Two Dimensional Frequency-Angle Domain Adaptive Combined Interpolation Method for Electromagnetic Scattering Analysis of Precipitation Particles

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Abstract ─ A combined 2-D interpolation method is proposed for the efficient electromagnetic scattering analysis of precipitation particles over a broad frequency and angular band. This method combines the cubic spline interpolation method and the Steor-Bulirsch model. The cubic spline interpolation method is applied to model the induced current over wide angular band. The Steor-Bulirsch model is applied to accelerate calculation over wide frequency band. In order to efficiently compute electromagnetic scattering, sparse-matrix/canonical grid method (SM/CG) is applied to accelerate the matrix vector multiplication in EFIE. Therefore, the calculation time of frequency and angular sweeps become shorter. Numerical results demonstrate that this combined method is efficient for wide-band scattering calculation of precipitation particles with high accuracy.

Index Terms ─ Electromagnetic scattering, frequency-angle domain interpolation, precipitation particles, sparse-matrix/canonical grid method.

I. INTRODUCTION

In recent years, for environmental applications in remote sensing, the research on characterization of the electromagnetic wave interaction with complex rainfall particles has become more and more important [1-4]. Several analytical methods based on wave theory, such as quasi-crystalline approximation, are frequently used [5]. However, these approaches are not able to capture the essential physics of many real world problems [6]. Alternatively numerical technologies can be used to deal with scattering of complex rainfall particles, for instance, the method of moment (MoM) [7]. While the MoM can provide accurate solution of complex media scattering [8-9], the main disadvantages of MoM are significant calculation time and large memory requirements for the storage of impedance matrix. In order to alleviate these bottlenecks, a series of accelerate methods are proposed, such as fast multiple method (FMM) [10], as well as sparse-matrix/canonical grid (SM/CG) [11] method, which is proposed as an efficient method for calculating the scattering from three-dimensional dense media [12-13].

The radar cross section (RCS) contains both frequency and angle information simultaneously. In many practical applications, it is desirable to predict the monostatic RCS of a target in both the frequency domain and spatial domain simultaneously. Although the computational complexity and memory requirement can be reduced by the SM/CG method, we still have to repeat the calculations at each frequency or angle of interest to obtain the RCS over a wide frequency-angle band. In order to alleviate this difficulty, many interpolation methods have been proposed and applied for acceleration, such as the asymptotic waveform evaluation (AWE) [14] method and the model-based parameter estimation (MBPE) [15] method. However,

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there are difficulties in implementing these methods. For the widely used AWE [16], sometimes it is difficult to obtain the derivatives of the impedance matrix and the induced current vector. Singularity problem is the main weakness of the MBPE method [15] as it needs matrix inversion to get the coefficients of the substitute model.

In this paper, we investigate a novel adaptive frequency-angle domain interpolation method based on Stoer-Bulirsch model [17-19] and cubic spline algorithm [20], combined with SM/CG method for fast analysis of precipitation particles scattering over broad frequency-angle band. In frequency domain, two approximate rational function models are required in Stoer-Bulirsch algorithm for each iterative step. Both models could be constructed by using functions with the same set of samples. With the increase of samples, the difference between the two models will decrease. Therefore, when the termination criterion is achieved, both of approximate models could be used as the interpolation model for final results. In angle domain, cubic Hermite interpolation formulation is the basic model of cubic spline (CS) method, which utilizes the information of C1-continuous to evaluate the first derivative of the incident current vector instead of solving the large linear equations. Compared to traditional extrapolation or interpolation methods, such as AWE and MBPE, this novel method needs no matrix inversion and avoids calculating derivatives. This advantage and SM/CG combined together virtually yield an extremely efficient technique that seems something of a novelty compared to both AWE and MBPE method.

This paper is structured as follows. In Section II we describe the EFIE formulation and the SM/CG method. Section III describes the basic theory of the novel combined 2-D adaptive interpolation method based on cubic spline interpolation method in angle domain and Stoer-Bulirsch model in frequency domain with coarse-to-fine hierarchy. Numerical results which demonstrate the accuracy and efficiency of the proposed method are given in Section IV. Conclusions and comments are provided in Section V.

II. EFIE FORMULATION AND SM/CG METHOD

When a finite body of arbitrary shape, with permittivity $\varepsilon(\mathbf{r})$ and conductivity $\sigma(\mathbf{r})$, is exposed in free space to a plane electromagnetic wave, the induced current could be accounted for by replacing the body with an equivalent free-space current density $\boldsymbol{J}_{\mathrm{eq}}$, which can be written as:

$$\boldsymbol{J}_{eq}=\left[\sigma(\mathbf{r})+j\omega\left(\varepsilon(\mathbf{r})-\varepsilon_0\right)\right]\boldsymbol{E}(\mathbf{r})=\tau(\mathbf{r})\boldsymbol{E}(\mathbf{r}). \quad (1)$$

The conduction current is the first term of (1), whereas the second term represents the polarization current. ε_0 is the free-space permittivity and $\boldsymbol{E}(\mathbf{r})$ is the total electric field inside the body. According to reference [9], the scattered field $\boldsymbol{E}^{\mathrm{s}}$ at an arbitrary point inside the body can be expressed as:

$$\boldsymbol{E}^{s}(\mathbf{r})=pv\int_V \boldsymbol{J}_{eq}(\mathbf{r}')\cdot\overline{\overline{\boldsymbol{G}}}(\mathbf{r},\mathbf{r}')dV'-\frac{\boldsymbol{J}_{eq}(\mathbf{r})}{3j\omega\varepsilon_0}, \quad (2)$$

where $\mathbf{r}'$ denotes a source point, and $\mathbf{r}$ denotes a field point:

$$\overline{\overline{\boldsymbol{G}}}(\mathbf{r},\mathbf{r}')=-j\omega\mu_0\left[\overline{\overline{I}}+\frac{\nabla\nabla}{k_0^{\,2}}\right]\psi(\mathbf{r},\mathbf{r}'), \quad (3)$$

$$\psi(\mathbf{r},\mathbf{r}')=\frac{\exp(-jk_0\left|\mathbf{r}-\mathbf{r}'\right|)}{4\pi\left|\mathbf{r}-\mathbf{r}'\right|}. \quad (4)$$

In (4), $k_0=\omega(\mu_0\varepsilon_0)^{1/2}$ and μ_0 is the permeability of free space. The symbol pv in (2) represents the principal value of the integral. The sum of the incident electric field $\boldsymbol{E}^{\mathrm{i}}$ and the scattering field $\boldsymbol{E}^{\mathrm{s}}$ can be written as:

$$\boldsymbol{E}(\mathbf{r})=\boldsymbol{E}^{i}(\mathbf{r})+\boldsymbol{E}^{s}(\mathbf{r}). \quad (5)$$

By substituting (2) into (5), the integral equation for $\boldsymbol{E}(\mathbf{r})$ can be demonstrated as:

$$\left[1+\frac{\tau(\mathbf{r})}{3j\omega\varepsilon_0}\right]\boldsymbol{E}(\mathbf{r})-pv\int_V\tau(\mathbf{r}')\boldsymbol{E}(\mathbf{r}')\cdot\overline{\overline{\boldsymbol{G}}}(\mathbf{r},\mathbf{r}')dV'=\boldsymbol{E}^{i}(\mathbf{r}), \quad (6)$$

where $\boldsymbol{E}^{\mathrm{i}}(\mathbf{r})$ is the known incident electric field, and $\boldsymbol{E}(\mathbf{r})$ is the unknown total electric field in (6). By using moment methods, (6) could be transformed into a matrix equation [9]:

$$\begin{bmatrix}[\boldsymbol{G}_{xx}] & [\boldsymbol{G}_{xy}] & [\boldsymbol{G}_{xz}]\\ [\boldsymbol{G}_{yx}] & [\boldsymbol{G}_{yy}] & [\boldsymbol{G}_{yz}]\\ [\boldsymbol{G}_{zx}] & [\boldsymbol{G}_{zy}] & [\boldsymbol{G}_{zz}]\end{bmatrix}\bullet\begin{bmatrix}[\boldsymbol{E}_x]\\ [\boldsymbol{E}_y]\\ [\boldsymbol{E}_z]\end{bmatrix}=-\begin{bmatrix}[\boldsymbol{E}^{i}_{x}]\\ [\boldsymbol{E}^{i}_{y}]\\ [\boldsymbol{E}^{i}_{z}]\end{bmatrix}. \quad (7)$$

In the following, we rewrite (7), let $\boldsymbol{G}$ denote the coefficient matrix in Equation (7), $\boldsymbol{E}=\{E_{\mathrm{n}}\}$, and $\boldsymbol{b}=\{E^{\mathrm{i}}_{\mathrm{m}}\}$ for simplicity. Then, the matrix Equation (7) can be symbolically rewritten as:

$$\boldsymbol{GE}=\boldsymbol{b}. \quad (8)$$

To employ sparse-matrix/canonical grid method (SM/CG) to accelerate the matrix vector multiplication, the whole structure is enclosed in a rectangular region at first and then we recursively subdivides it into small rectangular grids. The impedance matrix $\boldsymbol{G}$ is decomposed into the sum of a sparse matrix $\boldsymbol{G}^{\mathrm{s}}$ denoting the strong neighborhood interactions, which can be computed directly by MoM and a dense matrix $\boldsymbol{G}^{\mathrm{w}}$, denoting the weak far interactions:

$$\boldsymbol{G}=\boldsymbol{G}^{\mathrm{s}}+\boldsymbol{G}^{\mathrm{w}}. \quad (9)$$

The majority of computation of an iterative method is to perform the matrix-vector multiplication between $\boldsymbol{G}^{\mathrm{w}}$ and $\boldsymbol{E}$. We translate the original basis functions on the triangular elements to the rectangular grids in SM/CG. After we put Taylor expansion of the Green's functions [9] in here that the matrix $\boldsymbol{G}^{\mathrm{w}}$ is further written as:

$$\left[\boldsymbol{G}^{w}\right]=\sum_{i=0}^{K}\left[\boldsymbol{G}_{i}^{w}\right], \tag{10}$$

where K is the total number of terms of the expansion. Each term in the series corresponds to a Taylor series expansion term [11-12]. If the expansion order of Taylor series is selected too small, the uncertainty of calculation accuracy will be caused. However, when the expansion order is chosen too large, it will lead to a large consumption of computing resources, as well as calculation time. Through numerical experiments, the Taylor expansion order is chosen to be 2 in our experiment, as this choice is a tradeoff between accuracy and efficiency. Finally, the matrix-vector in SM/CG will be efficiently done using O(Nlog$N$) FFT-based methods, could be represented:

$$\left[\boldsymbol{G}^{w}\right]\boldsymbol{E}=\sum_{i}\left[\boldsymbol{T}_{ti}\right]\left[\boldsymbol{G}_{i}\right]\left[\boldsymbol{T}_{si}\right]\boldsymbol{E}. \tag{11}$$

Each term in the summation consists of a pre-multiplication of the current vector with a block-diagonal matrix [$\boldsymbol{T}_{si}$] followed by a multiplication with a block-Toeplitz matrix [$\boldsymbol{G}_i$] and a post-multiplication with another block-diagonal matrix [$\boldsymbol{T}_{ti}$].

III. TWO DIMENSIONAL ADAPTIVE COMBINED INTERPOLATION TECHNIQUE IN FREQUENCY-ANGLE DOMAIN

In wide frequency-angle band scattering analysis of precipitation particles, repeated solution of (8) is required at each incident direction and frequency. In order to improve efficiency, in this section, a novel combined 2-D adaptive interpolation technique is proposed to accelerate precipitation particles electromagnetic properties calculation. This adaptive strategy with the idea of coarse-to-fine hierarchy which considered as an iterative process is proposed to generate a set of nonuniform sampling nodes. There are two basic problems that need to be emphasized in the approach. One is how to keep the iterative process going on and the other is when to stop the process.

Both of problems are controlled by error. Correspondingly, there are two types of error that need to be defined. The error for judging whether or not more samples are required is called convergence error (CE), and the error used to locate the next possible sample is called maximum error (ME), which ensures a successively adaptive process. A good definition of CE leads to high precision and a good definition of ME leads to few samples.

A. Cubic spline method in angle domain

In angular domain, two interpolation models are applied, while one is linear interpolation model and the other is cubic spline interpolation model. The number of sampling nodes will be enough when linear model and cubic spline model obtain almost the same result. As a result, CE is defined as a tolerance between the two different models. The whole sampling process will end off until the error between the two models is smaller than CE.

Assuming that the sampling nodes in angular domain are φ_0, φ_1, … , φ_n which divide the whole range into several intervals. Within each interval such as [φ_{i-1}, φ_i], $I(\varphi)$ could be expanded as a third-order polynomial on φ. According to Hermite interpolation theory [20], the interpolation formula of $I(\varphi)$ can be described as:

$$\begin{aligned}I(\varphi)&=\frac{(\varphi-\varphi_i)^2\left[h_i+2(\varphi-\varphi_{i-1})\right]}{h_i^3}I(\varphi_{i-1})\\&+\frac{(\varphi-\varphi_{i-1})^2\left[h_i+2(\varphi_i-\varphi)\right]}{h_i^3}I(\varphi_i)\\&+\frac{(\varphi-\varphi_i)^2(\varphi-\varphi_{i-1})}{h_i^2}I'(\varphi_{i-1})+\frac{(\varphi-\varphi_{i-1})^2(\varphi-\varphi_i)}{h_i^2}I'(\varphi_i),\end{aligned} \tag{12}$$

where φ_i is the sampling point and $h_i=\varphi_i-\varphi_{i-1}$. Obviously, $I(\varphi_i)$ and $I'(\varphi_i)$ are needed in order to estimate the value of $I(\varphi)$ in the angle range [φ_{i-1}, φ_i]. Suppose the number of sampling points to be n, then n times of solution is needed to obtain the value of $I(\varphi_i)$ ($i=1,2,\ldots,n$), when another n times of solution is also required to get corresponding derivative value $I'(\varphi_i)$ ($i=1,2,\ldots,n$). Therefore, the times of solution of Equation (8) is 2n [20]. It is thus a waste of time to compute the first derivative of induced current vector of each sampling node.

Cubic-spline interpolation method applies another way to obtain the first derivative of each sampling node instead of solving the linear Equation (8) repeatedly [20]. This method just needs to compute the first derivative of φ_0 and φ_n by (1), which has been proved to be unnecessary. We can put them into zero under the natural boundary condition [20]. The first derivative of other sampling points are then given by:

$$\begin{bmatrix}2&\lambda_1&&&\\\mu_2&2&\lambda_2&&\\&\ddots&\ddots&\ddots&\\&&\mu_{n-1}&2&\lambda_{n-1}\\&&&\mu_{n-1}&2\end{bmatrix}\begin{bmatrix}I'(\varphi_1)\\I'(\varphi_2)\\\vdots\\I'(\varphi_{n-2})\\I'(\varphi_{n-1})\end{bmatrix}=\begin{bmatrix}g_1-\lambda_1I'(\varphi_0)\\g_2\\\vdots\\g_{n-2}\\g_{n-1}-\mu_{n-1}I'(\varphi_n)\end{bmatrix}, \tag{13}$$

where

$$\lambda_i=\frac{h_{i+1}}{h_i+h_{i+1}},\quad \mu_i=\frac{h_i}{h_i+h_{i+1}},$$

and

$$g_i=3\left[\mu_i\frac{I(\varphi_{i+1})-I(\varphi_i)}{h_{i+1}}+\lambda_i\frac{I(\varphi_i)-I(\varphi_{i-1})}{h_i}\right].$$

Due to the large time consumed in calculating the derivative, the cubic-spline interpolation approach is able to reduce a great deal of cost in angular domain.

B. Store-Bulirsch algorithm in frequency domain

In frequency domain, the interpolation function is described in the form of a fractional polynomial function with numerator of order N and denominator of order D for the frequency f by:

$$S(f)=\frac{a_0+a_1 f+a_2 f^2+\dots}{1+b_1 f+b_2 f^2+\dots}=\frac{a_0+\sum_{n=1}^{N} a_n f^n}{1+\sum_{d=1}^{D} b_d f^d}S. \qquad (14)$$

For given orders of N and D, the coefficients a_n and b_d can be determined from $k=N+D+1$ samples of $S(f)$ by solving a set of linear systems. The Store-Bulirsch algorithm does not require the inversion of matrix to get the coefficients of the rational function. The main idea of the algorithm is given as follows:

Suppose that there are a group of samples (f_i, $S(f_i)$), $i=1,\dots,k$ available for obtaining a rational function interpolation of function value $S(f)$ at any $f\in(f_1,f_2)$, whereas the explicit expression of rational function is unknown. The recursive process of Store-Bulirsch algorithm starts with the initial condition:

$$R_{i,1}=S(f_i), i=1,\dots,k, \qquad (15)$$

which constructs the first column of the triangle table. Starting from the second column, all elements are obtained using a recursive formula associated with two or three elements in the preceding columns.

Store-Bulirsch algorithm provides two ‘triangle rules’:

$$R_{j,k}=\frac{(f-f_j)R_{j+1,k-1}+(f_{j+k}-f)R_{j,k-1}}{f_{j+k}-f_j}, \qquad (16)$$

and

$$R_{j,k}=\frac{f_{j+k}-f_j}{\dfrac{f-f_j}{R_{j+1,k-1}}+\dfrac{f_{j+k}-f}{R_{j,k-1}}}, \qquad (17)$$

and one ‘rhombus rule’:

$$R_{j,k}=R_{j+1,k-2}+\frac{f_{j+k}-f_j}{\dfrac{f-f_j}{R_{j+1,k-1}-R_{j+1,k-2}}+\dfrac{f_{j+k}-f}{R_{j,k-1}-R_{j+1,k-2}}}. \qquad (18)$$

Based on these recursive rules, SB algorithm is organized as follow.

Let the whole frequency band to be defined from f_1 to f_2. The results recursively calculated by formula (16), (17), (18) are $S_1(f)$, $S_2(f)$, and $S_3(f)$ respectively. A testing point within the frequency band of interest is f_t and a sample is f_s. The true value from electromagnetic simulation or from experiment is denoted as $EM(f_s)$. In the interpolation process, a rational interpolation is implemented by using a distinct combination of the above three recursive rules. Pair one ((16) and (17)) and pair two ((16) and (18)) are alternately utilized to find the point at which the maximum sampling error occur, then add this sample into the sample group, until three recursive formulas, i.e., (16), (17) and (18), are sufficiently close to each other. In this paper, Equation (16) is used for interpolating frequency response at any given frequency f for final results. Specific algorithm steps are as follows.

Step 1: Set $f_{s1}=f_1$ and $f_{s2}=f_2$, and compute $EM(f_{s1})$ and $EM(f_{s2})$.

Step 2: Choose Equations (16) and (17) as the recursive formula and find the point at which the maximum sampling error occur, say at point f_{s3}, such that,

$$|S_1(f_{s3})-S_2(f_{s3})|=\max_i|S_1(f_{ti})-S_2(f_{ti})|. \qquad (19)$$

Step 3: Compute $EM(f_{s3})$. If a given convergence error (CE) is larger than the sampling error in step 2, that is $|S_1(f_{s3})-S_2(f_{s3})|<CE$, then the process switches to step 6 for termination. Otherwise, add the sample {f_{s3}, $EM(f_{s3})$} into the sample group and goes to next step for finding next samples.

Step 4: Change the recursive formula into (16) and (18), compare the sampling value at all testing points and find the point at which the maximum sampling error occurs, say at point f_{s4}, such that,

$$|S_1(f_{s4})-S_3(f_{s4})|=\max_i|S_1(f_{ti})-S_3(f_{ti})|. \qquad (20)$$

Step 5: Compute $EM(f_{s4})$, if $|S_1(f_{s4})-S_3(f_{s4})|<CE$, it means that the samples obtained are enough for the sampling for rule (16). Then the process goes to step 6. Otherwise, add the point {f_{s4}, $EM(f_{s4})$} into the sample group and go back to step 2 to find next samples.

Step 6: Suppose sufficient samples have been obtained for sampling. Saving the sample group {(f_{si}, $EM(f_{si})$), $i=1,\dots,N$}, which is the only required knowledge for rational function interpolation, and N is the number of samples in the last step.

C. Two-dimensional combined adaptive interpolation technique

For a desired frequency domain and angular domain, Fig. 1 shows the adaptive process for 2-D combined interpolation technique in details. The coarse-to-fine hierarchy strategy is applied to carry out 2-D adaptive sampling. Compared to 1-D adaptive strategy which is applied to decide the sampling node, 2-D adaptive strategy is used to select the “sampling line”. The dashed in Fig. 1 is the “sampling line” in adaptive sampling process, which divides the whole surface into many lattices. Specifically speaking, ME which defined by the difference between linear model and cubic spline model is applied to decide the location of “good sampling line” in angular domain, while Store-Bulirsch algorithm is utilized to obtain the location of “good sampling line” in frequency domain. As same as 1-D method, the process will end when there is no “sampling line” needed. When the value of CE is set appropriately, the meshing lines could avoid losing performance and accuracy.

It is assumed that the whole surface is defined by $[f_1, f_m] \times [\varphi_1, \varphi_m]$. A testing point within the range of interest is (f_{t1}, φ_{t2}) and a sampling point is (f_{s1}, φ_{s2}). The incident current vector for (f, φ) is $I(f, \varphi)$ and the electromagnetic properties from EM simulation is denoted as rcs(f, φ).

Step 1: Initialize four sampling points, (f_1, φ_1), (f_1, φ_n), (f_m, φ_1) and (f_m, φ_n). Then acquire $I(f_1, \varphi_1)$, $I(f_1, \varphi_n)$, $I(f_m, \varphi_1)$ and $I(f_m, \varphi_n)$.

Step 2: Assume there are $m \times n$ samples on the whole surface, and the number of rows is m and the number of columns is n. Applying linear interpolation method to obtain the approximate surface rcs_a1(f, φ) and applying cubic spline interpolation method to obtain the approximate surface rcs_a2(f, φ).

Step 3: For each interval $[\varphi_{sj}, \varphi_{sj+1}]$ (j=1,2,…,n), find the "sampling line" at which the ME occurs, say, at φ_{tj}, such that,

$$\max \sum_{si} \left| \mathrm{rcs\_a1}(f_{si}, \varphi_{tj}) - \mathrm{rcs\_a2}(f_{si}, \varphi_{tj}) \right|, \varphi_{sj} \le \varphi_{tj} \le \varphi_{sj+1}.$$

Step 4: If a given error tolerance, denoted as CE(φ), is larger than the ME in step 3, that means the samples obtained are enough; otherwise, adding the "sample line" of $I(f, \varphi_{sj})$ into the sample collection.

Step 5: For each interval $[f_{si}, f_{si+1}]$ (i=1,2,…,m), Applying Store-Bulirsch algorithm to obtain the "sampling line" at which the ME occurs.

Step 6: If a given error tolerance, denoted as CE(f), is larger than the ME in step 5, that means the samples obtained are enough; otherwise, adding the "sample line" of $I(f_{si}, \varphi)$ into the sample collection.

Step 7: If each interval $[\varphi_{sj}, \varphi_{sj+1}]$ satisfies the condition mentioned in step 4 and each interval $[f_{si}, f_{si+1}]$ satisfies the condition mentioned in step 6, the whole process terminates.

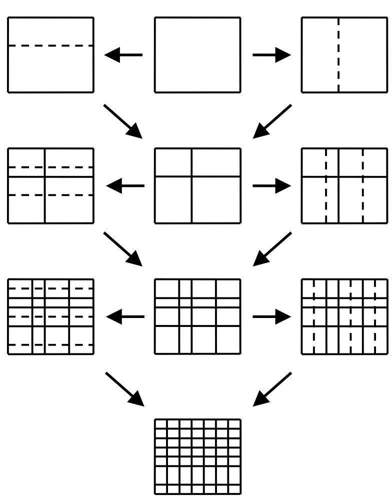


Fig. 1. Process of two dimensional frequency-angle adaptive combined sampling.

IV. NUMERICAL RESULTS

In this section, two numerical results are presented to demonstrate the efficiency of the proposed two-dimensional combined adaptive interpolation technique (SB-CS) method for fast calculation of wideband electromagnetic scattering of precipitation particles. In the implementation of the SB-CS method, we use GPBi-CG [21] algorithm to solve linear systems arising from electromagnetic wave scattering problems. In the simulation, θ and φ mean pitch angle and azimuth angle, respectively. Both experiments are conducted on an Intel Core i7 with 8 GB local memory and run at 3.6 GHz in single precision. The iteration process is terminated when the 2-norm residual error is reduced to 10^{-5}, and the limit of the maximum number of iterations is set as 10000. Two examples are applied to illustrate the performance of SB-CS method as follows.

Case I. 1000 spherical particles of arbitrary radius with random position

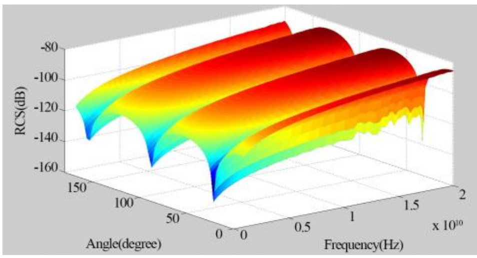
In the first simulation, scattering properties of 1000 spherical particles under random distribution in the space of 0.3*0.3*0.3m^3 are researched. Particle parameter for simulation in this part is set as follows: the rain group is constituted by 1000 homogeneous spheres with random diameters range from 0.1 to 4*mm*, with 35308 unknowns.

The complex refractive index m varies from 4+0.04*i* to 3.5245+0.08755*i*, when testing frequency band is from 1 GHz to 20 GHz. The direction of incident wave is fixed at $\theta = 0^\circ$, while φ is from 0° to 180°.

Case II. 500 precipitation particles with random position under certain axis ratio distribution [22]

In actual rainfall, the shape of precipitation particles is closely related to the size of water drops. In general, raindrop can be seen as sphere when volume is small. With increasing of raindrop size, the profile of precipitation particle is more close to the ellipsoid shape. This relationship has been studied by several researchers, and results of relevant research could be found in [22-24]. In this section, 500 precipitation particles under Keenan model [22] in the space of 0.5*0.5*0.5m^3 are investigated. Particle parameter for simulation in this part is set as follows: minor axis of the ellipsoidal particles is varied from 0.1 to 4*mm*. The number of unknowns is 27498 and the complex refractive index m varies from 4+0.04*i* to 3.3495+0.105*i*. The testing frequency band is from 8 GHz to 12 GHz. The direction of incident wave is fixed at at $\theta = 0^\circ$, while φ is from 0° to 180°.

Both the directly calculated and the interpolated results are presented in Figs. 2-3, while the relative error of SB-CS method for two examples has also been quantitative evaluated, shown in Fig. 4. These examples demonstrate that the SB-CS method is able to approximate the 3-D RCS precisely. As shown in Fig. 2, the real curve is obtained by using original SM/CG method with the interval of 100 MHz, which needs 34571 exact calculated sampling points. With the SB-CS technique, the sampling nodes reduce to 2267. For the second example depicted in Fig. 3, the standard sampling interval changes to 20 MHz. The total number of exact calculated nodes is 36381, when the proposed method also needs only 2070 sampling points to obtain the approximate simulation curve.



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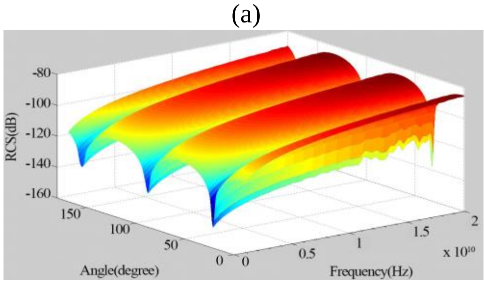
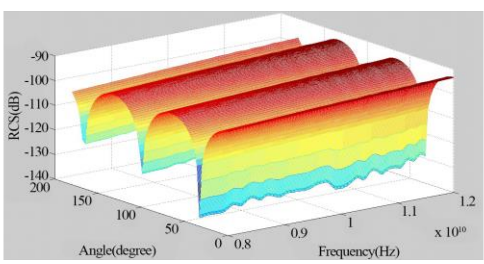


(b)

Fig. 2. 3-D RCS of 1000 spherical precipitation particles simultaneous versus frequency and angle. (a) SM/CG repeated solution, and (b) SB-CS method.



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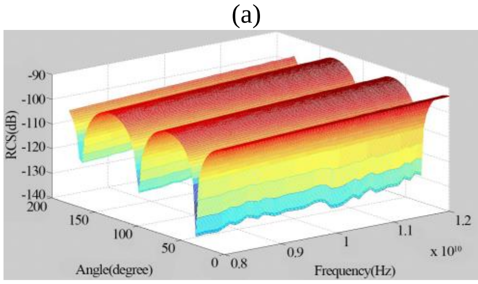
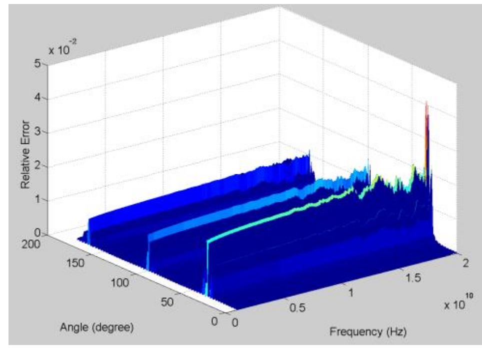
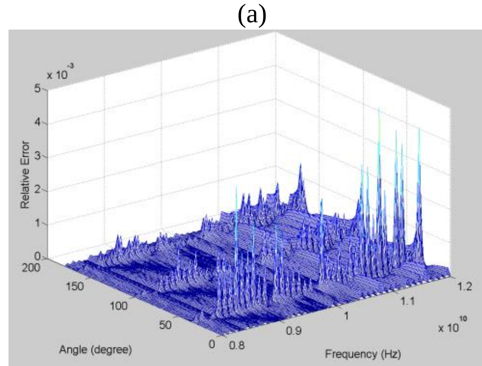


(b)

Fig. 3. 3-D RCS of 500 precipitation particles under Keenan model simultaneous versus frequency and angle. (a) SM/CG repeated solution, and (b) SB-CS method.



(a)



(b)

Fig. 4. The relative error of SB-CS method for two examples. (a) 1000 spherical precipitation particles, and (b) 500 precipitation particles under Keenan model.

Since a good interpolation method depends not only on its agreement performance, but also on its computational cost, Table 1 compares the number of calculated nodes, number of iterations and solution time between the traditional SM/CG method and the SB-CS method. Compared with the direct SM/CG method, it could be seen that the SB-CS method decreases the number of calculated nodes by a factor of 15.25 and 17.58 on both examples. Similar improvements could also be found in number of iterations, while solution time compression ratio is 14.68 and 17.06 for these two examples. AWE method is also applied on these two cases, corresponding solution time compression ratio is 11.25 and 13.93. In this paper, the order of AWE is 6 and 4, which means numerator is a sixth-order polynomial and denominator is a fourth-order polynomial. As the sampling points are selected adaptively across the broad frequency and angular band of interest, efficiency of SB-CS is better than AWE in some extent. Since the proposed method can determine the number and the location of sampling points automatically, more sampling nodes are required when the real curve is complex, as the simple curve needs less number of samples. These results demonstrate the effectiveness and flexibility of SB-CS method applying for wideband electromagnetic scattering calculation of precipitation particles in frequency-angle domain.

Table 1: Comparison of the cost and performance between direct SM/CG and SB-CS method

Object		Case I	Case II
Number of Unknowns		35308	27498
Calculated nodes	SM/CG	34571	36381
	AWE	3015	2558
	SB-CS	2267	2070
Number of iterations	SM/CG	187715	217284
	AWE	16224	15099
	SB-CS	12167	12262
Solution time (min)	SM/CG	30187	38873
	AWE	2683	2791
	SB-CS	2056	2278

V. CONCLUSIONS AND COMMENTS

In this paper, a novel two dimensional adaptive combined interpolation algorithm based on cubic spline technique and the Stoer-Bulirsch model is proposed for fast wideband electromagnetic computation of precipitation particles in frequency-angle domain. Using the proposed method, the frequency-angle domain response could be modeled by the adaptive sampling strategy to generate new sampling points automatically in both frequency and angle domain with quite less number of sampling points than traditional direct solvers. Compared to traditional AWE technique, this novel method needs no matrix inversion and avoids calculating derivatives, thus the derived fitting model could be applicable in an almost unlimited frequency and angle band. Numerical results indicate that this combined interpolation strategy performs well in terms of both simulation time and computation accuracy.

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REFERENCES

[1] G. J. Huffman, et al., "The TRMM multisatellite precipitation analysis (TMPA): Quasi-global, multiyear, combined-sensor precipitation estimates at fine scales," *Journal of Hydrometeorology*, vol. 8, pp. 38-55, 2007.

[2] R. F. Adler, et al., "Tropical rainfall distributions determined using TRMM combined with other satellite and rain gauge information," *Journal of Applied Meteorology*, vol. 39, no. 12, pp. 2007-2023, 2000.

[3] B. Yong, B. Chen, J. J. Gourley, L. Ren, Y. Hong, X. Chen, W. Wang, S. Chen, and L. Gong, "Intercomparison of the version-6 and version-7 TMPA precipitation products over high and low latitudes basins with independent gauge networks: Is the newer version better in both real-time and post-real-time analysis for water resources and hydrologic extremes?," *J. Hydrol.*, vol. 508, pp. 77-87, 2014.

[4] Z. Li, D. Yang, B. Gao, Y. Jiao, Y. Hong, and T. Xu, "Multiscale hydrologic applications of the latest satellite precipitation products in the Yangtze River Basin using a distributed hydrologic model," *Journal of Hydrometeorology*, vol. 16, pp. 407-426, 2015.

[5] M. I. Mishchenko, L. D. Travis, and A. A. Lacis, *Scattering, Absorption, and Emission of Light by Small Particles*. Cambridge University Press, 2002.

[6] D. W. Mackowski and M. I. Mishchenko, "Calculation of the T matrix and the scattering matrix for ensembles of spheres," *J. Opt. Soc. Am. A*, vol. 13, pp. 2266-2278, 1996.

[7] R. F. Harrington, *Field Computation by Moment Methods*. R. E. Krieger, Malabar, Fla., 1968.

[8] J. J. H. Wang, *Generalized Moment of Methods in Electromagnetics: Formulation and Computer Solution of Integral Equation*. John Wiley & Sons, New York, 1990.

[9] E. K. Miller, L. M. Mitschang, and E. H. Newman, *Computational Electromagnetics: Frequency-Domain Method of Moments*. IEEE Press, New York, 1992.

[10] J. M. Song, C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 10, pp. 1488-1493, 1997.

[11] M. Y. Xia, et al., "An efficient algorithm for electromagnetic scattering from rough surfaces using a single integral equation and multilevel sparse-matrix canonical-grid method," *IEEE Trans. Antennas Propagat.*, vol. 51, no. 6, pp. 1142-1149, 2003.

[12] P. Xu and L. Tsang, "Scattering by rough surface using a hybrid technique combining the multilevel UV method with the sparse matrix canonical grid method," *Radio Science*, vol. 40, no. 4, pp. 2227-2252, 2005.

[13] Q. Li, L. Tsang, K. S. Pak, and C. H. Chan, "Bistatic scattering and emissivities of random rough dielectric lossy surfaces with the physics-based two-grid method in conjunction with the sparse-matrix canonical grid method," *IEEE*

Trans. Antennas Propagat., vol. 48, no. 1, pp. 1-11, 2000.

[14] J. Ling, S. X. Gong, X. Wang, B. Lu, and W. T. Wang, "A novel two-dimensional extrapolation technique for fast and accurate radar cross section computation," *IEEE Antennas & Wireless Propagation Letters*, vol. 9, no. 1, pp. 244-247, 2010.

[15] E. K. Miller, "Model-based parameter estimation in electromagnetics. III. Applications to EM integral equations," *IEEE Antennas and Propagation Magazine*, vol. 40, no. 3, pp. 49-66, 1998.

[16] B. Y. Wu and X. Q. Sheng, "Application of asymptotic waveform evaluation to hybrid FE-BI-MLFMA for fast RCS computation over a frequency band," *IEEE Trans. Antennas Propagat*., vol. 61, no. 5, pp. 2597-2604, 2013.

[17] J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*. Spring-Verlag, Berlin, 1980.

[18] Y. Ding, K.-L. Wu, and D. G. Fang, "A broad-band adaptive frequency sampling approach for microwave-circuit EM simulation exploiting Stoer-Bulirsch algorithm," *IEEE Transactions on Microwave Theory and Techniques*, vol. 51, no. 3, pp. 928-934, 2003.

[19] J. Chen, Z. Liu, P. Shen, D. Ding, and H. Peng, "Adaptive frequency-sampling method for wideband electromagnetic scattering of precipitation particles," *IEEE Antennas and Wireless Propagation Letters*, vol. 10, pp. 835-838, 2011.

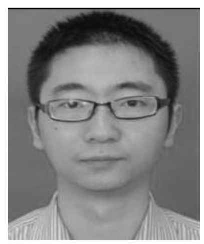
[20] Z. W. Liu, R. S. Chen, and J. Q. Chen, "Adaptive sampling cubic-spline interpolation method for efficient calculation of monostatic RCS," *Microwave & Optical Technology Letters*, vol. 50, pp. 751-755, 2008.

[21] S. L. Zhang, "GPBi-CG: Generalized product-type methods based on Bi-CG for solving nonsymmetric linear systems," *SIAM J. Sci. Comput.*, vol. 18, no. 2, pp. 856-869, 1997.

[22] V. N. Bringi and V. Chandrasekar, *Polarimetric Doppler Weather Radar: Principles and Application*. Cambridge University Press, 2004.

[23] H. R. Pruppacher and R. L. Pitter, "A semi-empirical determination of the shape of cloud and rain drops," *Journal of the Atmospheric Sciences*, vol. 28, pp. 86-94, 1970.

[24] A. W. Green, "An approximation for the shapes of large raindrops," *Journal of Applied Meterology*, vol. 14, pp. 1578-1583, 1975.



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