Calculation of the Magnetic Forces Between Planar Spiral Coils using Concentric Rings

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***Abstract*—** In this paper, the forces between current carrying planar spiral coils are calculated. In order to facilitate the calculation process, the coils have been replaced by concentric rings and using first and second order complete elliptic integrals, the forces between them have been calculated. The comparison of the calculations resulting from the replaced rings method and the direct method shows that the former is more effective in both simplicity and calculation time. To evaluate the precision of the calculations, planar spiral coils have been constructed and tested. The experimental results validate the results of the calculations.

***Index Terms*—** Planar spiral coils, magnetic force, vector magnetic potential, concentric rings.

I. INTRODUCTION

Planar spiral coils are used extensively in different applications such as communications, power electronics, and casting industries [1-3]. In these systems, to have a high inductance and flat configuration, spiral windings are employed. In DC/DC converters, because of flatness and special configuration, planar spiral coils are a better replacement for the ordinary inductances in order to reduce the volume of the converter. To calculate the magnetic force between these coils, some methods have been reported in literature. In [2] these forces are obtained just by test. In [3] the finite difference method is employed to calculate the force between them; furthermore, in this reference to calculate the magnetic force, spiral coils are replaced by concentric rings, but there is no study and discussion on the precision of the method. In [4] the force between circular coaxial coils has been investigated. Recently, the above authors employed mesh-matrix method in order to calculate the force between spiral coils [5]. In this paper, using concentric rings instead of spiral coils, an effective and simple procedure is developed to calculate the magnetic force between these coils. Using the results obtained from the numerical solution of the direct calculation method, the precision of the proposed method is investigated and finally compared with experimental results.

II. DIRECT CALCULATION METHOD

Consider a system of two spiral coils as shown in Fig. 1. To calculate the magnetic force between them, we should first calculate the vector magnetic potential resulting from one of the coils in any given point like P (see Fig. 2).

Vector magnetic potential of spiral coil 1 in any given point P is obtained by the following equation [6]:

$$A = \frac{\mu_0 I_1}{4\pi} \oint \frac{dl'}{R_1} \tag{1}$$

where I_1 is the current of the coil, dl' is the longitudinal differential component, and R_1 is the distance between this differential component and point P.

The coordinates marked by prime are related to the source. With suitable substitutions for dl', the following equation for vector magnetic potential is obtained:

$$A = \frac{\mu_0 I_1}{4\pi} \oint \frac{[-a_x \sin\phi' + a_y \cos\phi'] r' d\phi' + [a_x \cos\phi' + a_y \sin\phi'] dr'}{R_1}. \tag{2}$$

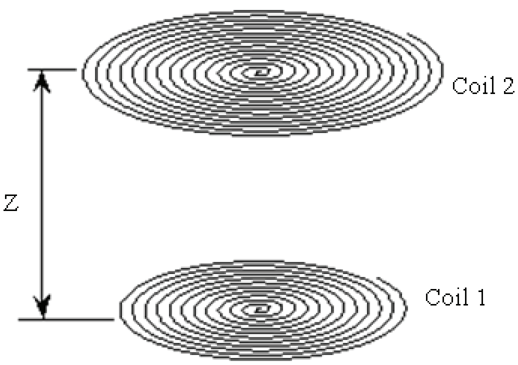

Fig. 1. The two spiral coils in z distance of each other.

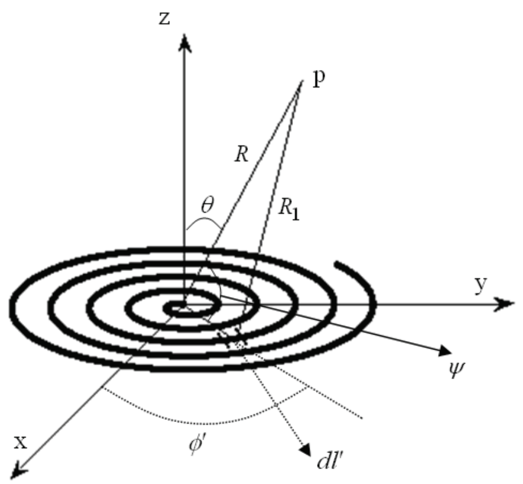

Fig. 2. Calculation of the vector magnetic potential of spiral coils in any given point like P.

To calculate the integral in (2), one of the integral variables must be replaced by another one according to the relations between them. The variables ϕ' and r' have a linear relation; consequently, we can write [7, 8]:

$$\phi' = K_1 r' \tag{3}$$

where K_1 is a constant coefficient that is called "compression factor" of coil 1. This factor depends on the diameter of the wire used and the structure of the coil and determines its compression. Having the vector magnetic potential, the magnetic field is calculated using the following equation [6]:

$$B = \nabla \times A \tag{4}$$

The force acted on the coil 2 is [9]:

$$F_{21} = I_2 \oint_{C_2} dl_2 \times B \tag{5}$$

In the above equation, dl_2 is longitudinal differential component on coil 2. Substituting proper expression for dl_2 and employing (4) in (5) and doing some mathematical calculations, we get:

$$F_{21} = a_x f_x + a_y f_y + a_z f_z \tag{6}$$

where f_x, f_y and f_z are the components of the force in directions x, y and z, respectively, and are equal to (7)-(9) at the bottom of the page.

In equations (7)-(9), the parameters r_1' and r_1 are the inner radii of coil 1 and 2, respectively, and r_2' and r_2 are the outer radii of coils 1 and 2, respectively. Also, the following equation has been used [7, 8]:

$$\phi = K_2 r \tag{10}$$

where K_2 is compression factor of coil 2 determined with regard to the compression of the coil and the diameter of the wire used in it.

III. CONCENTRIC RINGS METHOD

In the previous section, it was observed that to obtain the force between spiral coils, using the analytical method is slightly complex and time-consuming. Furthermore, the obtained integrands are not smooth functions, and we have some difficulties in the calculation of their integrals. Especially when the coils are compressively wounded, the problem is more acute.

$$f_x = \frac{\mu_0 I_1 I_2}{4\pi} \int_{r_1}^{r_2} \int_{r_1'}^{r_2'} \frac{[r\sin(K_2 r - K_1 r') - K_1 r r' \cos(K_2 r - K_1 r') + K_1 r'^2][\sin(K_2 r) + K_2 r \cos(K_2 r)]}{[(r\cos(K_2 r) - r'\cos(K_1 r'))^2 + (r\sin(K_2 r) - r'\sin(K_1 r'))^2 + z^2]^{3/2}} dr' dr \tag{7}$$

$$f_y = -\frac{\mu_0 I_1 I_2}{4\pi} \int_{r_1}^{r_2} \int_{r_1'}^{r_2'} \frac{[r\sin(K_2 r - K_1 r') - K_1 r r' \cos(K_2 r - K_1 r') + K_1 r'^2][\cos(K_2 r) + K_2 r \sin(K_2 r)]}{[(r\cos(K_2 r) - r'\cos(K_1 r'))^2 + (r\sin(K_2 r) - r'\sin(K_1 r'))^2 + z^2]^{3/2}} dr' dr \tag{8}$$

$$f_z = -\frac{\mu_0 I_1 I_2}{4\pi} z \int_{r_1}^{r_2} \int_{r_1'}^{r_2'} \frac{(1 + K_1 K_2 r r')\cos(K_2 r - K_1 r') - (K_2 r - K_1 r')\sin(K_2 r - K_1 r')}{[(r\cos(K_2 r) - r'\cos(K_1 r'))^2 + (r\sin(K_2 r) - r'\sin(K_1 r'))^2 + z^2]^{3/2}} dr' dr \tag{9}$$

To overcome this problem, accepting some errors, we can replace the spiral coils with concentric rings and then calculate the forces between them [7]. For this purpose, we first calculate the force between two concentric current carrying rings. Suppose rings 1 and 2 with radiuses a and b while carrying currents I_1 and I_2, respectively (see Fig. 3). To obtain the force exerted on the upper ring from the lower ring, we first calculate the magnetic field of lower ring in any given point P. To calculate the magnetic field, we use the vector magnetic potential concept. The vector magnetic potential of ring 1 in any point P on ring 2, using (1), is equal to (Fig. 4):

$$A = a_\phi \, f(R,\theta) \tag{11}$$

in which the function $f(R,\theta)$ is as follows:

$$f(R,\theta) = \frac{\mu_0 I_1}{4\pi}\int_0^{2\pi} \frac{a\sin\phi'}{\sqrt{R^2 + a^2 - 2aR\sin\theta\sin\phi'}}d\phi'. \tag{12}$$

In the above equation, a is the radius of ring 1, μ_0 is the permeability of vacuum, I_1 is the current of ring 1, and R is the distance between the origin and the field point P (Fig. 4). By obtaining the vector magnetic potential, magnetic field is calculated using (4). Substituting (11) and (12) in (4) and doing some mathematical calculations, we get:

$$B = a_R \, g_1(R,\theta) + a_\theta \, g_2(R,\theta) \tag{13}$$

where g_1 and g_2 are:

$$g_1(R,\theta) = \frac{1}{R\sin\theta}[\cos\theta \; f(R,\theta) + \sin\theta \frac{\partial}{\partial\theta} f(R,\theta)] \tag{14}$$

$$g_2(R,\theta) = -\frac{1}{R}[f(R,\theta) + R\frac{\partial}{\partial R} \; f(R,\theta)]. \tag{15}$$

The force exerted on ring 2 from ring 1 is calculated using (5). Applying (13)-(15) to (5) and also substituting an appropriate expression for dl_2 and doing some simple mathematical calculations, the following equation for the force is obtained:

$$F_{21} = -a_z \frac{\mu_0 \, abI_1 I_2 \, z}{2} \int_0^{2\pi} \frac{\sin\phi'}{[z^2 + a^2 + b^2 - 2ab\sin\phi']^{3/2}} d\phi'. \tag{16}$$

In the above equation, b is the radius of ring 2 and z is the axial distance between the two rings.

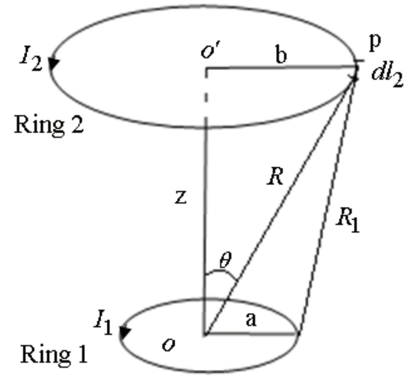

Fig. 3. Two concentric current carrying rings.

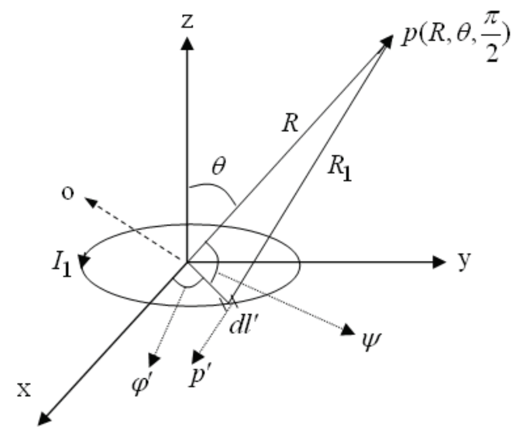

Fig. 4. Determination of vector potential of a current carrying ring with radius a in any given point P.

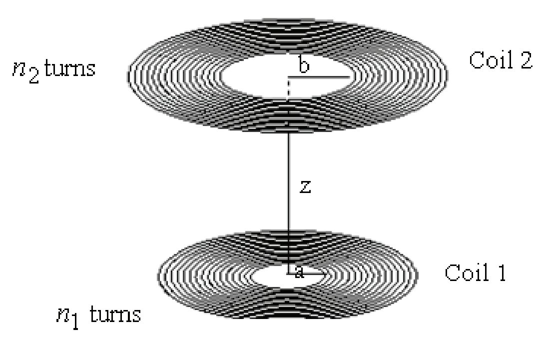

Fig. 5. Two coils with concentric rings.

The force obtained in (16) has no analytical solution, so we can use numerical integration methods to solve it. Changing the integral variable as $\phi' = (3\pi/2) + 2\theta$ in (16), the following equation for the force is obtained:

$$F_{21} = a_z \left(\frac{\mu_0 \quad I_1 I_2 \quad z \quad k}{2\sqrt{ab} \quad (1-k^2)} \right) \tag{17}$$
$$[(1-k^2)K(k) - (1-\frac{1}{2}k^2)E(k)]$$

where k is a constant parameter and is equal to:

$$k = \sqrt{\frac{4ab}{(a+b)^2 + z^2}} \tag{18}$$

and $K(k)$ and $E(k)$ are the first and the second order elliptic integrals, respectively, with the following definitions:

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1-k^2 \sin^2 \theta)^{1/2}} \tag{19}$$

$$E(k) = \int_0^{\frac{\pi}{2}} (1-k^2 \sin^2 \theta)^{1/2} \quad d\theta. \tag{20}$$

Now, having the force between the two rings, we can calculate the force between the two spiral coils after replacing them by concentric rings (Fig. 5). The magnetic force between the two coils (the force exerted on coil 2 from coil 1 in Fig. 5) will be as follows:

$$F_{21} = I_1 \, I_2 \sum_{j=0}^{n_2-1} \sum_{i=0}^{n_1-1} f_{21}(j,i) \tag{21}$$

where n_1 and n_2 are the number of turns of coil 1 and 2, respectively, and $f_{21}(j,i)$ is equal to:

$$f_{21}(j,i) = a_z \left(\frac{\mu_0 \, z k'}{2\sqrt{a_i \, b_j} \; (1-k'^2)} \right) [(1-k'^2)K(k')$$
$$-(1-\frac{1}{2}k'^2)E(k')] \; . \tag{22}$$

In the above equation, z is the distance between the two coils, and the parameters a_i, b_j and k' are defined as:

$$a_i = a_0 + (\frac{1}{2} + i) s_1 \tag{23}$$

$$b_j = b_0 + (\frac{1}{2} + j) s_2 \tag{24}$$

$$k' = \sqrt{\frac{4a_i b_j}{(a_i + b_j)^2 + z^2}} \tag{25}$$

where a_0 and b_0 are the inner radius of coils 1 and 2 and s_1 and s_2 are the distance between two neighboring turns in coils 1 and 2, respectively. If the coils are wound compressively, then s_1 and s_2 must be replaced by the diameter of the wires used in coils 1 and 2, respectively.

IV. CALCULATION RESULTS

In Section II, the force between two spiral coils was analytically obtained (equation (6)). Suppose that the compression factors of the coils are high. In this case, the force values in the x and y directions are almost zero, and the component of the force in the z direction is non- zero [7] which is given by equation (9). The force in this relation is the force exerted on coil 2 from coil 1 as it is shown in Fig. 1. Although we use precise analytical relations to obtain the force in (9), its integral has no analytical solution, and numerical integration techniques must be used to solve it. The integrand of the equation (9) has some "semi-poles" which depend on the value of the compression factors K_1 and K_2. The curve of the integrand versus variables r and r' is shown in Fig. 6 for different values of r and r' from 0 to 1. As seen in the figure, by increasing the values of r and r' from zero, the value of the integrand produces some sharp peaks (the semi-poles points). It is clear that integration of these surfaces is much more difficult because in order to obtain higher precisions, one needs to increase the number of iterations of numerical integration intensively which, in turn, requires much longer computational time to solve such a problem.

Now we compare the results of direct calculation of the force using equation (9) with that of the replaced concentric rings method. To calculate the integral in equation (9), we used recursive adaptive Simpson Quadrature method. In the replaced concentric rings method, the radius of each ring is assumed to be the average of the inner and the outer radii of each turn of spiral coils. In Tables 1 and 2 the results of calculation of the force using two methods for different values of turn number and different center to center distance of coils are compared. In these tables, the current in both coils is 20 Amperes, the diameter of the wires is 2 mm, and the compression factor for both coils is assumed to be $2\pi/d$, where d is the diameter of the wires in both coils; meaning that for each turn of coils or for change of 2π Radians in the value of variable ϕ in cylindrical coordinates, the change in the value of variable r (the radial growth of coils) is equal to diameter of the wires used in the coils. In Table 1, it is assumed that the coils start to grow from point (0, 0). Comparing the results of the two methods in this table, it is seen that for the fewer number of

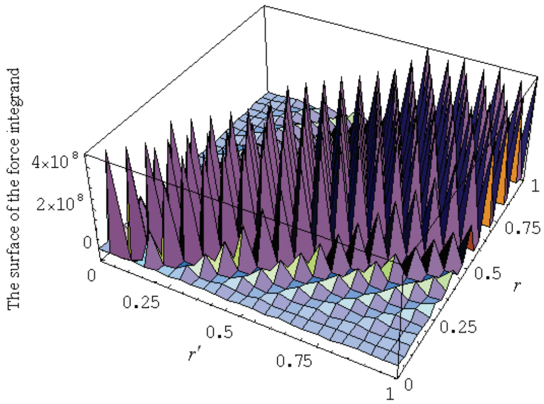

Fig. 6. The integrand in force equation for 500 turns in each coil.

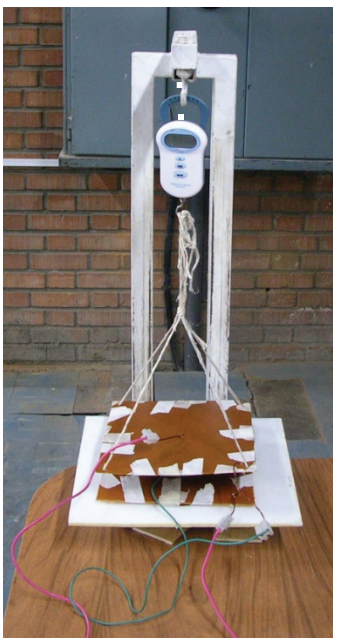

Fig. 7. Measurement of the magnetic force between the two spiral coils.

turns the error is high, but by increasing the number of turns, the error gradually decreases, and when the turn number approaches to 100, the error becomes zero. In Tables 1 and 2 the precision of the calculations is adjusted according to the numerical value of the results. For instance, for the first column of Table 1 the calculated numbers are in the range of 10^{-13} (their minimum value). To compare the calculation time in the two approaches, it suffices to mention that the required calculation time using the adaptive Simpson method for 100 turns in Table 1 for precision of 10^{-4} is 28000 times more than that of using replaced concentric rings method. As seen in the table, the results precisely coincide with each other. Another interesting point about Table 1 is that by increasing the distance between the two coils, the calculation error increases showing that in large distances, the replaced concentric rings method does not present a proper approximation of the force.

In Table 2, the comparison between the two methods is made for the case in which the inner radius of the two coils are equal to 2.5cm; in other words, the coils start to wind from $r = 2.5cm$. As seen from the results of the table, the errors in this case are less than the corresponding errors in Table 1. For example, the force error for 2 turn coils in distance of 8 cm reduced from 94.6% in Table 1 to 0.12% in Table 2. These fewer errors for lower turn numbers decrease expeditiously to zero by increasing the turn numbers.

According to the results of Tables 1 and 2, generally for turn numbers higher than 10 turns in each coil, using the replaced concentric rings presents good approximations while having much simpler and faster calculations compared with that of the direct method and using (9).

Now, suppose the case in which there is a smaller compression factor for the coils compared with the previous one, i.e. for each turn of coils or for change of 2π Radians in the value of variable ϕ in cylindrical coordinate, the change in the value of variable r is more than the diameter of the wires used in the coils. For example, suppose that the growth of r is equal to 6 mm; in this case, the compression factor for both coils will be:

$$K_1 = K_2 = \frac{2\pi}{0.006}.$$

The results of the calculations of the force with the above mentioned conditions using the methods of direct and replaced rings are presented in Table 3. In this table, like the previous cases, the current of the coils is 20 Amperes.

It is interesting to compare the results of Tables 1 and 3. In Table 3, the trend of increasing and decreasing of error with the increase of the distance between the two coils and the number of turns is the same as Table 1; but in this case, the

Table 1. Comparison of the force calculation methods between two spiral coils (inner radii and compression factor of the coils are 0 and $2\pi/0.002$, respectively).

	Number of Turns or Rings Per Coil	2	5	10	20	50	100
Z=2cm	Direct Method (N)	2.9854×10^{-6}	2.3725×10^{-4}	5.6356×10^{-3}	7.1760×10^{-2}	1.0079	5.4610
	Replaced Rings Method (N)	1.3416×10^{-6}	2.2745×10^{-4}	5.6091×10^{-3}	7.1713×10^{-2}	1.0078	5.4610
	Error (%)	55.1	4.1	0.47	0.07	0.01	0
Z=4cm	Direct Method (N)	4.9389×10^{-7}	2.3834×10^{-5}	9.2665×10^{-4}	2.2472×10^{-2}	5.8029×10^{-1}	4.0310
	Replaced Rings Method (N)	9.0210×10^{-8}	2.1251×10^{-5}	9.1686×10^{-4}	2.2445×10^{-2}	5.8024×10^{-1}	4.0310
	Error (%)	81.7	10.8	1.1	0.12	0.01	0
Z=8 cm	Direct Method (N)	1.0599×10^{-7}	2.1389×10^{-6}	8.8644×10^{-5}	3.6843×10^{-3}	2.1795×10^{-1}	2.3208
	Replaced Rings Method (N)	5.7461×10^{-9}	1.5053×10^{-6}	8.6061×10^{-5}	3.6745×10^{-3}	2.1791×10^{-1}	2.3208
	Error (%)	94.6	29.6	2.9	0.27	0.02	0

* Precision of the calculations in numerical integration for rings of 2 to 100 turns are $0.5\times10^{-13}, 0.5\times10^{-10}, 0.5\times10^{-9}, 0.5\times10^{-7}, 0.5\times10^{-5}$ and 0.5×10^{-4}, respectively.

Table 2. Comparison of the force calculation methods between two spiral coils (inner radii and compression factor of the coils are 2.5cm and $2\pi/0.002$, respectively).

	Number of Turns or Rings Per Coil	2	5	10	20	50	100
Z=2cm	Direct Method (N)	1.8844×10^{-3}	1.3230×10^{-2}	5.8839×10^{-2}	2.5524×10^{-1}	1.7033	7.0917
	Replaced Rings Method (N)	1.8851×10^{-3}	1.3234×10^{-2}	5.8849×10^{-2}	2.5526×10^{-1}	1.7033	7.0917
	Error (%)	-0.04	-0.03	-0.02	-0.01	0	0
Z=4cm	Direct Method (N)	5.1474×10^{-4}	4.0012×10^{-3}	2.0961×10^{-2}	1.1654×10^{-1}	1.0846	5.4020
	Replaced Rings Method (N)	5.1465×10^{-4}	4.0009×10^{-3}	2.0961×10^{-2}	1.1654×10^{-1}	1.0846	5.4020
	Error (%)	+0.02	+0.01	0.0	0.0	0	0
Z=8 cm	Direct Method (N)	7.6578×10^{-5}	6.6545×10^{-4}	4.1678×10^{-3}	3.1257×10^{-2}	4.8080×10^{-1}	3.2851
	Replaced Rings Method (N)	7.6487×10^{-5}	6.6492×10^{-4}	4.1660×10^{-3}	3.1252×10^{-2}	4.8079×10^{-1}	3.2851
	Error (%)	+0.12	+0.08	+0.04	+0.02	+0.002	0

* Precision of the calculations in numerical integration for rings of 2 to 100 turns are $0.5\times10^{-9}, 0.5\times10^{-8}, 0.5\times10^{-7}, 0.5\times10^{-6}, 0.5\times10^{-5}$ and 0.5×10^{-4}, respectively.

Table 3. Comparison of the force calculation methods between two spiral coils (inner radii and compression factor of the coils are 0 and $2\pi / 0.006$, respectively).

	Number of Turns or Rings Per Coil	2	5	10	20	50	100
Z=2cm	Direct Method (N)	7.3291×10^{-5}	3.0192×10^{-3}	2.7852×10^{-2}	1.7814×10^{-1}	1.5234	6.8511
	Replaced Rings Method (N)	5.9885×10^{-5}	2.9807×10^{-3}	2.7794×10^{-2}	1.7806×10^{-1}	1.5234	6.8511
	Error (%)	18.3	1.3	0.2	0.05	0.0	0
Z=4cm	Direct Method (N)	9.7945×10^{-6}	7.3240×10^{-4}	1.1940×10^{-2}	1.1105×10^{-1}	1.2302	6.0918
	Replaced Rings Method (N)	6.0653×10^{-6}	7.1382×10^{-4}	1.1902×10^{-2}	1.1100×10^{-1}	1.2301	6.0918
	Error (%)	38.1	2.5	0.3	0.05	0.01	0
Z=8 cm	Direct Method (N)	1.3593×10^{-6}	9.5316×10^{-5}	2.8855×10^{-3}	4.7627×10^{-2}	8.3326×10^{-1}	4.9196
	Replaced Rings Method (N)	4.4275×10^{-7}	8.9553×10^{-5}	2.8669×10^{-3}	4.7589×10^{-2}	8.3320×10^{-1}	4.9196
	Error (%)	67.4	6.1	0.65	0.08	0.01	0

* Precision of the calculations in numerical integration for rings of 2 to 100 turns are $0.5\times10^{-11}, 0.5\times10^{-9}, 0.5\times10^{-7}, 0.5\times10^{-6}, 0.5\times10^{-5}$ and 0.5×10^{-4}, respectively.

Table 4. Characteristics of the constructed spiral coils

	Number of turns	Inner radius(cm)	Diameter of wire used (mm)
Coil 1	54	2.15	1.6
Coil 2	55	2.0	1.6

Table 5. Experimental results and their comparison with calculation results of the replaced rings method

	Current of Coils (A)	4.8	5.4	7.3	10.6	14.1
Z=2 cm	Measured Force (N)	0.1079	0.1373	0.2453	0.5199	0.9123
	Calculated Force (N)	0.1067	0.1351	0.2469	0.5206	0.9211
Z=4 cm	Measured Force (N)	0.0687	0.0785	0.1472	0.3139	0.5592
	Calculated Force (N)	0.0640	0.0810	0.1480	0.3119	0.5520
Z=8 cm	Measured Force (N)	0.0196	0.0294	0.0589	0.1275	0.2158
	Calculated Force (N)	0.0256	0.0324	0.0592	0.1248	0.2208

calculated percentage of relative error of the force is lower than the corresponding values in Table 1. At first, it seemed that by decreasing the compression factor the calculation error increases, but this assumption is not true because by decreasing the compression factor, the relative error of calculations with replaced rings method decreases. This is also true for smaller compression factors [10].

V. THE EXPERIMENTAL RESULTS

In order to evaluate the precision of the replaced concentric rings method in calculating the force between coils, two coils with different radii were precisely constructed in the laboratory with the characteristics presented in Table 4.

To precisely measure the repelling and attracting forces between the coils, a test as illustrated in Fig. 7, is arranged. In this figure, one of the coils is placed on a fiber board isolator whose permeability is the same as air, and the other coil is connected to a digital force meter via four pieces of string and a fiber board isolator. So, by applying current to the circuit of the two coils, the force exerted on the higher coil, which is equal to the force on the lower coil, is precisely measured.

In Table 5, the calculation and experimental results for different distances are presented. To obtain the calculation results of this table, due to the large number of turns and inner radii for the coils, the replaced concentric rings method is employed. Regarding the results and the explanations of the previous section, using this method in this case causes no significant error. As Table 5 shows, the results of the force measurement are in good accordance with the results of the calculations, validating the precision of the proposed method.

VI. CONCLUSION

In this paper, the force between spiral coils is calculated using two methods: direct method and replaced concentric rings method. In the direct method, we face integrals with no analytical solutions. The numerical solution of these integrals, due to the fact that the integrands are not smooth, is difficult and time-consuming. To overcome this problem, we employed replaced concentric rings method which has simpler calculations and reduces the calculation time. Due to the obtained results, the calculation error of the replaced rings method for number of turns more than 10 is negligible, and the method is effective. These errors are reduced by increasing the inner radius of spiral coils, which is the case of many practical applications, and are acceptable values in lower turn numbers, too. According to the measurements done on the constructed coils, the calculation results are in good agreement with the experimental results, validating the effectiveness of the replaced rings method.

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