An Efficient Laguerre-FDTD Algorithm for Exact Parameter Extraction of Lossy Transmission Lines

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Abstract **─** This paper introduces a hybrid finite-difference time-domain (FDTD) method with weighted Laguerre polynomials to extract attenuation constants of lossy transmission lines. In the case of uniform lossy transmission lines, the complex variable compact two-dimensional (2-D) Laguerre-FDTD method is suitable for extracting attenuation constants exactly. To reduce memory requirements in this method, the divergence theorem is used to obtain a memory-efficient matrix equation. A lossy coplanar waveguide (CPW) example is presented to validate the accuracy and efficiency of the hybrid algorithm.

Index Terms **─** FDTD, Laguerre polynomials, memory reduction, skin depth.

I. INTRODUCTION

Although the finite-difference time-domain (FDTD) method has been widely used for electromagnetic simulation, it often results in a long solution time for the problems with fine grid division based on the Courant-Friedrichs-Lewy (CFL) stability condition [1-3]. In recent years, much attention has been paid to the unconditionally stable techniques, such as the alternating direction implicit (ADI) FDTD [4], Crank-Nicolson (CN) FDTD [5], Laguerre-FDTD [6], and locally one-dimensional (LOD) FDTD [7].

The Laguerre-FDTD method, based on weighted Laguerre polynomials and Galerkin's testing procedure, does not have to deal with time steps and separately computes the temporal and spatial variables. It may be much more efficient than the FDTD method with too many time steps to compute the solution. However, the Laguerre-FDTD method results in an implicit relation and has to perform the matrix inversion. Its memory storage requirements and computation time is dependent on the produced sparse matrix equation. Similar to the conventional FDTD case [8], an efficient Laguerre-FDTD method combined with a memory-reduced (MR) technique is introduced for electromagnetic modeling by substituting a Maxwell's divergence relationship into one of the curl difference equations [9-10].

In the past years, some numerical algorithms have been proposed to extract circuit parameters of lossy transmission lines [11-12]. To calculate more accurate results, an iterative process is applied to the compact 2-D Laguerre-FDTD method for the exact parameter extraction in this paper. A hybrid time-domain algorithm, which combines the MR Laguerre-FDTD method with the compact 2-D complex variable technique, analyzes a lossy coplanar waveguide (CPW). Starting from the three-dimensional (3-D) Maxwell's differential equations considering the divergence equation, the hybrid method analytically deals with the partial derivatives with respect to the propagation direction and time variable, respectively, and forms an implicit relation to obtain an order-marching scheme. Then, we use an iterative process with three steps for finding the exact solution of attenuation constants by using the compact 2-D complex variable technique.

II. THEORIES

A. A MR-Laguerre-FDTD algorithm with complex variables

In the conventional compact 2-D Laguerre-FDTD method [13], only a phase shift term $e^{-j_0\beta z}$ involved in the field expressions is not enough for

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lossy lines because a spatial attenuation term $e^{-\alpha z}$ is ignored, where z, α and β are the wave propagation direction, attenuation constant and phase constant, respectively. In general, however, the attenuated field components must not only vary with (x, y, t), but vary with z. Taking the spatial attenuation along z into account, the fields $U(x,y,z,t)$ can be expressed as

$$U(x,y,z,t) = u(x,y,t)\cdot e^{-(\alpha + j_0\beta)z}. \quad (1)$$

If the partial derivative with respect to z is replaced with $-(\alpha + j_0\beta)$, taking e_x and h_x for example the 3-D differential Maxwell's equations yield

$$\frac{\partial e_x}{\partial t} = \frac{1}{\varepsilon}\left[\frac{\partial h_z}{\partial y} + (\alpha + j_0\beta)h_y - \sigma e_x\right], \quad (2)$$

$$\frac{\partial h_x}{\partial t} = \frac{1}{\mu}\left[-(\alpha + j_0\beta)e_y - \frac{\partial e_z}{\partial y}\right]. \quad (3)$$

The other four equations can be constructed in a similar way.

Because of the explicit appearance of $e^{-\alpha z}$ in (1), the degenerated complex field components are not the functions of z anymore. It is apparent that, for single mode propagation, the temporal variations of field components are exactly steady oscillations.

In charge-free regions, the divergence of $\boldsymbol{D}$ can be chosen to replace (2)

$$\nabla\cdot\boldsymbol{D} = \frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} - (\alpha + j_0\beta)e_z = 0. \quad (4)$$

Since the Laguerre polynomials $L_n(t)$ are orthogonal with respect to the weighting function e^{-t}, an orthogonal set $\{\varphi_0, \varphi_1, \varphi_{2,} \cdots\}$ is chosen as the basis functions

$$\varphi_n(st) = e^{-st/2}L_n(st), \quad (5)$$

where $s > 0$ is a time scale factor. Using these entire-domain temporal basis functions, the electromagnetic fields $u(x,y,t)$ can be expanded as

$$\{u(x,y,t)\} = \sum_{n=0}^{N}\{u^n(x,y)\}\varphi_n(st). \quad (6)$$

The first derivative of field components $u(x,y,t)$ with respect to time t is [14]

$$\frac{\partial u(x,y,t)}{\partial t} = s\sum_{n=0}^{N}\left[\frac{u^n(x,y)}{2} + \sum_{\substack{k=0,\\ n>0}}^{n-1}u^k(x,y)\right]\varphi_n(st). \quad (7)$$

Using a Galerkin's testing procedure in time domain and central difference in space domain, and eliminating magnetic fields, with reference to [6], we get

$$\begin{aligned} &e_x^m|_{i,j} - e_x^m|_{i-1,j} + \frac{\Delta x_i}{\Delta y_j}e_y^m|_{i,j} - \frac{\Delta x_i}{\Delta y_j}e_y^m|_{i,j-1} \\ &\quad - \Delta x_i(\alpha + j\beta)e_z^m|_{i,j} = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} &-C_x^h|_{i-1,j}\,e_y^m|_{i-1,j} + \left[\frac{1 + 2\sigma_{i,j}/(s\varepsilon_{i,j})}{C_x^e|_{i,j}}\right. \\ &\quad \left. + C_x^h|_{i-1,j} + C_x^h|_{i,j} + \frac{2(\alpha + j\beta)^2\Delta\overline{x}_i}{s\mu_{i,j}}\right]e_y^m|_{i,j} \\ &\quad - C_x^h|_{i,j}\,e_y^m|_{i+1,j} + C_y^h|_{i-1,j}\,e_x^m|_{i-1,j} \\ &\quad - C_y^h|_{i-1,j}\,e_y^m|_{i-1,j+1} - C_y^h|_{i,j}\,e_x^m|_{i,j} \\ &\quad + C_y^h|_{i,j}\,e_x^m|_{i,j+1} + (\alpha + j\beta)\Delta\overline{x}_i C_y^h|_{i,j}\,e_z^m|_{i,j} \\ &\quad - (\alpha + j\beta)\Delta\overline{x}_i C_y^h|_{i,j}\,e_z^m|_{i,j+1} \\ &= 2(\alpha + j\beta)\Delta\overline{x}_i\sum_{k=0}^{m-1}h_x^k|_{i,j} - \frac{2}{C_x^e|_{i,j}}\sum_{k=0}^{m-1}e_y^k|_{i,j} \\ &\quad - 2\sum_{k=0}^{m-1}\left(h_z^k|_{i,j} - h_z^k|_{i-1,j}\right), \end{aligned} \quad (9)$$

$$\begin{aligned} &-C_x^e|_{i,j}\,C_x^h|_{i-1,j}\,e_z^m|_{i-1,j} - C_y^e|_{i,j}\,C_y^h|_{i,j-1} \\ &\quad \times e_z^m|_{i,j-1} + \left[1 + \frac{2\sigma_{i,j}}{s\varepsilon_{i,j}} + C_x^e|_{i,j}\,C_x^h|_{i,j}\right. \\ &\quad + C_x^e|_{i,j}\,C_x^h|_{i-1,j} + C_y^e|_{i,j}\,C_y^h|_{i,j} + C_y^e|_{i,j} \\ &\quad \left.\times C_y^h|_{i,j-1}\right]e_z^m|_{i,j} - C_x^e|_{i,j}\,C_x^h|_{i,j}\,e_z^m|_{i+1,j} \\ &\quad - C_y^e|_{i,j}\,C_y^h|_{i,j}\,e_z^m|_{i,j+1} - \frac{2(\alpha + j\beta)}{s\mu_{i-1,j}} \\ &\quad \times C_x^e|_{i,j}\,e_x^m|_{i-1,j} + \frac{2(\alpha + j\beta)}{s\mu_{i,j}}C_x^e|_{i,j}\,e_x^m|_{i,j} \\ &\quad - \frac{2(\alpha + j\beta)}{s\mu_{i,j-1}}C_y^e|_{i,j}\,e_y^m|_{i,j-1} \\ &\quad + \frac{2(\alpha + j\beta)}{s\mu_{i,j}}C_y^e|_{i,j}\,e_y^m|_{i,j} \\ &= -2C_x^e|_{i,j}\sum_{k=0}^{m-1}\left(h_y^k|_{i,j} - h_y^k|_{i-1,j}\right) + 2C_y^e|_{i,j} \\ &\quad \times\sum_{k=0}^{m-1}\left(h_x^k|_{i,j} - h_x^k|_{i,j-1}\right) - 2\sum_{k=0}^{m-1}e_z^k|_{i,j}, \end{aligned} \quad (10)$$

where

$$C_x^e|_{i,j} = \frac{2}{s\varepsilon_{i,j}\Delta\bar{x}_i}, \tag{11}$$

$$C_y^e|_{i,j} = \frac{2}{s\varepsilon_{i,j}\Delta\bar{y}_j}, \tag{12}$$

$$C_x^h|_{i,j} = \frac{2}{s\mu_{i,j}\Delta x_i}, \tag{13}$$

$$C_y^h|_{i,j} = \frac{2}{s\mu_{i,j}\Delta y_j}, \tag{14}$$

where, Δx_i and Δy_j are the lengths of the lattice edge where the electric fields are located; $\Delta\bar{x}_i$ and $\Delta\bar{y}_j$ are the distances between the adjacent center nodes where magnetic fields are located.

Compared with the traditional Laguerre-FDTD method, $e_x^m|_{i,j}$ in the MR form has a relationship only with adjacent four electric field components from (8), which results in a reduction of nonzero e_x element storage by four-ninth, and does not need to summate from order 0 to m-1.

Then, we have a matrix form equation

$$[A]\{e^m\} = \{g\} + \{\theta^{m-1}\}, \tag{15}$$

where, $\{e^m\} = \{e_x^m, e_y^m, e_z^m\}^{\mathrm{T}}$, $\{g\} = \{g_x, g_y, 0\}^{\mathrm{T}}$ is the excitation and $\{\theta^{m-1}\}$ is the summation of terms from orders 0 to m-1.

After obtaining $\{e_x^0, e_y^0, e_z^0\}^{\mathrm{T}}$, we can solve (8), (9) and (10) in an order-marching procedure recursively for the given α and β. Thus, we can obtain the time-domain electromagnetic fields from (5) and (6) from the calculation for the expansion coefficients.

B. Iterative process for parameter extraction

Based on the above Laguerre-FDTD equation (15), an iterative process for finding the exact attenuation constant α_{exact} of a lossy transmission line is suggested. The whole process has the following three steps.

Step One: Real-Variable Laguerre-FDTD Step. For a given phase constant β, set $\alpha = 0$, and then the complex-variable Laguerre-FDTD equations degenerate into the conventional real-variable Laguerre-FDTD equations. From [13], we can obtain an approximate attenuation constant α_{approx} corresponding to the given β. The late-time field distribution of the propagation mode is saved as the full-wave excitation in the next step to shorten the early-time period.

Step Two: *Complex-Variable Laguerre-FDTD Step*. For the same phase constant β, set $\alpha_{\mathrm{guess}} = \alpha_{\mathrm{approx}}$. If $\alpha_{\mathrm{guess}} = \alpha_{\mathrm{exact}}$, the late-time response will be a steady oscillation, i.e., its amplitude will keep constant. If $\alpha_{\mathrm{guess}} < \alpha_{\mathrm{exact}}$, the amplitude of the late-time response will decrease exponentially in time domain. If $\alpha_{\mathrm{guess}} > \alpha_{\mathrm{exact}}$, it increases exponentially. The phenomena, shown in Fig. 1, can be easily understood because the exponential variations in the late-time response's amplitude will compensate partly the insufficient or excessive losses made by α_{guess}.

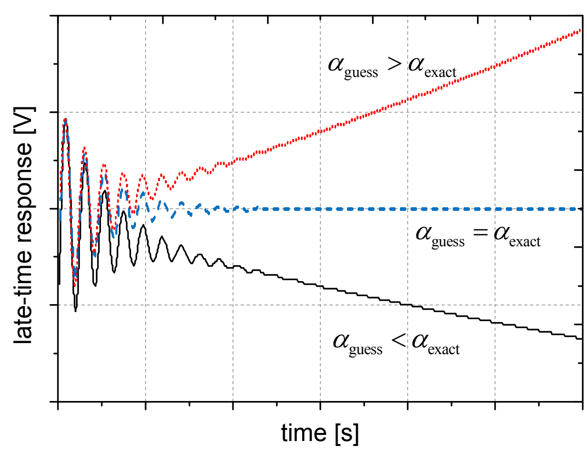

Fig. 1. Sketch of different late-time voltage responses for guessed attenuation constants.

In general, the late-time response's amplitude based on the 2-D Laguerre-FDTD equations will not keep constant. If the late-time response exponentially increases, that means $\alpha_{\mathrm{approx}} > \alpha_{\mathrm{exact}}$, and in this case we set $\alpha_{\min} = 0.5\alpha_{\mathrm{approx}}$ and $\alpha_{\max} = \alpha_{\mathrm{approx}}$. If the late-time response exponentially decreases, i.e., $\alpha_{\mathrm{approx}} < \alpha_{\mathrm{exact}}$, then we set $\alpha_{\min} = \alpha_{\mathrm{approx}}$ and $\alpha_{\max} = 1.5\alpha_{\mathrm{approx}}$. Thus, the search range $[\alpha_{\min}, \alpha_{\max}]$ for searching α_{exact} is determined.

Step Three: *Searching for* α_{exact}. For the same given β, a simple linear searching algorithm is combined with the proposed Laguerre-FDTD method to find α_{exact} between $\alpha_{\min}$ and $\alpha_{\max}$ iteratively. After several times of search, not more than twenty in general, the amplitude of the late-time response becomes constant, and the latest α is just the solution we want.

III. NUMERICAL EXAMPLE

In this example, the excitation source $\boldsymbol{J}$ is chosen as

$$\boldsymbol{J}(x, y, t) = \boldsymbol{g}(x, y)\delta(t), \tag{16}$$

where the temporal variation of the excitation $\delta(t)$ is a Dirac pulse, and the spatial variation $\boldsymbol{g}(x, y)$ is a quasi-static finite-difference solution of the transverse electric fields in the transmission line.

Figure 2 shows the cross structure a lossy coplanar waveguide (CPW). The anisotropy of LiNbO3 substrate and the finite conductivity of Au are taken into consideration. The parameters of the CPW are $W = 10.4\mu\text{m}$, $G = 9.6\mu\text{m}$, $t = 4.4\mu\text{m}$, $d = 400\mu\text{m}$, $M = 200\mu\text{m}$, $\sigma = 4.1\times10^{7}\,\text{S/m}$, $\varepsilon_{r//} = 43$ and $\varepsilon_{r\perp} = 28$. The perfect electric conductors (PECs) are used as the peripheral boundary condition.

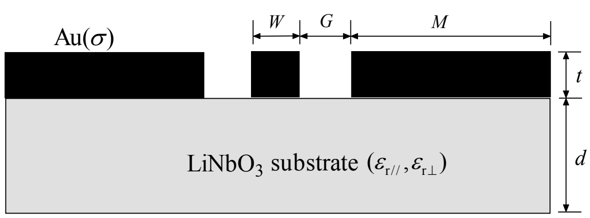

Fig. 2. Cross section of a lossy finite-ground CPW.

To consider the conductor loss in the CPW structure, the fields in conductors are analyzed and fine grid spacing is taken because of the influence of the skin depth. Graded grid division is adopted, and the minimum grid spacing is one-third of the skin depth corresponding to the maximum frequency f = 40GHz.

When only involving *Step One* of the whole three steps, the comparison of the computing time between the compact 2-D MR-Laguerre-FDTD method, conventional compact 2-D Laguerre-FDTD method and compact 2-D FDTD method for $\beta = 3.496$rad/cm is shown in Table 1. The two compact 2-D Laguerre-FDTD methods, which are free of stability constraint, show the significant improvement in computational efficiency. Moreover, the MR-Laguerre-FDTD method is more efficient than the conventional Laguerre-FDTD method because its memory storage of nonzero unknowns is reduced and 1/3 of electric field components do not need to summate from the order 0 to m-1. It is noted that it requires sixteen times of search to obtain the solution in *Step Three*.

Table 1: Comparison of the computing time when $\Delta x_{\min} = \Delta y_{\min} = 0.13\mu\text{m}$ and the whole lattice number is 51×27

Methods	**Total CPU time (s)**
MR-Laguerre-FDTD	213
Laguerre-FDTD	309
FDTD	945

The measured data in [15], numerical results with the compact 2-D Laguerre-FDTD method involving the whole three steps, and that only involving *Step One* is shown in Fig. 3, respectively. Compared with the results only involving *Step One*, the results with all three steps are in a better agreement with the measured data.

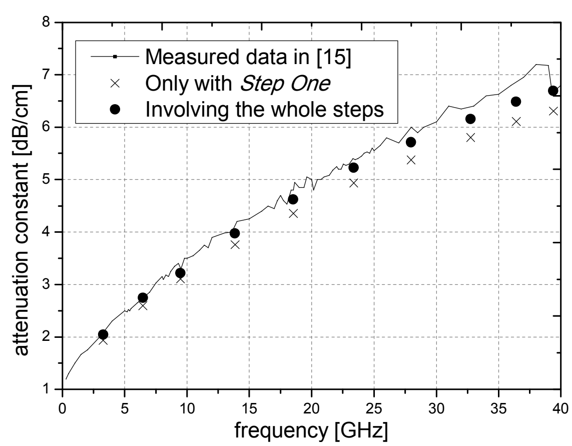

Fig. 3. Attenuation constants versus frequency for the lossy CPW.

IV. CONCLUSION

In this paper, an iterative process with complex variable technique is introduced for compact 2-D MR-Laguerre-FDTD method to analyze lossy transmission lines. With the divergence theorem, the memory storage of nonzero unknowns of e_x elements is reduced by 4/9 and 1/3 of electric field components do not need to summate from the order 0 to m-1. Under the condition of very fine grid spacing taken inside the lossy conductors, this unconditionally stable method shows improvement in computational efficiency compared with the FDTD method. Furthermore, an iterative process with complex variables is suggested to find the exact attenuation constants by using two additional steps, *Step Two* and *Step Three*. Although more CPU time is required, the hybrid method can obtain more accurate solutions than that only involving *Step One*, especially in the cases of heavily lossy lines.

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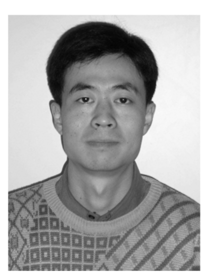

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