reported omnidirectional CP antennas with existing antenna

| Ref. | Dimensions $\left(\boldsymbol{\lambda}_{\mathbf{0}}\right)$ | Imp. Bandwidth (\%) | 3dB AR Bandwidth (\%) | Usable Bandwidth (\%) |
| :--- | :--- | :--- | :--- | :--- |
| $[7]$ | $1.63 \lambda_{0} \times 1.63 \lambda_{0} \times 0.28 \lambda_{0}$ | 21.60 | 25.3 | 21.60 |
| $[9]$ | $0.62 \lambda_{0} \times 0.62 \lambda_{0} \times 0.029 \lambda_{0}$ | 3.97 | - | 3.97 |
| $[10]$ | $0.52 \lambda_{0} \times 0.52 \lambda_{0} \times 0.026 \lambda_{0}$ | 3.50 | - | 3.50 |
| $[15]$ | $0.24 \lambda_{0} \times 0.24 \lambda_{0} \times 0.12 \lambda_{0}$ | 3.90 | 7.5 | 3.90 |
| This <br> work | $0.42 \lambda_{0} \times 0.42 \lambda_{0} \times 0.33 \lambda_{0}$ | 9.00 | 16.4 | 9.00 |

## V. CONCLUSION

A novel omnidirectional RHCP antenna is presented in this letter. The novel pagoda antenna loaded with the shorting pin and the lower substrate with two coupling patches achieves miniaturization in size with adequate bandwidth. The principle and experiment of the antenna are discussed in detail above. Finally, the impedance bandwidth of novel pagoda antenna is 0.53 GHz , AR bandwidth is 0.95 GHz and usable bandwidth is 0.53 GHz , which can cover the UAVs diagram transmission frequency of 5.8 GHz in China. With the final dimensions of $0.42 \lambda_{0} * 0.42 \lambda_{0} * 0.33 \lambda_{0}$, the antenna has advantages compared with the existing antennas. At the same time, it can be found that the AR bandwidth of the antenna is wider than the impedance bandwidth, which provides a potential for the following research. Impedance bandwidth can be further broadened to achieve miniaturized broadband omnidirectional CP pagoda antenna.

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# Performance of Aeronautical Mobile Airport Communications System in the Case of Aircraft at Final Approaching or Initial Climbing 

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#### Abstract

The aeronautical mobile airport communications system (AeroMACS) is proposed to support the communications between the tower and aircrafts or the service vehicles in the range of airport. In this paper, the working environment that the aircraft at final approaching or initial climbing (AFAIC) is researched. By considering the influence of Doppler frequency shift and the channel model in AFAIC case, we propose a transmission scheme which can obtain preferable transmission performance for low order modulation. Simulation results of the signal-to-noise ratio loss, the spectrum efficiency and the bit error rate are given, which indicate that the proposed scheme can meet the demand of AeroMACS and expand the zone of airport communication area.


Index Terms - Aeronautical mobile airport communications system, communication in aircraft at final approaching and initial climbing case, signal processing.

## I. INTRODUCTION

The increase of air traffic in civil airports has led to a growth of data loads needed for both air traffic control (ATC) and airline operational communications (AOC) services. The new generation of air traffic management (ATM) system needs the support of a broadband communication structure able to satisfy the increasing application and service requirements foreseen by the Communications Operating Concept and Requirements (COCR) [1].

To satisfy the communication order, both international civil aviation organization (ICAO) and Eurocontrol identified a solution to provide dedicated aeronautical communications on the airport surface.

This system is named aeronautical mobile airport communications system (AeroMACS), which supports the aircraft are in closest proximity to each other and to a wide variety of service and operational support aircrafts, vehicles, personnel, and infrastructures [2,3]. AeroMACS is based on the IEEE 802.16e standard, known as Mobile WiMAX [4].

The AeroMACS operates in the aeronautical Cband, which is with the frequency between 5091 MHz and 5150 MHz , and divided into 11 separated subchannels with the bandwidth of 5 MHz . The transmission characteristics and the antenna design of the aeronautical C-band has been widely studied [5,6]. In particular, non-line-of-sight (NLoS) communications are supported by means of orthogonal frequency division multiplexing access (OFDMA) technology, which provides robustness against detrimental propagation effects and efficient use of resources, thus enabling high-data-rate services.

According to the AeroMACS profile, OFDMA frames follow a time division duplexing (TDD) structure, in which the downlink (DL) and uplink (UL) transmissions occur at different times and share the same frequency channel, and the AeroMACS frame occupies the entire 5 MHz channel during its 5 milliseconds duration.

In [2] and [3], the concept and the basic scheme of AeroMACS is introduced. The transmission efficiency and performance is analyzed in [4] and [7]. The typical transmission case, which considers different communication environment and variety kind of access node (i.e., aircrafts, vehicles and sensors), is discussed in [8]. The MIMO antenna configuration scheme is proposed and tested in [9] and [10], and the multi-hop relay scheme based on IEEE 802.16j is proposed in [11].

To describe different work environments in the airport, two typical working cases, the aircraft at hangar taxiway or runway (AHTR) and the aircraft at the gate (AG) are defined [8]. In AHTR case, there is few obstacles between the access nodes, and the channels between different nodes contain one LOS path and some NLOS paths. In AG case, there are several aircrafts and tens of vehicles in one communication cell, and the channels between different nodes do not have LOS path.

Beside the two abovementioned cases, aircraft at final approaching or initial climbing (AFAIC) progress plays an important part of ATM communication in AeroMACS. The aircraft operation data in AFAIC case is also important to realizing the high level advanced surface movement guidance and control system (ASMGCS) or the automatic aircraft navigation such as automatic approaching or landing.

In AFAIC case, high approaching or climbing speed of aircraft leads to strong Doppler frequency shift in the communication system, which leads to intersubcarrier interference of the OFDMA scheme. Besides, the communication channel in the AFAIC case is with strong LOS path, which is different from the AG and AHTR cases.

To improve the transmission performance, many intersubcarrier interference cancellation schemes and transmission schemes are proposed [12-15]. In [16], a Doppler shift and timing error estimation and cancellation scheme is proposed for AeroMACS. However, it is with complicated decoding process, and the special channel between the base station and the aircraft in AFAIC case has not been considered.

In this paper, the transmission mode of AFAIC case in AeroMACS has been researched. By considering the influence of Doppler frequency shift and the channel model in AFAIC case, we propose a transmission scheme that the decoding process is similitude with the existing one in IEEE 802.16e. It is easy to be realized during the construction process of AeroMACS. Simulation results of the signal-to-noise ratio (SNR) loss, the spectrum efficiency and the bit error rate (BER) are also given at the end of this paper.

## II. SYSTEM MODEL

As shown in Fig. 1, the AeroMACS network architecture is based on IEEE 802.16e [17], which is an all-IP network that decouples the access architecture from IP connectivity and allows modularity and flexibility.

The mobile station (MS) and the subscriber station (SS) are the end user devices. The MS nodes are moving nodes such as aircrafts, vehicles, and personal nodes in the airport. The SS nodes are fixed nodes such as radar, weather stations, and sensors in the movement area of the airport.

The base station (BS) is the access point to the
network, implementing air interface and the access functionalities including UL and DL scheduler, radio resource management, and handover (HO) control. Each BS operates on an angular sector and is at a given frequency. During different transmission environments, the cover area of BS can be large (e.g., the macro cell for the AHTR scenery) or small (e.g., the micro cell for the AG scenery).

Access service network (ASN) is in charge of all the functions needed to provide radio access to AeroMACS subscriber, authentication, and resource management procedures. The connectivity service network (CSN) is a set of network functions that provide IP connectivity services to the AeroMACS subscriber. The ASN gateway, represents the link between the ASN and the CSN, described below, performing routing or bridging functions.


Fig. 1. Typical AeroMACS network, which includes BS node, MS nodes and SS nodes. The ASN, CSN and the ASN gateway realize the connection to the airport web.

In AFAIC case, the aircraft approaches to or climbs from the runway with the similar track. Considering that the minima taking-off interval or approaching interval in civil ATC rule is about 10 km , and the cover range of BS in macro cell reaches more than 10 km , as shown in Fig. 2, the number of MS node in AFAIC is limited (Normally, BS covers 2 aircrafts during both approaching and climbing cases).

Since the approaching speed or the climbing speed of aircraft is a dynamic value, which affected by aircraft type, load, oil quantity and wind speed, in [18], five categories, i.e., CAT A, B, C, D and E, of typical aircraft have been established by different approaching speed. In general, for normal civil aircraft, the speed range of AFAIC case is between $200 \mathrm{~km} / \mathrm{h}$ (kilometers per hour) and $400 \mathrm{~km} / \mathrm{h}$ [15]. The channel between BS and MS in AFAIC scenery is the fast fading channel with strong Doppler frequency shift. In the OFDM system, we take $\varepsilon$ as the normalized offset frequency:

$$
\begin{equation*}
\varepsilon=N f_{c} T_{S} v / c, \tag{1}
\end{equation*}
$$

where $f_{c}$ is the carrier frequency, $c$ is the light speed, and $v$ is the moving speed of MS in meter per second, $N$ is the number of the DFT point in OFDMA scheme and $T_{S}$ is the symbol period duration time.


Fig. 2. The AFAIC case. The aircrafts are approaching or climbing with the similar track.

In the typical AeroMACS, considering the abovementioned speed range in AFAIC case, and assuming that the carrier frequency is 5 GHz , the number of DFT point is 512 , and the symbol period duration time is $1.75 \times 10^{-7}$ seconds, the range of $\varepsilon$ is:

$$
\begin{equation*}
\varepsilon \in\left[4.97 \times 10^{-2}, 1.66 \times 10^{-1}\right] \tag{2}
\end{equation*}
$$

The channel model of AFAIC case is shown in Fig. 3, which is with one LOS path and some NLOS paths, and it is a typical Rice model.


Fig. 3. Channel model of AFAIC case, which contains one LOS path and several NLOS paths.

Define the amplitude of the LOS path is $a$, the variance of the diffuse process with zero-mean quadrature components of NLOS path is $c$, The Rice factor, which is the power ratio between the LOS and the diffuse components is given by:

$$
\begin{equation*}
K_{\mathrm{RICE}}=10 \lg \left(\frac{a^{2}}{c^{2}}\right) \mathrm{dB} \tag{3}
\end{equation*}
$$

Define the number of NLOS path is $N$. Let $f_{D}$ and
be the Doppler shifting frequency of the LOS path. The phase shift, the Doppler shifting frequency and the time delay of the $n$-th NLOS path are $\theta_{n}, f_{D_{n}}$ and $\tau_{n}$, respectively. The channel function of AFAIC is given by:

$$
\begin{equation*}
h(t)=a e^{j 2 \pi f_{D} t}+c \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j \theta_{n}} e^{j 2 \pi f_{D_{n}} t} e^{-j 2 \pi \tau_{n}} \tag{4}
\end{equation*}
$$

In (4), $a e^{j 2 \pi f_{D} t}$ denotes the channel function of the LOS path, and the rest parts denote the channel function of the NLOS paths where $e^{j \theta_{n}}, e^{j 2 \pi f_{D_{n}}}$ and $e^{-j 2 \pi \tau_{n}}$ represent the phase shifting component, the Doppler shift component and the time delay heft component of the $n$-th NLOS path, respectively.

As discussed in [15], in the typical AFAIC case, the parameter $K_{\text {RICE }}$ is between 9 dB and 20 dB , and the typical value of $K_{\text {RICE }}$ is 15 dB . The typical number of NLOS path is 20 .

## III. TRANSMISSION MODEL IN AFAIC

## CASE

From the physical layer specifications, the DL of OFDMA scheme is essentially equivalent to an OFDM system that in one symbol period, BS can occupy the all subcarriers. Let $\boldsymbol{d}_{i}$ be the transmitted symbols of the BS node during DL in the $i$-th symbol period and $N$ denotes the number of the subcarrier, where,

$$
\boldsymbol{d}_{i}=\left[\begin{array}{llll}
d_{i}(0) & d_{i}(1) & \cdots & d_{i}(N-1) \tag{5}
\end{array}\right]
$$

As noted in [12], the transmitted symbols are fed to a conventional OFDM modulator that consists of an $N$-point inverse discrete Fourier transform (IDFT) unit followed by the insertion of an $N_{g}$-point cyclic prefix (CP) to avoid interference between adjacent blocks. The output of IDFT unit is denoted as:

$$
s_{i}=\left[\begin{array}{llll}
s_{i}(0) & s_{i}(1) & \cdots & s_{i}(N-1) \tag{6}
\end{array}\right]
$$

where the CP is denoted as:

$$
\begin{equation*}
s_{i}(k)=s_{i}(k+N) \text { for }-N_{g} \leq k \leq-1 \tag{7}
\end{equation*}
$$

The modulated symbol on the $k$-th subcarrier is:

$$
\begin{gather*}
s_{i}(k)=\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_{i}(n) e^{\frac{j 2 \pi n k}{N}}  \tag{8}\\
\quad \text { for }-N_{g} \leq k \leq N-1
\end{gather*}
$$

The discrete-time downlink signal takes the form:

$$
\begin{equation*}
s^{T}(k)=\sum_{i} s_{i}\left(k-i N_{T}\right) \tag{9}
\end{equation*}
$$

where $N_{T}=N+N_{g}$.
By considering the time-variant frequency-selective fading channel between BS and MS, we denote the channel impulse response (CIR) of the $l$-th subcarriers as $h(l)$ and $l \in\{0,1, \cdots, L-1\}$. The received signal on the $k$-th subcarriers of MS in the $i$-th symbol period is:

$$
\begin{equation*}
r_{i}(k)=P_{B S} \sum_{l=0}^{L-1} h(l) s_{i}(l)+w(k) \tag{10}
\end{equation*}
$$

where $P_{B S}$ is the transmission power of BS, and $w(k)$ is the complex-valued additive white Gaussian noise (AWGN). The received signal of MS during contiguous symbol period is:

$$
\begin{equation*}
r(k)=P_{B S} \sum_{i} \sum_{l=0}^{L-1} h(l) s_{i}\left(k-l-i N_{T}\right)+w(k) \tag{11}
\end{equation*}
$$

At the receiver, $r(k)$ is divided into adjacent segments of length $N_{T}$, each corresponding to a transmitted OFDMA block. Assuming the CP is longer than the CIR duration, after removing the CP , the remaining $N$ samples are passed to an $N$-point discrete Fourier transform (DFT) unit, and the output of the DFT unit is:

$$
\begin{equation*}
R_{i, n}=P_{B S} H_{n} d_{i, n}+w_{i, n} \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
H_{n}=\sum_{l=0}^{L-1} h(l) e^{-\frac{j 2 \pi n l}{N}} \tag{13}
\end{equation*}
$$

and $w_{i, n}$ is the equivalent additive noise after DFT, which is with the complex Gaussian distribution.

By considering the frequency shift in AFAIC case calculated in (2), $\varepsilon$ is less than the bandwidth of the subcarrier, the frequency shift only leads to the intersubcarrier interference. The received signal of the MS in (10) can be rewritten as:

$$
\begin{equation*}
R_{i, n}=P_{B S} e^{j \varphi_{i}} H_{n} f_{N}(\varepsilon) d_{i, n}+I_{i, n}(\varepsilon)+w_{i, n} . \tag{14}
\end{equation*}
$$

Where

$$
\begin{gather*}
\varphi_{i}=\frac{2 \pi \varepsilon_{i} N_{T}}{N}  \tag{15}\\
f_{N}(\varepsilon)=\frac{\sin (\pi \varepsilon)}{N \sin \left(\frac{\pi \varepsilon}{N}\right)} e^{\frac{j \pi \varepsilon(N-1)}{N}}, \tag{16}
\end{gather*}
$$

and $I_{i, n}(\varepsilon)$ denotes the interference signal caused by the adjacent subcarriers. Since $I_{i, n}(\varepsilon)$ relates to the transmitted symbols on the adjacent subcarriers, we model it as a zero-mean random variable with its power denoted as

$$
\begin{equation*}
\sigma_{I}^{2}(\varepsilon)=P_{B S}\left[1-\left|f_{N}(\varepsilon)\right|^{2}\right] . \tag{17}
\end{equation*}
$$

By considering the Rice channel model of the AFAIC case in (3), formula (14) can be rewritten as:

$$
\begin{align*}
R_{i, n} & =P_{B S} a e^{j \varphi_{i}} f_{N}(\varepsilon) d_{i, n} \\
& +\sum_{l=0}^{N-1} P_{B S} c e^{\varphi_{i}} h(l) e^{-\frac{j 2 \pi n l}{N}} f_{N}(\varepsilon) d_{i, n} .  \tag{18}\\
& +I_{i, n}(\varepsilon)+w_{i, n}
\end{align*}
$$

During the UL transmission of AeroMACS, the flexible OFDMA spectrum allocation schemes are allowed. Normally, there are two UL transmission cases.

The first case is that in one symbol period, all the subcarriers are allocated to one MS node, and this scheme befits for the case that there are small number of MS nodes in the coverage zone of BS. The transmission model is similar with the DL mode. This scheme does not fit in the AG and the AHTR cases since large number of MS nodes results in lower spectrum allocation efficiency and more time delay.

Another case is that in one symbol period, the subcarriers are allocated to $u$ MS nodes $\left(\mathrm{MS}_{1}, \mathrm{MS}_{2}, \cdots\right.$, $\mathrm{MS}_{u}$ ), and we assume that the $N_{u s}$-th to the $N_{u e}$-th subcarriers are allocated to the $u$-th MS. After IDFT, the modulated signal of the $u$-th MS at the $k$-th subcarrier is:

$$
\begin{gather*}
s_{i, u}(k)=\frac{1}{\sqrt{N}} \sum_{n=N_{u s}}^{N_{u s}-1} d_{i, u, n} e^{\frac{j 2 \pi n k}{N}} .  \tag{19}\\
\text { for }-N_{u \mathrm{~s}} \leq k \leq N_{u \mathrm{e}}-1
\end{gather*}
$$

The received signal at the $k$-th subcarrier of BS can be denoted as:

$$
\begin{equation*}
r_{i}(k)=\sum_{u=0}^{U-1} P_{u} \sum_{k=N_{u 1}}^{N_{u z}-1} h_{u}(k) s_{i, u}(k)+w_{i}(k), \tag{20}
\end{equation*}
$$

where $h_{u}(k)$ is the CIR of the channel from $\mathrm{MS}_{u}$ to BS on the $k$-th subcarrier. The demodulated signal after DFT at BS node is:

$$
\begin{equation*}
R_{i, n}=\sum_{u=0}^{U-1} P_{u} \sum_{k=N_{u s s}}^{N_{u s}-1} h_{u}(k) e^{-\frac{j 2 \pi n k}{N}} d_{i, u}(k)+w_{i, n} . \tag{21}
\end{equation*}
$$

Let,

$$
\begin{equation*}
H_{n, u}^{\prime}=\sum_{k=N_{u s}}^{N_{u s}-1} h_{u}(k) e^{-\frac{j 2 \pi n k}{N}} d_{i, u}(k), \tag{22}
\end{equation*}
$$

and $\varepsilon_{u}$ be the nominalized frequency shift of the $u$-th MS, the received signal at the MS can be described as:

$$
\begin{equation*}
R_{i, n}=\sum_{u=0}^{U-1} P_{u} e^{j \varphi_{i, n}} f_{N}\left(\varepsilon_{u}\right) H_{n, u}^{\prime} d_{i, u}(k)+I_{i, n}\left(\varepsilon_{u}\right)+w_{i, n} \tag{23}
\end{equation*}
$$

By considering the Rice channel model in (3), formula (23) can be rewritten as":

$$
\begin{align*}
R_{i, n} & =\sum_{u=0}^{U-1} a_{u} P_{u} e^{j \varphi_{i, u}} f_{N}\left(\varepsilon_{u}\right) d_{i, u}(k) \\
& +\sum_{u=0}^{U-1} \sum_{k=N_{u s}}^{N_{u s}-1} c_{u} P_{u} e^{\varphi_{i}} h_{u}(k) e^{-\frac{j 2 \pi n k}{N}} f_{N}\left(\varepsilon_{u}\right) d_{i, u}(k)  \tag{24}\\
& +\sum_{u=0}^{U-1} I_{i, n}\left(\varepsilon_{u}\right)+w_{i, n}
\end{align*}
$$

A dramatic phenomenon is that, in the AFAIC case, the aircrafts are with the similar approaching speeds and the climbing speeds. The frequency shifts of different MS nodes are with the similar values.

During the decoding process at the receiver, we propose a rough CIR estimation and decoding (RCED) scheme. In this scheme, we estimate the affected CIR instead of the parameter $\varepsilon$ by one precursor symbol. Then, the receiver treats the interference signal $I_{i, n}(\varepsilon)$ as the additional noise. The process of RCED scheme is similar
with the normal decoding process, which can run under the IEEE 802.16e protocol receiver. So the RCED scheme can be realized by the limited improvement of the existing equipment.

In the DL process of AFAIC case, before transmission, we design a precursor symbol vector $\boldsymbol{d}_{p}$ which is known by the receiver. The precursor symbol is as follows:

$$
\boldsymbol{d}_{p}=[\underbrace{\left.\begin{array}{llll}
1 & 1 & \cdots & 1 \tag{25}
\end{array}\right] . . . . ~}_{N}
$$

The modulated symbol on the $k$-th subcarriers is:

$$
\begin{equation*}
s_{p}(k)=\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{\frac{j 2 \pi n k}{N}} \text { for }-N_{g} \leq k \leq N-1 \tag{26}
\end{equation*}
$$

The received signal of the precursor period at the $n$ th output of DFT unit of MS node is:

$$
\begin{equation*}
R_{p, n}=P_{B S} e^{j \varphi_{p}} H_{n} f_{N}(\varepsilon)+I_{i, n}(\varepsilon)+w_{i, n}, \tag{27}
\end{equation*}
$$

Let $T(n)=P_{B S} e^{j \varphi_{p}} H_{n} f_{N}(\varepsilon)$ be the transmission coefficient, it can be estimated during the precursor period as:

$$
\begin{equation*}
\hat{T}(n)=\underset{T(n)}{\arg \min }\left\{\left|R_{p}(n)-T(n)\right|\right\}, \tag{28}
\end{equation*}
$$

which also denoted as the received signal of the precursor slot at the $n$-th output of DFT unit.

The decoding process of the following symbol periods can be described as:

$$
\begin{equation*}
\hat{d}(n)=\underset{\hat{d}(n) \in \Delta}{\arg \min }\left\{\left|R_{i}(n)-\hat{T}(n) d(n)\right|\right\}, \tag{29}
\end{equation*}
$$

where $\Delta$ represents the possible symbol set of the transmitted signals.

The flow diagram of the RCED process is shown in Fig. 4. BS continual transmits one signal packet which includes $\left\{\boldsymbol{d}_{p}, \boldsymbol{d}_{1}, \boldsymbol{d}_{2}, \cdots, \boldsymbol{d}_{I}\right\}$. The packet length, which equals to $I+1$, depends on the time-variant characteristic of the transmission channel.


Fig. 4. Flow diagram of the RCED process.
In the UL process, to estimate the transmission
coefficient, the known precursor periods are allocated to each MS node, respectively, and the transmission coefficient estimating and the decoding process are similar with the DL case.

## IV. PERFORMANCES ANALYSIS

The performances of SNR loss, the spectrum efficiency and BER of the transmission scheme in AFAIC case are discussed in this part, and to simplify the analysis process, the DL transmission is considered.

Let $S N R_{\text {real }}$ be the real SNR of the receiver at MS, and the channel gain is equalized, the real SNR is:

$$
\begin{equation*}
S N R_{\mathrm{real}}=\frac{P_{B S} E\left\{\left|f_{N}(\varepsilon) \exp ^{j \varphi_{i}}\right|^{2}\right\}}{\sigma_{I}^{2}(\varepsilon)+\sigma_{w}^{2}} \tag{30}
\end{equation*}
$$

where $P_{B S}$ is the transmission power of the BS, $E\left\{\left|f_{N}(\varepsilon) \exp ^{j \phi_{i}}\right|^{2}\right\}$ is the coefficient of the transmitted signal, which represents the effect of the frequency shift of the signal, $\sigma_{I}^{2}(\varepsilon)$ and $\sigma_{w}^{2}$ are the power of the interference signal and the AWGN, respectively.

With the increase of the speed of MS node, the power of the signal, which equals to the primal signal power multiplied by the coefficient $E\left\{\left|f_{N}(\varepsilon) \exp ^{j g_{i}}\right|^{2}\right\}$, reduces and the power of intersubcarrier interference, which equals to $P_{B S}\left[1-\left|f_{N}(\varepsilon)\right|^{2}\right]$, increases.

According to the typical AeroMACS system in [2], we assume that the FFT point is 512 , the symbol period is $1.75 \times 10^{-7}$ seconds, and $N_{g}$ equals $64(1 / 8$ symbol period).

Without loss of generality, AWGN is assumed as a complex random variable, the mean and variance equal 0 and 1, respectively. Since the power of AWGN can be recognized as the variance of it, we set $\sigma_{w}^{2}=1$.

Let $S N R_{\text {sync }}=P_{B S} / \sigma_{w}^{2}$ be the ideal SNR of the synchronic communication system. Performances of SNR loss in AFAIC case with different $S N R_{\text {sync }}$ values, i.e., $5 \mathrm{~dB}, 10 \mathrm{~dB}, 15 \mathrm{~dB}, 20 \mathrm{~dB}$ and 30 dB , are shown in Fig. 5.

In the case that $S N R_{\text {sync }}$ is low, which means that MS is far away from BS, the real SNR loss is small. When $S N R_{\text {sync }}=5 \mathrm{~dB}$, the SNR loss is about 1 dB for the case that the MS node moves at $400 \mathrm{~km} / \mathrm{h}$.

With the increase of $S N R_{\text {sync }}$, which indicates that the MS approaches BS, the influence of Doppler frequency shift is obvious. When the speed of MS is $250 \mathrm{~km} / \mathrm{h}$, which is the most common approaching speed and the initial climbing speed of civil aircraft, the real SNR loss are $2 \mathrm{~dB}, 7 \mathrm{~dB}$ and 16 dB in the case that $S N R_{\text {sync }}$ equal 10 $\mathrm{dB}, 20 \mathrm{~dB}$ and 30 dB , respectively. For the case that the speed of MS is $400 \mathrm{~km} / \mathrm{h}, S N R_{\text {real }}$ reduces to less than 10 dB .


Fig. 5. SNR loss in AFAIC case, where $S N R_{\text {sync }}$ is set to $5 \mathrm{~dB}, 10 \mathrm{~dB}, 15 \mathrm{~dB}, 20 \mathrm{~dB}$ and 30 dB , and different move speeds of MS node are considered.

Simulation results of spectrum efficiency under different moving speeds of MS node are shown in Fig. 6. By considering that different modulation scheme will cause varying transmission performance, four common modulation schemes, i.e., binary phase shift keying (BPSK), quadrature phase shift keying(BPSK), 16quadrature amplitude modulation(QAM) and 64-QAM are considered. Since the channel gain is equalized, the gain of LOS path does not affect the simulation results.


Fig. 6. Spectrum efficiency for BPSK, QPSK, 16-QAM and 64-QAM with different moving speed of MS node.

For the BPSK modulation, to reach half spectrum efficiency ( $0.5 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ ), $S N R_{\text {sync }}$ are $5 \mathrm{~dB}, 7 \mathrm{~dB}$ and 15 dB for the speed of MS are $0 \mathrm{~km} / \mathrm{h}, 200 \mathrm{~km} / \mathrm{h}$ and $400 \mathrm{~km} / \mathrm{h}$, respectively. For the QPSK modulation, to reach half spectrum efficiency ( $1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ ), $S N R_{\text {sync }}$ are $6 \mathrm{~dB}, 8 \mathrm{~dB}$ and 20 dB for the speed of MS are $0 \mathrm{~km} / \mathrm{h}, 200 \mathrm{~km} / \mathrm{h}$ and $400 \mathrm{~km} / \mathrm{h}$, respectively.

For the $16-\mathrm{QAM}$ modulation, to reach the half
spectrum efficiency point ( $2 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ ), $S N R_{\text {sync }}$ are 14 dB and 23 dB for the speed of MS are $0 \mathrm{~km} / \mathrm{h}$ and $200 \mathrm{~km} / \mathrm{h}$, respectively. For the 64-QAM modulation scheme case, to reach the half spectrum efficiency point ( $3 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ ), $S N R_{\text {sync }}$ are 25 dB and 60 dB for the speed of MS are $0 \mathrm{~km} / \mathrm{h}$ and $200 \mathrm{~km} / \mathrm{h}$, respectively. For 16-QAM and 64QAM modulation schemes, $S N R_{\text {sync }}$ should be more than 100 dB to reach the half spectrum efficiency in the case that the speed of MS is $400 \mathrm{~km} / \mathrm{h}$, which is not realistic in AeroMACS.

From the results, the spectrum efficiency loss of the higher order modulation scheme (i.e., 16-QAM and $64-\mathrm{QAM})$ is greater than the lower cases. The reason of huge spectrum efficiency loss in the high order modulation cases is that the high order modulated code is with smaller Euclidean distance, which is much easier to be affected by the interference signal and the AWGN.


Fig. 7. BER performances of BPSK modulation.


Fig. 8. BER performances of QPSK modulation.
BER performances of the abovementioned 4 modulation schemes are shown in Fig. 7 to Fig. 10, and the $K_{\text {RICE }}$ is set to 15 dB as the typical value in AFAIC case.

For BPSK and QPSK modulations, as shown in Fig.

7 and Fig. 8, when the moving speed of MS is $200 \mathrm{~km} / \mathrm{h}$, the BER performances are similar with the case that MS node is stationary. In the case that the moving speed of MS reaches $400 \mathrm{~km} / \mathrm{h}$, the BER performance loss is less than 0.5 dB , which means that the proposed scheme can fit the AFAIC case for the low order modulation schemes.


Fig. 9. BER performances of 16-QAM modulation.


Fig. 10. BER performances of 64-QAM modulation.
As shown in Fig. 9, for 16-QAM modulation, compared with the case that MS node is stationary, the BER performance loss are 0.1 dB and 1.5 dB for the case that the moving speed of MS are $200 \mathrm{~km} / \mathrm{h}$ and $400 \mathrm{~km} / \mathrm{h}$, respectively.

For 64-QAM modulation shown in Fig. 10, the BER performance loss is about 0.3 dB for the case that the moving speed of MS is $200 \mathrm{~km} / \mathrm{h}$. When the moving speed of MS is $400 \mathrm{~km} / \mathrm{h}$, BER performance is with significant loss. The BER only reaches $10^{-1}$ when SNR is 14 dB .

We also show the BER performances in the AG case and the AHTR case where $K_{\text {RICE }}$ are set to 0 and 6.7 dB , respectively [19]. Benefit from the strong gain of LOS
path in AFAIC case, BER performances of BPSK, QPSK and 16-QAM modulation schemes in the AFAIC case with the moving speed of MS is $400 \mathrm{~km} / \mathrm{h}$ are better than the AG case and AHTR case significantly. For the 64QAM modulation, the BER performance of the $400 \mathrm{~km} / \mathrm{h}$ moving speed case is with mightily loss, which is worse than the AHTR case and reaches the same BER with the AG case when SNR reaches 14 dB .

Summarizing the simulation results in Fig. 5 to Fig. 10, we can get the transmission characteristics in AFAIC case. Firstly, the interference caused by the high moving speed of MS seriously affects the real SNR and the transmission efficiency, however, this interference signal is difficult to be cancelled. Secondly, the strong LOS path of the channel in AFAIC case can provide a stable transmission environment between BS and MS nodes, which can reduce BER of the transmission system. Third, increase the modulation order will lead to a sharp increase in BER. By using lower order modulation, the proposed RCED scheme can meet the communication requirements of AeroMACS in AFAIC case.

## V. CONCLUSION

In this paper, to combat the Doppler frequency shift in AFAIC case, we propose a RCED scheme which estimates the affected CIR instead of the nominalized frequency shift at the receiver. It can run under the IEEE 802.16e protocol receiver, which can be realized by the limited improvement of the existing equipment. Simulation results of BER performance show that the proposed RCED scheme can meet the requirements of AeroMACS. From our research, the coverage zone of AeroMACS can be expanded from the ground part (which include the AG case and the AHTR case) to the takeoff and approaching strip (the AFAIC case), which is benefit to improve the ATM communication link. The data of aircrafts in approaching can also be used to estimate the grounding point and grounding time, which provides data support for guidance, collision avoidance and path planning of ground vehicles in the A-SMGCS.

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# Broadband Circularly Polarized Antenna by Using Polarization Conversion Metasurface 

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#### Abstract

A compact and broadband circularly polarized antenna is proposed. A linear-to-circular polarization conversion metasurface is designed to broaden the $3-\mathrm{dB}$ axial ratio bandwidth and the impedance bandwidth of the proposed antenna, with the mechanism of the metasurface investigated. Different with the conventional metasurface antenna designed by using uniform unit cells, this design makes use of two metasurface arrays with different unit cells. Full wave simulations show that the $10-\mathrm{dB}$ impedance bandwidth of the proposed antenna is from 4.32 to $6.5 \mathrm{GHz}(40.3 \%)$, and the $3-\mathrm{dB}$ AR bandwidth is from 5 to 5.61 GHz ( $11.5 \%$ ). Compared with that using the uniform elemental array, this design leads to more than $10 \%$ improvement in the $10-\mathrm{dB}$ impedance bandwidth and more than $11.7 \%$ improvement in the axial ratio bandwidth. The proposed antenna has been fabricated and the simulated results have been verified with the measurements.


Index Terms - Circularly polarized antenna, linear-tocircular polarization conversion, metasurface.

## I. INTRODUCTION

Metasurface (MS) has attracted much attention due to its unique properties in the past few years [1]. It is usually formed by regular or irregular periodic planner arrays and presents EM properties not found in natural materials. With advantages of planar structure and strong capability of manipulating electromagnetic waves, MS can be easily integrated on traditional antennas and provide a promising approach for new antenna designs with improved performances [2-13].

For radar, wireless, and satellite communication applications, circularly polarized (CP) antennas [14, 15] are widely used because of their immunities to multipath distortion and polarization mismatch losses. MS can be utilized to improve the CP performance or to convert a linearly polarized radiation from antennas to a circularly polarized one without deteriorating the radiation
performance [3]. Dual-band conversion can also be realized by a single metasurface [4]. By properly combining the transmitted wave from the antenna and the reflected wave from the metasurface, broadband circular polarization can be obtained [5-7]. In [8-10], the metasurfaces work as polarization convertors and the Efield can be resolved into two orthogonal components. In this way, the circularly polarized wave can be possibly generated with wider bandwidth. In order to enhance the impedance-matching and the AR bandwidths, surface waves are excited on the MS to generate additional resonances with minimum AR points [11, 12]. Based on the mushroom antenna [1], a new wideband CP antenna can be realized by rotating the angle of the feed slot for a polarization-dependent MS superstrate [13].

The above MS antennas are all based on periodic MS arrays with the same unit cells. In this paper, we demonstrate by combining two linear-to-circular polarization conversion MS (PCMS) arrays, the 3-dB axial ratio (AR) bandwidth and impedance bandwidth can be further broadened. Simulation shows that the 10dB impedance bandwidth for the proposed MS antenna is $40.3 \%$ from 4.32 to 6.5 GHz , and the $3-\mathrm{dB}$ AR bandwidth is $11.5 \%$ from 5 to 5.61 GHz . Compared with the same element array, more than $10 \%$ improvement for $10-\mathrm{dB}$ impedance bandwidth and more than $11.7 \%$ improvement for $3-\mathrm{dB}$ AR bandwidth can be obtained. As the design verification, the proposed antenna is fabricated and measured. The measurement results agree with the simulation results.

## II. ANTENNA DESIGN

## A. Linear-to-circular polarization conversion MS

A linear-to-circular PCMS is made of arrays of the unit cell as shown in Fig. 1 (a). The unit cell consists of a metallic patch with a 45 degree-oriented rectangular slot and a substrate board with the thickness of 3 mm and the relative permittivity of 4.4. The dimensions of the unit cell specified in Fig. 1 (a) are as follows:
$p x=p y=11 \mathrm{~mm}, l=w=10 \mathrm{~mm}, l s=11 \mathrm{~mm}$, and $w s=$ 0.65 mm . Consider an x-polarized plane wave normally illuminated on the bottom of the PCMS. This means the incidence only contains Ex component. The amplitudes of the transmitted E-field components are simulated by commercial software Ansoft HFSS. Master/slave boundaries are utilized based on a unit-cell model. The simulation results show that the amplitude intersections of Ex and Ey can be obtained and the corresponding frequency of the matching point varies with the length of the slots on the patch (Ref. Fig. 1 (b)).


Fig. 1. (a) A unit cell of the PCMS, and (b) the amplitudes of the transmitted E-field components.

This indicates that, after the x-polarized plane wave transmits through the MS, both Ex and Ey components are generated with equal amplitudes in the transmitted wave. In this way, circularly polarized wave can be possibly generated when the MS array is properly designed. Accordingly, by considering the PCMS as a half-wavelength resonant cavity, the resonant length of slot can be qualitatively estimated by the following equation

$$
\begin{equation*}
l s=\lambda_{e} / 2=\lambda / 2 \sqrt{\varepsilon_{r}} \tag{1}
\end{equation*}
$$

where $l s$ is the wavelength in the dielectric substrate and $\varepsilon_{r}$ is the relative permittivity of the substrate.

As shown in Fig. 2, at 5.75 GHz , the currents on slot with $l s=11 \mathrm{~mm}$ are better excited than that on slot with $l s=10 \mathrm{~mm}$, which further verifies the linear-to-circular
polarization conversion function of the slot.


Fig. 2. Simulated surface current amplitudes of the PCMS at 5.75 GHz for different slot lengths.

## B. CP Antenna incorporating two CPMSs

Based on a MS-based broadband low-profile mushroom antenna as presented in [1], by simply replacing the mushroom EBG with the aforementioned PCMS arrays on a planar slot-coupling antenna, we obtain the proposed CP antenna. In order to obtain broader $3-\mathrm{dB}$ AR and impedance bandwidths, the 4 by 4 PCMS array are separated into two 2 by 4 arrays as depicted in Fig. 3. After optimization, the dimension parameters of the PCMS arrays are determined as follows (in unit of mm): $l=8.25, w=8, p x=9.45, p y=$ $9.2, l s 1=9.45$, $w s 1=0.65, l s 2=8.8$, and $w s 2=0.65$. The tilt angles for the two CPMSs are $q 1=52.5^{\circ}$ and $q 2=57^{\circ}$ respectively. The parameters of the feeding slot antenna are (in unit of mm ): $l e=25$, $w e=2, l f=27.5, w f=2.1$, $l g=16, w g=5$, and $g f=1.55$. Here, FR4 $\left(\varepsilon_{r}=2.2\right)$ is used as substrate for both the PCMS and the antenna. The thicknesses of the PCMS substrate and the antenna substrate are $h=3 \mathrm{~mm}$ and $t=1 \mathrm{~mm}$ respectively. A waveguide port is assigned to the coplanar waveguide as excitation in the HFSS model as shown in Fig. 3 (b). Figure 4 (a) shows the simulated S11 for the MS antenna. The simulated impedance bandwidth for $\mathrm{S} 11<-10 \mathrm{~dB}$ is $4.32-6.5 \mathrm{GHz}(40.3 \%)$. Figure 4 (b) shows the simulated axial ratio and gain in the boresight direction for the proposed antenna. It shows the PCMS array resulted in stable left-hand circular polarization (LHCP) radiation. The 3-dB AR bandwidth is from 5 to 5.61 GHz , about $11.5 \%$. In Fig. 4 (c) the simulated radiation patterns of the proposed antenna at 5.3 GHz is provided.

The simulation results show that, as the time changes, the surface currents located at the azimuth angle turn in a clockwise manner. Figure 5 shows snapshots of the surface currents of the proposed antenna at 5.2 GHz for three different time phases $(\omega t)$, from $0^{\circ}$ to $90^{\circ}$, with an interval of $45^{\circ}$. At $\omega t=0^{\circ}$, the dominant surface current can be found in the y-direction, while as $\omega t$ changes to $\omega t=45^{\circ}$ and then $\omega t=90^{\circ}$, the dominant surface currents can be observed in the diagonal direction and then in the x -direction. Hence, the polarization characteristic is the LHCP in +z-direction.

It also can be seen in Fig. 5 that the current intensities on PCMS on different locations are different. It is understood that the discrepancy in the unit cells of CPMSs will decrease the resonance of the CPMSs in the center zone where strong mutual coupling occurs, however, strong resonance can still be formed on the edge of the structure, rendering the improved antenna performance.


Fig. 3. Illustration of the geometry with the design parameters of the proposed antenna: (a) the top view; (b) the back view and the side view.

(a)


Fig. 4. (a) The simulated S11, (b) AR and broadside gain, and (c) the radiation pattern at 5.3 GHz of the proposed antenna.

(a) $\omega t=0^{\circ}$


Fig. 5. Simulated surface current distributions of the proposed antenna at 5.3 GHz for different time instants.

## III. EXPERIMENTAL RESULTS

In Fig. 6, the simulated axial ratio (AR), the broadside gain, and the S11 parameter of the proposed antenna based on two different dimensions of PCMS arrays are compared with those of the two antennas with uniform PCMS arrays. The key parameters which are different among the three antennas are specified in the figure. The rest parameters are the same with those in Section II. It can be seen that wider 3-dB AR bandwidth and impedance bandwidth can be achieved by properly adjusting the length and rotating the angle (in degrees) of the rectangular slots without deteriorating the radiation performance. Parametric studies are performed to identify the effect of separating to two PCMS arrays on the impedance bandwidth, AR bandwidth in the boresight direction and the results provide a useful strategy to broaden the bandwidth for practical design. Compared with the antenna used by same elements with the wider bandwidth, more than $10 \%$ improvement for $10-\mathrm{dB}$ impedance bandwidth and more than $11.7 \%$ improvement for 3-dB AR bandwidth can be obtained by adopting the combination of the two PCMS arrays.

The proposed antenna as shown in Fig. 7 is etched on FR4 substrate with a relative permittivity of 4.4 and a loss tangent of 0.02 . Its characteristics are measured as design verification. As depicted in Figs. 8 (a) and (b), the measured results show that the $10-\mathrm{dB}$ impedance bandwidth for the proposed MS antenna is $38.3 \%$ from 4.56 to 6.72 GHz , and the $3-\mathrm{dB}$ AR bandwidth is $11.3 \%$ from 5 to 5.6 GHz . Figures 8 (c) and (d) show the simulated and measured radiation patterns at 5.3 GHz . As can be found in Fig. 8, the measured results show good agreement with the simulated ones, and the deviations could be attributed to the fabrication and measurement tolerance.


Fig. 6. (a) The AR, broadside gain and (b) the S11 of the proposed antenna and the two reference antennas with uniform PCMS array (the proposed antenna: (ls $1, \mathrm{ls} 2, \mathrm{q} 1, \mathrm{q} 2)=\left(9.45 \mathrm{~mm}, 8.80 \mathrm{~mm}, 52.5^{\circ}, 57^{\circ}\right)$; Ref. Antenna 1: $(\mathrm{ls} 1, \mathrm{q} 1)=\left(9.45 \mathrm{~mm}, 52.5^{\circ}\right)$; Ref. Antenna 2 : (ls2, q2)=(8.80mm,57)).


Fig. 7. Photographs of the fabricated antenna prototype.


(c)


Fig. 8. (a) The S11, (b) the axial ratio, (c) the radiation patterns normalized by maxima at 5.3 GHz of the proposed antenna at $\varphi=0^{\circ}$ plan, and (d) the radiation patterns normalized by maxima at 5.3 GHz of the proposed antenna at $\varphi=90^{\circ}$ plan.

## IV. CONCLUSION

An efficient method is presented to broaden the 3dB AR bandwidth and the impedance bandwidth based on two PCMS arrays with different parameters in this paper. The proposed antenna design exhibits more than $10 \%$ improvement for $10-\mathrm{dB}$ impedance bandwidth and more than $11.7 \%$ improvement for the $3-\mathrm{dB}$ AR bandwidth compared with that composing the PCMS array with uniform elements. The simulated and measured results are in good agreement to verify the proposed antenna.

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# Numerical Analysis of Wideband and High Directive Bowtie THz Photoconductive Antenna 

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#### Abstract

This paper presents a novel wideband and high directivity Bowtie photoconductive antenna (PCA) for THz frequency applications. The radiation properties of proposed PCA were analyzed by varying important design parameters, such as substrate thickness, conductor thickness, bowtie antenna width, length and gap. The optimized values of these parameters are then used to design a wideband PCA THz antenna which exhibits impedance bandwidth of $3 \mathrm{THz}, 3 \mathrm{~dB}$ AR bandwidth of 6 THz , peak directivity of about 18.2 dBi and peak radiation efficiency of $98 \%$ within the operating band. To improve the directivity of the proposed antenna, a silicon-based lens is added in the structure and the effect of silicone lens on THz antenna directivity is also studied for enhanced directivity of proposed antenna. The proposed THz antenna can be a prospective candidate for future THz applications such as spectroscopy, imaging, sensing and indoor communication.


Index Terms - Bowtie antenna, high directivity, photoconductive THz antenna, wideband.

## I. INTRODUCTION

Due to an increased demand for high data rate and wide bandwidth applications there has been a consistent trend of moving towards higher frequency bands in the electromagnetic spectrum. The recent emergence of 5 G technology has commercialized the use of millimeter waves $(30 \mathrm{GHz}$ to 300 GHz ) for high data rate communication [1-3]. Now, there has been an ongoing attempt to use THz waves or submillimeter waves having
frequency from 0.3 THz to 10 THz for specialized applications like brain imaging, tumor detection, security screening, material characterization using spectroscopy, bio-sensing, high data rate indoor communication and non-invasive imaging [4-9].

However, antenna design for THz is a big challenge in itself. Antenna design techniques used at low frequency are not applicable at THz range. Due to very small operating wavelength, the size of antenna becomes very small making it impossible to feed such an antenna using conventional feeding mechanism [3]. However, loss in THz signal power as it propagates is a serious challenge. This necessitates high gain THz antenna to compensate the losses and polarization insensitivity characteristics, particularly for imaging and sensing applications.

THz antennas working on the principle of photoconduction are getting attention by researchers working in the field of THz systems. However, their limitation is reduced radiated THz power and thus low efficiency [10-12]. Also, as compared to microwave antennas THz antennas require different excitation and current generation process that involve losses. A wideband on-chip dielectric resonator antenna (DRA) operating at sub-THz frequencies was proposed with a bandwidth of 65 GHz [4]. Low directivity is another constraint in THz antenna design. A bowtie-shaped antenna combined with a silicon-based lens and with an artificial magnetic conductor (AMC) was proposed to enhance its directivity properties [5]. The other reported examples of THz PCA designs are dipole planner array [13], Yagi-Uda [14], bow tie [11, 12, 15, 16], conical
horn [17] and spiral-shaped [18]. The authors used the antireflection coatings [11, 19], integration of lens [11, 12, 20-23] and metasurfaces [5, 13, 15] with antenna structures to enhance THz antennas directivity.

In this paper we have proposed a novel ultra wideband (UWB) and high directivity bowtie photoconductive antenna for THz band. The radiation properties of proposed antenna have been optimized after parametric analysis of critical design parameters. From full-wave EM simulations, the proposed antenna achieves the -10 dB impedance bandwidth of 3 THz with peak gain of about 15 dBi and radiation efficiency of $98 \%$ within the operating band. The antenna exhibit impedance matched behavior across 2 THz to 5 THz band. The directivity of antenna has also been enhanced using lens with optimized design parameters. The design, shape and structure of the added lens is selected carefully to ensure the wideband operation as well as improved directivity from the realized design. The designed antenna shows superior performance in terms of impedance bandwidth and efficiency as compared to the designs proposed in [10-18]. The detailed analysis of the design procedure, parametric study, effect of the added lens on the antenna performance and discussion about the obtained results is given in the following sections.


Fig. 1. Design of bowtie THz antenna.
Table 1: Initial design dimensions of Bowtie THz antenna

| Parameters | Values $(\boldsymbol{\mu m})$ |
| :---: | :---: |
| $L_{s}$ | 347 |
| $h$ | 1.3 |
| $t$ | 0.02 |
| $W_{s}$ | 340 |
| $L_{t}$ | 103 |
| $W_{t}$ | 220 |
| $W_{r}$ | 7 |
| $L_{r}$ | 12 |
| $g$ | 7.6 |

## II. DESIGN PROCEDURE

## A. Antenna design

The proposed antenna design is shown in Fig. 1. It is basically a bowtie antenna printed on a photoconductive substrate. DC bias is applied across the bowtie antenna. In order to generate THz radiation from this bowtie antenna, optical pulse is incident on the antenna gap ( $g$ ), which propagates into the photoconductor and generate photocarriers inside the photoconductor. The generated photocarriers are accelerated in the DC bias field, producing a transient photocurrent, which drives the bowtie antenna and ultimately re-emits as a THz frequency pulse [24]. The metal conductor of the designed bowtie antenna has thickness of ' $t$ ' and is made of gold conductor whereas the substrate is made of quartz. The relative permittivity and loss tangent of the used substrate material is 3.78 and 0.0001 , respectively. The full-wave numerical analysis of the proposed bowtie THz antenna is performed in CST MWS. The initial dimensions of the designed antenna are given in Table 1.

In order to increase the directivity of this antenna a hemispherical lens is placed on bottom side as shown in Fig. 2.


Fig. 2. Design of Bowtie THz antenna with lens.

## B. Optimization of the UWB THz antenna

Firstly, the parametric analysis of the antenna design parameters of Fig. 1 is performed to obtain the optimized design parameters for the ultra-wideband (UWB) impedance matching characteristics. The major issues in the modeling are about the extensive analysis of the effect of the different antenna design parameters on its performance. For this purpose, a parametric analysis was performed to obtain the optimized design parameters for the ultra-wideband impedance matching characteristics. The variation ranges of the various design parameters for the parametric study are selected carefully keeping in view the fabrication constrains to minimize the extensive memory requirements of the numerical models with various design parameters

The upper and lower limit of each parameter is chosen by covering the maximum range of values without affecting the shape of the proposed antenna. In all parametric analysis we present only the important parameters values which give us significant behavior keeping in view the fabrication constrains.

## 1) Effect of Substrate Thickness ( $h$ )

Because the thickness of the substrate ( $h$ ) is usually larger than the wavelength of THz waves, surface/ substrate modes may be generated and the effect of the substrate cannot be ignored. For this purpose, parametric study was conducted by varying the substrate thickness from $1.3 \mu \mathrm{~m}$ to $4.3 \mu \mathrm{~m}$. The effect on $S_{l l}$ is shown in Fig. 3. As thickness increases the impedance matching behavior slightly degrades at upper operating frequencies. The results depict that for the optimal value of substrate thickness, i.e., $2.3 \mu \mathrm{~m}$, the -10 dB impedance bandwidth of the designed antenna is around 3 THz ( 2 to 5 THz ).


Fig. 3. $S_{l l}$ of THz antenna without lens with the variations in substrate thickness ( $h$ ).

Normally, the antennas having -10 dB impedance bandwidth of greater than equal to 500 MHz or fractional bandwidth (FBW) of more than $20 \%$ are referred as ultra-wideband (UWB) antennas [25] [26]. As depicted in Fig. 3, the -10 dB impedance bandwidth $\left(\left|S_{11}\right| \leq\right.$ -10 dB ) and FBW of our proposed antenna is around 3 THz and $50 \%$ respectively, that's why we are using the term of UWB THz antenna for the proposed antenna structures.

## 2) Effect of Gold Thickness ( $t$ )

The effect of metal thickness $(t)$ of top layer of proposed THz antenna is studied parametrically as shown in Fig. 4. The metal thickness is varied from 0.02 $\mu \mathrm{m}$ to $0.05 \mu \mathrm{~m}$ and only slight variation in the impedance matching characteristics across the band is observed.

## 3) Effect of $L_{s}$

The parametric study on $L_{s}$ is also conducted and the variations in $S_{l l}$ are shown in Fig. 5 which depicts that
this parameter has also no effect on $S_{l l}$ as like metal thickness variations.


Fig. 4. $S_{l l}$ of THz antenna without lens with the variations in metal thickness $(t)$.


Fig. 5. $S_{l l}$ of THz antenna without lens with the variations in substrate length $\left(L_{s}\right)$.

## 4) Effect of $L_{t}$

The effect of variation in $L_{t}$ on antenna $S_{1 l}$ is shown in Fig. 6. This parameter is varied from $103 \mu \mathrm{~m}$ to 124 $\mu \mathrm{m}$ and it is observed that matching behavior slightly drift towards lower frequency with increasing $L_{t}$ values.


Fig. 6. $S_{l l}$ of THz antenna without lens with the variations in $L_{t}$.

## 5) Effect of $W_{r}$

As this parameter is varied between $7 \mu \mathrm{~m}$ to $19 \mu \mathrm{~m}$, significant variation in S11 is observed as shown in Fig. 7. The best result is obtained for $W_{r}=13 \mu \mathrm{~m}$.


Fig. 7. $S_{l l}$ of THz antenna without lens with the variations in $W_{r}$.

## 6) Effect of Bowtie Aram Width ( $W_{t}$ )

The parameter $W_{t}$ (bowtie arm width or flare angle) is varied from $220 \mu \mathrm{~m}$ to $280 \mu \mathrm{~m}$. Figure 8 shows the effect of this variations on the reflection coefficient of the antenna. It can be noted from Fig. 8 results that the increase in $W_{t}$ brings a minor shift (towards lower frequencies) in the resonance frequencies of the antenna.


Fig. 8. $S_{l l}$ of THz antenna without lens with the variations in $W_{t}$.

## 7) Effect of Growth $W_{s}$

The change in reflection properties of the designed THz bowtie antenna when the parameter $W_{s}$ (width of the substrate) is varied from $340 \mu \mathrm{~m}$ to $370 \mu \mathrm{~m}$ is depicted in Fig. 9. The change in the width of the substrate mainly effects the dips of the resonance frequencies without any significant change in the impedance bandwidth of the antenna.

## 8) Effect of $g$

The gap between the bowtie antenna poles is also varied from $1.6 \mu \mathrm{~m}$ to $7.6 \mu \mathrm{~m}$ to analyze its impact on
$S_{11}$ characteristics as shown in Fig. 10. As the gap width is increased from $1.6 \mu \mathrm{~m}$ to $3.6 \mu \mathrm{~m}$ the impedance matching improved. However, when $g$ is increased further the impedance matching started degraded again.

Table 2 lists the optimized design parameters of the proposed bowtie THz antenna based on the conducted parametric study. The performance of the antenna is further analyzed with the addition of the lens in the antenna structure with the optimized antenna parameters of Table 2.


Fig. 9. $S_{l l}$ of THz antenna without lens with the variations in $W_{s}$.


Fig. 10. $S_{l l}$ of THz antenna without lens with the variations in $g$.

Table 2: Optimized design dimensions of Bowtie THz antenna

| Parameters | Values $(\boldsymbol{\mu m})$ |
| :---: | :---: |
| $L_{s}$ | 347 |
| $h$ | 2.3 |
| $t$ | 0.03 |
| $W_{s}$ | 350 |
| $L_{t}$ | 117 |
| $W_{t}$ | 260 |
| $W_{r}$ | 13 |
| $L_{r}$ | 8 |
| $g$ | 3.6 |

## III. ANALYSIS OF UWB THZ ANTENNA WITH LENS

This section describes the analysis of the change in the diameter $\left(R_{L}\right)$ of the added silicon lens on the antenna performance. The performance of the antenna is analyzed in terms of the impedance matching, directivity, axial ratio and efficiency of the antenna. Following sections illustrates the conducted analysis.

## A. Impedance matching

Figure 11 shows the variations in the reflection coefficient of the antenna when the diameter of the added silicon lens is changed from $80 \mu \mathrm{~m}$ to $160 \mu \mathrm{~m}$ with a step size of $40 \mu \mathrm{~m}$. The results depict that the -10 dB impedance bandwidth of the proposed antenna improves with the increase in the diameter of the silicon lens.


Fig. 11. Effect of variations in $R_{L}$ on $S_{11}$ of THz antenna with lens.

## B. Directivity

In order to increase the directivity of proposed THz antenna, hemispherical silicone lens is placed over the bowtie antenna. The effect of radius of silicone lens on directivity is shown in Fig. 12. The best performance is observed for $R_{L}=120 \mu \mathrm{~m}$ with consistently high directivity across the whole operating band. The directivity of the antenna decreases in higher frequency band when the diameter of the silicon lens is increased to $160 \mu \mathrm{~m}$.


Fig. 12. Effect of variations in $R_{L}$ on directivity of THz antenna with lens.

## C. Axial ratio

In order to study the polarization behavior of proposed THz antenna, the axial ratio is observed for different values of $R_{L}$ as shown in Fig. 13. The antenna generally exhibits linear polarization behavior. However, for $R_{L}=160 \mu \mathrm{~m}$ circular polarization behavior is observed near 5.5 THz . It can be observed from the Fig. 13 that the 3 dB AR bandwidth of the proposed antenna is 6 THz .

The presence of the lens affects the polarization of the radiated electromagnetic fields because this lens generates a phase shift due to the wave propagation within it. This phase shift depends on the shape and the permittivity of the propagation medium constituting the lens, and thus explains the variation of the antenna polarization with the radius of the added lens. However, the polarization changes from linear to circular for a specific size of the lens means that the lens acts as a Quarter-Wavelength Plate (QWP) at certain frequency ranges, and the introduced phase shift is equal to $\pi / 2$.


Fig. 13. Effect of variations in $R_{L}$ on axial ratio of THz antenna with lens.

## D. Efficiency

The effect of $R_{L}$ on proposed THz antenna efficiency is also studied. Figure 14 shows the total efficiency results of the analyzed antenna for the three different diameters of silicon lens. The overall efficiency of the proposed antenna is more than $55 \%$ in the entire analyzed frequency band of 1 to 6 THz . The best efficiency is observed for $R_{L}=160 \mu \mathrm{~m}$ with peak value of $98 \%$.

## IV. RESULTS AND DISCUSSIONS

This section discusses the radiation pattern characteristics of the designed antenna with and without lens. The radiation pattern of proposed antenna was simulated for the three different values of RL. Figure 15 shows the 2D polar radiation pattern for $R_{L}=80 \mu \mathrm{~m}$ for proposed THz antenna with and without lens. It can be observed that lens significantly increased the directivity by focusing the radiation pattern towards it. The addition of the lens increases the level of the distributed current around the source location of the antenna which results in the enhancement of the infrared (IR) coupling and thus
the increase of the directivity of the antenna. The peak directivity for $R_{L}=80 \mu \mathrm{~m}$ lens in both planes is given in Table 3.


Fig. 14. Effect of variations in $R_{L}$ on efficiency of THz antenna with lens.

Table 3: Increase of the antenna directivity (in dB ) at different frequencies for $R_{L}=80 \mu \mathrm{~m}$

| Frequency (THz) | $\mathbf{P h i}=\mathbf{0}^{\mathbf{0}}$ | $\mathbf{P h i}=\mathbf{9 0}^{\mathbf{0}}$ |
| :---: | :---: | :---: |
| 2 | 13.4 | 13.4 |
| 2.95 | 17.5 | 17.5 |
| 3.5 | 14.4 | 14.4 |
| 4.05 | 13.1 | 13.1 |

Figure 16 shows the 2D polar radiation pattern of the proposed THz antenna with and without lens with $R_{L}=120 \mu \mathrm{~m}$. Table 4 depicts the peak directivities of the designed antenna with lens which has diameter of 120 $\mu \mathrm{m}$.

Table 4: Increase of the antenna directivity (in dB ) at different frequencies for $R_{L}=120 \mu \mathrm{~m}$

| Frequency (THz) | $\mathbf{P h i}=\mathbf{0}^{\mathbf{}}$ | $\mathbf{P h i}=\mathbf{9 0}^{\circ}$ |
| :---: | :---: | :---: |
| 1.75 | 18.2 | 18.2 |
| 2.1 | 15.7 | 15.7 |
| 3.8 | 16.6 | 16.6 |
| 4.5 | 11 | 11 |

The 2D polar radiation pattern of the analyzed antenna with $\left(R_{L}=160 \mu \mathrm{~m}\right)$ and without lens is shown in Fig. 17 for different frequencies in both azimuth and elevation planes. Table 5 illustrates the increase in the peak directivity of the antenna with the addition of the lens in its structure.

The 3D patterns of proposed THz antenna with different size lens are given in Fig. 18. It can be noted from Fig. 18 that the radiation pattern is completely opposite-pointed. To increase the performance of the
proposed antenna, a silicon lens is added on the backside of the structure. The structure was excited from the upper side and the target from the installed lens is to concentrate the main beam in the back side of the structure whereas reduce the lobes on the upper side. The highly directive main beam can easily be observed in all cases. It can be noted from Tables 3, 4 and 5 results that the maximum peak directivity of the designed antenna with lens diameter of $80 \mu \mathrm{~m}, 120 \mu \mathrm{~m}$, and $160 \mu \mathrm{~m}$ is 17.5 $\mathrm{dBi}(@ 2.95 \mathrm{THz}), 18.2 \mathrm{dBi}$ (@ 1.75 THz ) and 16.6 dBi (@ 2.15 THz ). The directivity of the antenna increases with the enhancement of the lens diameter from $80 \mu \mathrm{~m}$ to $120 \mu \mathrm{~m}$. However, a slight degradation in the directivity performance is observed with the further enhancement of the lens diameter to $160 \mu \mathrm{~m}$.

## V. COMPARISON WITH LEGACY DESIGNS

The comparison of the proposed wideband bowtie THz antenna with the available PCA designs in the literature is performed. The comparison of the proposed design is performed in terms of the antenna type, antenna substrate, antenna electrode material, lens type, -10 dB impedance bandwidth, maximum directivity, and 3 dB AR bandwidth of the reported designs. The summary of the performed comparison with legacy designs in illustrated in Table 6. The authors in [10-12, 15, 17] used the hemispherical based silicon lenses to enhance the directivity of the proposed different kinds of antenna types. Malhotra et al. [13] and Zhu et al.[15] integrated the frequency selective surface (FSS) with the antenna electrodes to increase the directivity of the proposed dipole planner array and bow-tie PCAs, respectively. However, as can be noted from Table 6 that the reported maximum directivity of all compared legacy designs is less than the achieved peak directivity of proposed THz antenna. In addition, the -10 dB impedance and 3 dB AR bandwidth of the proposed bowtie antenna design is highest among all the compared legacy designs [10-18]. This comparison reflects that the proposed THz antenna can be a prospective candidate for future THz applications such as spectroscopy, imaging, sensing and indoor communication due to its wideband impedance and AR bandwidth characteristic as well as high directivity.

Table 5: Increase of the antenna directivity (in dB ) at different frequencies for $R_{L}=160 \mu \mathrm{~m}$

| Frequency (THz) | Phi $=\mathbf{0}^{\mathbf{0}}$ | Phi $=\mathbf{9 0}^{\mathbf{0}}$ |
| :---: | :---: | :---: |
| 1.6 | 16.4 | 16.4 |
| 2.15 | 16.6 | 16.6 |
| 2.45 | 14.8 | 14.8 |
| 3.75 | 15.7 | 15.7 |



Fig. 15. 2D radiation pattern for $R_{L}=80 \mu \mathrm{~m}$ : (a) 2 THz , (b) 2.95 THz , (c) 3.5 THz , and (d) 4.05 THz .


Fig. 16. 2D radiation pattern for $R_{L}=120 \mu \mathrm{~m}$ : (a) 1.6 THz , (b) 2.15 THz , (c) 2.45 THz , and (d) 3.75 THz .


Fig. 17. 2D Radiation patter for $R_{L}=160 \mu \mathrm{~m}$ : (a) 1.75 THz , (b) 2.1 THz , (c) 3.8 THz , and (d) 4.5 THz .


Fig. 18. 3D radiation patterns for different $R_{L}$ values: (a) $R_{L}=80 \mu \mathrm{~m}$ at $f=2 \mathrm{THz}$, (b) $R_{L}=120 \mu \mathrm{~m}$ at $f=2.1 \mathrm{THz}$, and (c) $R_{L}=160 \mu \mathrm{~m}$ at $f=2.15 \mathrm{THz}$.

Table 6: Comparison of proposed UWB Bowtie PCA design with the legacy design

| References | Antenna Type | Substrate | Antenna Electrode Material | Lens/FSS | $\begin{gathered} -10 \mathrm{~dB} \\ \text { Impedance } \\ \text { Bandwidth } \\ \text { (THz) } \\ \hline \end{gathered}$ | Maximum Directivity (dBi) | 3dB AR Bandwidth (THz) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Jyothi <br> [12] | Bow-tie PCA | GaAs | $\begin{gathered} \text { TiAu } / \mathrm{AuGe} \\ / \mathrm{AuCr} \end{gathered}$ | Si hemispherical lens | 0.20 | 10.85 | - |
| Malhotra [13] | Dipole planner array | LT-GaAs | Ti-Au | FSS | 0.37 | 13.2 | - |
| Gupta et al.[11] | Bow-tie PCA with dielectric coating | SI-GaAs | AuGe | HRFZ-Si lens | - | - | - |
| Zhu [15] | Bow-tie PCA | LT-GaAs | Ti-Au | No lens | 0.18 | 8.0 | - |
|  | Bow-tie PCA |  |  | Si hemispherical lens |  | 11.8 |  |
|  | Bow-tie PCA with lens and combined with metasurface superstrate |  |  | FSS |  | 11.9 |  |
| Han [14] | Yagi-uda | GaAs | Ti-Au | No lens | 0.02 | 10.9 |  |
| Singh et al. [18] | Spiral-shaped | Si | Al | No lens | 0.25 | - | - |
| Formanek [10] | Dipole-type PCA | GaAs | Gold | Aspheric lens | 0.80 | - | - |
| Deva [17] | Conical horn | GaAs | - | Si-lens | - | 18.5 | - |
| Park [16] | Nanoplasmonic bow-tie PCA | GaAs | $\mathrm{Cr} / \mathrm{Au}$ | No lens | 1.00 | - | - |
| Proposed Work | Bowtie | Quartz | Gold | Si hemispherical lens | 3.00 | 18.2 | 6.00 |

## IV. CONCLUSION

This research study has presented the full-wave numerical results of a wideband and high directivity bowtie PCA for THz frequency sensing and imaging applications. The detailed parametric analysis of the antenna design parameters was performed in CST MWS to obtain the optimal design parameters of the proposed antenna. The designed optimized bowtie PCA exhibits wideband impedance matching characteristics of 3 THz as well as 3 dB axial ratio bandwidth of 6 THz . The addition of the silicon lens with different diameters in the antenna structures increases its peak directivity to 18.2 dBi and overall efficiency of more than $55 \%$ with peak value of $98 \%$ in the analyzed frequency band of 1 to 6 THz.

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# Mathematical Modelling on the Effects of Conductive Material and Substrate Thickness for Air Substrate Microstrip Patch Antenna 

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#### Abstract

The use of microstrip patch configuration in the 5 th generation ( 5 G ) wireless network is expected to fulfill the demands of smartphone users by significantly increasing the capacity of the communication technology. The main aim of this paper is to disclose the development of a mathematical model on the effects of conductive material and substrate thicknesses on the centre frequency for the performance evaluation of a low profile, costeffective antenna in 5 G devices applications. This mathematical model is proposed for an antenna system operated with air substrate resonating at a bandwidth range of $5 \mathrm{GHz}-38 \mathrm{GHz}$. The effects of different thickness of conductive material and substrate on the antenna's bandwidth, gain, and efficiency for 5 G applications were studied. Antennas were fabricated and tested in this study to evaluate the robustness of the proposed mathematical model at $28 \mathrm{GHz}, 24 \mathrm{GHz}$, and 10 GHz . Gains of 9.55 $\mathrm{dBi}, 9.53 \mathrm{dBi}$ and 10.1 dBi , impedance bandwidths of $2.12 \mathrm{GHz}, 2.14 \mathrm{GHz}$ and 0.41 GHz , with input reflection coefficients of $42.75 \mathrm{~dB}, 25.33 \mathrm{~dB}$ and 21.51 dB , and performance efficiencies of $98.91,87.4$ and $83.2 \%$ were obtained for the respective resonances. For validation of results, the experimental results and the simulation results from the proposed mathematical model were made into comparison, and excellent correlation between the measured and simulated results was obtained.


Index Terms - 5G technology, conductive material thickness, mathematical model, microstrip patch insetfed antenna.

## I. INTRODUCTION

The fifth-generation (5G) wireless technology is expected to overcome the drawbacks of the previous generations of networks by supporting higher frequency bands as well as providing more benefits. Considering the end user's demand, it is necessary to design novel antenna systems for modern compact devices. To support the expected requirements for a higher data traffic, intensive researches were carried out on the fifthgeneration (5G) cellular system [1]. Since 5 G cellular systems are anticipated to work at a frequency band of $30-300 \mathrm{GHz}$ close to millimeter-wave, it will become available in the future technologies [1-4]. The 26 GHz , $28 \mathrm{GHz}, 38 \mathrm{GHz}$, and 50 GHz bands are the four frequency bands currently being investigated for 5 G applications by academia and industry worldwide [5]. The extreme free space path loss that occurs at these frequencies is one of the drawbacks in using mm-wave frequencies for mobile communication. To solve potential path loss issues, a highly directional arrayed antenna is needed. A 5 G antenna must have two distinct characteristics - high gain and wide bandwidth. Both properties depend strongly on the thickness of radiating patch and substrate used in fabricating antennas. Recent trends lead to the development of an antenna that transmits and receives the broadband characteristics and high gains that can be operated at high frequencies. In this way, size reduction and bandwidth enhancement have become major design issues for sensible applications of microstrip antennas [4-6].

Much research has been focused on modeling, designing and optimizing 5 G antennas in the last eight years. However, most of the published work focused on a) investigating different configurations of (massive) Multiple Input Multiple Output (MIMO) antennas [7-12] as well as methods for reducing antenna element isolation [13-14]; b) new 5G mm-wave antennas with a wide range of polarizations like circular polarization [15-17] dual polarization [18-19] and polarization configurability [20]; c) study of multiple dual-band antenna structures (covering the proposed 28 GHz and 38 GHz band for 5 G application) like rectangular slot patch [21], integrated substrate waveguide [22], printed slot [23-25], slot waveguide [26] and PIFA [27] are proposed.

Based on available literatures in this domain, no research findings have reported on the effects of the thickness of conductive material and substrate on the production of 5 G antennas for their extensive performance. Therefore, this paper demonstrates the formulation of a mathematical model based on the effects of conductive material thickness and substrate height on a single rectangular microstrip patch inset-fed antenna operates from $1-40 \mathrm{GHz}$ resonance for 5 G wireless communication applications. The influences of these parameters were quantified on the antennas' impedance bandwidth, efficiency and gain. The use of air substrate was integrated in the designs to greatly reduce the cost of antenna manufacturing. The approach used in this work involved three phases summarizing the following: data acquisition and analysis, parameter substitution and optimization, and model development and optimization. The detailed explained in section (iv). The antennas were designed and tested, and the performance results are summarized in Table 1. An excellent correlation between the measured and mathematical modelled (simulation) results was obtained.

Table 1: Performance of the proposed mathematical model antennas

| Frequency <br> (GHz) | Gain <br> (dB) |  | Bandwidth <br> (GHz) |  | Return Loss <br> (dB) |  | Efficiency <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sim. | Mea. | Sim. | Mea. | Sim. | Mea. |  |
| $\mathbf{2 8}$ | 9.58 | 9.45 | 2.12 | 2.00 | 42.75 | 38.53 | 83.2 |
| $\mathbf{2 4}$ | 9.53 | 9.40 | 2.14 | 2.01 | 25.33 | 25.13 | 87.4 |
| $\mathbf{1 0}$ | 10.1 | 9.87 | 0.41 | 0.39 | 21.51 | 21.73 | 98.9 |

## II. ANTENNA CONFIGURATION AND DESIGN

The material used for this proposed antenna's radiation patch is copper. Typically, the dimensions of an inset-fed microstrip patch antenna are determined
using the equations regarded to the microstrip antenna as provided in references [28-29]. Air substrate which has a dielectric constant $\left(\varepsilon_{\mathrm{r}}\right)$ of 1 and a negligible loss tangent was used. To achieve the goals, optimization of the antenna dimensions is necessary. The optimized design parameters at 28 GHz are listed in Table 2.

Table 2: Dimensions of the proposed antenna

| Parameter | Value (mm) |
| :--- | :---: |
| Length of patch $\left(\mathrm{L}_{\mathrm{p}}\right)$ | 4.54 |
| Width of patch $\left(\mathrm{W}_{\mathrm{p}}\right)$ | 5.34 |
| Length of ground plane $\left(\mathrm{L}_{\mathrm{g}}\right)$ | 9.08 |
| Width of ground plane $\left(\mathrm{W}_{\mathrm{g}}\right)$ | 10.68 |
| Thickness of substrate $\left(\mathrm{h}_{\mathrm{s}}\right)$ | 0.50 |
| Conductive thickness $\left(\mathrm{h}_{\mathrm{t}}\right)$ | 1.00 |
| Length of inset-fed $\left(\mathrm{f}_{\mathrm{i}}\right)$ | 1.45 |
| Width of feedline $\left(\mathrm{W}_{\mathrm{f}}\right)$ | 2.45 |
| Gap between patch and inserted-fed $\left(\mathrm{G}_{\mathrm{pf}}\right)$ | 0.50 |
| Feedline length $\left(\mathrm{L}_{\mathrm{f}}\right)$ | 2.68 |

The geometric configurations of the proposed antennas with the following necessary dimensions; width of the patch $\left(\mathrm{W}_{\mathrm{p}}\right)$, length of the feeder $\left(\mathrm{L}_{\mathrm{f}}\right)$, inset depth (d), gap width (g), and feeder length from the left edge of a patch $\left(L_{1}\right)$ are illustrate in Fig. 1.


(c)

Fig. 1. Geometry of the proposed antennas with required dimensions at: (a) 28 GHz , (b) 24 GHz , and (c) 10 GHz .

## III. PARAMETRIC ANALYSIS ON THE PERFORMANCE OF ANTENNA

Several design parameters affecting the bandwidth of the operating impedance were studied to comprehend the design guidelines of the proposed antenna. The radiating rectangular patch was designed using a standard formula applied in any conventional resonant frequency design. The key design parameters, which are useful in maximizing the bandwidth of an antenna, are the thickness of conductive material ( $\mathrm{h}_{\mathrm{c}}$ ) and the substrate height $\left(\mathrm{h}_{\mathrm{s}}\right)$. Parametric analysis was carried out at a thickness variation of 0.1 mm to 2 mm with varying antenna parameters. The simulated results are as shown in Table 3, and it is observed that with increasing conductor thickness ( $\mathrm{h}_{\mathrm{c}}$ ) and constant width of patch and length of inserted-fed, the bandwidth increases but the length of the patch slightly decreases from the thickness of 1.1 to 2.0 , as well as the centre operating frequency which shifts away from the desired resonant frequency. The resonant frequency resonated at 28 GHz at thickness of 0.1 mm to 1 mm . The most appropriate conductor thickness for this proposed antenna is 1 mm .

The selection of a proper substrate thickness is another important task in the development of microstrip patch antennas. In choosing the most appropriate substrate thickness ( $\mathrm{h}_{\mathrm{s}}$ ), a developer needs to have knowledge on the effect of substrate thickness variation on the resonant frequency. In this case, $\mathrm{h}_{\mathrm{s}}$ is varied from 0.4 mm to 1 mm with other varying antenna design parameters and the simulated results are as shown in Table 4. From the results, it is observed that when the air substrate thickness is increased while keeping the dimensions of the other parameters as in Table 2, there is a shift in the resonant frequency and the effective dielectric constant changes; which leads to a change in the effective dimensions of the patch. In this set of observation, as the height increases, a volume of fringing effect occurs, and this leads to the increase in the bandwidth. With greater height of substrate, higher
amount of modes is excited, resulting in the degradation of the gain. The most appropriate height for this proposed antenna design at 28 GHz is 0.5 mm . However, surface waves are generated as the height of the substrate increases. These waves extract power in the direction of radiation from the total available power. Reduction in the parameters of the antenna design is therefore observed.

## IV. DEVELOPMENT OF MATHEMATICAL MODEL

In this work, the effect of the variation of the resonance frequency $\left(\mathrm{F}_{\mathrm{r}}\right)$; patch Length $\left(L_{p}\right)$; patch width $\left(W_{p}\right)$; and dielectric permittivity $\left(\varepsilon_{\mathrm{r}}\right)$ on the antenna parameters (gain, directivity, impedance bandwidth and input reflection coefficient), were studied with respect to conductor thickness ( $\mathrm{h}_{\mathrm{c}}$ ) and substrate height $\left(\mathrm{h}_{\mathrm{s}}\right)$, using computer simulation technology (CST) studio 2016 Software package However, the simulated data obtained where used to develop mathematical models which are father described in this manuscript MATLAB V19 Software package was used as the modeling environment.

In order to develop a mathematical model for the conductor thickness $\left(h_{c}\right)$ and substrate height $\left(h_{s}\right)$, simulated data were obtained from the CST studio suit 2016 for different center frequencies based rectangular microstrip inset patch antenna which are shows in Table 3 and Table 4 respectively. However, due to the limited amount of data, artificial neural networks (ANNs) toolbox in MATLAB was used to train a model and used to generate the desired amount of data to enable model creation. The steps involved in the proposed model development can be summarized as follows:
i. Data Acquisition and Preparation;
ii. Parameter Substitution and Optimization;
iii. Model Development and Optimization [30].

## i. Data Acquisition and Preparation

This stage can be described using the following block diagram.


Fig. 2. Data acquisition and preparation block diagram.

## ii. Parameter Substitution and Optimization

However, since the width and length ( $\mathrm{W}_{\mathrm{p}}$ and $\mathrm{L}_{\mathrm{p}}$ ) of patch are dependent on other parameters of the model, based on antenna theory formulations, these parameters can be substituted for. To achieve that, the following stages were further executed.


Fig. 3. Parameter substitution and optimization.

## iii. Model Development and Optimization

This represents the main stage of this work, and it presents the strategy used in developing the set of mathematical function that could be used to evaluate a suitable value for $h_{c}$ and $h_{s}$ for a given antenna design specification. In this work, $h_{s}$ is chose to be defined using the following simplified mathematical expressions in equation (1):

$$
\begin{equation*}
h_{S}=\left(\frac{\alpha F_{r} \sqrt{\varepsilon_{r}+1}+\beta \sqrt{2} C_{o}}{\sqrt{2} C_{o}+\gamma F_{r} \sqrt{\varepsilon_{r}+1}}\right) \tag{1}
\end{equation*}
$$

where the $\alpha, \beta$, and $\gamma$ can be found using an optimization technique. However, the particle swarm optimization (PSO) techniques was chosen. The objective function of the optimization was to minimize the difference between the estimation and true value of the $\mathrm{h}_{\mathrm{s}}$. After the optimization, it was found that the model was inaccurate when $\alpha$ and $\beta$ are fixed at constant values, however, $\gamma$ can be fixed at 0.5053 . In order to make $\alpha$ and $\beta$ vary with change in parameter specification, let $\alpha$ and $\beta$ be as shown in equations (2) and (3), respectively:

$$
\begin{gather*}
\alpha=X_{1}\binom{\mathrm{X}_{2}\left(W_{p}\right)^{X_{3}}-X_{4}\left(F_{r}\right)^{X_{5}}-X_{6}\left(F_{r}\right)^{X_{7}}}{+X_{8}\left(E_{r}\right)^{X_{9}}-X_{10}\left(E_{r}\right)^{X_{11}}+X_{12}},  \tag{2}\\
\beta=Y_{1}\binom{\mathrm{Y}_{2}\left(W_{p}\right)^{X_{3}}-Y_{4}\left(F_{r}\right)^{X_{5}}-Y_{6}\left(F_{r}\right)^{X_{7}}}{+Y_{8}\left(E_{r}\right)^{X_{9}}-Y_{10}\left(E_{r}\right)^{X_{11}}+Y_{12}} . \tag{3}
\end{gather*}
$$

The set of parameters $X$ and $Y$ are unknown variables that can be determined using PSO. The resulting mathematic model for substrate height was determined as presented in the following equations (4) and (5) respectively:

## A. Substrate height model (hs)

The substrate height of a rectangular patched antenna can be calculated using equation (4),

$$
\begin{equation*}
h_{S}=\left(\frac{7.6904 \alpha F_{r} \sqrt{\varepsilon_{r}+1}+1.4142 \beta C_{O}}{1.4142 C_{o}+0.5053 F_{r} \sqrt{\varepsilon_{r}+1}}\right), \tag{4}
\end{equation*}
$$

where, $\alpha$ and $\beta$ are represented in equation (5) and (6) respectively:

$$
\begin{align*}
& \alpha=\left(\begin{array}{l}
0.8525\left(\frac{\mathrm{C}_{\mathrm{O}}}{2 \mathrm{~F}_{\mathrm{r}} \sqrt{\varepsilon_{r}+1}}\right)^{1.041} \\
-0.7464\left(F_{r}\right)^{0.3993}-0.3902\left(F_{r}\right)^{0.6572} \\
+0.0817\left(E_{r}\right)^{0.3679}-0.5217\left(E_{r}\right)^{-0.5611} \\
+4.4398
\end{array}\right),  \tag{5}\\
& \beta=\left(\begin{array}{l}
2.4846\left(\frac{\mathrm{C}_{\mathrm{O}}}{2 \mathrm{~F}_{\mathrm{r}} \sqrt{\varepsilon_{r}+1}}\right)^{-0.1202} \\
-0.3713\left(F_{r}\right)^{0.4138}+3.1339\left(F_{r}\right)^{-0.223} \\
-0.4411\left(E_{r}\right)^{-0.3029}-1.2403\left(E_{r}\right)^{-8.1344} \\
+0.6557
\end{array}\right) . \tag{6}
\end{align*}
$$

Based on this model, the mathematical model for computing the conductive material thickness can be generated. Finally, the mathematical model of the conductor thickness was found to be in equation (7).

## B. Conductive material thickness model ( $\mathrm{h}_{\mathrm{c}}$ )

The height of conductive mat Height of conductive material thickness of a rectangular patch antenna can be calculated using equation (7):

$$
\begin{equation*}
h_{C}=\binom{\left(\frac{\mathrm{F}_{\mathrm{r}} \varepsilon_{r} \sqrt{2 \varepsilon_{r}+2}}{\mathrm{C}_{\mathrm{O}}}\right)^{\left(\varepsilon_{r}-1\right)} \times}{\binom{ 1.3703 \mathrm{~h}_{\mathrm{S}}+83.1872\left(h_{S}\right)^{8.2891}}{-172.286\left(h_{S}\right)^{11.07824}-0.3501}^{2}}, \tag{7}
\end{equation*}
$$

where, $\mathrm{C}_{0}$ is the speed of light in air ( $\mathrm{mm} / \mathrm{s}$ ); $\mathrm{F}_{\mathrm{r}}$ is the desired center frequency $(\mathrm{GHz})$ and $\mathrm{h}_{\mathrm{S}}$ is the substrate height (mm) and it can be evaluated using the mathematical formulation. Table 5 shows the optimized dimensional parameters of the proposed mathematical modelled antennas.

## V. RESULTS AND DISCUSSION

## A. Input reflection coefficients

In reference to the optimized antennas dimensions as well as the developed mathematical model, prototypes of the proposed antennas were fabricated and tested to validate their operational performances through the mathematical model. The antennas were made excited using a long pin SMA connector by connecting its coaxial probe to the rectangular patch. Figure 4 displays
a photograph of the fabricated antenna prototype. The Sparameters of the antennas were tested using the Agilent Vector Analyser (N5245A). On the other hand, Fig. 5 presents the input reflection coefficient characteristics of the simulated and measured results. The simulated input reflection coefficients are lower than -10 dB (VSWR< 2), measurably around $42.75 \mathrm{~dB}, 25.33 \mathrm{~dB}$ and 21.51 dB at frequencies of $28 \mathrm{GHz}, 24 \mathrm{GHz}$ and 10 GHz with bandwidths of $2.12 \mathrm{GHz}, 2.14 \mathrm{GHz}$ and 0.41 GHz , respectively. The measured return losses of 38.53 dB ,
25.13 dB and 21.73 dB with bandwidths of 2.00 GHz , 2.01 GHz and 0.39 GHz respectively are found enough for proper impedance matches. However, it is pointed out that the bandwidth differences for $0.12 \mathrm{GHz}, 0.13$ GHz and 0.02 GHz constitute about $5.83 \%, 6.27 \%$ and $5.00 \%$ respectively between the simulated and measured bandwidth. These differences arise from the manufacturing sensitivity and the effect of the coaxial feed connector. An excellent correlation was observed between the measured and simulated results.

Table 3: Variation of antenna parameters with thickness of conductive material (simulated)

| $\mathbf{h}$ <br> $(\mathbf{m m})$ | $\mathbf{d}$ <br> $\mathbf{( m m})$ | $\mathbf{W}_{\mathbf{p}}$ <br> $(\mathbf{m m})$ | $\mathbf{L}_{\mathbf{p}}$ <br> $(\mathbf{m m})$ | $\mathbf{F}_{\mathbf{r}}$ <br> $(\mathbf{m m})$ | $\mathbf{D}$ <br> $\mathbf{( d B})$ | $\mathbf{G}$ <br> $\mathbf{( d B})$ | $\mathbf{R}$ <br> $(\mathbf{d B})$ | $\mathbf{B W}$ <br> $\mathbf{( G H z})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 1.45 | 5.34 | 4.38 | 28.00 | 10.150 | 10.10 | 24.47 | 1.0930 |
| 0.2 | 1.45 | 5.34 | 4.78 | 28.00 | 10.130 | 10.10 | 25.69 | 1.1269 |
| 0.3 | 1.45 | 5.34 | 4.73 | 28.00 | 10.090 | 10.10 | 27.57 | 1.2092 |
| 0.4 | 1.45 | 5.34 | 4.70 | 28.00 | 10.040 | 10.00 | 26.70 | 1.2586 |
| 0.5 | 1.45 | 5.34 | 4.67 | 28.00 | 9.970 | 9.97 | 27.86 | 1.3566 |
| 0.6 | 1.45 | 5.34 | 4.63 | 28.00 | 9.880 | 9.88 | 35.94 | 1.4225 |
| 0.7 | 1.45 | 5.34 | 4.61 | 28.00 | 9.790 | 9.79 | 33.88 | 1.5158 |
| 0.8 | 1.45 | 5.34 | 4.57 | 28.00 | 9.694 | 9.69 | 33.54 | 1.6091 |
| 0.9 | 1.45 | 5.34 | 4.55 | 28.00 | 9.589 | 9.59 | 32.65 | 1.7025 |
| 1.0 | 1.45 | 5.34 | 4.54 | 28.002 | 9.474 | 9.47 | 31.49 | 1.7245 |
| 1.1 | 1.45 | 5.34 | 4.54 | 27.907 | 9.373 | 9.37 | 30.17 | 1.8852 |
| 1.2 | 1.45 | 5.34 | 4.51 | 27.809 | 9.211 | 9.22 | 26.78 | 1.9895 |
| 1.3 | 1.45 | 5.34 | 4.50 | 27.850 | 9.086 | 9.06 | 25.06 | 2.0609 |
| 1.4 | 1.45 | 5.34 | 4.46 | 27.880 | 8.940 | 8.94 | 20.63 | 2.0983 |
| 1.5 | 1.45 | 5.34 | 4.44 | 27.800 | 8.766 | 8.77 | 18.69 | 2.0928 |
| 1.6 | 1.45 | 5.34 | 4.42 | 27.744 | 8.620 | 8.62 | 17.47 | 2.1642 |
| 1.7 | 1.45 | 5.34 | 4.41 | 27.728 | 8.473 | 8.47 | 16.66 | 2.2411 |
| 1.8 | 1.45 | 5.34 | 4.40 | 27.728 | 8.306 | 8.31 | 16.03 | 2.2840 |
| 1.9 | 1.45 | 5.34 | 4.39 | 27.760 | 8.164 | 8.16 | 15.50 | 2.3830 |
| 2.0 | 1.45 | 5.34 | 4.36 | 27.760 | 7.992 | 7.99 | 14.36 | 2.3940 |

Table 4: Variation of antenna design parameters with substrate thickness (simulated)

| $\mathbf{h}_{\mathbf{s}}$ <br> $(\mathbf{m m})$ | $\mathbf{L}_{\mathbf{p}}$ <br> $\mathbf{( m m})$ | $\mathbf{W}_{\mathbf{p}}$ <br> $(\mathbf{m m})$ | $\mathbf{d}$ <br> $(\mathbf{m m})$ | $\mathbf{F}_{\mathbf{r}}$ <br> $\mathbf{( \mathbf { G H z } )}$ | $\mathbf{D}$ <br> $(\mathbf{d B i})$ | $\mathbf{G}$ <br> $(\mathbf{d B})$ | $\mathbf{R}$ <br> $(\mathbf{d B})$ | $\mathbf{B W}$ <br> $(\mathbf{G H z})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.4 | 4.54 | 5.34 | 1.45 | 28.576 | 9.585 | 9.58 | 19.323 | 1.5471 |
| 0.5 | 4.54 | 5.34 | 1.45 | 28.024 | 9.451 | 9.45 | 42.648 | 2.1180 |
| 0.6 | 4.54 | 5.34 | 1.45 | 27.696 | 9.310 | 9.31 | 28.001 | 2.3009 |
| 0.7 | 4.54 | 5.34 | 1.45 | 26.696 | 9.188 | 9.19 | 25.616 | 2.3512 |
| 0.8 | 4.54 | 5.34 | 1.45 | 26.467 | 9.064 | 9.06 | 28.118 | 2.3725 |
| 0.9 | 4.54 | 5.34 | 1.45 | 26.008 | 8.932 | 8.93 | 40.162 | 2.4405 |
| 1.0 | 4.54 | 5.34 | 1.45 | 25.656 | 8.785 | 8.79 | 32.600 | 2.5075 |



Fig. 5. Comparison of the measured and simulated results "input reflection coefficient" of a fabricated air substrate patch at: (a) 28 GHz , (b) 24 GHz , and (c) 10 GHz resonances.

## B. Radiation pattern

Radiation patterns were measured using a swept frequency measurement conducted in an anechoic chamber. The measured radiating patterns of the proposed antennas were plotted at $28 \mathrm{GHz}, 24 \mathrm{GHz}$ and 10 GHz resonances and are presented in Fig. 6. The measured input impedance was $50.401-\mathrm{j} 1.02$, which is approximate to $50 \Omega$. Since both the real and imaginary parts of the measured input impedance were close to the Smith chart centre, it therefore indicates a maximum power transfer has occurred. Figures 6 (a), (b) and (c) show the normalized measured and simulated radiation patterns of the proposed linearly omnidirectional polarized antennas at $28 \mathrm{GHz}, 24 \mathrm{GHz}$ and 10 GHz resonances respectively in the E- and H-planes obtained using a swept frequency
measurement in an anechoic chamber. The simulated patterns are reasonably in agreement with the measured patterns, and this shows that the antenna is rather of the directional type. Nonetheless, the contrast between the measured and simulated radiation patterns indicates some variations between these patterns. The measured Eplane radiation pattern indicates a reasonable degree of variation. This is probably due to the coaxial feed probe's effect on the height variability. It is important to note that the pattern of a radiation is mainly generated due the current of excitation from a coaxial probe to the radiating element and the current in the probe. Regarding the H-plane, both the simulated and measured patterns of radiation show a constant good agreement, but with a minor difference and a good directivity in both frequencies. The actual measured radiation patterns and gains of the proposed antennas are in close agreement with the simulated results.


Fig. 6. Measured radiation patterns (E \& H planes) of the proposed antennas at: (a) 28 GHz ; (b) 24 GHz , and (c) 10 GHz .

## C. Comparison of the proposed mathematical model

Table 6 provides a contrast between the published works performance profile comparisons with proposed antennas in terms of resonant frequency overall size and measured values, gain, radiation efficiency, return loss as well as bandwidth. This comparison shows that the proposed mathematical model antennas operating at $28 \mathrm{GHz}, 24 \mathrm{GHz}$ and 10 GHz has a wider bandwidth and a high gain compared to other antennas published previously.

The proposed mathematical model has the following advantages:
i. Ability to predict resonance frequency faster and more accurately compared to many advanced simulation tools that are available commercially
ii. Simplifies and save the simulation time to analyses parameters and process parametric result data compared to many advanced simulation tools that are available commercially
iii. It reduced the cost of optimum performance requirements for electromagnetic simulation (such as high clock speed and core count CPU, an efficient GPU workstation, and fast RAM.

Table 6: Publish works performance profile comparison with proposed antennas

| Frequency <br> $(\mathbf{G H z})$ | Size <br> $\left(\mathbf{m m}^{\mathbf{2}}\right)$ | Gain <br> $(\mathbf{d B})$ | Bandwidth <br> $(\mathbf{G H z})$ | Efficiency <br> $(\%)$ | Return loss <br> $(\mathbf{d B})$ | Ref. Antenna |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 28 | $04.42 \times 3.47$ | 8.58 | 0.63 | $\mathrm{~N} / \mathrm{A}$ | 44.46 | $[31]$ |
| 28 | $04.40 \times 4.20$ | 4.47 | 1.55 | 94.00 | $\mathrm{~N} / \mathrm{A}$ | $[32]$ |
| 28 | $20.00 \times 16.50$ | 9.33 | 0.45 | $\mathrm{~N} / \mathrm{A}$ | 23.50 | $[33]$ |
| $\mathbf{2 8}$ | $\mathbf{0 5 . 3 4 \times \mathbf { 4 . 5 4 }}$ | $\mathbf{9 . 4 5}$ | $\mathbf{2 . 0 0}$ | $\mathbf{8 3 . 2 0}$ | $\mathbf{3 8 . 5 3}$ | Proposed Antenna |
| 24 | $07.27 \times 7.27$ | 3.24 | 1.87 | 90.00 | 32.50 | $[34]$ |
| 24 | $13.80 \times 11.40$ | 3.00 | 0.40 | $\mathrm{~N} / \mathrm{A}$ | 32.00 | $[35]$ |
| 24 | $28.00 \times 20.00$ | 8.20 | 2.00 | 93.00 | 23.00 | $[36]$ |
| $\mathbf{2 4}$ | $\mathbf{5 . 2 0} \times 7.00$ | $\mathbf{9 . 4 0}$ | $\mathbf{2 . 0 1}$ | $\mathbf{8 7 . 4 0}$ | $\mathbf{2 5 . 1 3}$ | Proposed Antenna |
| 10 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | 1.18 | 66.30 | 41.50 | $[37]$ |
| 10 | $50 \times 25$ | 8.76 | 1.14 | 82.32 | 27.77 | $[38]$ |
| 10 | $20 \times 18$ | 3.14 | 0.80 | $\mathrm{~N} / \mathrm{A}$ | 34.50 | $[39]$ |
| $\mathbf{1 0}$ | $\mathbf{1 3 . 6} \times \mathbf{0 7}$ | $\mathbf{9 . 8 7}$ | $\mathbf{0 . 3 9}$ | $\mathbf{9 8 . 9 0}$ | $\mathbf{2 1 . 7 3}$ | Proposed Antenna |

## VI. CONCLUSION

Evaluation of the effects of conductive material and substrate thickness, as well as the development of a mathematical model of a single element rectangular microstrip patch inset-fed antenna for 5 G wireless communication application are presented in this paper. Microstrip patch antennas operated using air substrate at $28,24 \& 10 \mathrm{GHz}$ resonance were designed, simulated, optimized and analysed accordingly. The conductive patch materials and substrate techniques used to produce mm -wave integrated antennas have a significant impact on the antennas' impedance and radiation characteristics. Therefore, the effects of conductive patch material and substrate technology on the performance of an antenna must be thoroughly defined and properly understood prior to the designing of a 5 G antenna. The validity of the proposed model equations is verified by comparison with the measured results. The measured results obtained from the fabricated antennas prototypes are in good agreement with the simulated (model equations) results. The implementation of this proposed mathematical modelling approach will minimize the time required to obtain the best resonant frequency design compared to
the parametric studies using a simulation software. The proposed antennas support a very low profile, which is an excellent in the integrated low-cost millimeter-wave applications.

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# A Dual-polarized UWB Antenna Fed by a New Balun for Electronic Reconnaissance System Application 

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#### Abstract

For a practical engineering application, a printed ultra-wide band (UWB) antenna with dual polarization is presented in this paper. An integrated Balanced-to-unbalanced (Balun) transformer from microstrip line to coplanar stripline (CPS) is designed, which is employed to feed the symmetrically coplanar Vivaldi radiator. Two substrates are used to support the Vivaldi radiators which are orthogonally mounted. The proposed antenna can build two orthogonal UWB channels and receive two polarization components of incident electromagnetic signal. By bending the balanced coplanar striplines, the cross placement of two polarization radiators can be easily realized, which is suitable for the engineering application. A dualpolarized Vivaldi antenna with the frequency range from 2.5 GHz to 6 GHz was simulated and fabricated. The measured indicated that the average return loss of designed antenna is less than -10 dB and the port isolation degree is about 20 dB with the operational frequency range, the gains are about larger than 3 dB and the average cross polarization levels are about -20 dB . The research results verify the feasibility of the proposed antenna scheme. The presented dual-polarized antenna has the advantages of easy design, high port isolation and convenient engineering application, which can be used in the fields such as electronic reconnaissance system.


Index Terms - Balun, CPS, dual polarization, electronic reconnaissance system, UWB, Vivaldi.

## I. INTRODUCTION

Radar electronic reconnaissance system usually adopts ultra-wideband antenna to receive various radar signals, and uses signal processing algorithms to realize radar signal detection, feature extraction, target recognition and other functions[1-2]. Ultra-wideband antennas play an important role in radar electronic reconnaissance systems. In order to obtain more comprehensive information on the radar signal, the
polarization-sensitive reconnaissance system is the future development trend. The dual-polarization UWB antenna is a key component of the polarization-sensitive radar reconnaissance system. Research on dualpolarization UWB antenna suitable for engineering application has important practical significance [3-4]. The dual-polarized antenna unit provides two polarized signal receiving ports, and simultaneously receives two orthogonal polarization components of the incident electromagnetic wave, thereby sensing the fully polarized information of the electromagnetic wave [5-7]. For dual-polarized UWB antennas, good polarization port isolation and low cross-polarization levels are important considerations, which require antennas with lower return loss over the ultra-wideband range and smooth radiation pattern [8-10]. In the design and development of dual-polarized UWB antennas, antenna types of various types of structures have emerged, such as dual-polarized ridged horn antennas, dual-polarized sinusoidal antennas [11-12], and dual-polarized logarithmic periodic antennas [13-14], dual-polarized slot antenna [15-16]. The dual-polarized Vivaldi antenna has been paid more and more attention and widely used due to its advantages of low cost and simple structure. Vivaldi antennas are broadband tapered slot antennas with ultra-wideband impedance radiation pattern performance. The dual-polarized UWB Vivaldi antenna is usually composed of a printed medium substrate. Two symmetric dielectric substrates are placed at the intersection, and two polarization ports are formed by mechanical assembly. In actual operation, the output port is usually a coaxial connector. In order to solder easily, the output port is generally a microstrip transmission line on the printed medium substrate. Since the microstrip transmission line is an unbalanced structure, it is necessary to introduce an unbalanced device to balanced conversion feeding structure, this unbalanced device is balun [17-19]. Balun is usually used as the conversion from microstrip line to slot line, based on
electromagnetic coupling, the excitation of tapered slots and space electromagnetic radiation are realized. The traditional microstrip-slot broadband balun structure is complex and has many design parameters, which results in a large amount of design work. For the cross structure of two dielectric substrates, it is difficult to assemble the antenna, it is easily results in the occlusion of two polarization ports and poor assembly consistency. In addition, due to the cross between polarization ports, the isolation degree of polarization ports becomes worse, and the electromagnetic coupling between polarization channels becomes stronger, which makes the measurement error larger [21-22].

Based on the above considerations, this paper proposes an integrated balun-fed dual-polarization UWB antenna based on microstrip line to coplanar stripline conversion. The balun design is simple and easy to connect and match with the Vivaldi radiator. The occlusion of orthogonal polarization ports bends and vertically isolates the coplanar strip lines, which effectively reduces the assembly difficulty and improves the isolation of the polarization ports, which lays a good foundation for engineering applications. In this paper, the structure of the specific dual-polarized antenna is designed and implemented for the frequency range of $2.5 \mathrm{GHz} \sim 6 \mathrm{GHz}$. According to the working frequency range and technical indicators, the size parameters of the antenna were preliminarily calculated. Finally, the full wave electromagnetic simulation software was used for numerical calculation and performance optimization, and then the structure and size that meet the requirements were determined to complete the design and implementation of the antenna. The research results of antenna processing and test work show that the design of the polarization diversity antenna is effective.

The paper is organized as follows. Section 2 gives the structure of the designed dual-polarized UWB antenna and discusses the simulation and optimization of the dual-polarized UWB antenna. Section 3 describes the test results. Finally, the research conclusion is provided.

## II. ANTENNA DESIGN

The schematic diagram of the ultra-wideband dualpolarized Vivaldi antenna designed in this paper is shown in Figure 1. The dielectric substrate of the entire antenna is selected from FR4 plates, and the thickness of the dielectric substrate is 1 mm . Two dielectric substrates of the same size are placed orthogonally, each polarized port corresponds to an ultra-wideband Vivaldi radiator, the feed transmission line is a coplanar strip line, and the Vivaldi radiator uses two exponential curves to constrain its shape in the middle region. An exponentially graded gap is formed to realize an ultra-wideband radiation
field. In order to facilitate assembly of the dielectric substrate, the excited coplanar strip line is bent, and in the vertical direction, the two coplanar strip lines are kept at a certain interval, which can improve the port isolation. The input terminal of the antenna is a microstrip line, which is suitable for soldering with the coaxial line. An integrated printing balun is added between the microstrip line and the coplanar strip line to complete the conversion between the unbalanced and balanced structures, at the same time, it can achieve impedance matching. The Balun is a kind of microstrip line-coplanar stripline conversion Balun.


Fig. 1. The structure diagram of dual-polarized UWB antenna.

The geometrical models of the two dielectric substrates of the dual-polarized Vivaldi antenna are shown in Fig. 2 and Fig. 3 respectively. The designed Vivaldi radiator is a symmetrical structure, and the spacing between the excited coplanar strip lines is $a_{1}$, the width of the dielectric substrate is $w_{-} g n d$. The designed microstrip line-coplanar stripline ultrawideband Balun consists of the microstrip line at the input end, the coplanar strip line at the output end, the fan-shaped mating branch and the partially printed ground plane. The radius of the fan-shaped branch is $r_{-} f a n$.


Fig. 2. The structure model of the first antenna radiator: (a) front view and (b) back view.


Fig. 3. The structure model of the second antenna radiator: (a) front view and (b) back view.

At the output of the Balun, the coplanar stripline excites the balanced Vivaldi radiator, the radiation impedance of the radiator is matched with the output impedance of the Balun, and the full-wave electromagnetic simulation technique is used to complete the parameter design. The curve equation of Vivaldi antenna as follows:

$$
\begin{equation*}
x=a e^{r y} . \tag{1}
\end{equation*}
$$

The characteristic impedance of CPS can be calculated by the following formula [20]:

$$
\begin{gather*}
Z_{0}=\frac{120 \pi}{\sqrt{\varepsilon_{r e}}} \frac{K\left(k_{1}\right)}{K^{\prime}\left(k_{1}\right)}(\Omega),  \tag{2}\\
\varepsilon_{r e}=1+\frac{\varepsilon_{r}-1}{2} \frac{K^{\prime}\left(k_{1}\right)}{K\left(k_{1}\right)} \frac{K\left(k_{2}\right)}{K^{\prime}\left(k_{2}\right)},  \tag{3}\\
k_{1}=\frac{s}{s+2 W}=\frac{a}{b},  \tag{4}\\
k_{2}=\frac{\sinh \left(\frac{\pi a}{2 h}\right)}{\sinh \left(\frac{\pi b}{2 h}\right)},  \tag{5}\\
W_{S}=W_{C}=W_{0} . \tag{6}
\end{gather*}
$$

Where, $\varepsilon_{r}$ is the dielectric constant of the dielectric plate, $\varepsilon_{r e}$ is the relative dielectric constant, $K$ is the elliptic integral function, $h$ is the thickness of the dielectric plate, $W$ and $g$ are the width of the stripline and the gap between them, respectively.

According to formula (2)-(6), the characteristic impedance of coplanar stripline is calculated, and the structure of Balun and impedance matching is designed and optimized.

The simulated performance curves of polarization isolation for the designed antenna varying with the parameter of $r_{-} f a n$ are shown in Fig. 4. It can be seen that the radius of the fan-shaped branch has little effect on port isolation of the low frequency band. In the high frequency band, as the radius of the fan-shaped branch becomes larger, the isolation between the ports is greatly
undulated, and the size is preferably 10 mm .
The simulated curves of polarization isolation for the antenna varying with the parameter of $a_{1}$ are shown in Fig. 5. It can be seen that with the increase of $a_{1}$, the polarization port isolation of the two ports both show a good trend. When $a_{1}$ becomes 1.5 mm , it continues to increase by $a_{1}$.


Fig. 4. Port isolation curves varying with $r_{-}$fan .


Fig. 5. Port isolation degree curves varying with $a_{1}$. 5

The dimensions of the dual-polarized UWB Vivaldi antenna are determined as follows: $a_{1}$ is 1.5 mm , $r_{-} f a n$ is 10 mm , and $w_{-} g n d$ is 80 mm . The simulation results of the return loss and port isolation of the dualpolarized Vivaldi antenna designed in this paper are shown in Figure 6. In the frequency range of 3 GHz to 4 GHz , the average return loss of the two polarization ports of the antenna is about 8 dB , and the average isolation of the polarization port of the antenna is about 20 dB . The performance of the two port circuits is basically symmetrical.


Fig. 6. The simulated return loss and port isolation degree of dual-polarized antenna.

Figure 7 shows the simulation results of the impedance characteristics of the two polarization ports. It can be seen that the average value of the resistance part of the input impedance of the two polarization ports is about 50 ohms in the working frequency band, and the reactance part is small. The mutual impedance value fluctuates near zero, and the average value is close to zero, which reveals the impedance characteristics of ultra wideband.



Fig. 7. The simulation results of the impedance characteristics of the two polarization ports: (a) the self impedance of port $1,(b)$ the self impedance of port 2, and (c) the mutual impedance between the port 1 and the port2.

Figure 8 shows the current distributions on the surface at 3 GHz and 6 GHz , respectively. It can be seen that the current distribution on the surface of the antenna changes smoothly with the frequency, the current distribution on the two polarization ports is symmetrical, and the current distribution is concentrated on the slot edge of the radiation dipole. The radiation fields of the two polarization ports are orthogonal, which shows the effectiveness of the design scheme.


Fig. 8. The current distributions of the impedance characteristics of the two polarization ports.

Figure 9 and Fig. 10 show the simulated three dimension gain patterns of two polarized ports at four typical frequencies, respectively. According to Fig. 9 and Fig. 10, the wide beam performances of two polarized ports within the whole working frequency range are observed.


Fig. 9. The simulated three dimension gain patterns of port 1: (a) pattern at 3 GHz , (b) pattern at 4 GHz , (c) pattern at 5 GHz , and (d) pattern at 6 GHz .


Fig. 10. The simulated three dimension gain patterns of port 2: (a) pattern at 3 GHz , (b) pattern at 4 GHz , (c) pattern at 5 GHz , and (d) pattern at 6 GHz .

## III. RESULTS AND DISCUSSIONS

According to the design result of the dual-polarized UWB Vivaldi antenna, an antenna prototype was fabricated and tested. The antenna performance experiment was carried out in the microwave anechoic chamber. The processed antenna photo is shown in Fig. 11. The test results of return loss and port isolation of the dual-polarized Vivaldi antenna in this paper are shown in Fig. 12. The return loss in the frequency range of $3 \mathrm{GHz} \sim 6 \mathrm{GHz}$ is less than 10 dB , and the port isolation is less than 20 dB . The return loss and isolation index meet the expected specifications of the dual-polarized antenna.


Fig. 11. The photo of the fabricated dual-polarized UWB antenna.


Fig. 12. The measured return loss and port isolation degree of dual-polarized antenna.

Figure 13, Fig. 14, Fig. 15 and Fig. 16 show the radiation pattern tested results at $3 \mathrm{GHz}, 4 \mathrm{GHz}, 5 \mathrm{GHz}$ and 6 GHz , respectively. At each frequency point, the radiation directions of the E and H planes are given respectively. All test results are summarized in Table 1. The test results show that the test direction of the antenna is basically consistent with the simulation results, which verifies the effectiveness of the design. The tested gains are lower than the simulated results. The radiation pattern has a certain fluctuation, which is due to the environmental interference of the test site and machining errors.


Fig. 13. The measured patterns of the antenna at 3 GHz : (a) Port 1 at E plane, (b) Port 1 at H plane, (c) Port 2 at E plane, and (d) Port 2 at H plane.


Fig. 14. The measured patterns of the antenna at 4GHz: (a) Port 1 at E plane, (b) Port 1 at H plane, (c) Port 2 at E plane, and (d) Port 2 at H plane.


(c)

(d)

Fig. 15. The measured patterns of the antenna at 5 GHz : (a) Port 1 at E plane, (b) Port 1 at H plane, (c) Port 2 at E plane, and (d) Port 2 at H plane.


Fig. 16. The measured patterns of the antenna at 6 GHz : (a) Port 1 at E plane, (b) Port 1 at H plane, (c) Port 2 at E plane, and (d) Port 2 at H plane.

Table 1: The test results of designed antenna

| Frequency $(\mathbf{G H z})$ | 2.5 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|S_{11}\right\|$ for port $1(\mathbf{d B})$ | -19 | -16 | -10 | -12 | -16 |
| Gain for port $1(\mathbf{d B})$ | 4 | 3.5 | 4.5 | 6 | 7.5 |
| $\left\|S_{22}\right\|$ for port $2(\mathbf{d B})$ | -9 | -16 | -11 | -11 | -12 |
| Gain for port $2(\mathbf{d B})$ | 3.5 | 3 | 4.6 | 6.7 | 7.6 |
| Isolation degree $(\mathbf{d B})$ | -22 | -23 | -29 | -30 | -23 |

## IV. CONCLUSION

This paper researched a dual-polarized UWB Vivaldi antenna, which employs an integrated Balun from microstrip line to coplanar stripline to feed the Vivaldi antenna radiator. The bended coplanar strip-lines were used to improve isolation degree of polarization ports. The traditional dual-polarized of the Vivaldi antenna
isolation features have greatly improved. The design and fabrication difficulty of dual-polarized antenna have been reduced. It is easy to be used in the practical engineering. A UWB Vivaldi antenna working at $2.5 \mathrm{GHz} \sim 6 \mathrm{GHz}$ is designed by full-wave electromagnetic simulation software. The simulation and experimental results verified the proposed dual-polarized UWB Vivaldi antenna scheme.

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# A Wideband Wide-beam Dual Polarized Dipole Antenna and its Application in Wideband Wide-angle Scanning Array 

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#### Abstract

A wideband wide-beam dual polarized dipole antenna is proposed in this paper. The antenna has a compact size $\left(0.48 \lambda_{0} \times 0.48 \lambda_{0} \times 0.154 \lambda_{0}\right)$ and can operate in a wide frequency range from 1.7 GHz to 2.4 GHz with a half power beam-width more than $100^{\circ}$ in the H-plane for dual polarization. Furthermore, the proposed antenna is employed in two linear arrays. The main beam of the configured arrays can scan from $-60^{\circ}$ to $+60^{\circ}$ with a gain fluctuation less than 4 dB over the entire band for dual polarization. The antenna is fabricated and measured in an anechoic chamber. The measured results have a good agreement with the simulated results.


Index Terms - Dipole antenna, dual polarization, wideband, wide beam, wide-angle scanning.

## I. INTRODUCTION

In recent years, with the rapid development of modern wireless communication technology, high performance antennas have attracted more and more attentions. In order to improve the communication network capacity, achieve the high rate communication and minimize the multiple fading, the antenna should have the characteristics of the wide bandwidth, dual polarization, high cross polarization discrimination (XPD) and high front to back ratio (FBR) [1-3]. What is more, the emergence, progress and development of new technologies, such as 5G technology, Internet of things, low orbit satellite communication and so on, have stimulated researchers' enthusiasm on the large-scale electronically scanned array (ESA) [4-6]. As a result, a wide beam-width antenna is demanded for phased array to improve the gain at the wide scanning angle [7].

To meet the above requirements, various kinds of antennas have been proposed [8-10]. Owing to its merits of low profile, easy fabrication and light weight, the microstrip antenna has been used widely. Many methods involving wide band [11], dual polarization [12], wide
beam-width [13] and wide scanning angle [14,15] have been reported, but only one or some aspect is referred. [14] proposes a wide-beam microstrip element by optimizing a parasitic pixel layer and achieves a 2-D planar wide-angle scanning from $-75^{\circ}$ to $+75^{\circ}$. But it only works at 5.2 GHz for single polarization. A wide-angle scanning linear phased array antenna is proposed in [15]. By employing a wide-beam microstrip antenna element, the proposed array can achieve $\pm 75^{\circ}$ scanning with a gain fluctuation less than 3 dB in a frequency band from 3.2 GHz to 3.8 GHz for single polarization, indication a fractional bandwidth of $17 \%$. The planar dipole antenna is another popular radiator and it has advantages of wide bandwidth, dual polarization and low cost, which makes it suitable for $2 \mathrm{G} / 3 \mathrm{G} / 4 \mathrm{G}$ base station [16,17]. But the narrow half power beam-width (generally $65^{\circ} \pm 5^{\circ}$ ) and high profile (generally $0.25 \lambda$ ) make the traditional planar dipole antenna mismatch the requirement of ESA, such as the large-scale ESA for 5G base station. The magnetoelectric (ME) dipole antenna proposed by Luk and Wong is an attractive antenna owing to its excellent electrical characteristics, such as wideband, symmetric radiation patterns, low cross polarization, low back radiation and stable gain over the entire operating band [18]. [19] shows a wide-beam circularity polarized (CP) microstrip magnetic-electric dipole antenna and it is applied to achieve a wide-angle CP scanning from $-66^{\circ}$ to $66^{\circ}$. But it is only effective at 5.6 GHz . As a result, to the knowledge of the author, an antenna, which can achieve wide-angle scanning in a wide bandwidth for dual polarization, is rarely presented.

In order to address problems mentioned above, a wideband wide-beam dual polarized dipole antenna is proposed. By employing the proposed antenna as an element, two wide-angle scanning linear arrays can be obtained. The main beam of arrays can scan from $-60^{\circ}$ to
$+60^{\circ}$ with a gain fluctuation less than 4 dB from 1.7 GHz to 2.4 GHz for dual polarization, which makes it a promising candidate for 5 G base station, radar and satellite communication systems.

## II. DESIGN OF PROPOSED ANTENNA ELEMENT

## A. Antenna configuration

The configuration of the proposed antenna is shown in Fig. 1. It consists of four layers from up to down. On the top layer, it is a Taconic TLY-5A substrate with the permittivity of 2.2 and the loss tangent of 0.0009 . The size is $70 \mathrm{~mm} \times 70 \mathrm{~mm} \times 1.524 \mathrm{~mm}$. Two shaped dipoles are printed on the bottom of the substrate along the $\pm 45^{\circ}$ diagonal direction to form dual polarized radiation, while two microstrip lines and loop-structures are printed on the other side of the substrate to excite dipoles by coupling. Two vias are employed to avoid the intersection between the two feeding microstrip lines. There are two $50 \Omega$ coaxial cables are used to feed the antenna. The inner conductors are connected to the microstrip lines, while the outer conductors are soldered with the dipoles on the bottom of the substrate. As we all know, the height between the dipole and the PEC reflector is usually set as $0.25 \lambda_{0}$ to form a directional radiation with high gain. In order to obtain a lower profile, here we make a compromise between bandwidth and profile and two layers are employed in the middle of the antenna. The upper one is an 8 mm -thick foam layer, while the lower one is a Taconic FR-60 substrate with dielectric constant of 6.15 and thickness of 13 mm . They are used to support the dipole antenna and reduce the overall height of proposed antenna lower than $0.25 \lambda_{0}$. The bottom layer is a metal plane, which is as a reflector to form directional radiation. There are four plastic screws on the periphery of the antenna to form a solid structure.

Table 1: The parameters of proposed antenna (unit:mm)

| Parameter | W1 | W2 | W3 | W4 |
| :---: | :---: | :---: | :---: | :---: |
| Value | 4 | 8 | 3 | 3.5 |
| Parameter | W5 | W6 | L1 | L2 |
| Value | 1 | 9.32 | 15 | 21.5 |
| Parameter | L3 | L4 | L5 | L6 |
| Value | 3 | 4.75 | 7.07 | 2 |
| Parameter | r1 | r2 | r3 | G |
| Value | 1 | 1.13 | 0.5 | 70 |
| Parameter | H1 | H2 | H3 |  |
| Value | 1.524 | 8 | 13 |  |


(c) Top view and bottom view of the top layer

Fig. 1. Configuration of the dual polarized antenna.


Fig. 2. Prototype of the proposed antenna.

The overall size of the proposed antenna is $70 \mathrm{~mm} \times 70 \mathrm{~mm} \times 22.524 \mathrm{~mm}$ and is corresponding to $0.48 \lambda_{0} \times 0.48 \lambda_{0} \times 0.154 \lambda_{0}$ (where $\lambda_{0}$ is the wavelength of the center operating frequency), which indicates a simple, compact structure and a low profile.

## B. Numerical study and discussion of parameters

The proposed antenna is modeled in High Frequency Structure Simulator (HFSS) and optimized with the Finite Element Method (FEM). The detailed parameters of the proposed antenna are listed in Table 1. A prototype based on the optimized parameters is fabricated to verify the validity of the proposed antenna and it is exhibited in Fig. 2.


Fig. 3. Numerical study of parameters.
Because the hybrid substrate has a great influence on the impedance bandwidth, here the key parameter H 2 and H 3 are numerically studied and shown in Fig. 3, while the other parameters remain unchanged. It can be seen that as H2 goes down, the single resonance gradually turns into the dual resonance and the
bandwidth is enhanced. When the H 2 gets smaller, the two resonant points are far apart, which leads to a dual band antenna. As a result, H2 is selected as 8 mm to obtain a relatively wide bandwidth.

It can be seen from Fig. 3 (b) that the variation of parameter H3 will lead to the different resonance strength of two resonant points. Considering the balance between them, bandwidth and the profile, the parameter H 3 is chosen as 13 mm here.

## C. Performances of the proposed antenna

The simulated S-parameters of the proposed antenna are illustrated in Fig. 4. It can be seen that a -10 dB frequency band covering 1.7 GHz to 2.4 GHz can be observed, which means a fractional bandwidth of $34 \%$. The isolation between two ports is greater than 25 dB over the entire frequency band, indicating a good isolation. The measured S11 and S22 are consistent with the simulated ones and a lower port-to-port measured isolation than -25 dB can be observed over the entire operating band.


Fig. 4. S-parameters versus frequency.



(c) E-plane $(2 \mathrm{GHz})$

(d) H-plane (2GHz)

(e) E-plane ( 2.4 GHz )

Fig. 5. Radiation patterns at $1.7 \mathrm{GHz}, 2 \mathrm{GHz}$ and 2.4 GHz for $-45^{\circ}$ polarization.

Considering the symmetry of the proposed antenna, only E-plane and H-plane radiation patterns with Port 1 excited at different operating frequency are exhibited in Fig. 5. It can be seen that a stable radiation pattern can be obtained over the entire operating band. The beamwidth of the proposed antenna in the E-plane is $84^{\circ}$ at $1.7 \mathrm{GHz}, 86^{\circ}$ at 2 GHz , and $92^{\circ}$ at 2.4 GHz , while the beam-width of the proposed antenna in the H-plane is $113^{\circ}$ at $1.7 \mathrm{GHz}, 112^{\circ}$ at 2 GHz , and $106^{\circ}$ at 2.4 GHz . It is obvious that the beam-width of the proposed antenna is wider than that of the reported planar dipole antennas used in base station (The beam-width is usually $65^{\circ} \pm 5^{\circ}$ ). The measured results agree with the simulated ones, especially for the principle polarization component. The discrimination between them may come from the fabrication tolerance and imperfection of measurement environment.

## III. THE PROPOSED ELEMENT FOR WIDEANGLE SCANNING ARRAY

## A. Geometry of scanning linear array

In order to verify the effect of the proposed antenna in the ESA, two scanning linear arrays are designed as shown in Fig. 6. Each array consists of eight proposed elements and the spacing between the adjacent element is set as 64 mm , which approximately corresponds to $0.5 \lambda$ at 2.4 GHz .


Fig. 6. Configuration of scanning linear array.

## B. Simulation of the scanning linear array

The simulated performances of two scanning linear arrays are exhibited in Fig. 7 and Fig. 8, respectively. It can be seen that the main beam of two arrays can scan from $-60^{\circ}$ to $+60^{\circ}$ with a gain fluctuation less than 4 dB over the entire frequency band, especially less than 3 dB at 1.7 GHz and 2 GHz . Compared with the tradition phased array antenna, which usually can scan from $-45^{\circ}$ to $+45^{\circ}$ with a gain fluctuation of $4-5 \mathrm{~dB}$ [7], the proposed array has an excellent wide-angle scanning performance. The relevant scanning performances are concluded in Table 2 and Table 3.

## C. Measured performances of the scanning linear array

The proposed array is fabricated and its prototype is shown in Fig. 9. Because of the similar scanning performances for configuration array 1 and array2, only the performances of configuration array 1 are measured in an anechoic chamber. A power divider with one input port and eight output ports and eight analog phase shifters are employed to achieve beam-steering. The measured results for configuration array 1 are presented in Fig. 10.

The measured results show that the main beam can steer from $-60^{\circ}$ to $60^{\circ}$ at $1.7 \mathrm{GHz}, 2 \mathrm{GHz}$ and 2.4 GHz . Compared with the normal direction, the gain degradation at $\pm 60^{\circ}$ elevation angle is $1.9 \mathrm{~dB}, 2 \mathrm{~dB}$ and 3.6 dB , which is basically consistent with the simulation. The measured results are concluded in Table 4.

The difference between simulated and measured results may come from the fabrication and assembly error. Meantime, the imperfect measured environment and errors introduced by work divider and phase shifter also make the simulated results deviate from measured ones.

(a) 1.7 GHz


Fig. 7. Scanning performances of configuration array 1 for $-45^{\circ}$ polarization at different frequency.

(a) 1.7 GHz


Fig. 8. Scanning performances of configuration array 2 for $-45^{\circ}$ polarization at different frequency.

Table 2: The scanning performances of configuration array 1 for $-45^{\circ}$ polarization at different frequency in YOZ plane

| Array 1 |  | $-60^{\circ}$ | $-30^{\circ}$ | $0^{\circ}$ | $30^{\circ}$ | $60^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.7 <br> GHz | Gain <br> $(\mathrm{dB})$ | 10.2 | 11.6 | 12.4 | 11.6 | 10 |
|  | SLL <br> $(\mathrm{dB})$ | -8.8 | -12 | -13.5 | -11.7 | -9 |
| 2 | Gain <br> $(\mathrm{dB})$ | 10.7 | 11.8 | 13 | 11.8 | 10.9 |
| GHz | SLL <br> $(\mathrm{dB})$ | -10.3 | -11.5 | -12.8 | -11.6 | -10.8 |
| 2.4 | Gain <br> $(\mathrm{dB})$ | 10.1 | 11.4 | 13.9 | 11.4 | 10.3 |
| GHz | SLL <br> $(\mathrm{dB})$ | -5.8 | -10 | -13 | -10.1 | -6.4 |

Table 3: The scanning performances of configuration array 2 for $-45^{\circ}$ polarization at different frequency in XOZ plane

| Array 2 |  | $-60^{\circ}$ | $-30^{\circ}$ | $0^{\circ}$ | $30^{\circ}$ | $60^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.7 <br> GHz | Gain <br> $(\mathrm{dB})$ | 10.1 | 11.7 | 12.5 | 11.7 | 10.1 |
|  | SLL <br> $(\mathrm{dB})$ | -9.2 | -11.7 | -13 | -11.1 | -8.8 |
| 2 | Gain <br> $(\mathrm{dB})$ | 11.1 | 11.9 | 13.2 | 11.9 | 11 |
| GHz | SLL <br> $(\mathrm{dB})$ | -10.9 | -11.4 | -13.1 | -11.4 | -10.5 |
| GHz <br> GHz | Gain <br> $(\mathrm{dB})$ | 10.9 | 11.3 | 13.8 | 11.2 | 10.4 |
|  | SLL <br> $(\mathrm{dB})$ | -7.6 | -10.2 | -13.2 | -10 | -5.4 |



Fig. 9. Prototype of the scanning array.

(a) 1.7 GHz


Fig. 10. Measured scanning performances of configuration array 1 for $-45^{\circ}$ polarization at different frequency.

Table 4: The measured scanning performances of configuration array 1 for $-45^{\circ}$ polarization at different frequency in YOZ plane

| Array 1 |  | $-60^{\circ}$ | $-30^{\circ}$ | $0^{\circ}$ | $30^{\circ}$ | $60^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.7 <br> GHz | Gain <br> $(\mathrm{dB})$ | 10.3 | 10.8 | 12.2 | 12.5 | 11.2 |
|  | SLL <br> $(\mathrm{dB})$ | -8.4 | -8.9 | -13.5 | -12.6 | -10.5 |
| 2 | Gain <br> $(\mathrm{dB})$ | 11.6 | 13.7 | 13.6 | 13.7 | 11.6 |
| GHz | SLL <br> $(\mathrm{dB})$ | -7.7 | -11.5 | -13.8 | -11.2 | -9.3 |
| 2.4 | Gain <br> $(\mathrm{dB})$ | 10.4 | 14 | 14 | 12.6 | 10.7 |
|  | SLL <br> $(\mathrm{dB})$ | -6.4 | -10.7 | -12.4 | -11 | -6.5 |

## IV. CONCLUSION

In this paper, a novel dipole antenna is designed. Owing to its merits of wide bandwidth, wide beam-width and dual polarization, the proposed antenna is employed as an element and two linear arrays with eight elements are proposed. Compared with the traditional phased array, the main beam of the proposed arrays can scan from $-60^{\circ}$ to $+60^{\circ}$ with a gain fluctuation less than 4 dB in a wide frequency band, covering from 1.7 GHz to 2.4 GHz with a fractional bandwidth of $34 \%$. The element and array are fabricated and measured for validity and measured results agree well with the simulated ones. The antenna has a simple, compact structure and a low profile, and can be applied to 5 G base station, radar and satellite communication systems.

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# Metamaterial Based Compact Branch-Line Coupler with Enhanced Bandwidth for Use in 5G Applications 

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#### Abstract

A novel compact 5G branch-line coupler (BLC) based on open-circuit coupled-lines and interdigital capacitor structure is presented in this paper. The proposed BLC shows the composite right/left handed (CRLH) metamaterial transmission-line (TL) operation. The proposed BLC is designed and simulated using CST microwave studio. The designed BLC is then fabricated using the FR4 substrate ( $\varepsilon_{r}=4.3$ and $h=1.66 \mathrm{~mm}$ ). The proposed 5G BLC with coupled-lines and the interdigital capacitor has achieved the fractional bandwidth of $\sim 40.2 \%$ and the size reduction of $54 \%$ as compared to the conventional BLC. The fabricated BLC operates at $2.74-4.15 \mathrm{GHz}$ frequency band with a coupling factor of $-3 \pm 0.2 \mathrm{~dB}$ and the phase difference of $88^{\circ}$ between the output ports. The BLC measurements are performed at the operating frequency of 3.5 GHz . The simulated and measured scattering parameters and phase difference results are in good agreement with each other. The proposed design is suitable for use in future butler-matrix based beamforming networks for antenna array systems in 5 G wireless applications.


Index Terms - 5G, beamforming network, branch-line coupler, butler-matrix, Composite Right/Left Handed (CRLH) Transmission-Line (TL), interdigital capacitor.

## I. INTRODUCTION

In the 5G wireless communication systems, the multi-beam and beam-scanning smart antennas [1] are playing a crucial role to get the desired output. The use of multi-beam antenna array systems and beamforming network (BFN) is an important means of achieving high
directivity and improved coverages for 5G [2]-[3]. The BFNs are used for the adjustment of the amplitude and phase distribution of the multi-beam antenna array system [4]-[5]. Most studies so far have concentrated on the BFNs as a significant part of the 5 G system. The butler-matrix (BM) is the most commonly used BFNs, due to its compact design, easy and low-cost fabrication process. There is no requirement of external bias in the BM operation. Moreover, it works like a reciprocal network (used for receiving/transmitting signals) and can be used for the BFN for 5G communication system. The development of the conventional BM consists of three main components, which are; 1) 3-dB couplers, 2) Crossovers and 3) $45^{\circ}$ phase shifters. So, this paper focusses on the design of a compact branch-line coupler (BLC) with enhanced bandwidth that can achieve improved coverage for 5 G application.

A quadrature coupler is one in which the input is split into two signals (usually with a goal of equal magnitudes) that are 90 degrees apart in phase. Types of quadrature couplers include BLCs (also known as quadrature-hybrid couplers), lange couplers and overlay couplers [6]. The BLCs are used in several 5G communication applications which include BM based antenna arrays, and radio frequency (RF) transceiver systems, and wideband six-port reflectometer design formed by enhanced BLCs [7]-[9]. The 5G technology has many advantages for the end-user, like providing wide coverage, high data-rates and improved spectral efficiency compared to their predecessors [9]. In addition to the above challenges, the 5 G technologies have reported limited resources and the paucity of the new
structure for antenna and passive devices like BLCs, power divider, etc. [1].

In [10], the use of coupled-line unit-cells shows the same properties as dual composite right/left-handed (D-CRLH) unit-cells, and it reduces the BLC size by $52 \%$ and improves the bandwidth by $18.9 \%$. In [11], a miniaturized BLC is developed by using the square-split ring resonators that reduces the overall area of $39.82 \%$ without much improvement in the bandwidth. In [12], a novel miniaturized BLC based on CRLH-TL structure is presented. Interdigital structure and double-spiraldefected resonant cell are used to get the properties of CRLH. In [13], the authors designed a compact structure of BLC using open stubs and meandered TL.

This paper, therefore, aims to propose, design, fabricate and characterize a new BLC, which is considered one of the main components for the construction of the 5G BM. It is envisioned that the BLC proposed in this paper can be used in the future 5G antenna array BFNs. The 3.5 GHz frequency band has been chosen in this study for 5 G technology [14]. Moreover, the 700 MHz , 3.5 GHz and $26 / 28 \mathrm{GHz}$ have also been identified as potential frequency bands for the initial deployment of 5G technology in Malaysia [15]. In the next section, the design methodology of the proposed BLC will be discussed. The configuration is approximately similar to a basic BLC with the $\lambda / 4$ length with additional open-circuits coupled-lines technique, and interdigital capacitor (IDC) achieves metamaterial properties. Lai et al. [16] first proposed the CRLH-TL structure. The proposed 5G coupler is designed by using the CST microwave studio software which uses the finitedifference time-domain (FDTD) techniques for the 3D electromagnetic field analysis. The proposed coupler is designed using the low-cost FR4 substrate [17]. The agreement between the simulation and measured results suggest that the designed coupler performs well and can be used in the future 5 G systems.

The rest of the paper is organized as follows: Section II gives a brief overview of the proposed BLC geometry and design process. The BLC fabrication and characterization are discussed in Section III. The simulated and measured results in terms of BLC parameters like reflection coefficient ( $S_{l 1}$ ), insertion loss ( $S_{12}, S_{13}$ ) and isolation loss ( $S_{14}$ ) are summarized in Section IV, and then a comparison of the results is presented. Finally, Section V draws conclusions.

## II. BRANCH-LINE COUPLER DESIGN

The proposed design of the new BLC is depicted in Figs. 1 (a-b). As shown in Fig. 1 (a), the BLC consists of four ports [18]. The port-1, port-2 and port-3 are inputport, through port and the coupled-port, respectively. Whereas, the port-4 is an isolated port that has no or very negligible output power. The impedance of the horizontal and vertical TL is $Z_{0} / \sqrt{2}$ and $Z_{0}$, respectively.

Whereas, $Z_{0}$ indicate the characteristic impedance. In the proposed BLC, the value of $Z_{0}$ is chosen as $50 \Omega$. The length of the line impedance at each branch is shown in Eq. (1). The IDC unit-cell used in the vertical arm of the BLC is shown in Fig. 1 (b). The width ( $W$ ) and feed length of the BLC can be calculated by using the following equations [18] shown in Eqs. (1-7), respectively. The substrate is chosen to be FR4 with the relative permittivity $\left(\varepsilon_{r}\right)$ of 4.3 , and the thickness $(h)$ of 1.66 mm , respectively.

$$
\begin{gather*}
\text { Feed length }=\frac{\lambda}{4 \sqrt{\varepsilon_{r}}} .  \tag{1}\\
A=\frac{Z_{o}}{60}\left(\frac{\varepsilon_{r}+1}{2}\right)^{1 / 2}+\frac{\varepsilon_{r}-1}{\varepsilon_{r}+1}\left(0.23+\frac{0.11}{\varepsilon_{r}}\right)=1.5,  \tag{2}\\
B=\frac{60 \pi^{2}}{Z_{o} \sqrt{\varepsilon_{r}}},  \tag{3}\\
W / h<2,  \tag{4}\\
W / h=\frac{8 \exp (A)}{\exp (2 A)-2},  \tag{5}\\
W / h>2, \tag{6}
\end{gather*}
$$

$W / h=\frac{2}{\pi}\{B-1-\ln (2 B-1)\}+\frac{\varepsilon_{r}-1}{2 \varepsilon_{r}}\left[\ln (B-1)+0.39-\frac{0.61}{\varepsilon_{r}}\right]$.

The value of $A$ is 1.5 and 1.1 by substituting impedance equal to $50 \Omega$ and $35 \Omega$, respectively for vertical and horizontal arms of the BLC. Based on Eq. (1-7), the BLC-TL width $\left(W_{35}\right)$ and $\left(W_{50}\right)$ is optimized to 5 mm and 2.8 mm for the fabrication of BLC, as shown in Fig. 1 (a), respectively. The dimensions of the horizontal and vertical IDC unit-cell are summarized in Table 1. The difference of dimensions in the horizontal and vertical unit-cell finger length is due to the impedance matching and the S-matrix requirements for the four-port BLC [18]. Figure 1 (b) also shows that the gap ' $s$ ' between the fingers and at the end of the fingers are same. The fingers have a width of ' $m$ ', and length ' $l$ ' which are also specified. The top surface of the substrate is a copper conductor with a thickness of 0.035 mm .

The substrate properties will also affect the BLC performance. Due to the ease of design and fabrication, the IDC often uses planar microstrip TL. Many authors have studied the properties of this type of capacitor [19]-[23]. There are many assumptions made for the calculation of capacitance, some of which are: (1) finger
and gap width are considered (2) capacitance also depends on the number of fingers (3) the capacitor dimensions are much less than a quarter wavelength; (4) metallization thickness and (5) the capacitance is
neglected at the end of the finger. An enhanced model where the capacitor is divided into its core components is studied briefly in [24].


Fig. 1. (a) Proposed BLC showing Interdigital capacitor unit-cell and open-circuits coupled-lines, and (b) zoom-in of IDC unit-cell.

Table 1: Dimensions of horizontal and vertical IDC unitcell

| Description | Notation | Dimension (mm) |
| :---: | :---: | :---: |
| Horizontal Arm |  |  |
| Finger Width | $m$ | 0.4 |
| Finger Length | $l$ | 3.9 |
| Width of IDC | $w$ | 5.2 |
| Finger Gap | $s$ | 0.4 |
| Exterior Finger Width | $t$ | 0.4 |
| Vertical Arm |  |  |
| Finger Width | $m$ | 0.4 |
| Finger Length | $l$ | 2.2 |
| Width of IDC | $w$ | 5.2 |
| Finger Gap | $s$ | 0.4 |
| Exterior Finger Width | $t$ | 0.4 |

Figures 2 (a-b) shows the proposed equivalent circuit model of the IDC. Alley et al. [19] have developed the most popular model. The effect of interdigital fingers capacitance is taken into account in this model. The total capacitance is given by Eq. (8) [24]:

$$
\begin{equation*}
C=\left(\varepsilon_{r}+1\right) l\left[(N-3) A_{1}+A_{2}\right](\mathrm{pF}) \tag{8}
\end{equation*}
$$

In Eq. (8), $N$ represents the number of fingers and the relative permittivity of substrate material represented as $\varepsilon_{r}$. The constant $A_{l}$ and $A_{2}$ in Eq. (8) are represented by Eqs. (9-10) [25], respectively:

$$
\begin{align*}
& A_{l}=4.409 \tanh \left[0.55\left(\frac{h}{w}\right)^{0.45}\right] \times 10^{-6}(\mathrm{pF} / \mu \mathrm{m})  \tag{9}\\
& A_{2}=9.92 \tanh \left[0.52\left(\frac{h}{w}\right)^{0.5}\right] \times 10^{-6}(\mathrm{pF} / \mu \mathrm{m}) \tag{10}
\end{align*}
$$

The constant $A_{1}$ and $A_{2}$ represented IDC interior and exterior finger in terms of $h$ and $w$. The $w$ is the width of the overall IDC conductor, and the $h$ represents the thickness of the substrate material. Based on Eqs. (8-10), the capacitance of IDC is equal to 0.14 pF . Figures 2 (a-b) represent the IDC's lumped equivalent circuits (EC) for the low- and high-frequency applications, respectively. Finally, the series parasitic resistance due to conductor loss is given by Eq. (11):

$$
\begin{equation*}
R=\frac{4}{3} \frac{l}{m N} R_{S}(\Omega) \tag{11}
\end{equation*}
$$

Where, $R_{s}$ is the sheet resistivity of the conductor. The capacitance $\left(C_{s}\right)$ and inductance ( $L$ ) in eq. (12-13) are calculated for the $m / h \ll 1$ approximations. The magnetic field lines are also considered not to loop around each finger, but to loop around the cross-section of the interdigital width, as shown previously in Fig. 2(a). The $c$ indicates the velocity of light in free space. The $L$ and $C_{s}$ are represented as [22]:

$$
\begin{align*}
& L=\frac{z_{0} \sqrt{\varepsilon_{e f f}}}{c} l(\mathrm{H}),  \tag{12}\\
& C_{S}=\frac{1}{2} \frac{\sqrt{\varepsilon_{e f f}}}{z_{0} c} l(\mathrm{~F}) . \tag{13}
\end{align*}
$$



Fig. 2. EC of the IDC: (a) low frequency and (b) high frequency model, respectively.

The parameters $l$ and $w$ of the IDC unit-cell, as shown in Fig. 1 (b) are calculated in multiple steps which are described below by using Eqs. (14-15), respectively [26]:

Step-1: Choose center frequency $\left(f_{o}\right)=3.5 \mathrm{GHz}$.
Step-2: Calculate the width of IDC required to achieve BLC horizontal arm impedance of $Z_{o} / \sqrt{ } 2$. Based on Eqs. (2-7), the value of ' $w$ ' is found to be 5.2 mm .

Step-3: Set the number of fingers, ' $N=7$ ' (as per BLC design shown in Figs. 1 (a-b)). Then, by using Eq. (14), determine the required parameter ' $m$ ' and ' $s=2 \mathrm{~m} / 3$ ':

$$
\begin{equation*}
m \approx \frac{w}{\left(\frac{5 N}{3}-\frac{2}{3}\right)} \tag{14}
\end{equation*}
$$

Step-4: Optimize the value of ' $m$ ' and ' $s$ ' to 0.4 mm .
Step-5: Calculate the value of ' $l$ ' and optimize it to 3.9 mm by using Eq. (15):

$$
\begin{equation*}
l \approx \frac{\lambda_{g}}{8} \approx \frac{c}{8 f_{o} \sqrt{\varepsilon_{r}}} . \tag{15}
\end{equation*}
$$

The different techniques such as double quarterwave transformer, series open-circuited stubs, opencircuited capacitively coupled lines, and arbitrary power division ratio have been implemented in conventional BLC design to overcome the bandwidth limitation. From these techniques, capacitively coupled $\lambda / 4$ opencircuited lines achieve a much improved fractional bandwidth of $63 \%$ [27]. So, in this work, it was decided to insert four capacitively coupled open-circuited $\lambda / 4$ lines at each port of the proposed BLC by adopting
proper characteristic impedances for arbitrary power coupling. This will provide wide bandwidth with DC block capability and flat coupling characteristics [28]. The wideband characteristics of BLC can be analyzed by the equivalent admittance approach of the matched BLC and by the characteristics impedance calculation of the coupled lines as described in [29]. The dimensions of the coupled-lines are summarised in Fig. 3. The IDC make a shunt capacitor, and the coupled lines generate series gap capacitance with respect to the TL, which is acting as an inductor. Therefore, it has series and parallel LC circuit that makes a metamaterial TL.


Fig. 3. Geometry of an open-circuit coupled-line used in the proposed BLC design with its dimensions

## III. BLC FABRICATION AND CHARACTERISATION

The proposed and designed BLC is then fabricated using the FR4 substrate. Figure 4 (a) shows the fabricated BLC PCB and its dimensions, which are $27 \mathrm{~mm} \times 23 \mathrm{~mm}$, respectively and highlight the compact size of the BLC. The SMA connectors are then used with the fabricated BLC prototype for measurements. The fabricated BLC with detailed port information and dimensions are shown in Fig. 4 (b). It is important to note in Fig. 4 (b) that the central conductor of the SMA connector is only soldered to the transmission line for the measurement purposes. The S-parameter measurements of the fabricated BLC are then performed using Keysight (Agilent Technologies) FieldFox N9925A vector network analyzer (VNA).

(a)

(b)

Fig. 4. FR4 substrate based fabricated prototype of the proposed BLC: (a) without connectors and (b) corresponding BLC ports and PCB dimensions are shown with SMA connectors

## IV. RESULTS AND DISCUSSION

This section discusses the simulated and measured results of the BLC. Figure 5 (b) and Fig. 6, illustrate the performance of the proposed 5G BLC in terms of S-parameters and phase differences between the output ports, respectively. The simulation results in Fig. 5 (b), show that the proposed BLC is operating between 2.87 GHz to 4.17 GHz frequency band with a coupling factor of -3 dB , respectively. The proposed simulated BLC return-loss ( $S_{I I}$ ) and isolation loss ( $S_{l 4}$ ) characteristics were achieved better than 10 dB and the bandwidth of $37.2 \%$ at 3.5 GHz as compared to the conventional BLC result shown in Fig. 5 (a). Meanwhile, the proposed BLC design consideration in terms of insertion loss ( $S_{12}$ ) and the coupling loss ( $S_{13}$ ) should be -3 dB to achieve equal power splitting across the port- 2 and port-3. The simulated $S_{l 2}$ and $S_{l 3}$ results, as shown in Fig. 5 (b) are -3.04 dB and -3.17 dB , respectively, also lying in the same frequency range. The insertion and coupling loss error (LE) is 0.04 dB and 0.17 dB , respectively, for the desired value of -3 dB .

The phase difference between port-3 and port-2 should be $90^{\circ}$ as per the design consideration of the proposed BLC. The comparison of simulated and measured results are shown in Fig. 6. It can be observed from Fig. 6 that when port-1 is excited, the simulated phase difference between port-3 and port-2 are $169.37^{\circ}$ $\left(S_{13}\right)$ and $78.47^{\circ}\left(S_{12}\right)$, respectively, at 3.5 GHz frequency band. In the simulation results, the phase difference between port-3 and port-2 is $90.9^{\circ}$ as per the Eq. (16). So, the phase error is $0.9^{\circ}$ with respect to the desired value of $90^{\circ}$ :

$$
\begin{equation*}
\angle S_{13}-\angle S_{12}=\varphi . \tag{16}
\end{equation*}
$$



Fig. 5. S-Parameter response of: (a) conventional BLC (simulated response), and (b) proposed designed and fabricate BLC (both simulated and measured response).

The designed BLC has an overall area of $21.8 \mathrm{~mm} x$ $26.65 \mathrm{~mm}=580.97 \mathrm{~mm}^{2}$. Since, the conventional BLC area is $30 \mathrm{~mm} \times 41.7 \mathrm{~mm}=1251 \mathrm{~mm}^{2}$ at 3.5 GHz , without degradation in performance, the proposed BLC only occupies $46 \%$ of the conventional design area. Figure 5 (a) shows the conventional BLC S-Parameter response. Compared to the conventional BLC [18], the proposed design is highly competitive, as shown in Table 2.


Fig. 6. Simulated and measured phase difference between port-2 and port-3.

Table 2: Results comparison of simulated conventional BLC design with proposed simulated and measured BLC Response at 3.5 GHz

| Parameter | Conventional <br> BLC <br> (Simulated) | Proposed <br> BLC Design <br> (Simulated) | Proposed <br> BLC Design <br> (Measured) |
| :---: | :---: | :---: | :---: |
| $S_{I I}$ | -24 dB | -19 dB | -12 dB |
| $S_{12}$ | -2.5 dB | -3.04 dB | -3.19 dB |
| $S_{I 3}$ | -3.6 dB | -3.17 dB | -2.8 dB |
| $S_{I 4}$ | -17 dB | -19 dB | -15.2 dB |
| Phase <br> Difference | $89^{\circ}$ | $90.9^{\circ}$ | $88^{\circ}$ |
| Bandwidth <br> @ $\boldsymbol{S}_{11}$ below <br> -10dB | 3.1 GHz to <br> 4.06 GHz | 2.87 GHz to <br> 4.17 GHz | 2.74 GHz to <br> 4.15 GHz |
| Size | 30 mm x |  |  |
| 41.7 mm | 21.8 mm x | 26.65 mm | 27 mm x |
| 27 mm |  |  |  |
| Percentage <br> Reduction |  |  |  |

A comparison between the simulated and measured results is shown in Fig. 5 (b) for the $S_{11}, S_{12}, S_{13}$ and $S_{14}$ magnitudes, and in Fig. 6 for the $S_{12}$ and $S_{13}$ phases, respectively. The proposed BLC shows good agreement between the simulated and measured results. The measured $S_{1 l}$ is better than -10 dB between 2.74-4.15 GHz frequency band, with a $40.2 \%$ relative bandwidth. The isolation loss $\left(S_{14}\right)$ is also better than -10 dB between 2.5 GHz to 4.1 GHz frequency band. Meanwhile, the performance of the insertion loss $\left(S_{12}\right)$ and the coupling $\left(S_{13}\right)$ are at -3.2 dB and -2.8 dB respectively, also lying in the same frequency range. So that the insertion and coupling LE is $\pm 0.2 \mathrm{~dB}$ with respect to the desired value. As depicted in Fig. 6, the measured phase of port-3 and port-2 at 3.5 GHz is $166.4^{\circ}\left(S_{13}\right)$ and $78.4^{\circ}\left(S_{12}\right)$ respectively, when port-1 is excited. The BLC has a measured phase difference of $88^{\circ}$ between Port-2 and Port-3 as per the Eq. (14). So, the phase error is $2^{\circ}$ with
respect to the desired value.
In the comparison between the simulated and measured results shown in Fig. 5 (b) and Fig. 6, there is a small shift in the centre frequency, but the range of frequency lies in the same frequency band between 2.74 GHz to 4.15 GHz , respectively. The dielectric loss tangent $(\tan \delta)$ of the simulated FR-4 substrate is around 0.025 , whereas for the fabricated FR-4 substrate based prototype is around 0.019 . Because of this variation in the lossy FR4 substrate properties [30], there is a shift in the centre frequency, coupling and insertion LE of 0.2 dB with respect to the standard value of 3 dB , and the phase difference error (PDE) of $2^{0}$. The dielectric constant of the FR-4 is also frequency-dependent and varies with the frequency. In the future, this frequency shift problem, LE, and the PDE can be improved by using a low-loss substrate.

A comparison of the proposed BLC with the previously published work related to BLC is summarized in Table 3. The researchers are attempting to either enhance the bandwidth or to minimize the size of the BLC in most of the design described in Table 3. The proposed work shows the bandwidth enhancement and minimize the size of the BLC together, which is considered as the main requirement for the design of 5 G system. Moreover, the LE and PDE are also very small as compared to the previous designs.

Table. 3: Comparison of Proposed BLC design with the existing designs available in the literature

| Operating <br> Freq. <br> (GHz) | Bandwidth <br> $\left(\boldsymbol{S}_{I I}=\mathbf{- 1 0 d B}\right)$ | Size <br> Reduction | Loss <br> Error <br> $(\mathbf{d B})$ | PD <br> Error <br> $($ Deg $)$ | Ref. / <br> Year |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{3 . 2 2}$ | $5 \%$ | $54.7 \%$ | $\pm 1 \mathrm{~dB}$ | $-3^{\circ}$ | $[36] /$ <br> 2013 |
| $\mathbf{3 . 5}$ | $6.57 \%$ | $67 \%$ | $\pm 1.7 \mathrm{~dB}$ | $-4^{\circ}$ | $[33] /$ <br> 2015 |
| $\mathbf{3 . 5}$ | $34 \%$ | $63 \%$ | $\pm 0.9 \mathrm{~dB}$ | $\pm 5^{\circ}$ | $[34] /$ <br> 2015 |
| $\mathbf{3 . 5}$ | $14.6 \%$ | $61.76 \%$ | $\pm 0.9 \mathrm{~dB}$ | $\approx 1^{\circ}$ | $[31] /$ <br> 2016 |
| $\mathbf{3 . 5}$ | $\approx 17 \%$ | $\approx 75 \%$ | $\pm 0.5 \mathrm{~dB}$ | $-1.7^{\circ}$ | $[37] /$ <br> 2016 |
| $\mathbf{3}$ | $13.3 \%$ | $55 \%$ | $\pm 0.3 \mathrm{~dB}$ | $1^{\circ}$ | $[32] /$ <br> 2018 |
| $\mathbf{3 . 8}$ | $15.7 \%$ | $40 \%$ | $\pm 0.5 \mathrm{~dB}$ | $-2.5^{\circ}$ | $[35] /$ <br> 2019 |
| $\mathbf{3 . 5}$ | $40.2 \%$ | $54 \%$ | $\pm 0.2 \mathrm{~dB}$ | $-2^{\circ}$ | $[$ This <br> work]/ <br> 2020 |

From Table 3, it is important to note that LE refers to the difference between the insertion loss/coupling loss with the desired value of -3 dB . Also, the PDE refers to the phase difference between port-3 and port-2 of the BLC with the desired value of $90^{\circ}$.


Fig. 7. Dispersion diagram of the proposed BLC.
Figure 7 shows the dispersion diagram of the proposed BLC. The dispersion diagram is plotted for the frequency and absolute value of $\beta$. The value of $\beta$ is calculated using the S-parameter value, and it clearly shows that high frequency supports the forward wave and low-frequency support backward wave. Hence, the proposed BLC is designed to work in the right-hand (RH) region from above 3.2 GHz and left-hand (LH) region from below 3.2 GHz . This indicates that the proposed BLC is a CRLH metamaterial TL [19],[38]. The main requirement for 5 G technology is wide bandwidth and compact size of the components [39]. The CRLH-TL exhibit a bandpass behavior between the RH and LH regions, which shows improved bandwidth, as shown in Fig. 7 and the compact size of the proposed BLC.

## V. CONCLUSION

A new BLC was proposed in this paper based on the interdigital capacitor unit-cell and open coupled lines technique that shows metamaterial TL properties. The simulated and measured results of the proposed BLC showed that the coupler has excellent S-parameter and phase difference performances at the desired operating frequency of 3.5 GHz . The BLC is operating at the frequency band of $2.74 \mathrm{GHz}-4.15 \mathrm{GHz}$. The proposed BLC design is then fabricated, and it demonstrates a high fractional bandwidth of up to $40.2 \%$. This article explained the compact and wide-band BLC has a good profile that is $54 \%$ less in size than conventional BLCs. The proposed BLC is potentially suitable to be used later in the butler-matrix based array antenna system for the 5G applications [40]. The structure can be integrated to form a compact and wideband butler-matrix for future array antenna beamforming systems.

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# Ultra-Low Loss and Flat Dispersion Circular Porous Core Photonic Crystal Fiber for Terahertz Waveguiding 

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#### Abstract

A novel design of circular porous core photonic crystal fiber (CCPCF) is proposed and studied for Terahertz propagation with an ultra-low-loss. The effective index, effective mode area, dispersion, material and bending losses of the suggested design are studied using full vectorial finite element method. The CCPCF with high cladding air filling factor and porous core exhibits ultra-low material absorption loss of $0.022 \mathrm{~cm}^{-1}$ at a frequency of 1.0 THz . Further, very low bending losses of $2.2 \times 10^{-18} \mathrm{~cm}^{-1}$ can be achieved for 1.0 cm bending radius at 1.0 THz with low confinement loss of $1.37 \times 10^{-5} \mathrm{~cm}^{-1}$. Additionally, an ultra-flat low dispersion of $0.61 \pm 0.035 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ can be obtained within the frequency range of $0.8-1.0 \mathrm{THz}$. Therefore, the reported CCPCF has a strong potential for transmission in the Terahertz regime.


Index Terms - Bending losses, dispersion, photonic crystal fiber, porous core, terahertz waveguiding.

## I. INTRODUCTION

Recently, THz photonic crystal fibers (PCFs) have granted a great interest in the scientific community because of their promising applications in telecommunication, medical science, spectroscopy, security, imaging, and sensing [1]-[7]. Currently, the THz waveguide is still under extensive research due to the material absorption losses. Additionally, there are some problems associated with transmitter-receiver alignment and atmosphere-dependent losses [8]. Therefore, long-distance communication in the THz regime is a big challenge. The PCF is suggested to overcome such high absorption losses. However, the solid core of PCF still absorbs an intolerable amount of
the transmitted signal energy. The porous core with air holes is then introduced to PCF to reduce the effective material loss (EML) [9]-[11]. In this context, Bao et al. [12] have proposed a honeycomb terahertz fiber with an absorption loss of $1.5 \mathrm{~cm}^{-1}$ at 1.0 THz . Additionally, a hexagonal porous core PCF with an absorption loss of $0.17 \mathrm{~cm}^{-1}$ has been presented with Teflon as a background material [13]. Further, a porous core photonic bandgap fiber was proposed with EML of 0.432 $\mathrm{cm}^{-1}$ and dispersion loss of $2.5 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ [14]. Kaijage et al. [15] have also suggested an octagonal PCF for terahertz wave guidance with an EML of $0.070 \mathrm{~cm}^{-1}$ at $\mathrm{f}=1.0 \mathrm{THz}$. In 2015, Islam et al. [16] have proposed a rotated hexagonal porous core single-mode fiber with an EML of $0.066 \mathrm{~cm}^{-1}$ and a core power fraction of $40 \%$. Additionally, a circular cladding with a circular porous core was introduced [17] with an absorption loss of 0.053 $\mathrm{cm}^{-1}$. A kagome lattice and hexagonal core were also reported [18] with EML of $0.034 \mathrm{~cm}^{-1}$, low confinement loss at 1.0 THz frequency, and near-zero flattened dispersion of $0.60 \pm 0.14 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ within the frequency range of $0.75-1.15 \mathrm{THz}$. However, the fabrication of kagome-structured fiber is more challenging than regular fiber with circular-shaped air holes. Islam et al.[19] have also presented a circular cladding with hexagonal porous core with an absorption loss of $0.053 \mathrm{~cm}^{-1}$. Furthermore, hexagonal cladding PCF with a circular porous core was proposed with an absorption loss of $0.057 \mathrm{~cm}^{-1}$ [20]. Using the same structure, Islam et al. [1] have reduced the absorption loss to $0.043 \mathrm{~cm}^{-1}$. An ultra-low loss hybrid porous core fiber was introduced for broadband applications [21] with small EML of $0.043 \mathrm{~cm}^{-1}$ and near-zero flattened dispersion properties.

In this paper, a novel design of PCF is reported for

Terahertz waveguiding with circular lattice and circular porous core. The geometrical parameters are studied to minimize the material, confinement and bending losses with a nearly zero and flat dispersion over the studied frequency range. The simulation results are obtained using the full vectorial finite element method (FVFEM) via the Comsol Multiphysics software package [22]. The suggested design has a very low absorption loss of $0.022 \mathrm{~cm}^{-1}$ at $\mathrm{f}=1.0 \mathrm{THz}$ with low confinement loss of $1.37 \times 10^{-5} \mathrm{~cm}^{-1}$. Further, the proposed PCF exhibits low bending loss and small flat dispersion of $0.61 \pm 0.035$ $\mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ within the frequency range $0.8-1.0 \mathrm{THz}$. The achieved EML and bending loss are smaller than those introduced in the literature [1], [15]-[17], [19], [20], [23]-[29]. Further, the guiding properties of the proposed PCF are better than the previously reported circular PCF with EML of $0.053 \mathrm{~cm}^{-1}$, confinement loss of $1 \times 10^{-2} \mathrm{~cm}^{-1}$, and dispersion of $1.23 \pm 0.09 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ [17].

The suggested PCF with low EML, confinement losses and bending losses can be used for telecommunication applications, medical spectroscopy and sensing purposes in the promising terahertz band. Additionally, the flat dispersion through the frequency range of $0.8-1.0 \mathrm{THz}$ will offer an undegraded signal at the receiver terminal. This will in turn influence the speed and bandwidth of the transmitted signal.

## II. DESIGN CONSIDERATIONS

The proposed CCPCF with the porous core is shown
in Fig. 1. The cladding region consists of 8 successive rings of air holes. The first ring contains 10 holes with a diameter (d) of $90 \mu \mathrm{~m}$. The total number of cladding air holes is 226 . The spacing between two successive rings is called hole pitch ( $\Lambda$ ) which depends directly on the cladding air filling factor (AFF). The AFF is the ratio between the hole diameter $(d)$ and the hole pitch ( $\Lambda$ ) which is kept constant at 0.96 to provide low effective material loss (EML). The second cladding ring holes' center is located at a distance of $\left(d_{s c}\right)$ which depends on the hole pitch ( $\Lambda$ ) and the core diameter ( $D_{\text {core }}$ ). Therefore, $\left(d_{s c}\right)$ could be changed automatically during the simulation with the core diameter and the hole pitch as well. In this study, $d_{\mathrm{sc}}=\left(D_{\text {core }} / 2\right)(1+\rho)+\Lambda$ where $(\rho)$ is a constant factor. Therefore, we could obtain an explicit quasi TE mode and quasi TM mode as well [30]. The core has a diameter $D_{\text {core }}$ of $460 \mu \mathrm{~m}$ and a total number of 37 holes whose diameter $\left(d_{c o}\right)$ depends directly on the porosity $(P)$ of the core region. The porosity is the ratio between the total area of the air holes in the core region to the total core area [31]. In this study, 3 successive rings are used with 6,12 , and 18 air holes with hole pitch ( $\Lambda_{\text {core }}$ ) of $66 \mu \mathrm{~m}$. Additionally, a diameter $d_{c o}=75.6 \mathrm{P} \mu \mathrm{m}$ is used for the reported design. The background material is TOPAS with refractive index $n=1.53$ and bulk material absorption loss $\alpha_{\text {mat }}=0.2 \mathrm{~cm}^{-1}$ at a frequency of 1.0 THz [15]. In this study, the effective index, EML, confinement losses, effective modal area, bending losses, fractional power, and dispersion of the quasi TE mode of the reported design are studied.


Fig. 1. Cross-section of the CCPCF with TOPAS as a background material.

## III. NUMERICAL TECHNIQUE

The modal analysis of the suggested CCPCF is made by the full vectorial finite-element method (FVFEM) [32], [33] via the Comsol Multiphysics 5.2 commercial software package [22]. Starting from Maxwell's equations, the magnetic field-based vector wave equation can be obtained as:

$$
\begin{equation*}
\nabla \times\left(\varepsilon^{-1} \nabla \times \mathbf{H}\right)-\omega^{2} \boldsymbol{\mu}_{0} \mathbf{H}=\mathbf{0} . \tag{1}
\end{equation*}
$$

where $\boldsymbol{\omega}$ is the angular frequency, and $\mu_{0}$ is the free space permeability. Further, $\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}_{\boldsymbol{0}} \boldsymbol{\varepsilon}_{\mathbf{r}}$ is the permittivity of the waveguide material where $\varepsilon_{0}$ is the free space permittivity and $\boldsymbol{\varepsilon}_{\mathbf{r}}$ is the relative permittivity of the waveguide material. The following eigenvalue equation can be obtained by applying the standard finite element method (FEM) procedure to equation (1):

$$
\begin{equation*}
[K]\{\mathbf{H}\}-\boldsymbol{\beta}^{2}[\mathbf{M}]\{\mathbf{H}\}=\{\mathbf{0}\} \tag{2}
\end{equation*}
$$

where [K] and [M] are the global stiffness and mass matrices, $[\mathrm{H}]$ is the global magnetic field vector, $\{\mathbf{0}\}$ is the null vector and $\boldsymbol{\beta}$ is the propagation constant. The eigenvalue equation can be solved to obtain the eigenvector $\mathbf{H}$ and the corresponding eigenvalue $\boldsymbol{\beta}$. Additionally, the effective index of the propagation mode is calculated from $\boldsymbol{\beta}$ since $N_{\text {eff }}=\boldsymbol{\beta} / \boldsymbol{k}$, where $\boldsymbol{k}$ is the free space wavenumber.

Through the modal analysis, a set of modes is calculated by the FVFEM and the dominant mode is defined as the mode with the highest real effective index value. Since $\boldsymbol{\nabla} . \mathbf{H}=\mathbf{0}$ and interface boundary conditions are automatically satisfied in the formulation, then there is no chance for spurious (nonphysical) modes to appear in the spectrum of the solution. The cross-section of the waveguide structure is discretized using the VFEM. In this study, a minimum mesh element size of $1.19 \mu \mathrm{~m}$ is taken with a total number of degrees of freedom of 640381, with 65748 total number of elements. Additionally, A circular-shaped perfectly-matched layer is employed to calculate the leakage loss of the studied modes. An Intel Core i5 computer processor at 2.5 GHz with 8.0 GB RAM and 64-bit operating system was utilized to run the software with an average runtime of 89 seconds per run.

## IV. RESULTS AND DISCUSSION

In order to calculate the confinement losses accurately, the PML thickness is studied. Figure 2 shows the confinement losses variation with the PML radius-tocladding radius ratio. It may be seen from this figure that the confinement losses of the quasi TE and TM modes are nearly constant at $1.37 \times 10^{-5} \mathrm{~cm}^{-1}$ and $3.65 \times 10^{-6} \mathrm{~cm}^{-1}$ at a ratio of 1.03 and $f=1.0 \mathrm{THz}$. Therefore, the PML radius is kept at 1.07 of the cladding radius throughout the whole study.

Figure 3 shows the frequency-dependent effective index $N_{\text {eff }}$ of the fundamental quasi TE and quasi TM modes of the CCPCF for three porosity cases $20 \%, 30 \%$, and $40 \%$ in the Terahertz range of $0.8-1.3 \mathrm{THz}$. It may be seen that $N_{\text {eff }}$ increases with increasing the frequency
while it decreases with the porosity of the core region. As the frequency increases, the confinement of the light through the core region increases which increases the effective index of the supported modes. Further, the leakage of the mode toward the cladding region increases by increasing the porosity percentage as may be seen in the field plot in Fig. 3. Therefore, the maximum porosity of $40 \%$ has been used. If the porosity is further increased, the distance between the adjacent holes becomes small which will be a challenge for the fabrication process. Additionally, the mode will have high leakage to the cladding region which increases the confinement losses. As a result, the effective index of the quasi TE and TM modes decreases by increasing the porosity percentage as shown in Fig. 4. Further, the effective index of the quasi TE mode at a specific porosity percentage is very close to that of the quasi TM mode due to the symmetry of the proposed design.


Fig. 2. Confinement loss of the quasi TE/TM mode versus the PML radius-to-cladding radius ratio.

The effective material loss (EML) $\boldsymbol{\alpha}_{e f f}$ could be considered as the main factor of the power dissipation in the THz band and can be calculated by the expression [34]:

$$
\begin{equation*}
\alpha_{e f f}=\frac{\sqrt{\varepsilon_{0} / \mu_{0}} \int_{\text {mat }} n_{\text {mat }}|E|^{2} \alpha_{\text {mat }} d A}{\left|\int_{\text {all }} S_{z} d A\right|} \tag{3}
\end{equation*}
$$

where $\boldsymbol{n}_{\text {mat }}$ is the material refractive index, $\boldsymbol{a}_{\text {mat }}$ is the bulk material absorption loss, $\varepsilon_{0}$ and $\mu_{0}$ are the permittivity and permeability of the free space, respectively, and $\boldsymbol{S}_{z}$ is the z-component of the pointing vector defined as $\mathbf{S}_{\mathbf{z}}=\mathbf{1} / \mathbf{2}(\mathbf{E} \times \mathbf{H})$.z. The EML of the proposed fiber for different porosity values $20 \%, 30 \%$, and $40 \%$ is shown in Fig. 5. It may be seen that the EML decreases by increasing the porosity percentage. The figure depicts that the EML values have a maximum of $0.031 \mathrm{~cm}^{-1}$ at a porosity of $20 \%$ at $f=1.3 \mathrm{THz}$ and a minimum of $0.021 \mathrm{~cm}^{-1}$ at a porosity of $40 \%$ at $f=0.8$

THz. At the operating frequency of 1.0 THz , the EML is equal to $0.022 \mathrm{~cm}^{-1}$ which is far better than those reported in Table 1. If a frequency of 0.8 THz is used, the EML will not exceed $\approx 0.021 \mathrm{~cm}^{-1}$. Therefore, the proposed design is highly qualified for many Terahertz applications. The high density of the cladding air holes and the porosity of the core region are responsible for achieving low EML. This is due to the reduced amount of the bulk material in the whole design, especially through the core region.


Fig. 3. The effective index of the fundamental quasi TE/TM mode versus the frequency at different porosity percentages. The inset shows the main electric field component of the quasi TM mode at different porosities.


Fig. 4. The effective index of the fundamental quasi TE and quasi TM modes versus the porosity percentage (\%).


Fig. 5. EML versus the operating frequency at different porosity percentages.

The confinement loss of the supported modes is also implemented as given by [35]-[37]:

$$
\begin{equation*}
L_{c}=8.686 K_{0} \operatorname{Im}\left(N_{e f f}\right) \tag{4}
\end{equation*}
$$

where $K_{0}$ is the propagation constant of free space and $\boldsymbol{I m}\left(\boldsymbol{N}_{\text {eff }}\right)$ is the imaginary part of the effective index $N_{\text {eff }}$ of the studied mode. Figure 6 shows the calculated confinement loss of the quasi TE mode supported by the proposed CCPCF versus the frequency at different porosity percentages. The mode field starts to leak out the core region by increasing the porosity percentage which increases the confinement losses. Additionally, as the frequency increases, the confinement of the mode through the core region increases. Further, the confinement losses are smaller than the material loss with a value of $1.37 \times 10^{-5} \mathrm{~cm}^{-1}$ at $\mathrm{f}=1.0 \mathrm{THz}$ and porosity of $40 \%$. Therefore, the confinement losses can be neglected if compared to the material loss. It may be seen that the proposed structure enjoys a low confinement loss due to the high AFF value of the cladding. Therefore, a high index contrast is obtained between the cladding and the core regions with better field confinement in the core region.

The effective modal area ( $\boldsymbol{A}_{\text {eff }}$ ) of the quasi TE mode of the proposed CCPCF is also analyzed through the frequency band of $0.8-1.3 \mathrm{THz}$. The $\boldsymbol{A}_{\text {eff }}$ could show the mode confinement through the core region and is given by [1], [10],[37]:

$$
\begin{equation*}
A_{e f f}=\frac{\left[\int I(r) r d r\right]^{2}}{\int I^{2}(r) r d r} \tag{5}
\end{equation*}
$$

where $\boldsymbol{I}(\boldsymbol{r})=\left|\boldsymbol{E}_{\boldsymbol{t}}\right|^{2}$ is the transverse electric field intensity distribution in the fiber cross-section. Figure 7 shows the variation of the frequency-dependent effective mode area of the quasi TE mode. It is evident from this figure that the effective mode area is proportional to the porosity percentage and is inversely proportional to the operating frequency. This is due to the good confinement
of the studied mode at high frequency and small porosity. The effective mode area at 1.0 THz and $40 \%$ porosity is equal to $1.22 \times 10^{5} \mu m^{2}$, which is very comparable to most of the reported values in Table 1.


Fig. 6. Frequency-dependent confinement loss of the quasi TE mode at different porosity percentages.


Fig. 7. Frequency-dependent effective mode area of the quasi TE mode at different porosity percentages.

The PCFs can also suffer from bending losses ( $\boldsymbol{\alpha}_{B L}$ ) which can lead to the distortion of the optical mode. The $\boldsymbol{\alpha}_{B L}$ can be expressed as [38]:

$$
\begin{equation*}
\alpha_{B L} \cong\left(\frac{\sqrt{\pi}}{8 A_{\text {eff }}}\right)\left(\frac{1}{\beta\left(\beta^{2}-\beta_{c l}^{2}\right)^{\frac{1}{4}}}\right)\left(\frac{\exp \left(-\frac{2}{3} R_{b}\left(\beta^{2}-\beta_{c l}^{2}\right)^{\frac{3}{2}} \boldsymbol{\beta}^{-2}\right)}{\sqrt{R_{b}\left(\beta^{2}-\beta_{c l}^{2}\right) \beta^{-2}+R_{c}}}\right) \tag{6}
\end{equation*}
$$

where $\beta$ is the propagation constant as $\beta=\mathbf{2 \pi} \boldsymbol{N}_{\text {eff }} / \boldsymbol{\lambda}, \boldsymbol{R}_{c}$ is the fiber core radius and $\boldsymbol{R}_{\boldsymbol{b}}$ is the bending radius. In this study, it is assumed that the cladding refractive
index is approximately unity due to the high filling factor of the cladding region. Further, $\boldsymbol{\beta}_{\boldsymbol{c l}}$ is calculated for each frequency as $\boldsymbol{\beta}_{\boldsymbol{c l}}=\mathbf{2} \boldsymbol{\pi} \times \mathbf{1} / \boldsymbol{\lambda}$. Figure 8 shows the frequency-dependent $\boldsymbol{\alpha}_{\boldsymbol{B L}}$ at different porosity percentages. It may be seen that the bending losses increase with increasing the porosity while it decreases with increasing the frequency. This is due to the good confinement of the light through the core region at high frequency. Higher bending loss of $1.96 \times 10^{-13} \mathrm{~cm}^{-1}$ is achieved at $f=0.8 \mathrm{THz}$ and porosity of $40 \%$. However, a small value of $1.0 \times 10^{-37} \mathrm{~cm}^{-1}$ is obtained at a frequency of 1.3 THz and a porosity of $20 \%$. The bending loss at $f=1.0 \mathrm{THz}$ is equal to $2.2 \times 10^{-18} \mathrm{~cm}^{-1}$ with a porosity of $40 \%$.


Fig. 8. Frequency-dependent bending loss of the quasi TE mode at different porosity percentages.

It is aimed to maximize the traveled power through the porous core region for efficient THz regime transmission. Therefore, we calculated the fractional propagated power through the proposed CCPCF using the expression [39], [40]:

$$
\begin{equation*}
\varphi=\frac{\int_{\text {core } e} S_{z} d A}{\int_{\text {all }} S_{z} d A} \tag{7}
\end{equation*}
$$

where the numerator represents the integration of the z-component of the pointing vector over the core holes only while the denominator shows the integration all over the whole design. Figure 9 shows the fractional power of the quasi TE mode through the proposed CCPCF versus the frequency at different porosity percentages. The figure shows that high fractional power of $41.7 \%$ is achieved at $f=1.3 \mathrm{THz}$ and porosity of $40 \%$. Further, a fractional power of $41.16 \%$ is obtained at $\mathrm{f}=1.0 \mathrm{THz}$ and porosity of $40 \%$, which is a relatively high ratio. It should be also noted that the proposed CCPCF provides a nearly constant fractional power at low porosity value ( $20 \%$ and $30 \%$ ) through the frequency
range of $0.8-1.3 \mathrm{THz}$ which increases accordingly with the porosity percentages.

Next, the dispersion characteristics of the proposed CCPCF are analyzed. It is worth noting that the refractive index of the TOPAS material is nearly constant in the frequency range of $0.1-1.5 \mathrm{THz}$ where the material dispersion can be totally neglected [12]. Then, the waveguide dispersion is only analyzed. The dispersion coefficient $\left(\boldsymbol{\beta}_{2}\right)$ can be calculated by [41]:

$$
\begin{equation*}
\beta_{2}=\frac{1}{c}\left(2 \frac{d N_{e f f}}{d \omega}+\omega \frac{d^{2} N_{e f f}}{d \omega^{2}}\right) \tag{8}
\end{equation*}
$$

where $\boldsymbol{\omega}=\mathbf{2} \boldsymbol{\pi} \boldsymbol{f}$ is the radial frequency.


Fig. 9. The fraction of power percentage of the quasi TE mode through the core holes versus the frequency at different porosity percentages.


Fig. 10. The effective index $N_{\text {eff }}$ of the quasi TE mode versus the radial frequency.

Figure 10 shows the relationship between $\boldsymbol{N}_{\text {eff }}$ and $\boldsymbol{\omega}$ for three different porosity percentages as derived using the least square method (LSM) with a polynomial of the fourth order. The three equations have been mathematically differentiated and applied to the formula in Eq. (8). Figure 11 shows the frequency-dependent dispersion of the quasi TE mode of the proposed CCPCF for the three porosity cases. The average dispersion in the range of $0.8-1.3 \mathrm{THz}$ is $0.15 \pm 0.13 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ for $20 \%$ porosity, $0.12 \pm 0.05 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ for $30 \%$ porosity and $0.97 \pm 0.39 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ for $40 \%$ porosity. It may be also seen from Fig. 11 that at $40 \%$ porosity, the maximum value of dispersion is $1.36 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ at 1.3 THz and the minimum value is $0.58 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ at 0.9 THz (both are very small). It should be also noted that the proposed CCPCF has a nearly zero flat dispersion in the frequency range of $0.8-1.0 \mathrm{THz}$ with an average value of $0.61 \pm 0.035 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ at $40 \%$ porosity. Such a very small dispersion is obtained over a wide range of frequencies which is very comparable to those reported in Table. 1. The calculated dispersion is directly related to the effective index $N_{\text {eff }}$ of the supported modes. Such modes are confined through the porous core and the innermost cladding air holes. Therefore, the $N_{\text {eff }}$ and hence the dispersion can be controlled by the geometrical parameters of the PCF.


Fig. 11. The frequency dependent dispersion $\boldsymbol{\beta}_{2}$ at different porosity percentages.

The proposed design can be fabricated using the most common stack-and-draw method [42]. Our proposed CCPCF is assumed to be simple-to-stack as the whole structure enjoys a high degree of symmetry around the center of the core with relatively low core porosity percentages. Therefore, the stacking process will be easy and will provide a high degree of accuracy by the end of the drawing process.

A comparison between the proposed CCPCF and the previously-published PCF for THz guiding is shown in Table 1. It is revealed from this table that the proposed CCPCF has better comprehensive characteristics than those reported previously. The suggested design has EML of $0.022 \mathrm{~cm}^{-1}$ at $f=1.0 \mathrm{THz}$ while the other PCFs have EML within the range from 0.034 to $0.070 \mathrm{~cm}^{-1}$. Further, the confinement loss of the CCPCF of $1.37 \times 10^{-5} \mathrm{~cm}^{-1}$ which is very comparable to all of the mentioned papers including [23], [24], and [25]. It is also noticed that the obtained effective mode area has a value of $1.22 \times 10^{5} \mu \mathrm{~m}^{2}$ which is also very comparable to the previous results. The bending loss of the suggested design is only $0.2 \%$ of the minimum achieved result [20] as shown in Table 1. The fractional power is also comparable to the other values [27] with a nearly constant value over the studied frequency range of $0.8-1.3 \mathrm{THz}$ at porosity percentages of $20 \%$ and $30 \%$. Further, an ultra-flattened nearly zero dispersion of $(0.61 \pm 0.035 \mathrm{Ps} / \mathrm{THz} / \mathrm{cm})$ is achieved over a frequency range from 0.8 to 1.0 THz which is very comparable to the minimum dispersion of $(0.14 \pm$ $0.07 \mathrm{Ps} / \mathrm{THz} / \mathrm{cm}$ ) reported in [23]. Therefore, the proposed design would be an efficient transmission structure for the THz regime. It may be noted that the suggested
design has overall better guiding characteristics at the same time than those reported in the literature as shown in Table 1. However, short manufacture length and high price are the main disadvantages of using PCF as transmission media for telecommunications in THz regime. Additionally, the coupling of PCF with with other waveguides and devices is not straight forward.

## V. CONCLUSION

In this paper, we proposed a CCPCF for the THz regime with an ultra-low EML of $0.022 \mathrm{~cm}^{-1}$, low confinement losses of $1.37 \times 10^{-5} \mathrm{~cm}^{-1}$ and low bending losses of $2.2 \times 10^{-18} \mathrm{~cm}^{-1}$ at 1.0 THz . Further, a wide-band flat low dispersion of $0.61 \pm 0.035 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$ is achieved within the frequency range of $0.8-1.0 \mathrm{THz}$. The structure has almost a constant fractional power for each porosity percentage through the frequency range of $0.8-1.3 \mathrm{THz}$ with a comparable value of $41.16 \%$ at 1.0 THz . The reported CCPCF has a superior guiding mechanism than the previous circular PCF [17] with EML of $0.053 \mathrm{~cm}^{-1}$, a confinement loss of $1 \times 10^{-2} \mathrm{~cm}^{-1}$, and dispersion of $1.23 \pm 0.09 \mathrm{ps} / \mathrm{THz} / \mathrm{cm}$. Therefore, the reported CCPCF has a strong potential for transmission in the Terahertz regime.

Table 1: A comparison between the CCPCF and the previously published papers

| Structure | Year | EML $\left(\mathrm{cm}^{-1}\right)$ at $f=1.0 \mathrm{THz}$ | Confinement Losses ( $\mathrm{cm}^{-1}$ ) at $f=1.0 \mathrm{THz}$ | $\begin{gathered} \text { Effective Mode } \\ \text { Area }\left(\mu m^{2}\right) \text { at } \\ f=1.0 \mathrm{THz} \\ \hline \end{gathered}$ | BendingLosses $\left(\mathrm{cm}^{-1}\right)$ <br> at $f=1.0 \mathrm{THz}$ | Fraction of Power (\%) at $f=1.0 \mathrm{THz}$ | Dispersion ( $\mathrm{Ps} / \mathrm{THz} / \mathrm{cm}$ ) | Flat Dispersion Bandwidth (THz) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CCPCF |  | 0.022 | $1.37 \times 10^{-5}$ | $1.22 \times 10^{5}$ | $2.2 \times 10^{-18}$ | 41.16 | $0.61 \pm 0.035$ | 0.8-1.0 |
| [23] | 2019 | 0.062 | $4.7 \times 10^{-7}$ | ------------ | ------------ |  | $0.14 \pm 0.07$ | 0.6-0.85 |
| [26] | 2019 | 0.07 | $1.66 \times 10^{-3}$ | $9 \times 10^{4}$ | $2.61 \times 10^{-14}$ | ----------- | $1.2 \pm 0.32$ | 1.1-1.9 |
| [27] | 2019 | 0.04 | $1 \times 10^{-4}$ | $1.65 \times 10^{5}$ | ------------- | 40 | $0.98 \pm 0.09$ | 1.0-2.0 |
| [24] | 2018 | 0.065 | $3.8 \times 10^{-9}$ | $1.8 \times 10^{5}$ | ------------ |  | $1.3 \pm 0.03$ | 1.0-1.8 |
| [25] | 2018 | 0.05 | $1 \times 10^{-9}$ | ------------- | ------------- | ----------- | $0.53 \pm 0.07$ | 0.5-1.48 |
| [1] | 2017 | 0.043 | $1 \times 10^{-2.5}$ | $2.1 \times 10^{5}$ | ------------ | 47 | $1.14 \pm 0.09$ | 1.0-1.3 |
| [20] | 2017 | 0.057 | $6.9 \times 10^{-3}$ | $1.1 \times 10^{5}$ | $1.07 \times 10^{-15}$ | 42 | $1.3 \pm 0.55$ | 0.7-1.2 |
| [28] | 2017 | 0.034 | $1 \times 10^{-3.7}$ | $6 \times 10^{5}$ | ------------ | 44 | $0.94 \pm 0.09$ | 0.7-1.3 |
| [19] | 2016 | 0.053 | $6.3 \times 10^{-4}$ | ------------- |  | 47 | $1.2 \pm 0.25$ | 1.0-1.55 |
| [17] | 2015 | 0.053 | $1 \times 10^{-2}$ | ------------- | ------------- | 46 | $1.23 \pm 0.09$ | 0.9-1.3 |
| [16] | 2015 | 0.066 | $4.73 \times 10^{-4}$ | ------------ | $\approx 1.310^{-12}$ | 40 | $1.06 \pm 0.12$ | 0.5-1.08 |
| [15] | 2013 | 0.070 | $1 \times 10^{-1}$ | $1.5 \times 10^{5}$ | $1.2 \times 10^{-9}$ | ----------- | -------------- | ----------- |
| [29] | 2011 | 0.042 | ------ | $1.04 \times 10^{5}$ | ------ | 50 | $\begin{array}{\|l\|} \hline \approx 3 \text { with } 0.0373 \\ \mathrm{ps} \mathrm{~m}^{-1} \mu \mathrm{~m}^{-1} \mathrm{SD} \\ \hline \end{array}$ | ----- |

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# Fast Rise-Time Electromagnetic Pulse Protection Characteristics of $\mathbf{Z n O}$ Varistors 

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#### Abstract

In order to study the response of ZnO varistors under the radiation of fast rise-time electromagnetic pulse, an experiment system is built composed of square wave pulse source, coaxial cable, coaxial fixture, attenuator, oscilloscope and insulating gas vessel. Electromagnetic pulse protection characteristics of ZnO varistors are tested and analyzed. Results show that: appearance of negative pulse in the responsive waveform means the completion of field-induced insulatorconductor phase transition for ZnO varistors. Peak value of responsive positive pulse decreases after phase transition, and the residual voltage is generally constant for different pulse strength and widths. The phenomenon of overshoot voltage is observed. Negative pulse is caused by the reflection of electromagnetic wave after the phase transition. The resistance of ZnO after phase transition is less than $50 \Omega$ and decreases linearly along with the increase of incident voltage or field, which leads to a highly nonlinear current-voltage characteristic of ZnO varistors in the radiating electromagnetic environment. Sum of energy of positive and negative pulse keeps constant, which indicates that the weakened positive pulse is converted into negative pulse. So impact of negative pulse needs to be taken into consideration when ZnO varistors are used to protection against strong electromagnetic pulse.


Index Terms - Coaxial fixture, fast rise-time electromagnetic pulse, grain boundary theory, insulatorconductor phase transition, nonlinear current-voltage characteristic, quantum tunnelling effect, ZnO varistor.

## I. INTRODUCTION

Traditional materials of electromagnetic shielding, such as metal sheet or fiber [1], dope [2], fabric [3, 4],
have constant electrical properties and shielding effectiveness. So they don't distinguish useful but weak signals from purposive strong electromagnetic disturbance, which are both shielded. Therefore they are failed to solve the contradiction between strong filed protection and normal operation of equipment. Alternatively, materials such as Zener diodes [5-7], junction field-effect transistors (FET) [8-10], heterogeneous composites filled with conductive particles [11, 12], ZnO -based [13, 14] or $\mathrm{VO}_{2}$-based [15, 16] metal-oxides and so on, have changeable essential electromagnetic parameters and this change is adaptive and reversible. This type of materials will exhibit highly nonlinear current-voltage resistance switching characteristics or field-induced insulator-conductor phase transition, so they are expected to be used as repeatable transient suppression devices against the effect of strong electromagnetic pulse while some of them are already be used to absorb the voltage surges for circuit protection like metal ZnO arrester (MOA).

For the purpose of protection from fast rise-time electromagnetic pulses like nuclear electromagnetic pulse (NEMP), FET and diodes are unsuited because of their relatively slow response speed. According to IEC61000-2-9 [17], the rise time of NEMP lies in $1.8 \mathrm{~ns} \sim 2.8 \mathrm{~ns}$, so response time must be less than that values otherwise the peak of NEMP pulse is hard to be rejected. Meanwhile, the materials with the micro-structure of "conducting grains surrounded by thin insulating barriers" like heterogeneous composites filled with conductive particles and some kinds of metal-oxides, can satisfy the requirement of response time and endure the high incident energy, cause that their electrical properties are determined by the theory of grain boundary or quantum tunneling process [18], which is inherently fast as to the
scale of $\sim 10^{-15} \mathrm{~s}$. So these materials are widely applied in the field of integrated circuit, non-volatile resistive switching memories, sensors and so on [19]. One representative of these materials is ZnO -based metaloxide varistors which are ceramic semiconductor devices and most used commercially for the protection of power or other low-frequency circuits against lightning or switching surges [20].

However, ZnO varistors are mostly analyzed and applied in the field of circuit protection. Research on the application of ZnO in electric field protection is inadequate, especially for fast rise-time electromagnetic pulse field. In some certain literatures about the response of ZnO against pulses [21-24], ZnO varistors are generally inserted in the circuits as circuit elements, based on the fact that ZnO can absorb the pulse voltage energy to make over-voltage dismiss or decrease rather than to shield pulse using the property of field-induced insulator-conductor phase transition. The former is injecting voltage or current pulses into ZnO varistors, and the latter means electromagnetic pulses are radiated onto ZnO varistors directly.

In this paper we build an experiment system to test the response of ZnO varistors for shielding fast rise-time electromagnetic pulses. A coaxial fixture with ZnO varistor fitted in is designed and manufactured to be an element of the system in order to make ZnO varistor exposed into the radiating fast rise-time electromagnetic pulses environment through coaxial lines. The results are compared with that of [21]. Some differences are found between responsive waveforms by conducting over-voltage pulses in [21] and waveforms by radiating electromagnetic pulses in our work.

## II. EXPERIMENTAL PROCEDURES

## A. Experiment system

The configuration of coaxial fixture is shown schematically in Fig. 1 and photographically in Fig. 2. The whole fixture is composed of outside conductor (marked by 1 in Fig. 1) and inside conductor (marked by 2 in Fig. 1). The outside and inside conductors both conclude four components connected with each other through thread. The insulating supports (marked by 3 in Fig. 1) are made by Teflon to isolate the conductors and have been grooved annularly to compensate discontinuous capacitance. Due to the reason that the coaxial fixture would be used to propagate very strong electromagnetic pulse and discharge must be refused, the air inlet and outlet (marked by 4 in Fig. 1) are designed to inject and outflow the insulating gas. And also the four zones marked by 5 in Fig. 1 are all chamfered to avoid discharge at sharp points. ZnO varistor with annular shape (diameter not bigger than 43 mm ) and certain thickness ( $1 \mathrm{~mm} \sim 5$
mm ) should be placed in the middle of fixture just like that in Fig. 2. According to (1) and (2) [25], the diameters of outside and inside conductors of coaxial fixture are set as 13 mm and 5.65 mm respectively, which makes the characteristic resistance as $50 \Omega$ matched and the theoretical working frequency range from DC to 10.23 GHz:

$$
\begin{align*}
f_{\mathrm{c}} & =\frac{2 c_{0}}{\pi\left(R_{1}+R_{2}\right)}  \tag{1}\\
Z_{0} & =\frac{60}{\sqrt{\varepsilon_{\mathrm{r}}}} \ln \left(\frac{R_{2}}{R_{1}}\right) \tag{2}
\end{align*}
$$

where $f_{\mathrm{c}}$ is the cut-off frequency, $c_{0}$ is the velocity of light, $R_{1}$ and $R_{2}$ indicate respectively the diameters of inside and outside conductor, $Z_{0}$ means the characteristic resistance of coaxial line and $\varepsilon_{\mathrm{r}}$ is the relative dielectric constant of air as 1 .

Based on the schematic diagram of Fig. 3, and using the coaxial fixture to clamp the ZnO varistor, the experiment system is built by more other elements like NOISEKEN INS-4040 noise generator, coaxial cables, attenuators, TEK TDS7404B oscilloscope and insulating gas (SF6) vessel as shown in Fig. 4. The NOISEKEN INS-4040 noise generator is used as square wave pulse source and can produce a variety of pulse width such as 50 ns up to $1 \mu \mathrm{~s}$ with the peak value of strength from 10 V up to 4 kV ideally and the rise time from 500 ps to 1 ns, which makes the max value of pulse field strength at the place of fitting materials in the coaxial fixture as almost $1.70 \mathrm{MV} / \mathrm{m}$ theoretically calculated by (3) [26]:

$$
\begin{equation*}
|\boldsymbol{E}|_{\max }=\frac{2 U_{0}}{R_{1} \ln \left(\frac{R_{2}}{R_{1}}\right)}, \tag{3}
\end{equation*}
$$

where $U_{0}$ is the output peak value of square wave pulse source. Although this value is less than the air breakdown field strength $3 \mathrm{MV} / \mathrm{m}$, we still have found the discharge phenomenon in the coaxial fixture sometimes in the experiment process. So insulating gas such as SF6 is adopted as the standby. The whole system has the matching resistance of $50 \Omega$ for consistent.


Fig. 1. Schematic diagram of the coaxial fixture.


Fig. 2. Photograph of the coaxial fixture.


Fig. 3. Schematic diagram of experiment system.


Fig. 4. Photograph of experiment system.

## B. Analyses of propagation performance of system

Good propagation performance should be satisfied for avoiding wave distortion, which can be reflected by S11 parameters of system. So S11 curves were obtained through vector network analyzer (VNA) in the frequency range of 300 kHz to 1.5 GHz and 10 MHz to 10.23 GHz respectively, as shown in Fig. 5 and Fig. 6. We can see that S11 of this coaxial fixture is less than -10 dB and the standing-wave ratio (SWR) will be less than 2 in the frequency range of 300 kHz to 7.36 GHz , which is enough to ensure the well transmission performance of fast rise-time pulse in this paper. Propagation performance was proved from the output waveforms of system with
no material tested shown in Fig. 7, which demonstrates good shape of $1 \mu \mathrm{~s}$ square waveforms. $U$ represents the waveforms displayed by oscilloscope. The discrepancy in the frequency range of 7.36 GHz to theoretic 10.23 GHz maybe comes from the machining error, which can be ignored for that most of pulse energy are widely distributed throughout low frequencies.


Fig. 5. S11 of coaxial fixture (partially shown in 300 $\mathrm{KHz} \sim 1.5 \mathrm{GHz}$ ).


Fig. 6. S11 of coaxial fixture (partially shown in 10 MHz $\sim 10.23 \mathrm{GHz}$ ).


Fig. 7. Output waveforms of system without ZnO varistor fitted in.

## C. Analyses of output error of system

Output error of system must be eliminated to assure the accuracy of experiment results. Fig. 8 shows the errors between input and output waveforms. $U_{\mathrm{i}}$ represents the ideal value of input voltage and $U_{\mathrm{o}}$ is the average peak value of output pulses of system in five times triggering. The black solid line with block symbols describes average output peak value of square wave pulse source. The blue dashed line with block symbols describes average error between practical values and ideal values. The black solid line with circular symbols describes the average output peak value of system under the situation that the coaxial fixture is connected with the pulse source and no ZnO varistor is fitted in. The blue dashed line describes average error between practical values and ideal values. It is obviously that errors introduced by the coaxial fixture are very little and can be dismissed. Good propagation performance has been satisfied, so errors introduced by coaxial cables and attenuators can be dismissed. Most of the errors come from the pulse source itself. As the practical output peak value of square wave pulse source is 3.621 kV , the max value of pulse field strength at the place of fitting materials in the coaxial fixture has to be modified from $1.70 \mathrm{MV} / \mathrm{m}$ to 1.54 MV/m.


Fig. 8. Average output value and error of square pulse source and coaxial fixture respectively.

## D. Analyses of experiment results

A type of ZnO varistors with the diameter as 34 mm and thickness as 3 mm , which are bought as commercial products with a standard sensitive voltage as 700 V in circuits, were fitted in the coaxial fixture and tested by this experiment system. Before testing, the silver plated on both sides of ZnO have been erased, as the silver layer would cause the electromagnetic wave to be completely reflected. The responsive pulse waveforms and exciting pulse waveforms are both shown in Fig. 9 with pulse width as $1 \mu \mathrm{~s}$ and a series of input pulse voltages. As we can see from the figures, the output responsive pulse is same as the input exciting pulse when the input voltage is less than 2 kV mainly. This result means ZnO varistor is still a good insulator and doesn't affect the propagation
of electromagnetic pulse before the input voltage comes bigger than 2 kV . When the input voltage increases over 2 kV mainly (the max electric field in the coaxial fixture is correspondingly almost $850 \mathrm{kV} / \mathrm{m}$ ), the peak value of output responsive pulse becomes lower than input exciting pulse and negative pulse occurs. As the input voltage increases, the negative pulse increases too. This phenomenon suggests that ZnO varistor has change to be a conductor with certain resistance, which will be explained in detail by Section III of this paper. Under the condition that insulating gas has been injected to avoid discharging along the face of material, the responsive pulse can be observed repeatedly and similarly, which means ZnO varistor maintains the stable performance and is not broken down by strong electric field during the process of experiment. So conclusion can be gain as that appearance of negative pulse in the responsive waveforms indicates that field-induced insulatorconductor phase transition has happened simultaneously.


(d) $U_{\mathrm{i}}=4 \mathrm{kV}$

Fig. 9. Comparison of exciting pulse waveforms and responsive pulse waveforms in a series of input voltages.

In contrast with the waveforms of input square pulses, there are three changes happened for the responsive waveforms: 1) the peak value of pulse decreases after phase transition; 2) the overshoot voltage exists with the phase transition; 3) the negative pulse emerges and increases as the input increases. Correspondingly, for the purpose of revealing the changing rules, we have observed three aspects of results which are 1) the voltage drops $\Delta U$ (shown in Fig. 9) from $U_{\mathrm{o}}$ to $U_{\mathrm{r}}$, which is the residual voltage after phase transition; 2) the overshoot voltage $\delta U$ (shown in Fig. 9) from the peak value of responsive pulse $U_{\mathrm{p}}$ to $U_{\mathrm{r}}$; and 3) the radio distribution of positive pulse energy and negative pulse energy to the whole pulse energy respectively. Relevant values are listed by Table 1, from which we can find that: $U_{\mathrm{r}}$ is a basically constant value as almost $1.7 \mathrm{kV}, \Delta U$ and $\delta U$ increases linearly as input voltage increases. Their respective radio distributions are shown in Fig. 10.

For the first aspect: $\Delta U$, based on Table I and black solid line with block symbols in Fig. 10, we can know that the capability of ZnO to suppress the peak of radiated strong electromagnetic pulse is approximately enhanced linearly as the input voltage increases. The second one: overshoot voltage $\delta U$, whose change is shown as red dashed line with circular symbols in Fig. 10, has relationship with the mechanism of phase transition of ZnO and will be discussed in the next section. The last one is demonstrated by Fig. 11, in which energy is calculated with pulse amplitude multiplied by time. We can see that the positive pulse energy decreases and the negative pulse energy increases as the input pulse voltage increases. However, the sum of energy of positive and negative pulses is found to be a constant approximately. This fact means the suppression of input positive pulse is transferred to the increment of negative pulse by ZnO , which need to be taken sincerely into consideration when ZnO is used to protect against fast rise-time electromagnetic pulse.

Table 1: Changes of voltage in the field-induced insulator-conductor phase transition (all in kV )

| $U_{\mathrm{i}}$ | $U_{\mathrm{o}}$ | $U_{\mathrm{p}}$ | $U_{\mathrm{r}}$ | $\Delta U$ | $\delta U$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1.835 | 1.843 | 1.655 | 0.21 | 0.188 |
| 2.5 | 2.28 | 2.138 | 1.753 | 0.527 | 0.385 |
| 3 | 2.723 | 2.304 | 1.761 | 0.962 | 0.543 |
| 3.5 | 3.169 | 2.59 | 1.763 | 1.406 | 0.827 |
| 4 | 3.621 | 2.635 | 1.772 | 1.849 | 0.863 |



Fig. 10. The radio distribution of $\Delta U$ and $\delta U$.


Fig. 11. The radio distribution of positive pulse energy and negative pulse energy to the whole pulse energy respectively.

## III. DISCUSSION

For protection of fast rise-time electromagnetic pulse, the electric field threshold of phase transition and the responsive time are two most important indicators that we are concerned about. In the above experiments, the practical voltage threshold of phase transition is almost 1.835 kV (the ideal value is 2 kV ), which means the electric field threshold is almost $780 \mathrm{kV} / \mathrm{m}$ calculated by (3), for the input pulses with $1 \mu \mathrm{~s}$ width. Father experiments indicate that the threshold of field strength is fixed to this value even though different pulse widths were injected. This is because in the electric field conduction region of ZnO nonlinear resistance (that is, the current-voltage characteristic region with high
resistance to low resistance phase transition), the tunneling effect to generate tunnel current is related to the electric field or voltage applied on the grain boundary layer, and have nothing to do with pulse width [18]. Moreover, the quantum tunneling effect is basically in femtosecond, so under the action of electromagnetic pulse with fast risetime in nanosecond, ZnO can respond in time when the electric field reaches the threshold. This has been proved by the fact that there is no delay between the response waveforms and the input excitation waveforms in all experimental results. The responsive speed is same as that in [21], which is less than or equal to 500 ps. From the above analyses, we can get a conclusion that even though this type of ZnO varistors can meet the requirement of response speed, they may be not suitable for NEMP protection due to their high threshold of electric field strength, which is much bigger than $50 \mathrm{kV} / \mathrm{m}$. According to previous research, the threshold electric field of phase transition has a positive correlation to the thickness of ZnO . Obviously the ZnO varistors we test are too thick to be used as NEMP shielding materials.

Same as the responsive pulses in [21], there exists a constant residual voltage $U_{\mathrm{r}}$ with an overshoot voltage $\delta U$ in our work. Figure 12 shows that the residual voltage $U_{\mathrm{r}}$ is generally constant even for different pulse widths. $U_{\mathrm{r}}$ represents the shielding capability of ZnO . In the process of experiments, the fact that $U_{\mathrm{r}}$ doesn't change with the increase of input electric field has proved that the shielding effectiveness of ZnO is enhanced and adaptable along with the change of incident field. Note that no unified standard about the shielding effectiveness for electromagnetic pulse in time domain has been defined [27, 28], so this paper would not give the exact value of it for ZnO varistors. In addition, the phenomenon of overshoot voltage in ZnO test can also be observed in the circuit experiments. There is no unified view on the cause of its formation in the academic world. An opinion generally accepted is that: when the applied voltage rises, the transition from capacitive current to resistive current passing through ZnO plate needs a certain time delay, which leads to an overshoot in the pulse waveform [18].

However, different from the responsive pulses obtained by injecting fast square wave voltage into ZnO in [21], in which only the peak of pulses decreases and the waveform does not change, for the response pulses of ZnO irradiated by fast square electromagnetic pulse, not only the peak deceases, but also the negative pulse emerges. In order to verify the reason of the negative pulse, a 20 m coaxial transmission line was adopted to substitute the original one (almost 0.2 m ) to propagate the pulse. From the result shown by Fig. 13, we can see that the part of negative pulse has a time delay from the part of positive pulse, and the time delay is just the propagation time of electromagnetic pulse forward and backward along with the 20 m transmission line ( time
delay $\Delta t=2 \times 20 \mathrm{~m} / v ; v$ is the velocity of electromagnetic wave in the coaxial line and $v=c / \sqrt{\varepsilon_{r}}$ [29]; $c$ is the velocity of light in the vacuum; $\varepsilon_{r}$ is the permittivity of insulating medium in the coaxial line and $\varepsilon_{r} \approx 2.5$ ). So, the negative pulse indicates that ZnO varistors has occurred high-to-low resistance switching phase transition. It is considered to be a reflected signal generated by the mismatch between the low resistance of ZnO and the system impedance $50 \Omega$. According to the transmission line theory [26], the low resistance of ZnO after phase transition can be calculated by below equations:

$$
\begin{gather*}
\Gamma=\frac{Z_{\mathrm{L}}-Z_{0}}{Z_{\mathrm{L}}+Z_{0}}=\frac{U_{\mathrm{p}}^{\prime}}{U_{\mathrm{p}}}  \tag{4}\\
Z_{\mathrm{L}}=Z_{0} \frac{1+\Gamma}{1-\Gamma} \tag{5}
\end{gather*}
$$

where $U_{\mathrm{p}}$ is the peak value of positive pulse and $U_{\mathrm{p}}^{\prime}$ is the peak value of negative pulse; $\Gamma$ is the reflection coefficient in the end of transmission line; $Z_{0}$ is the equivalent impedance of coaxial line and $Z_{0}=50 \Omega ; Z_{L}$ is the equivalent impedance of ZnO varistors. Note that for the purpose of simplifying calculation, here the dispersive capacitances of transmission line and ZnO varistors are dismissed and all the impedance is seemed as pure resistance. The calculated results are listed in Table 2, from which we can see that: the resistance of ZnO after phase transition is less than $50 \Omega$ and decreases linearly along with the increase of incident voltage or field. Though the current of coaxial transmission line cannot be generally measured directly and is also not easily calculated by the equation in [21] due to the emergence of negative pulse, it is still clear to conclude that the current-voltage characteristic of ZnO varistors in the radiating electromagnetic environment is nonlinear based on the relationship of current, voltage and resistance, which agrees with the conductive mechanism of ZnO based on double Schottky barrier model [18].


Fig. 12. The responsive waveforms of ZnO varistor with the 2.5 kV voltage but different pulse widths inputted.


Fig. 13. The responsive waveforms of ZnO varistor with the 3 kV voltage and 150 ns pulse width inputted using the coaxial cable with different lengths.

Table 2: Impedance calculation of ZnO varistors (voltage in kV , impedance in $\Omega$ )

| $U_{\mathrm{i}}$ | $U_{\mathrm{p}}$ | $U_{\mathrm{p}}^{\prime}$ | $\Gamma$ | $Z_{\mathrm{L}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 1.843 | -0.09 | -0.049 | 45 |
| 2.5 | 2.138 | -0.41 | -0.192 | 34 |
| 3 | 2.304 | -0.792 | -0.344 | 24 |
| 3.5 | 2.59 | -1.155 | -0.446 | 19 |
| 4 | 2.635 | -1.425 | -0.541 | 15 |

## IV. CONCLUSION

In this paper, an experimental system to test the response of ZnO for shielding fast rise-time electromagnetic pulses is built by using a coaxial fixture with gas insulation design. The electromagnetic pulse protection characteristics of ZnO varistors are measured and analyzed. The results show that:

1) The coaxial fixture has good transmission performance, and the actual S11 parameter is less than -10 dB in the range of 300 kHz to 7.36 GHz , introducing a very small test error to the experiment system. Analyses of output error shows that most of the errors come from the pulse source itself.
2) The insulator-conductor phase transition occurs when the 1.835 kV pulse is irradiated onto the surface of ZnO varistor, which means the threshold of electric field strength is $780 \mathrm{KV} / \mathrm{m}$ approximately. A negative pulse waveform appears after phase transition, which is different from that in the past studies when ZnO is injected with an overvoltage pulse.
3) There is no delay between the responsive waveforms and the input excitation waveforms, which indicates that the responsive speed of ZnO is less than or equal to 500 ps , so ZnO can respond in time when the electric field reaches the threshold.
4) The peak value of positive pulse decreases after phase transition, and the residual voltage is generally constant for different pulse strength and widths, which
has proved that the shielding effectiveness of ZnO is enhanced and adaptable along with the increase of incident field. The phenomenon of overshoot voltage is observed, and overshoot voltage increases linearly as the input radiating pulse field strength increases.
5) The appearance of negative pulse in the waveform means the completion of field-induced insulatorconductor phase transition. Negative pulse is caused by the reflection of electromagnetic wave after the phase transition. The resistance of ZnO after phase transition is less than $50 \Omega$ and decreases linearly along with the increase of incident voltage or field, which leads to the highly nonlinear current-voltage characteristic of ZnO varistors in the radiating electromagnetic environment.
6) Sum of energy of positive and negative pulses keeps constant, which means that the weakened positive pulses are converted into negative pulses. So impact of the negative pulse needs to be taken into consideration when ZnO varistors are used to protection against strong electromagnetic pulse.

In conclusion, there are some similarities and some differences between the results obtained by injecting fast square wave voltage into ZnO in previous works and that obtained by radiating square wave pulse onto ZnO surface in our work. This may possess certain reference value for solving the problem of electromagnetic pulse defense. What need to be added is that: due to the similarity in frequency distribution and fast rise time between square wave and NEMP, we use square waveform pulse source to substitute NEMP generator. Continuous wave usually lacks enough strength to induce phase transition of ZnO . The response for pulse with other waveforms can be observed by using corresponding pulse source to replace the pulse source in the existing system.

We wish to emphasize that the results presented here are preliminary. The varistors materials we used were not optimized for specific application of NEMP protection. The relationship between the thickness of ZnO varistors and the threshold field strength requires clarification. Meanwhile, the impact of negative pulse on protective effectiveness for electromagnetic pulse also needs further studies. These questions may be answered in following papers.

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# Magneto-Mechanical Behavior Analysis using an Extended Jiles-Atherton Hysteresis Model for a Sheet Metal Blanking Application 

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#### Abstract

Manufacturing processes affect the magnetic properties of the ferromagnetic components of electrical equipment. The optimization of the designed devices depends on two factors: the mechanical state of the material of a blanked part, especially near the cutting edge, and the magneto-mechanical behavior of the material used. In this paper we investigate the magnetic induction degradation of a blanked stator fabricated using fully processed, non-oriented $\mathrm{Fe}-(3 \mathrm{wt} \%) \mathrm{Si}$ steel sheet. Owing to the geometric symmetry, we first simulated a half-stator teeth blanking using the Abaqus software. Subsequently, a magneto-mechanical extended Jiles-Atherton hysteresis model was used to determine the magnetic induction distribution on the blanked teeth stator. The numerical results show that the magnetic induction degradation can reach $25 \%$ upon applying moderate magnetic field, i.e., $1000 \mathrm{~A} / \mathrm{m}$, and $8 \%$ upon applying magnetic field close to the magnetic saturation, i.e., $3500 \mathrm{~A} / \mathrm{m}$. The depth of the affected region was approximately 1.25 mm before the material regained its initial magnetic state.


Index Terms - Blanking, finite element method, JilesAtherton hysteresis, magnetic properties, strain-magnetic coupling.

## I. INTRODUCTION

Rotating electrical machines comprise ferromagnetic parts, which are manufactured using either conventional processes (blanking, drilling, shearing, etc.) or nonconventional ones (laser cutting or wire cutting). These processes, especially in ferromagnetic sheet metal forming, introduce local changes in the microstructure and generate internal stresses, thereby affecting the magnetic properties in the region of the cutting edge [1, 2] and increasing the magnetic losses near the cutting edge [3, 4]. Cao et al. [5] described the residual-stress distribution and magnetic domain structure on the
cutting edge, demonstrating that the decrease in the width of the residual-stress-variation zone reduced the deterioration of magnetic properties. For punched ring structures, Lewis et al. [6] developed a simple model to represent the power loss as a function of the damagedregion width; The results showed that the power loss increased with decrease in the damaged width. Notably, elastic and plastic deformations significantly affect the magnetic properties of materials and alter the shape of the hysteresis curves. The elastic strain modifies the local internal nergy of the material, thereby affecting its magnetic behavior. However, compared with internal elastic stain, plastic deformations induce higher disorder in the magnetic behavior; therefore, the deterioration in magnetic properties is less noticeable over the elastic tensile stress range than that because of plastic deformation [7]. The adverse effects of plastic tension on magnetic properties are manifested in the range of the low plastic strain levels and in the low and medium magnetic field amplitudes [8-11]. The aforementioned phenomenon is more noticeable at high tensile stress, which results in increased losses [12]. Furthermore, the mechanical stresses affect magnetic anisotropy, decrease permeability, and increase power losses [10]. Blanking parameters (i.e., punch velocity and clearance) have also been investigated [13] to determine their contribution to the degradation of magnetic properties. Xiong et al. [14] demonstrated the mechanical-cutting-induced changes in the microstructure and texture of non-oriented $\mathrm{Fe}-\mathrm{Si}$, deterioration in its magnetic properties, and a significant change in the resulted hysteresis loop. They also reported that these variations were more apparent upon increasing the cutting length per mass. Wang et al. [15] and Kuo et al. [16] revealed the importance of punching clearance, which deteriorates the magnetic performance. In addition, Weiss et al. [17] demonstrated that smaller cutting clearances could result in higher residual stresses near the cutting surface. They further showed that both
important cutting speed and wear of the tool increased the losses and deteriorated magnetization. Furthermore, material characteristics of electrical steels, such as thickness, grain size, and crystallographic texture strongly affect the magnetic properties of non-oriented electrical steel sheets [18]. Kuo et al. [16] indicated that a material with small grain size had low deformation during the punching process. Omura et al. [19] and Toda et al. [20] studied the effect of the hardness and thickness of non-oriented electrical steels on iron-loss deterioration by blanking process and, consequently, observed that material iron loss was low in thin or hard specimens.

Establishing a description for multiple physical coupling processes is essential. Several models have been developed and implemented into a numerical model to define the magneto-mechanical behavior of ferromagnetic materials in electrical devices. Coupling was employed to study the effect of blanking on the magnetic response [10] and the contribution of multiaxial stress on magnetization [10,11,13,21]. A certain amount of coupling was proposed as an extension of existing classical magnetic hysteresis models [13,22-24]. The numerical modeling of such problems requires sufficient computer capacity, powerful software for the simulations, and a well-defined and efficient approach. However, all of the cited studies have limitations. The coupled model of Ossart et al. [10] is based on the conversion of the local microhardness into an equivalent plastic strain, which cannot describe the complex state induced by blanking. Bernard and Daniel [13] extended the Jiles-Atherton (J-A) hysteresis model by introducing mechanical stress via anhysteretic magnetization and by modifying the pinning factor. This model was then implemented into a time-stepping finite element method. However, this approach suffers from critical convergence difficulties. The multiscale approach used by Aydin et al. [21] and based on the free energy in the domain scale is also insufficient, as it relies on only a few material parameters and provides a description of the coupled magneto-mechanical anhysteretic behavior only.

In this work, we present a coupled experimental and numerical analysis of magnetic behavior degradation for stator teeth blanking. Magnetic experiments on ferromagnetic sheet steel were carried out at different levels of plastic strain to develop an extended J-A hysteresis model considering the strain-magnetic field coupling. A sheet metal blanking model of a half-teeth stator was developed and validated using the Abaqus software. Then, a coupling analysis using Python and Abaqus was performed. The Python code obtained the plastic strain map from the finite element simulation and calculated the corresponding magnetic induction for a given magnetic field. These values were inserted in the teeth stator part to visualize the magnetic induction and distribution of degradation. An analysis performed close
to the cutting edge revealed the "magnetic dead zone" depth.

## II. THE JILES-ATHERTON HYSTERESIS MODEL

Jiles and Atherton [25] defined the total magnetization M as the sum of two components:

$$
\begin{equation*}
\mathrm{M}=\mathrm{M}_{\mathrm{rev}}+\mathrm{M}_{\mathrm{irr}} \tag{1}
\end{equation*}
$$

where $\mathrm{M}_{\mathrm{rev}}$ is the reversible component resulting from the deformation of the walls on the coupling sites under the action of an external field and $\mathrm{M}_{\mathrm{irr}}$ represents the irreversible magnetization. They can be analyzed by the set of differential equations:

$$
\left\{\begin{array}{l}
\frac{d M_{r e v}}{d H}=c\left(\frac{d M_{a n}}{d H}-\frac{d M_{i r r}}{d H}\right)  \tag{2}\\
\frac{d M_{i r r}}{d H}=\frac{M_{a n}-M_{i r r}}{\delta k-\alpha\left(M_{a n}-M_{i r r}\right)}
\end{array}\right.
$$

where c is the coefficient of reversibility in the range of $0-1, \mathrm{H}$ is the applied magnetic field, $\delta$ is the directional parameter, taking the value of +1 or $-1, \alpha$ is the domain coupling parameter, and k is the pinning factor. The anhysteretic magnetization is denoted by $\mathrm{M}_{\mathrm{an}}$ and it can be expressed by the modified Langevin function:

$$
\begin{equation*}
M_{a n}=M_{s}\left(\operatorname{coth}\left(\frac{H_{e}}{a}\right)-\frac{a}{H_{e}}\right) \tag{3}
\end{equation*}
$$

where a represents the domain density, $\mathrm{M}_{\mathrm{s}}$ is the saturation magnetization and $\mathrm{H}_{\mathrm{e}}$ is the Weiss effective field given by:

$$
\begin{equation*}
\mathrm{H}_{\mathrm{e}}=\mathrm{H}+\alpha \mathrm{M} . \tag{4}
\end{equation*}
$$

Considering all these expressions, the $\mathrm{J}-\mathrm{A}$ hysteresis model can be expressed by the differential equation:

$$
\begin{equation*}
\frac{d M}{d H}=(1-c) \frac{M_{a n}-M_{i r r}}{\delta k-\alpha\left(M_{a n}-M_{i r r}\right)}+c \frac{d M_{a n}}{d H} \tag{5}
\end{equation*}
$$

which is a function of five parameters $\left\{\mathrm{M}_{\mathrm{s}}, \mathrm{a}, \alpha, \mathrm{c}, \mathrm{k}\right\}$.

## III. EXPERIMENTAL ASPECTS OF MAGNETO-MECHANICAL COUPLING

In this section, we present the experimental arrangement for the magnetic measurements, the obtained magnetic hysteresis at different levels of plastic strain, and the developed theoretical model based on the J-A hysteresis model, which considers the magnetomechanical behavior coupling. It should be noted that a more detailed description of the magnetic measurement test apparatus is available in Refs. [22,26].

## A. Arrangement for magnetic measurements

The samples were obtained from a fully processed, non-oriented $\mathrm{Fe}-(3 \mathrm{wt} \%) \mathrm{Si}$ steel sheet with a thickness of 0.35 mm . The magnetic measurements were carried out in the "initial state" (i.e., without residual stress) and in the "loaded state" (i.e., different ranges of plastic strain applied up to $10 \%$ using a universal testing machine). The applied stress and the magnetic field were
both in the rolling direction.
Figure 1 shows the obtained experimental hysteresis loop at different loaded states. The total strain is assumed to be plastic strain under a static regime (frequency of 1 Hz ). As can be seen in Fig. 1, the magnetic properties are affected even at a low range of deformation. For example, it is noticeable that the magnetization at saturation and the remanence decrease with the increase in the deformation. Saturation is the state reached when an increase in the applied external magnetic field cannot increase the magnetization of the material and remanence is the magnetic flux density remaining in a material following the removal of the magnetizing field. However, the coercive field and hysteresis losses increase with the increase in the deformation. Magnetic coercivity is a measure of the ability of a ferromagnetic material to withstand an external magnetic field without becoming demagnetized and the hysteresis loss is proportional to the area of the hysteresis loop.


Fig. 1. Experimental hysteresis loops as functions of the applied elastoplastic deformation [23].

## B. Hysteresis simulation

Several efforts have been made to develop hysteresis models suitable for different materials and frequencies. In our study, we employed the J-A model [22,25]. The implementation of such a model requires the identification of five parameters: the saturation magnetization, the domain-coupling parameter, the pinning factor, the coefficient of reversibility, and the domain density. The J-A parameters were successfully identified using a genetic algorithm procedure for each plastic strain range [22]. A parameter sensitivity investigation allowed us to assume that parameter a had the greatest effect on the shape of the hysteresis curve. The study of the evolution of the J-A model parameters for different elastoplatic deformations reveals that only the domain density a depends on the deformation. The results of the hysteresis curve simulation for nondeformation, low deformation, and even at high deformation, with $\alpha, \mathrm{c}$, and k constants, are in agreement with the measurements. Accordingly, we assume that
they are independent of the elastoplastic deformation. Then, only parameter a is expected to be a function of $\varepsilon p$ and follows the law:

$$
\begin{equation*}
a=\frac{119+650 \cdot \varepsilon_{p}}{1+\varepsilon_{p}} . \tag{6}
\end{equation*}
$$

For the three other J-A parameters, we averaged the centered values of ten genetic algorithm simulations: $\alpha=150 \times 10^{-6}, \mathrm{c}=31 \times 10^{-3}$, and $\mathrm{k}=498 \mathrm{~A} / \mathrm{m}$. Figure 2 shows the experimental and simulated hysteresis for the initial state and $10 \%$ of mechanical deformation. Compared with the experimental results, all simulation results show a mean square error of less than $5 \%$.

(a)

(b)

Fig. 2. Measured and simulated hysteresis loops for $\mathrm{Fe}-$ (3wt\%)Si at different elastoplastic deformations (a) $0 \%$ and $(b)=10 \%)$ [22].

## IV. NUMERICAL ASPECTS OF STATOR TEETH BLANKING

In this section, we present the finite element analysis of a half-teeth stator blanking by using the Abaqus software.

## A. Finite element model

Figure 3 depicts the different tools employed for the simulation of the stator teeth blanking, where, owing to its geometric symmetry, only a half part is included. The
half blank, which was rectangular, with dimensions $115 \mathrm{~mm} \times 45 \mathrm{~mm} \times 0.8 \mathrm{~mm}$, was positioned between a die and blank holder. All the process tools had a fitting radius of 0.2 mm . The radial clearance between the punch and die, relative to the sheet thickness, was $10 \%$ (sheet thickness equal to 1.2 mm ). The model of the tools was based on the application of rigid bodies. The die and blank holder were fixed, and the punch moved at $100 \mathrm{~mm} / \mathrm{s}$. The contact was described using a Coulomb friction model with a friction coefficient, $\mu$, of 0.18 .


Fig. 3 Geometric description of the blanking tools.
The material investigated was a $1.2-\mathrm{mm}$-thick sheet of isotropic non-oriented full-process $\mathrm{Fe}-(3 \mathrm{wt} \%) \mathrm{Si}$ steel. Various tensile tests were performed at the strain rate of $10^{-5} \mathrm{~s}^{-1}$. This value of strain rate is, generally, used to describe the "quasi-static" behavior of the material. The material work hardening can be described using a conventional Ludwik's law. The quasi-static yield stress is given as follows:

$$
\begin{equation*}
\bar{\sigma}_{0}\left(\bar{\varepsilon}_{p}\right)=770\left(\bar{\varepsilon}_{p}\right)^{0.26} . \tag{7}
\end{equation*}
$$

While assuming the quasi-static material behavior, we do not consider the high magnitude of the punch velocity. Therefore, it is better to improve the materialbehavior description using a strain-rate-behavior law. In such a model, the rate of the effective plastic strain is related to the difference between the current stress and yielding stress, as proposed by Peirce et al. [27]. For onedimensional rate-dependent plasticity and isotropic work hardening, the effective plastic strain rate is given as:

$$
\begin{equation*}
\dot{\bar{\varepsilon}}^{p}=\dot{\bar{\varepsilon}}_{0}^{p}\left(\frac{|\sigma|}{\bar{\sigma}\left(\bar{\varepsilon}^{p}\right)}\right)^{m} \tag{8}
\end{equation*}
$$

where $\bar{\varepsilon}^{p}$ is the equivalent plastic strain, $\dot{\bar{\varepsilon}}^{p}$ is the equivalent plastic strain rate, which is the reference strain rate used to measure the quasi-static yield stress, and $m$ is the rate sensitivity parameter ( $\mathrm{m}>0$ ).

As the blanking process is not time-dependent, the
dependence on the strain rate is considered using the rate-dependent yield. When the dependences on the strain and strain rate are assumed to be separable and isotropic work hardening is considered, the strain rate dependent yield-stress $\bar{\sigma}$ is defined by:

$$
\begin{equation*}
\bar{\sigma}\left(\bar{\varepsilon}^{p}, \dot{\bar{\varepsilon}}^{p}\right)=\bar{\sigma}_{0}\left(\bar{\varepsilon}^{p}\right)\left(\frac{\dot{\bar{\varepsilon}}^{p}}{\dot{\bar{\varepsilon}}_{0}^{p}}\right)^{m}, \tag{9}
\end{equation*}
$$

where $\bar{\sigma}_{0}\left(\bar{\varepsilon}^{p}\right)$ is the quasi-static yield stress at the quasistatic strain rate $\dot{\bar{\varepsilon}}_{0}^{p}$.

To study the strain rate sensitivity, the true stressstrain curves were measured using an Instron test machine equipped with a charge-coupled device (CCD) camera and a data acquisition system controlling the prescribed displacement to maintain a constant strain rate in the center of the specimen. The technique used for these video controlled tests is based on the procedure developed by G'sell and Jonas [28]. The experimental arrangement was used to perform tensile tests at different strain rates ranging from $10^{-5}$ to $5 \times 10^{-3} \mathrm{~s}^{-1}$, with a reference strain rate set as $10^{-5} \mathrm{~s}^{-1}$. The result obtained for the investigated material shows a significant strain rate sensitivity value $(\mathrm{m}=0.0085)$.

The numerical simulation of the sheet metal blanking process has been reported by several studies [2,29,30] Different approaches have been proposed to simulate the shearing process and to treat ductile fracture. In this work, we use the non-iterative explicit approach to model the high nonlinearity associated with the blanking process. An arbitrary Euler-Lagrange formulation was employed to treat the large mesh distortion occurring during the calculation and leading to strain localization and mesh degradation, resulting in significant errors. The blank is meshed using 22610 hexahedral elements with reduced integration (type C3D8R), as shown in Fig. 4.


Fig. 4. Finite element model for the blanking test simulation.

Among the several existing sheet metal-forming processes, the blanking process is special in the respect that plastic straining is followed by ductile fracture and material separation. We used the well-known

Gurson-Tvergaard-Needleman model [31,32] to treat ductile fracture. The nine Gurson-Tvergaard-Needleman parameters are summarized in Table 1 [2,29], where term $\mathrm{f}_{0}$ is the initial void fraction, $\left\{\mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{q}_{3}\right\}$ are adjustable material parameters, $f_{N}$ is the volume fraction of the nucleating void, $\varepsilon_{\mathrm{N}}$ is the mean strain for void nucleation, S is the standard deviation, $\mathrm{f}_{\mathrm{c}}$ is the critical void volume fraction, and $f_{F}$ is the void volume fraction at failure.

Table 1: Gurson-Tvergaard-Needleman parameters

| $f_{0}$ | $q_{1}$ | $q_{2}$ | $q_{3}$ | $\varepsilon_{\mathrm{N}}$ | $f_{\mathrm{N}}$ | $S$ | $f_{\mathrm{c}}$ | $f_{\mathrm{F}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.01 | 1.5 | 1 | 2.25 | 0.3 | 0.04 | 0.1 | 0.11 | 0.12 |

## B. Finite element results

Because of the lack of experimental tests on stator teeth blanking, the validation of our numerical approach is based only on the numerical punch displacement as a function of the punch load. According to the literature, the maximum blanking load, $\mathrm{F}_{\text {max }}$, is expressed by [33]:

$$
\begin{equation*}
\mathrm{F}_{\max }=\text { k.P.e. } \sigma_{\max }, \tag{10}
\end{equation*}
$$

where k is a calibration factor $(0.8<\mathrm{k}<0.95)$, P is the perimeter of the punch, e is the thickness of the blank, and $\sigma_{\max }$ is the maximum tensile stress of the material.

According to equation (10), the maximum load needs to be in the range of $13.0-15.5 \mathrm{kN}$. In addition, for steel case blanking, fracture typically occurs between 85 and $95 \%$ of the thickness of the sheet [29]. These findings are confirmed by the curve of the simulated penetration as a function of load (Fig. 5). A reference point was assigned to the punch (rigid body) which enabled the acquisition of the tool displacement and total punch force required to penetrate the blank. The relative punch displacement is defined as the ratio of punch displacement and the blank thickness.

Figure 6 shows the von Mises stress distribution and the "equivalent plastic strain" (PEEQ) distribution.


Fig. 5. Teeth stator load as a function of displacement (maximum load $=13685 \mathrm{~N}$; relative displacement at fracture $=0.88$ ).


Fig. 6. (a) von Mises stress distribution, and (b) PEEQ distribution.

## V. APPLICATION OF THE MAGNETOMECHANICAL COUPLING

The magneto-mechanical coupling is realized following different steps and using the Abaqus software and a model in Python language. Figure 7 shows the applied numerical procedure. First, a blanking finite element simulation of the half-teeth stator is performed as described in the previous section. Then, the value of the equivalent plastic strain for each element of the specimen mesh is obtained. For a given value of magnetic field, H , we calculate the corresponding value of the magnetic induction using the extended $\mathrm{J}-\mathrm{A}$ formulation (see section "Materials and Methods"). Two new variables are created and saved in the odb file used by Abaqus. The first variable is denoted as "magnetic induction," describing the magnetic distribution through the blanked part [Fig. 8 (a) and 9 (a)]. The second variable is denoted as "degradation," representing the relative degradation of the magnetic induction, and it corresponds to the percentage rate of corresponding magnetic induction and maximum magnetic induction [Fig. 8 (b) and Fig. 9 (b)].

Figure 8 and Fig. 9 show that far from the cutting edge, the material regains its original magnetic properties. For the better analysis of the magnetic efficiency and to provide guidance for electrical parts designers, it is important to characterize the magnetic properties near the cutting edge and to determine the width of the affected area. For this purpose, we defined two "paths," shown in Fig. 10, to analyze the evolution of the magnetic properties near the cutting edge. The results summarized in Fig. 11 show that the magnetic degradation is more substantial for the low and medium range magnetic fields: the magnetic induction degradation reached $23.8 \%$ for an applied magnetic field of 1000 $\mathrm{A} / \mathrm{m}$. Close to the magnetic saturation $\left(\mathrm{H}_{\mathrm{s}}=4500 \mathrm{~A} / \mathrm{m}\right)$, the degradation is less severe: the magnetic induction degradation reached $7.4 \%$ for an applied magnetic field
of $3500 \mathrm{~A} / \mathrm{m}$. The affected area is approximately 1.25 mm for all cases.


Fig. 7. Flowchart of the magneto-mechanical coupling process.

Figure 8 and Fig. 9 represent the magnetic flux distribution and the corresponding degradation for two magnetic field values, $\mathrm{H}=1000 \mathrm{~A} / \mathrm{m}$ and $\mathrm{H}=3500 \mathrm{~A} / \mathrm{m}$, respectively.

(b)

Fig. 8. Simulation at magnetic field of $\mathrm{H}=1000 \mathrm{~A} / \mathrm{m}$ : (a) Magnetic induction distribution and (b) degradation.


Fig. 9. Simulation at magnetic field of $\mathrm{H}=3500 \mathrm{~A} / \mathrm{m}$ : (a) Magnetic induction distribution and (b) degradation.


Fig. 10. Paths for monitoring the magnetic induction evolution.


Fig. 11. Magnetic induction evolution along different paths.

## VI. CONCLUSION

We performed both experimental and numerical studies on the magneto-mechanical coupling and applied them to stator blanking. The finite-element simulation of a teeth stator using the Gurson-Tvergaard-Needleman constitutive model and an arbitrary Euler-Lagrange mesh description were performed to obtain the accurate plastic strain distribution on the blanked part. A static model of a fully processed, non-oriented $\mathrm{Fe}-(3 \mathrm{wt} \%) \mathrm{Si}$ steel sheet under plastic strain was developed on the basis of the classical J-A hysteresis model. In addition, genetic algorithms were used to identify the hysteresis model parameters and formulate an extended description of the J-A model. Using Python language, a simulation procedure was developed. First, the plastic strain state was obtained using the Abaqus process. Subsequently, we analyzed the magnetic induction corresponding to a given value of magnetic field. Finally, both the magnetic induction and magnetic degradation were described using the Abaqus software. From the results, it was evident that the magnetic degradation could reach $25 \%$ for a moderate value of the applied magnetic field, and that the width of the "magnetic dead zone" approached 1.25 mm near the cutting edge before the material regained its initial magnetic state. The developed procedure is a helpful tool for engineers and electrical equipment designers to optimize the performance of their electrical designs.

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