

# Transient Analysis of Thin-Wire Antennas over Debye Media

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**Abstract** — This paper presents a numerical procedure to calculate the time-domain response of thin-wire antennas over Debye media. The method is based in an expansion of the electric-field integral-equation in the time-domain (EFIE-TD), which accounts for the Debye media by using a reflection-coefficient approach. Resulting extended integral equation is subsequently solved by the method of moments. Numerical examples including Debye soils show not only the accuracy of the method but also a higher computational efficiency in comparison with other hybrid numerical techniques.

**Index Terms** — Reflection coefficient approximation, time-domain integral equation, thin-wire antennas.

## I. INTRODUCTION

Early formulated in the eighties [1], time-domain integral equations provide an accurate and computationally efficient solution of Maxwell's equations. Their main drawback is a lack of versatility, because in most practical cases is not possible to infer valid integral-equations for complex media, which limits their role as part of the electromagnetic numerical solvers. However, in those feasible cases, the abovementioned superior computational performance compared with differential formulations [2] makes them an interesting line of research. This paper is focused in the extension of the EFIE-TD to account for the

presence of a half-space Debye media. This a problem of interest in different practical cases in which Debye-dispersive materials appear —e.g., wet soils at ground-penetrating radar (GPR) [3] or ceramic materials for nondestructive evaluation purposes [4].

There are different approaches to account for half-space dispersive media applying both differential and integral equations. Regarding integral equations, good results have been reported by using procedures based in exact Green function's for the considered media [5], or methods which derive surface equivalent currents in the boundary of the half-space [6]. Both choices share the advantage of providing accurate solutions for antennas placed near above the dispersive media, but also pay a high price in terms of computational cost in comparison to their non-dispersive counterpart. However, most practical cases include antennas placed above the Debye media at a height enough to apply the reflection coefficient approach [7, 8], leading thus to alternative integral equations which lead to computationally efficient codes [9]. A key point to implement this reflection-coefficient (RC) method is the availability of analytical equations for the reflection coefficients in the time domain. Recently, several papers [10, 11] have presented these coefficients for Debye media both in TE and TM formulations, enabling so the application of the RC approach to model thin-wire antennas in front of Debye media. At this point, it can be pointed out that the problem of thin-wires antennas in front of complex media has

been also addressed by applying the FDTD method. For instance, a ground penetrating radar antenna was modeled using a hybrid FDTD-MoM technique in [12], and examples including thin-wire subcell model based on modified telegrapher's equations were firstly presented in [13] and later applied for the analysis of thin wire antennas in a loaded cavity in [14].

This paper is structured as follows. Taking as starting point the EFIE-TD for non-dispersive case, Section II presents the proposed numerical integral-equation procedure to account for the half-space Debye media, including both a brief formulation of the reflection coefficients in the time domain and numerical expressions to solve the integral equation by the method-of-moments. Section III proceeds to validate the code, and also presents some additional results including water soil for test purposes.

## II. EFIE-TD FOR THIN WIRES ABOVE DEBYE MEDIA

Thin-wire structures are those whose radius  $a$  is negligible compared with its length, and thus two-dimensional surface currents  $\vec{J}_s(\vec{r}', t')$  can be approximated as one-dimensional total currents  $\vec{I}(\vec{r}', t') = 2\pi a \vec{J}_s(\vec{r}', t')$ , placed at the center of the thin-wire structure and flowing along its axis. PEC thin-wire antennas, widely employed in practice because of its portability, are a class of antennas which performance is very well-known [15], and constitutes a good choice for testing the accuracy and computational performance of numerical algorithms solving EFIE. When located inside a dielectric space with permittivity  $\epsilon$  and velocity of propagation  $v$ , EFIE-TD for PEC thin-wires is:

$$\begin{aligned} \left(\vec{E}^i(\vec{r}, t)\right)_{\tan} &= \left(\frac{1}{4\pi\epsilon} \int_C \frac{1}{v^2 R} \frac{\partial}{\partial t} \vec{I}(\vec{r}', t') ds'\right)_{\tan} \\ &+ \left(\frac{1}{4\pi\epsilon} \int_C \frac{\vec{R}}{R^3} \left(\int_0^{t'} \frac{\partial}{\partial r'} I(\vec{r}', \tau) d\tau\right) ds'\right)_{\tan}, \quad (1) \\ &+ \left(\frac{1}{4\pi\epsilon} \int_C \frac{\vec{R}}{vR^2} \frac{\partial}{\partial r'} I(\vec{r}', t') ds'\right)_{\tan} \end{aligned}$$

where  $\vec{r}$ ,  $\vec{r}'$  and  $\vec{R}$  accounts for field, source and distance vectors, respectively, and  $t' = t - R/v$  is the retarded time which assures the causality of the

system.  $C$  corresponds to the contour following the axis of the wire, where  $s'$  and  $s$  note the positions located in the axis and on the surface of the wire, respectively. Making use of  $\hat{s}$  as the tangential unit vector on the surface of the wire, equation (1) can be expressed as:

$$\begin{aligned} \hat{s} \cdot \vec{E}^i(\vec{r}, t) &= \frac{1}{4\pi\epsilon} \int_C \frac{\hat{s}}{v^2 R} \cdot \frac{\partial}{\partial t} \vec{I}(\vec{r}', t') ds' \\ &+ \frac{1}{4\pi\epsilon} \int_C \frac{\hat{s} \cdot \vec{R}}{R^3} \left(\int_0^{t'} \frac{\partial}{\partial r'} I(\vec{r}', \tau) d\tau\right) ds' \quad . \quad (2) \\ &+ \frac{1}{4\pi\epsilon} \int_C \frac{\hat{s} \cdot \vec{R}}{vR^2} \frac{\partial}{\partial r'} I(\vec{r}', t') ds' \end{aligned}$$

If a ground plane is present, equation (2) is no longer valid [16, 17], because contributions from the reflected electromagnetic field in the surface are not considered. RC approximation [7] holds that total electric field in any point on the dielectric media surrounding the antenna can be expressed as a sum of a direct wave, corresponding to the scattered field in the dielectric  $\vec{E}^d(\vec{r}, t)$ , and a reflected wave, calculated as the convolution of the time domain reflection coefficient  $\Gamma^D(t)$  of the dielectric-Debye interface and the electric field coming from an image source  $\Psi$  located into the Debye media  $\vec{E}^r(\vec{r}, t)$ :

$$\hat{s} \cdot \vec{E}^i(\vec{r}, t) = \hat{s} \cdot \left(\vec{E}^d(\vec{r}, t) + \Gamma^D(t) * \vec{E}^r(\vec{r}, t)\right), \quad (3)$$

where

$$\begin{aligned} \vec{E}^d(\vec{r}, t) &= \frac{1}{4\pi\epsilon} \int_C \frac{1}{v^2 R} \frac{\partial}{\partial t} \vec{I}(\vec{r}', t') ds' \\ &+ \frac{1}{4\pi\epsilon} \int_C \frac{\vec{R}}{R^3} \left(\int_0^{t'} \frac{\partial}{\partial r'} I(\vec{r}', \tau) d\tau\right) ds', \quad (4) \\ &+ \frac{1}{4\pi\epsilon} \int_C \frac{\vec{R}}{vR^2} \frac{\partial}{\partial r'} I(\vec{r}', t') ds' \end{aligned}$$

and

$$\begin{aligned} \vec{E}^r(\vec{r}, t) &= \frac{1}{4\pi\epsilon} \int_{C_\psi} \frac{1}{v^2 R} \frac{\partial}{\partial t} \vec{I}(\vec{r}', t') ds' \\ &+ \frac{1}{4\pi\epsilon} \int_{C_\psi} \frac{\vec{R}}{R^3} \left(\int_0^{t'} \frac{\partial}{\partial r'} I(\vec{r}', \tau) d\tau\right) ds', \quad (5) \\ &+ \frac{1}{4\pi\epsilon} \int_{C_\psi} \frac{\vec{R}}{vR^2} \frac{\partial}{\partial r'} I(\vec{r}', t') ds' \end{aligned}$$

where contour image wires  $C_\psi$  are located at positions according to classical image theory,

involving only straightforward geometrical equations. Thus, the Debye half-space is removed of the original problem and it is substituted by an equivalent problem placed at free-space where equations based on Green's functions remain valid. It is important to remark that equation (3) is only applicable in those cases where the antenna is in the vicinity of the ground at heights  $h$  accomplishing [8]:

$$h > \frac{0.25\lambda}{\epsilon_r \sqrt{1 + \frac{\sigma}{j\omega\epsilon_r\epsilon_0}}} \quad (6)$$

Analytical equations of  $\Gamma^D(t)$  are available [10], expressed in terms on the polarization of the plane-wave incident to the ground plane. Naming  $\hat{n}$  as the unit vector normal to the ground plane in the point of incidence, and  $\Gamma_{TM}^D(t)$  and  $\Gamma_{TE}^D(t)$  as the time-domain RC in the case of vertically and horizontally polarized waves, respectively, equation (5) can be decomposed as [18]:

$$\begin{aligned} \hat{s} \cdot \vec{E}^i(\vec{r}, t) &= \hat{s} \cdot \vec{E}^d(\vec{r}, t) + \hat{s} \cdot \left\{ \Gamma_{TE}^D(t) * (\vec{E}^r(\vec{r}, t) \cdot \hat{n}) \hat{n} \right\} \\ &+ \hat{s} \cdot \left\{ \Gamma_{TM}^D(t) * \left[ \vec{E}^r(\vec{r}, t) - (\vec{E}^r(\vec{r}, t) \cdot \hat{n}) \hat{n} \right] \right\} \\ \hat{s} \cdot \vec{E}^d(\vec{r}, t) &+ \hat{s} \cdot \left\{ \Gamma_{TM}^D(t) * \vec{E}^r(\vec{r}, t) \right\} \\ &+ \hat{s} \cdot \left\{ \left[ \Gamma_{TE}^D(t) - \Gamma_{TM}^D(t) \right] * (\vec{E}^r(\vec{r}, t) \cdot \hat{n}) \hat{n} \right\} \end{aligned} \quad (7)$$

which is, by substituting of equations (4) and (5), the thin-wires EFIE-TD for half-space Debye media.

### A. TD-RC for half-space Debye media

The TD-RC for the incidence of electromagnetic plane-waves onto an interface separating dielectric and Debye media has been presented in [10], and further computationally improved by [11, 19]. In this work, the formulation achieved by applying directly an inverse Laplace transform to the Fresnel RCs is applied, because 1) do not require of additional parameters related to series representing the analytical equations, and 2) the effect of the savings in the computational burden in a method-of-moments (MoM) code for solving equation (7) is limited. Naming  $\tau$ ,  $\epsilon_s$ , and  $\epsilon_\infty$ , the relaxation time, static and infinite permittivity, respectively, of the Debye media, and  $\theta$  the angle of incidence, the TD-RC for TE polarized waves is:

$$\begin{aligned} \Gamma_{TE}^D(t) &= \frac{1 - K_{TE}}{1 + K_{TE}} \delta(t) + F_{TE} s_B g_1(t) u(t) \\ &- F_{TE} 2s_B^2 \frac{K_{TE}}{1 - K_{TE}^2} \left[ e^{-s_A t} u(t) * g_2(t) u(t) \right] \end{aligned} \quad (8)$$

where:

$$K_{TE} = \frac{\sqrt{\epsilon_\infty - \sin^2 \theta}}{\cos \theta} \quad (9)$$

$$F_{TE} = \frac{2K_{TE}}{(1 + K_{TE})(1 - K_{TE}^2)} \quad (10)$$

$$s_A = \frac{1}{\tau} \frac{\epsilon_s - 1}{\epsilon_\infty - 1} \quad (11)$$

$$s_B = \frac{1}{2\tau} \frac{\epsilon_s - \epsilon_\infty}{\epsilon_\infty - \sin^2 \theta} \quad (12)$$

and:

$$g_1(t) = e^{-\frac{t}{\tau}} \left[ (K_{TE} - 1) \bar{I}_0(s_B t) - (K_{TE} + 1) \bar{I}_1(s_B t) \right] \quad (13)$$

$$g_2(t) = e^{-\frac{t}{\tau}} \left[ (K_{TE} - 1) \bar{I}_0(s_B t) + (K_{TE} + 1) \bar{I}_1(s_B t) \right] \quad (14)$$

with  $\bar{I}_n(x)$  corresponding to the exponentially modified Bessel function of the first kind and order  $n$ , and  $u(x)$  to the unit step function.

Additionally, TD-RC for TM polarized waves can be written as:

$$\begin{aligned} \Gamma_{TM}^D(t) &= \frac{1 - K_{TM}}{1 + K_{TM}} \delta(t) + F_{TM} K_{TM} [f_2(t) - f_1(t)] u(t) \\ &- F_{TM} (1 + K_{TM}) s_B [g_3(t) u(t) + f_1(t) u(t) * g_3(t) u(t)] \end{aligned} \quad (15)$$

where:

$$K_{TM} = \frac{\epsilon_\infty \cos \theta}{\sqrt{\epsilon_\infty - \sin^2 \theta}} \quad (16)$$

$$F_{TM} = \frac{2K_{TM}}{(1 + K_{TM})(1 - K_{TM}^2)} \quad (17)$$

$$Q = \frac{\epsilon_\infty - 2 \sin^2 \theta}{\epsilon_\infty} \quad (18)$$

and:

$$f_1(t) = \left[ A_1 e^{-s_E t} + B_1 e^{-s_F t} \right] \quad (19)$$

$$f_2(t) = \left[ A_2 e^{-s_E t} + B_2 e^{-s_F t} \right] \quad (20)$$

$$g_3(t) = e^{-\frac{t}{\tau}} \left[ Q\bar{I}_0(s_B t) + \bar{I}_1(s_B t) \right], \quad (21)$$

where  $(s_E, s_F)$  are the negative roots of the second-order polynomial  $P(s)$ :

$$P(s) = s^2 + s \left( \frac{s_0 + s_1 - 2s_2 K_{TM}^2}{1 - K_{TM}^2} \right) + \left( \frac{s_0 s_1 - 2s_2^2 K_{TM}^2}{1 - K_{TM}^2} \right), \quad (22)$$

and additional constants included in equations (19), (20), and (22) are:

$$s_0 = 1/\tau, \quad (23)$$

$$s_1 = \frac{1}{\tau} \frac{\varepsilon_s - \sin^2 \theta}{\varepsilon_\infty - \sin^2 \theta}, \quad (24)$$

$$s_2 = \frac{1}{\tau} \frac{\varepsilon_s}{\varepsilon_\infty}, \quad (25)$$

and:

$$A_1 = \frac{(s_0 - s_E)(s_1 - s_E)}{(s_F - s_E)}, \quad (26)$$

$$B_1 = -\frac{(s_0 - s_F)(s_1 - s_F)}{(s_F - s_E)}, \quad (27)$$

$$A_2 = \frac{(s_2 - s_E)(s_2 - s_E)}{(s_F - s_E)}, \quad (28)$$

$$B_2 = -\frac{(s_2 - s_F)(s_2 - s_F)}{(s_F - s_E)}. \quad (29)$$

## B. MoM for thin-wires EFIE-TD including half-space Debye media

The computational implementation of equation (7) can be made through the MoM [20]. In this work, unknown currents  $I(s', t')$  of equations (4) and (5) are expanded using lagrangian sub-sectional basis functions [21], defined in  $N_s$  spatial and  $N_T$  temporal segments along a rectilinear uniform segmentation of the contour of the thin-wires. Weight functions applied are point-matching delta functions  $\delta(\vec{r} - \vec{r}_u)$  and  $\delta(t - t_v)$  chosen, respectively, along a set of  $N_s$  points  $\vec{r}_u$  located the surface of the wire, and a discrete set of time  $N_T$  instants  $t_v$ .

Therefore, it can be named  $\Delta_i$  as the size of the  $i$ -th segment of the wire,  $\Delta_i$  as the duration of the

time intervals in the marching-on-time procedure,  $s_i'' = s' - s_i$  as the distance of a position  $s'$  located at any  $i$ -th segment of the wire from its center  $s_i$ , and  $t_j'' = t' - t_j$  as the time distance referred to a chosen  $j$ -th time  $t_j$ . Using this notation and the point-matching functions, a discrete form of equation (7) arises:

$$\hat{s}_u \cdot \vec{E}^i(\vec{r}_u, t_v) = \hat{s}_u \cdot \vec{E}^d(\vec{r}_u, t_v) + \hat{s}_u \cdot \left\{ \Gamma_{TM}^D(t_v) * \vec{E}^r(\vec{r}_u, t_v) \right\} + \hat{s}_u \cdot \left\{ \left[ \Gamma_{TE}^D(t_v) - \Gamma_{TM}^D(t_v) \right] * \left( \vec{E}^r(\vec{r}_u, t_v) \cdot \hat{n} \right) \hat{n} \right\}, \quad (30)$$

in which the direct and reflect waves are given by:

$$\begin{aligned} \vec{E}^d(\vec{r}_u, t_v) &= \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N_s} \int_{\Delta_i} \frac{\hat{s}_i}{v^2 R_{iu}} \frac{\partial I_{ij}(s_i'', t_j'')}{\partial t_j''} ds_i'' \\ &+ \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N_s} \int_{\Delta_i} \frac{\vec{R}_{iu}}{R_{iu}^3} \left( \int_0^{t_j''} \frac{\partial I_{ij}(s_i'', \tau)}{\partial s_i''} d\tau \right) ds_i'' \\ &+ \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N_s} \int_{\Delta_i} \frac{\vec{R}_{iu}}{v R_{iu}^2} \frac{\partial I_{ij}(s_i'', t_j'')}{\partial s_i''} ds_i'' \end{aligned}, \quad (31)$$

and

$$\begin{aligned} \vec{E}^r(\vec{r}_u, t_v) &= \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N_s} \int_{\Delta_{\psi,i}} \frac{\hat{s}_i}{v^2 R_{iu}} \frac{\partial I_{ij}(s_i'', t_j'')}{\partial t_j''} ds_i'' \\ &+ \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N_s} \int_{\Delta_{\psi,i}} \frac{\vec{R}_{iu}}{R_{iu}^3} \left( \int_0^{t_j''} \frac{\partial I_{ij}(s_i'', \tau)}{\partial s_i''} d\tau \right) ds_i'' \\ &+ \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N_s} \int_{\Delta_{\psi,i}} \frac{\vec{R}_{iu}}{v R_{iu}^2} \frac{\partial I_{ij}(s_i'', t_j'')}{\partial s_i''} ds_i'' \end{aligned}, \quad (32)$$

where  $\hat{s}_u$  and  $\hat{s}_i$  stand for the tangential vectors, respectively, to the contour of the wire at the field point  $\vec{r}_u$  and to the axis of the wire at the source point  $\vec{r}_i$ .  $\vec{R}_{iu}$  corresponds to the vector between source and field points, and accomplishes the relation  $\vec{R}_{iu} = \vec{r}_u - \vec{r}_i - s_i'' \hat{s}_i$ .

In order to keep an affordable computational burden in the solution of equation (30) it is required to pay a special attention to the terms involving the convolution operation. As it is shown in [10, 11], and similarly to the transient response from lossy grounds [22], the TD-RC from a Debye soil shows a highly decreasing form for realistic soils (see Figure 1 where TD-RC of typical ground and water

half-space, chosen as examples of very different Debye media, are depicted). Based in this property, it can be established a useful approximation which allows to neglect late-time responses of the TD-RC and, thus, undesirable long computational times for convolutions. This approximation consists in a temporal truncation of the TE and TM TD-RCs to  $0 < t < t_{max}$ , where  $t_{max}$  correspond to that time where  $\Delta t \cdot \Gamma_{\{TE, TM\}}^D(t_{max}) < 0.1 \cdot \Gamma_{\{TE, TM\}}^D(0)$  with  $\Delta t$  corresponding to the time interval of analysis. In practice, no significant loss of accuracy is made by applying this condition, because in those cases where the rate of decrease in the TD RC response is lower, the impulsive part of equations (8) or (15) outweighs their non-impulsive counterpart. For those Debye media where the non-impulsive part predominates, its decreasing rate is high enough to only consider a limited set of terms of the time response. It worth to remark that the former approximation is compromised for high angles of incidence, which should be considered for thin-wire antennas very near to the interface. However, this fact does not imply a restriction in practice, because the accuracy of the RC approach is only guaranteed for heights of the thin-wires high enough above the ground [8].

### III. RESULTS

#### A. Validation of the code

Numerical results for the validation of the code have been achieved by simulating the scenario shown at Figure 2. A separate validation of TE and TM cases can be performed by considering only one wire above ground—where only TM TD-RCs plays a role in the simulation, and two wires—where both TM and TE are placed under consideration. Figure 2 also shows the image thin-wires below ground, which correspond to the term of contour  $C_\psi$  in equation (32).

Therefore, a typical slightly wet Debye soil with parameters  $\epsilon_s=2.5220$ ,  $\epsilon_o=2.4725$ , and  $\tau=21.5 \cdot 10^{-12}$  s [23] is chosen for the validation. First example under consideration validates the code for TM-incidence by considering a thin-wire antenna of total length of 1 m and a diameter of 5 mm placed at a height of 0.25 m above the soil. The antenna is fed at its central point by a normalized derivative gaussian pulse in the form:

$$v(t) = e^{0.5} g \sqrt{2} (t - t_{max}) e^{-g^2 (t - t_{max})^2}, \quad (33)$$

with parameters  $g=1.5 \cdot 10^9$  s<sup>-1</sup> and  $t_{max}=4/g$ . Figure 3 shows the current at the feed point, in comparison to a full-wave simulation coming from a FDTD code including thin-wire approach [24]. Accuracy of the results is in the range of previously reported differences between TDIE and FDTD methods [25, 26]. A minor time delay is appreciated by comparing the calculated waveforms of MoM-TD RC and FDTD. Reasons for this delay are associated to the longer wavelengths of the feeding pulse, which does not accomplish the required height to wavelength ratio corresponding to equation (6).

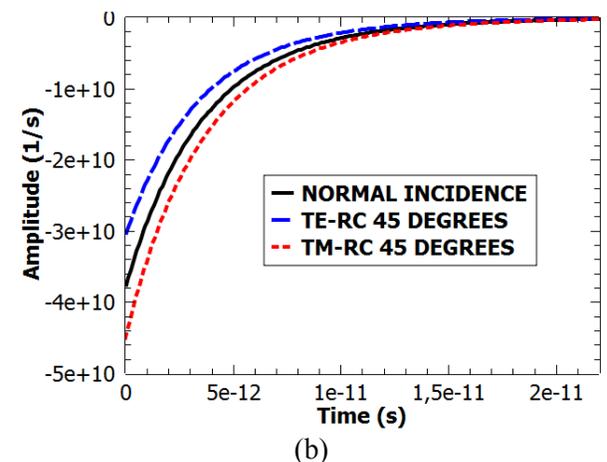
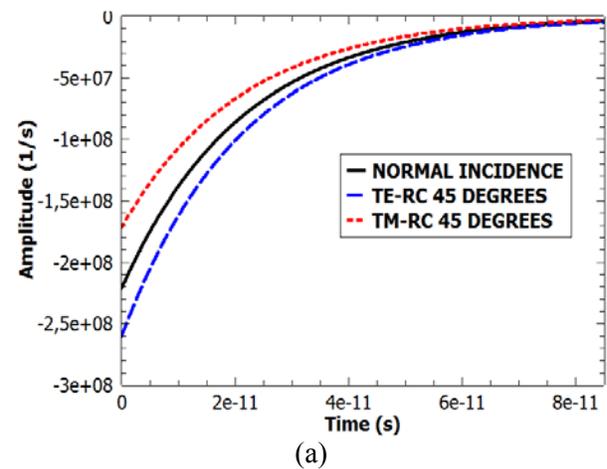


Fig. 1. TD-RC at different angles of incidence for a free space-Debye interface with parameters corresponding to (a) typical soil and (b) water.

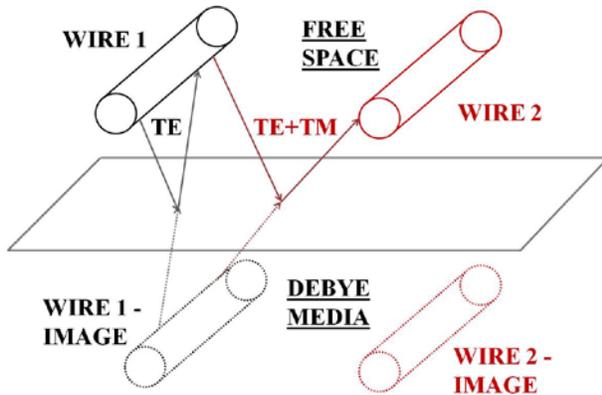


Fig. 2. Simulated scenario of horizontal thin-wires in a free space-Debye media. The results section shows different examples by considering only one thin-wire (black) as transmitter and a second thin-wire (red) as receiver.

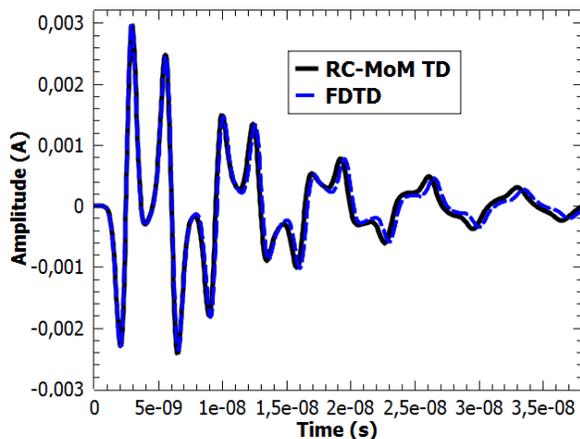


Fig. 3. Current at the center of a single horizontal thin-wire antenna above ground.

Figure 4 depicts the results of the validation for the two horizontal thin-wires case, in a configuration resembling the bistatic mode of a GPR equipment. Thus, a thin-wire acts as transmitting antenna, and it is fed as its center with a pulse of waveform following equation (33), while the other thin-wire is the receiving antenna in which measurements of the incoming electric field are made through the current at its center. Parameters of the feeding pulse are  $g=1.5 \cdot 10^9 \text{ s}^{-1}$  and  $t_{\max}=4/g$ , and thin-wires are located at a horizontal distance of 0.25 m, and both placed 0.25 m above ground. The conclusions regarding the accuracy of the method are similar to those mentioned for the single thin-wire case.

## B. Thin-wire antennas over Debye soils

Numerical methods have been developed for the simulation of GPR scenarios [12, 27, 28]. Performance of GPR antennas, e. g. mismatch of input impedance, above a Debye soil can be degraded as a consequence of the coupling effect of the reflecting wave produced by the ground. Controlling this effect is important in GPR applications, because mismatches can result in dispersive waveforms non-related to the presence the objects, and thus can lead to false positives in the detections. As an example of the usefulness of the proposed algorithm, a monostatic GPR equipment has been modelled by placing a single horizontal thin-wire in front of both slightly wet soil of Section 3A, as well as in front of a water soil (which has been chosen as an extreme case of fully-saturated water soil). Debye parameters of water-soil  $\epsilon_s=81.83$ ,  $\epsilon_\infty=23.46$ , and  $\tau=9.41 \cdot 10^{-12} \text{ s}$  [23], and feeding pulse and geometrical parameters are identical to the single-wire case of Figure 3.

Time-domain currents at the center of the antenna for these soils are shown at Figure 5. The case of a thin-wire embedded in free space is also plot as reference signal. It can be appreciated the strong effect of the water soil in the reflected pulse, which can mask reflected signals produced by buried objects, and thus disable the effectiveness of the equipment.

Figure 6a and 6b show another effect which should be considered at the design stage of the antennas. When placed in front of Debye media, input impedance of the antenna can change abruptly, leading to undesirable mismatches which could affect to the life cycle of the equipment. For example, first resonances of the dipoles depicted in Figure 6 are placed at 143 MHz, 142 MHz, and 139 MHz for free space, wet soil, and water media, respectively. So, a minor frequency shift is produced by the presence of the Debye media. However, the real part of the input impedance at the first resonance is 70  $\Omega$ , 60  $\Omega$ , and 17  $\Omega$  for free space, wet soil, and water, respectively. Potential damages could happen in front of water soils with such mismatches at the input impedance.

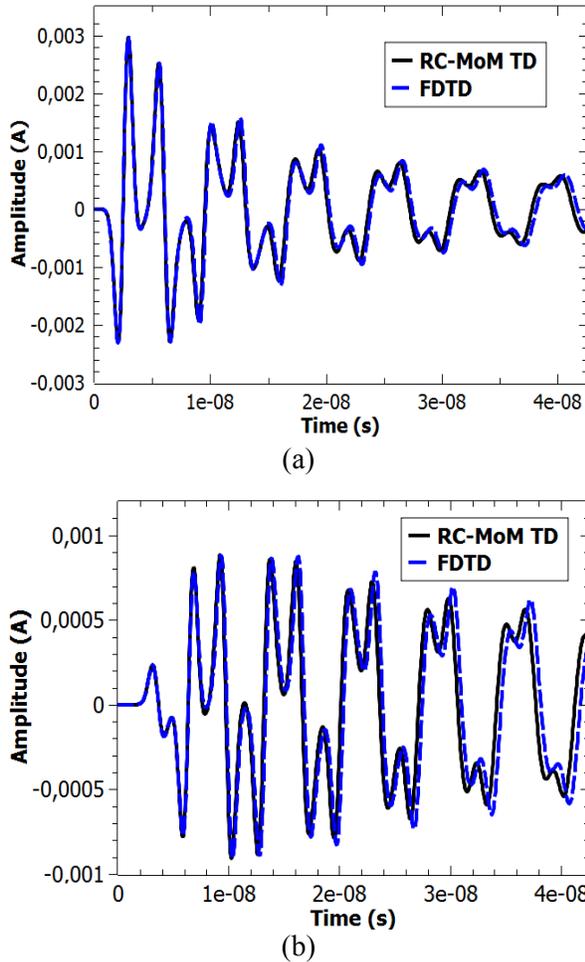


Fig. 4. Current at the center of the (a) transmitting and (b) receiving antenna for the case of two horizontal thin-wires above ground.

A final aspect to be considered is the reduction of the computational time achieved by using the proposed algorithm. Compared to finite difference schemes, it is roughly 400 times for the slightly wet soils and near to 800 for the water soil. It is very interesting to remark that the presence of the water half-space also increase the computational time for the RC-TDIE approach, because the shorter RC waveforms of the water soil (see Figure 1) require for a decrease of the time interval in order to provide an adequate sampling of the RC. For this reason, and following numerical guidelines of [1] according to the spectra of the feeding pulse, 101 spatial segments have been used to model any of the thin-wires in the case of free-space and slightly wet soils, leading to a time interval of 33.02 picoseconds, while a total of 202 segments are needed for the

case of the water soils. Spite this fact, a finer spatial discretization is also necessary in FDTD for the water half-space and thus even higher computational savings in time are achieved by the RC-TDIE algorithm.

## VI. CONCLUSION

This paper has presented a numerical method based on a hybrid RC-TDIE approach for the simulation of thin-wire antennas placed over Debye soils. Numerical results have shown the accuracy of the method as well as a low-cost computational performance compared to full-wave procedures. Examples have also shown the utility of the numerical simulations for the design of thin-wire antennas for GPR applications.

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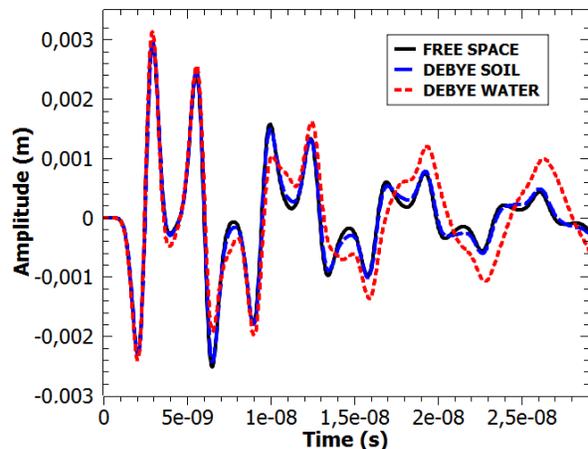


Fig. 5. Current at the center of the thin-wire antenna for the case of different Debye soils.

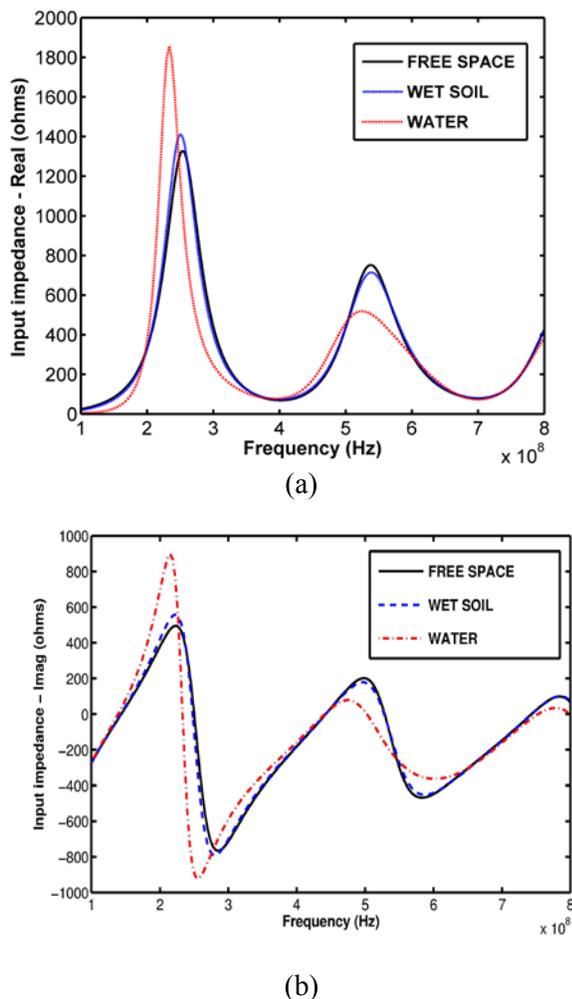


Fig. 6. Input impedance of the thin-wire antenna for the case of different Debye soils.

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