EM Scattering by a Lossy Dielectric-Coated Nihility Elliptic Cylinder

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***Abstract*—** The problem of electromagnetic wave scattering by a lossy dielectric coated nihility elliptic cylinder is analyzed by solving the analogous problem of scattering by two lossy dielectric layers elliptic cylinder using the method of separation of variables. The incident, scattered, and transmitted fields are expressed in terms of complex Mathieu functions. Numerical results are obtained for the scattered fields of lossy and lossless dielectric coated nihility circular and elliptic cylinders to show their effect on the backscattering widths.

***Index Terms*—** Nihility, lossy dielectric, elliptic cylinder, Mathieu functions.

I. INTRODUCTION

Analytical solution to the problem of a plane electromagnetic wave scattering by a lossless dielectric coated PEC elliptic cylinder has been investigated by many authors [1-2], and the solution was later extended to the nonconfocal dielectric case [3]. Axial slot antenna on a dielectric-coated elliptic cylinder was solved by [4]. Sebak obtained a solution to the problem of scattering from dielectric-coated impedance elliptic cylinder [5]. The scattering by mutilayered dielectric elliptic cylinders has been studied by many authors [6-12]. Recently, scattering by nihility circular cylinders and spheres were studied by many authors by letting the refractive index of the dielectric medium approach zero [13-16].

In this paper, the solution of the electromagnetic wave scattering by a lossy dielectric coated nihility elliptic is obtained by solving the problem of scattering by two lossy dielectric layered elliptic cylinders and letting the refractive index of the inner dielectric layer approach zero (relative permittivity and relative permeability of the inner cylinder are approximately null-valued, i.e., the electromagntic waves cannot propagate in that region). Nihility is unachievable, but it may be approximately simulated in some narrow frequency range [13]. The analysis and the software used for obtaining the numerical results have been validated by calculating the normalized backscattering widths for dielectric coated nihility elliptic cylinder of axial ratios approximately 1, and showing that these results are in full agreement with the same obtained for a lossless dielectric coated nihility circular cylinder analyzed using cylindrical wave functions.

II. FORMULATION OF THE SCATTERING PROBLEM

Consider the case of a linearly polarized electromagnetic plane wave incident on a two lossy dielectric layered elliptic cylinder at an angle ϕ_i with respect to the positive x axis, as shown in Fig. 1.

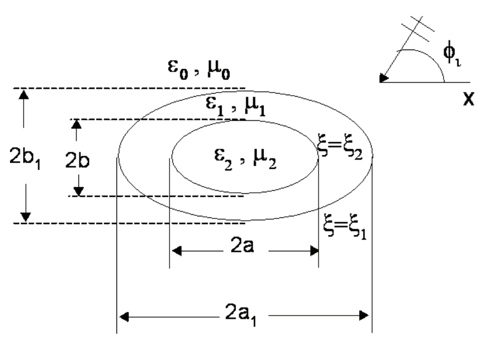

Fig. 1. Geometry of the scattering problem.

The outer dielectric layer has permittivity ε_1 and permeability μ_1 while the inner layer has permittivity ε_2 and permeability μ_2 .The semi-major and semi-minor axes for the inner dielectric layer are a and b and for the outer dielectric layer are a_1 and b_1.

It is convenient to define the x and y coordinates of the Cartesian coordinate system in terms of the u and v coordinates of an elliptical coordinate system also located at the centre of the cylinder in the form of $x = F\cosh u \cos v$ and $y = F\sinh u \sin v$. A time dependence of e^{jwt} is assumed throughout the analysis, but suppressed for convenience.

The electric field component of the TM polarized plane wave of amplitude E_0 is given by

$$E_z^i = E_0 e^{jk_0\rho\cos(\varphi-\varphi_i)} \tag{1}$$

where k_0 is the wave number in free space and $j=\sqrt{-1}$. The incident electric field may be expressed in terms of complex angular and radial Mathieu functions as follows

$$E_z^i = \sum_{m=0}^{\infty} A_{em} R_{em}^{(1)}(c_0,\xi) S_{em}(c_0,\eta) + \sum_{m=1}^{\infty} A_{om} R_{om}^{(1)}(c_0,\xi) S_{om}(c_0,\eta) \tag{2}$$

where,

$$A_{\substack{em\\om}} = E_0 j^m \frac{\sqrt{8\pi}}{N_{\substack{em\\om}}(c_0)} S_{\substack{em\\om}}(c_0,\cos\phi_i) \tag{3}$$

$$N_{\substack{em\\om}}(c) = \int_0^{2\pi} [S_{\substack{em\\om}}(c,\eta)]^2 dv \tag{4}$$

while $c_0 = k_0 F$, F is the semifocal length of the elliptical cross section, $\xi = \cosh u$, $\eta = \cos v$, S_{em} and S_{om} are the even and odd angular Mathieu functions of order m, respectively, $R_{em}^{(1)}$ and $R_{om}^{(1)}$ are the even and odd radial Mathieu functions of the first kind, and N_{em} and N_{om} are the even and odd normalized functions.

The scattered electric field outside the two dielectric layered elliptic cylinder ($\xi > \xi_1$) may be expressed in terms of Mathieu functions as follows

$$E_z^s = \sum_{m=0}^{\infty} B_{em} R_{em}^{(4)}(c_0,\xi) S_{em}(c_0,\eta) + \sum_{m=1}^{\infty} B_{om} R_{om}^{(4)}(c_0,\xi) S_{om}(c_0,\eta) \tag{5}$$

where B_{em} and B_{om} are the unknown scattered field expansion coefficients, $R_{em}^{(4)}$ and $R_{om}^{(4)}$ are the even and odd Mathieu functions of the fourth kind. The transmitted electric field into the outer dielectric layer ($\xi_2 < \xi < \xi_1$) may be written as

$$E_z^I = \sum_{m=0}^{\infty} \left[C_{em} R_{em}^{(1)}(c_1,\xi) + D_{em} R_{em}^{(2)}(c_1,\xi) \right] S_{em}(c_1,\eta) + \sum_{m=1}^{\infty} \left[C_{om} R_{om}^{(1)}(c_1,\xi) + D_{om} R_{om}^{(2)}(c_1,\xi) \right] S_{om}(c_1,\eta) \tag{6}$$

where $c_1 = k_1 F$, $k_1 = \omega\sqrt{\mu_1\varepsilon_1}$, $\varepsilon_1 = \varepsilon_1^{'} - j\varepsilon_1^{''}$, C_{em}, C_{om}, D_{em}, D_{em} are the unknown transmitted field expansion coefficients, and $R_{em}^{(2)}$ and $R_{om}^{(2)}$ are the radial Mathieu functions of the second type.

Similarly, the transmitted electric field into the inner dielectric layer ($\xi < \xi_2$) may be written as

$$E_z^{II} = \sum_{m=0}^{\infty} F_{em} R_{em}^{(1)}(c_2,\xi) S_{em}(c_2,\eta) + \sum_{m=1}^{\infty} F_{om} R_{om}^{(1)}(c_2,\xi) S_{om}(c_2,\eta) \tag{7}$$

where $c_2 = k_2 F$, $k_2 = \omega\sqrt{\mu_2\varepsilon_2}$, $\varepsilon_2 = \varepsilon_2^{'} - j\varepsilon_2^{''}$, and F_{em}, F_{om} are the unknown transmitted field expansion coefficients.

The magnetic field components inside and outside the elliptic cylinder can be obtained using Maxwell's equation, i.e.,

$$H_u = \frac{-j}{\omega\mu\, h} \frac{\partial E_z}{\partial v} \tag{8}$$

$$H_v = \frac{-j}{\omega\mu h} \frac{\partial E_z}{\partial u} \tag{9}$$

where $h = F\sqrt{\cosh^2 u - \cos^2 v}$. The unknown expansion coefficients in equation (5) can be obtained by imposing the boundary conditions at the various interfaces. Continuity of the tangential field components at $\xi = \xi_1$ and $\xi = \xi_2$ require that

$$E_z^i + E_z^s = E_z^I \qquad \xi = \xi_1 \qquad (10)$$

$$H_v^i + H_v^s = H_v^I \qquad \xi = \xi_1 \qquad (11)$$

$$E_z^I = E_z^{II} \qquad \xi = \xi_2 \qquad (12)$$

$$H_v^I = H_v^{II} \qquad \xi = \xi_2. \qquad (13)$$

Substituting equations (2), (5)-(7), into equations (10)-(13) and applying the orthogonality property will lead to a system of equations which may be solved numerically for the unknown scattered field coefficients B_{em} and B_{om}.

The expressions for the incident, scattered and transmitted electromagnetic fields for TE case can be obtained using the duality principle of the TM case.

III. NUMERICAL RESULTS

The obtained numerical results are presented as normalized echo pattern widths for lossy dielectric coated nihility circular and elliptic cylinders of different sizes, axial ratios, incident angles and permittivities, for both TM and TE polarizations of the incident wave. To validate the analysis and the software used for calculating the results, we have computed the normalized echo pattern widths for lossy $(\varepsilon_1 = 9.8 - j0.5)$ and lossless $(\varepsilon_1 = 9.8 - j0.0)$ dielectric coated nihility elliptic cylinders of axial ratio 1.001, $k_o a = 1.05$, $k_o a_1 = 2.1$, $\varepsilon_1 = 9.8 - j0.0$, and $\varphi_i = 180^o$. The numerical results have been shown in Fig. 2, and they are in full agreement for lossless case, verifying the accuracy of the analysis as well as the software used for obtaining the results [15].

Figure 3 shows the echo width pattern for a lossy dielectric coated nihility elliptic cylinder for both TM and TE cases with an incident angle of $\varphi_i = 180^o$. The numerical results are plotted for $\varepsilon_1 = 4.0 - j0.0$ and $\varepsilon_1 = 4.0 - j0.5$. The electrical dimensions of the scatterer are $k_o a = 2.50$, $k_o a_1 = 3.51$, $k_o b = 1.25$, and $k_o b_1 = 2.76$. It can be seen that the TM nihility lossless case has higher echo width values in the backward directions compared to the TE case. The lossy TM and TE cases have lower values compared to the losseless case in the forward and backscattering directions. Figure 4 shows a similar case with an incident $\phi_i = 90^o$.

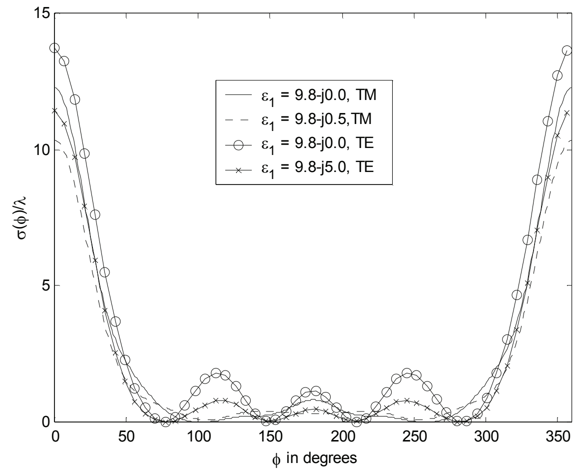

Fig. 2. Echo width pattern against the scattering angle ϕ of a lossy dielectric coated nihility circular cylinder with $k_o a = 1.05$, $k_o a_1 = 2.1$ and $\varphi_i = 180^o$.

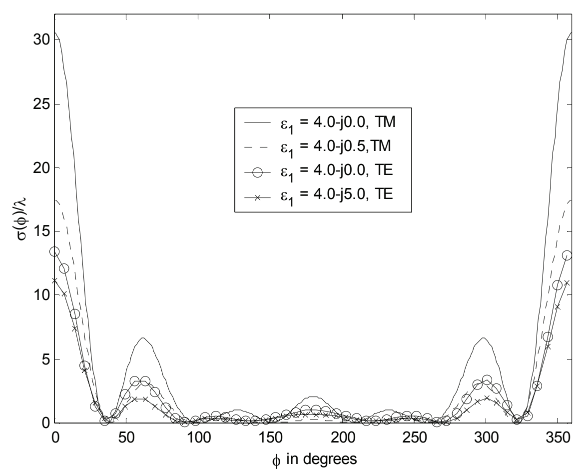

Fig. 3. Echo width pattern against the scattering angle ϕ of a lossy dielectric coated nihility elliptic cylinder with $k_o a = 2.50$, $k_o a_1 = 3.51$, $k_o b = 1.25$, and $k_o b_1 = 2.76$, and $\varphi_i = 180^o$.

Figure 5 shows the backscattering echo width pattern against $\varepsilon_1^{'}$ of a lossy and lossless dielectric coated nihility elliptic cylinder. The electrical dimensions are $k_o a = 1.25$, $k_o a_1 = 2.18$, $k_o b = 0.62$, and $k_o b_1 = 1.88$, and the incident angle is $\phi_i = 0^o$. It can be seen that the backscattering echo width for nihility coated lossless case is higher than the coated lossless PEC case for higher values of $\varepsilon_1^{'}$, namely more than 8.

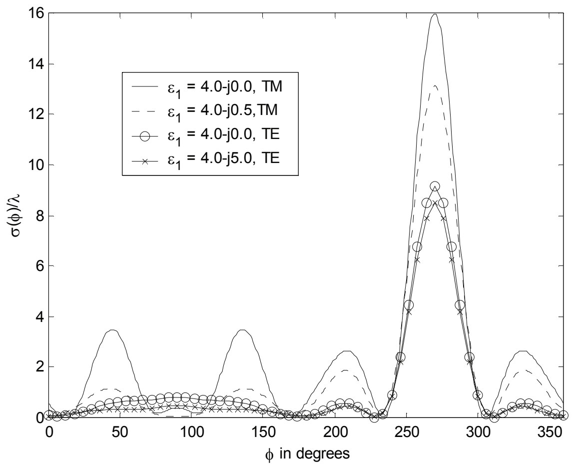

Fig. 4. Echo width pattern against the scattering angle ϕ of a lossy dielectric coated nihility elliptic cylinder with $k_o a = 2.50$, $k_o a_1 = 3.51$, $k_o b = 1.25$, and $k_o b_1 = 2.76$, and $\varphi_i = 90^o$. Fig.

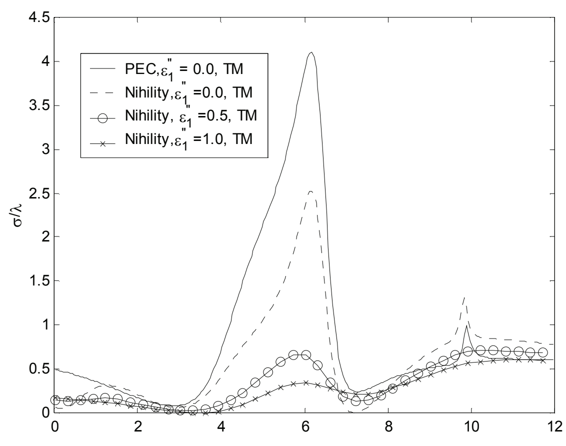

Fig. 5. Backscattering echo width pattern against $\varepsilon_1^{'}$ of a lossy dielectric coated nihility elliptic cylinder with $k_o a = 1.25$, $k_o a_1 = 2.18$, $k_o b = 0.62$, and $k_o b_1 = 1.88$, and $\phi_i = 0^o$.

Figure 6 shows the backscattering echo width pattern against $k_o a_1$ of a lossy and lossless dielectric coated nihility elliptic cylinder. The electrical dimensions are $k_o a = 0.6$, $k_o a_1 = 2.18$, $k_o b = 0.5$, and $k_o b_1$ changes simultaneously with $k_o a_1$ for incident angle $\phi_i = 0^o$. It can be seen that the backscattering echo width for nihility coated lossless case is lower than the coated lossless PEC case for $k_o a_1$ greater than 2.5.

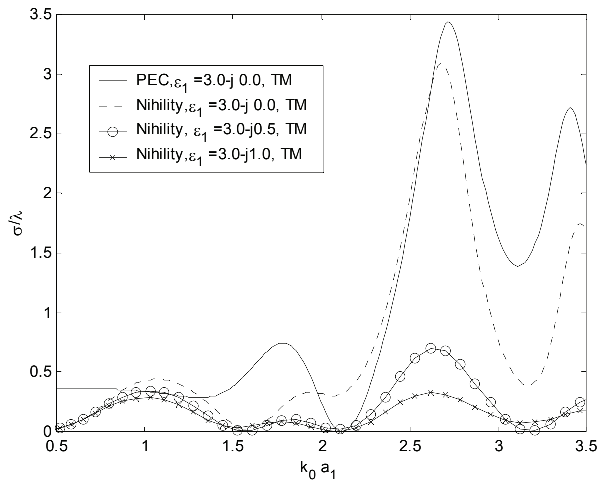

Fig. 6 Backscattering echo width pattern against $k_o a_1$ of a lossy dielectric coated nihility elliptic cylinder with $k_o a = 0.6$, $k_o b = 0.5$ and $\phi_i = 0^o$.

IV. CONCLUSIONS

Analytical solution of the electromagnetic wave scattering by lossy and lossless dielectric coated nihility circular and elliptic cylinders is obtained for TM and TE polarizations. The validity and accuracy of the obtained numerical results were verified against the case of lossless dielectric coated nihility circular cylinder using cylindrical wave functions and the agreement was excellent. Numerical results were presented as a function of the geometrical parameters of the cylinder and can be used to design scatterers with reduced or enhanced backscattering echo widths. Further, the solution is general where the special case of coated nihility circular cylinder may be obtained by letting the axial ratios approximately equal to 1.0 while the special case of coated nihility strip may be obtained by letting the thickness of the inner cylinder vanishes (letting the minor axis of the inner cylinder approaches zero). The nihility medium as well as different loss tangents considered in the numerical results showed a significant change in echo width patterns, i.e.

reduced at some geometrical parameters while enhance at others. It is worth mentioning that nihility material may be released from metamaterial where the refractive index approaches zero, i.e., metamaerial is made of mixture of helices and resonant dipoles [17], and nihility could happen at frequencies higher than optical frequencies.

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