A New Method for Estimating the Direction-of-Arrival Waves by an Iterative Subspace-Based Method

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***Abstract*—** In this paper, a new subspace-based method for high-resolution direction-of-arrival (DOA) estimation of multiple plane waves in a noisy environment is proposed. This method called Iterative-Subspace-Decomposition (ISD) involves an iterative decomposition into two blocks of the matrices corresponding to the noise and the source subspaces. The proposed algorithm provides enhanced estimation performance of the DOA. It also significantly improves the resolution capability with respect to existing algorithms. The method can be applied to low signal-to-noise ratio (SNR) environment and is suitable for arrays with arbitrary sensor geometries, including linear arrays. Several numerical simulations are presented to assess the proposed method enhanced performance in comparison to that obtained by some classical algorithms. This comparative study has shown that the ISD leads to a significant reduction in the Root Mean Square Errors (RMSE) and resolution rate capabilities of the DOA estimates. As though, it is shown that the ISD method is superior in resolving closely spaced signals with a small number of snapshots and at low SNR.

***Index Terms*—** Subspace-based methods, DOA estimation, iterative decomposition, high-resolution capability, estimation performance, arbitrary sensor geometries, linear arrays.

I. INTRODUCTION

Direction of Arrival (DOA) estimation of narrowband wave fronts impinging on an array of sensors has long been of great interest in several applications [1]. Numerous techniques have been developed to determine the angle of arrival of signals incident on an antenna array [2]. These methods typically are based on the phase difference of the signal at adjacent elements in the antenna array since this phase difference is proportional to the angle of arrival of the incoming signal.

In the literature, the classical subspace based methods have been investigates extensively [3]. Super-resolution techniques have also been developed that take advantage of the structure of the input data model. These methods, including MUSIC [4], ESPRIT [5] and SUMWE [6], fall into a class of algorithms known as subspace-based techniques [7]. Nevertheless, the existing linear operation based methods, e.g., Propagator [8], find the signal or noise subspace from the array data by partitioning the array response matrix or exploiting the array geometry and its shift invariance property, and then estimate the directions of arrival (DOAs) of incident signals by the way similar to the classical MUSIC estimator. However, it is shown in [6] that the estimation accuracy of the linear operation based methods is generally poorer than the classical subspace based methods (e.g., MUSIC) from the statistical viewpoint. That's why MUltiple SIgnal Classification (MUSIC) is a popular high-resolution technique for estimating the DOA of

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multiple plane waves in a noisy environment, using an array of M sensors [9]. The method involves eigendecomposition of the spectral covariance matrix Γ of the M-dimensional data vector to determine the noise and the source subspace. The matrix Γ is estimated from a finite number of samples of the data vector. For a given data size K, reduction of the signal-to-noise ratio (SNR) at the sensor array output causes an increase in the covariance matrix estimation error and a corresponding increase in the DOA estimation error [10]. The estimation errors may be reduced by increasing K, but requirements of temporal coherence and speed impose an upper limit on the permissible value of K.

Inevitably, the performance of the MUSIC estimator suffers a progressive degradation as the SNR is reduced. In the case of finite data samples, it cannot resolve adjacent sources with large power level differences between them. In this work, the possibility of using an Iterative-Subspace-Decomposition (ISD) technique is explored to improve the performance of the MUSIC in low SNR environment. The ISD method is based on the invariance property of noise and source subspace after an iterative decomposition into two blocks. This method not only has a higher resolution than the MUSIC but also can resolve very weak sources in the vicinity of strong ones. The proposed algorithm can handle all array configurations and like the MUSIC method its DOA estimates are asymptotically exact, i.e., exact estimates are obtained asymptotically as the number of measurements goes to infinity irrespective of the SNR and angular separations of the sources [11].

The paper is organized as follows. First, the signal and noise model is presented and DOA estimation problem is formulated. Next, the new method is presented and its iterative process is investigated. Simulation results for comparison between the proposed algorithm and the MUSIC method are given in Section 4, followed by a summary of the main conclusions arising from this work.

II. DATA MODEL AND MUSIC ALGORITHM

Consider a Uniform Linear Array (ULA) of M sensors with intersensor spacing d. If plane waves from N narrowband far field sources, with the same known center frequency f_0, arrive at the array at angles θ_1, θ_2, …, θ_N, with respect to the array normal, the complex received signal of the mth sensor at time t, can be written as

$$\begin{aligned} r_m(t) &= \sum_{i=1}^{N} a_m(\theta_i) s_i\left(t - \tau_m(\theta_i)\right) + b_m(t) \\ &= \sum_{i=1}^{N} a_m(\theta_i) s_i(t) exp\left(-j2\pi f_0 \tau_m(\theta_i)\right) + b_m(t) \end{aligned} \tag{1}$$

where $\tau_m(\theta_i)$ is the propagation delay between a reference point and the mth sensor for the ith wavefront impinging on the array from direction θ_i as shown in Fig. 1, $a_m(\theta_i)$ is the corresponding sensor element complex response (gain and phase) at frequency f_0, and $b_m(t)$ is additive noise at the mth sensor element.

With the narrow band assumption, $s_i(t)$ is the ith signal complex envelop representation which can be shown as

$$s_i(t) = u_i(t) exp\left(j\left(2\pi f_0 t + \varphi_i(t)\right)\right), \quad i = 1,\ldots,N \tag{2}$$

where $u_i(t)$ and $\varphi_i(t)$ are slowly varying functions of time that define the amplitude and phase of ith signal, respectively. Slowly varying means $u_i(t) \approx u_i(t-\tau)$ and $\varphi_i(t) \approx \varphi_i(t-\tau)$ for all possible propagation delays τ between array sensors, and as a result of this, the effect of a time delay on received waveforms is simply a phase shift, i.e.,

$$s_i(t-\tau) \approx s_i(t) exp(-j2\pi f_0 \tau) \tag{3}$$

Using vector notation for the received signals of M sensors, the data model can be presented as

$$\underline{r}(t) = \sum_{i=1}^{N} \underline{a}(\theta_i) s_i(t) + \underline{b}(t) \tag{4}$$

where

$$\begin{aligned} &\underline{r}(t) = [r_1(t),\ldots,r_M(t)]^{\mathrm{T}},\ \underline{b}(t) = [b_1(t),\ldots,b_M(t)]^{\mathrm{T}}, \\ &\underline{a}(\theta_i) = [a_1(\theta_i) exp(-j2\pi f_0 \tau_1(\theta_i)),\ldots, \\ &\qquad a_M(\theta_i) exp(-j2\pi f_0 \tau_M(\theta_i))]^{\mathrm{T}} \end{aligned} \tag{5}$$

$i = 1,\ldots,N$.

and superscript T denotes transpose of a matrix. The $M\times 1$ vector $\underline{a}(\theta_i)$ is known as array response or array steering vector for direction θ_i. With defining the $M\times N$ matrix $A=[\underline{a}(\theta_1),\ldots,\underline{a}(\theta_N)]$ and $N\times 1$ vector $\underline{s}(t)=[s_1(t),\ldots,s_N(t)]^{\mathrm{T}}$, relation (4), can be written as

$$\underline{r}(t)=A\underline{s}(t)+\underline{b}(t). \quad (6)$$

The problem of determining the DOAs can now be reduced to the problem of estimating the vector parameter $\theta=[\theta_1,\ldots,\theta_N]^{\mathrm{T}}$ given K observations or *snapshots* $\{x(t)\}_{t=1}^{K}$. In order to make the estimation problem tractable, some assumptions have to be made about the model (6). The following assumptions are common in the literature on DOA estimation:

A1) The number of sensors M is larger than the number of emitting sources N, i.e., $M > N$. The number of sources N is assumed to be known.

A2) The steering vectors $\underline{a}(\theta)$ is known for all θ and the array is configured in such a way that the matrix A has full column rank, i.e., rank (A) = N. This also implies the source directions to be different in space, i.e., $\theta_i\neq\theta_j$.

A3) The additive noise at each sensor is a zero-mean stationary complex Gaussian random vector which is both temporally and spatially white. The noise processes of different sensors are uncorrelated and with the covariance matrix

$$\Gamma_b=E\{\underline{b}(t)\underline{b}(t)^{\mathrm{H}}\}=\sigma_b^2 I_M, \quad (7)$$

where σ_b^2 is the noise power at each sensor and I_M is an $M\times M$ identity matrix.

It is further assumed that the noise is uncorrelated with the N source signals.

A4) the sources are uncorrelated zero mean stationary processes with the $N\times N$ diagonal covariance matrix

$$P_S=E\{\underline{s}(t)\underline{s}(t)^{\mathrm{H}}\}=diag\{\eta_1^2,\eta_2^2,\ldots,\eta_N^2\}, \quad (8)$$

where $\eta_i^2=E\{|s_i(t)|^2\}$ denote the power (variance) of the ith source.

From the above assumptions and (6), the $M\times M$ covariance matrix of received data can be expressed as

$$\Gamma=E\{\underline{r}(t)\underline{r}(t)^{\mathrm{H}}\}=AP_S A^{\mathrm{H}}+\sigma_b^2 I_M=\Gamma_s+\Gamma_b \quad (9)$$

where $\Gamma_s=AP_S A^{\mathrm{H}}$ denote the signal covariance matrix.

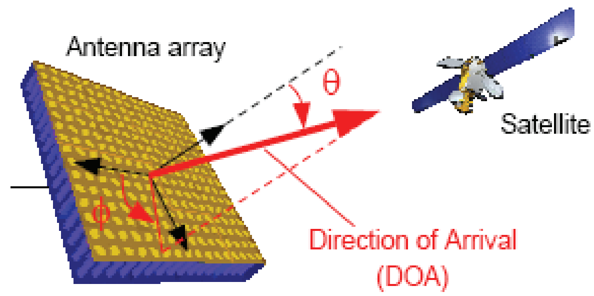


Fig. 1. Geometry of the array for DOA estimation.

Let Γ be eigen decomposed as

$$\begin{aligned}\Gamma&=[\mathrm{U_s V_b}]diag\{P_S,0\}[\mathrm{U_s V_b}]^{\mathrm{H}}+\sigma_b^2 I_M\\&=[\mathrm{U_s V_b}]diag\{P_S+\sigma_b^2 I_N,\sigma_b^2 I_{M-N}\}[\mathrm{U_s V_b}]^{\mathrm{H}} \quad (10)\\&=[\mathrm{U_s V_b}]\begin{bmatrix}\Lambda & 0\\ 0 & \sigma_b^2 I_{M-N}\end{bmatrix}\begin{bmatrix}\mathrm{U_s^H}\\ \mathrm{V_b^H}\end{bmatrix}\end{aligned}$$

where $\Lambda=diag\{\eta_1^2+\sigma_b^2,\eta_2^2+\sigma_b^2,\ldots,\eta_N^2+\sigma_b^2\}$,

$\mathrm{U_s}$ and $\mathrm{V_b}$ are the signal subspace and noise subspace eigen vector matrices, respectively. It can be shown that $A^{\mathrm{H}}\mathrm{V_b}=0$. Or equivalently,

$$\underline{a}^{\mathrm{H}}(\theta)\mathrm{V_b V_b^H}\underline{a}(\theta)=0 \quad (11)$$

at the true DOAs.

In practice, the data covariance matrix Γ is not available but a maximum likelihood estimate $\hat{\Gamma}$ based on a finite number (K) of data samples can be obtained as

$$\hat{\Gamma}=\frac{1}{K}\sum_{k=1}^{K}\underline{r}(t_k)\underline{r}(t_k)^{\mathrm{H}} \quad 12)$$

where $t_k=kT_s$, T_s is the sampling period and estimation of DOA of sources is based on this sample covariance matrix.

If $\hat{\Gamma}$ is eigen decomposed as in (10), one would arrive at the estimate of the noise subspace eigen vector matrix as $\hat{\mathrm{V}}_{\mathrm{b}}$. Since $\hat{\mathrm{V}}_{\mathrm{b}}$ is only an estimate, the left-hand side of (11), would be minimum, and not zero, at the true DOAs if V_{b} is replaced by $\hat{\mathrm{V}}_{\mathrm{b}}$. Spectral MUSIC utilizes this fact, so that the ambiguity function,

$$F_{MUSIC}(\theta) = \left(\frac{1}{\underline{a}^{\mathrm{H}}(\theta)\hat{\mathrm{V}}_{\mathrm{b}}\hat{\mathrm{V}}_{\mathrm{b}}^{\mathrm{H}}\underline{a}(\theta)} \right) \qquad (13)$$

peaks at the true DOA, whereas Root MUSIC simply roots the polynomial $\underline{a}^{\mathrm{H}}(\theta)\hat{\mathrm{V}}_{\mathrm{b}}\hat{\mathrm{V}}_{\mathrm{b}}^{\mathrm{H}}\underline{a}(\theta)$ to find the DOA.

III. PROPOSED DOA ESTIMATION ALGORITHM: Iterative-Subspace-Decomposition (ISD) Method

A set of basis vectors that span the signal subspace S are the eigenvectors corresponding to the N largest eigenvalues of the measurement covariance matrix Γ. This follows from the definition of the eigenvalues and eigenvectors of Γ given by

$$\Gamma \underline{v}_m = \lambda_m \underline{v}_m, \quad m = 1,2,\ldots,M, \qquad (14)$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M$ are the eigenvalues (in decreasing order) of Γ and $\underline{v}_i$ are the corresponding eigenvectors. Inserting for Γ into (14), gives

$$\Gamma_s \underline{v}_m = \left(\lambda_m - \sigma_b^2\right)\underline{v}_m, \qquad (15)$$

which is the definition of the eigenvalues and the eigenvectors of the signal covariance matrix $\Gamma_s = APA^{\mathrm{H}}$, where $\left(\lambda_m - \sigma_b^2\right)$ is the mth eigenvalue and $\underline{v}_m$ is the mth eigenvector.

The $M \times M$ matrix Γ_{s} is by construction positive semidefinite with rank equal to N (under the assumption that the signals are not fully correlated). This means that Γ_{s} has N positive, nonzero eigenvalues and M-N eigenvalues that are equal to zero. The eigenvalues of Γ are then $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N > \lambda_{N+1} = \ldots = \lambda_M = \sigma_b^2$ and the eigenvectors of Γ corresponding to the N largest eigenvalues are the same as the eigenvectors of Γ_{s} that correspond to the only nonzero eigenvalues. These eigenvectors span the same subspace as Γ_{s} and hence also A. The signal subspace can then be written as:

$$S = span\{A\} = span\{\mathrm{U}_{\mathrm{s}}\}, \qquad (16)$$

where $\mathrm{U}_{\mathrm{s}} = \left[\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_N\right]$. Similarly, the noise subspace is

$$S^{\perp} = span\{\mathrm{V}_{\mathrm{b}}\}, \qquad (17)$$

where $\mathrm{V}_{\mathrm{b}} = \left[\underline{v}_{N+1}, \underline{v}_{N+2}, \ldots, \underline{v}_M\right]$. For an infinite number of snapshots K, the sample covariance matrix is equal to the measurement covariance matrix, that is, $\lim_{\mathrm{M}\to\infty}\hat{\Gamma} = \Gamma$, and the signal subspace can be found as described above.

In practice, however, there is a finite amount of data available, which implies that $\hat{\Gamma} \neq \Gamma$. This means that the exact signal subspace cannot be found. Instead, estimates of the signal and noise subspaces can be made from the eigenvectors of $\hat{\Gamma}$, i.e., $\hat{S} = span\{\hat{\mathrm{U}}_{\mathrm{s}}\}$ and $\hat{S}^{\perp} = span\{\hat{\mathrm{V}}_{\mathrm{b}}\}$, where $\hat{\mathrm{U}}_{\mathrm{s}} = \left[\hat{\underline{v}}_1, \hat{\underline{v}}_2, \ldots, \hat{\underline{v}}_N\right]$, $\hat{\mathrm{V}}_{\mathrm{b}} = \left[\hat{\underline{v}}_{N+1}, \hat{\underline{v}}_{N+2}, \ldots, \hat{\underline{v}}_M\right]$, and $\hat{\underline{v}}_m$, $m = 1,2,\ldots,M$ are the eigenvectors corresponding to the eigenvalues of $\hat{\Gamma}$, which are $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \ldots \geq \hat{\lambda}_M$.

A. Signal and Noise Subspace

The $M \times M$ sample covariance matrix $\hat{\Gamma}$ given in (12), is a positive definite matrix. We denote its eigenvalues (in decreasing order) and their corresponding eigenvectors by $\hat{\lambda}_m$ and $\hat{\underline{v}}_m$, i.e.,

$$\hat{\Gamma} = \mathrm{VDV}^{\mathrm{H}} = \sum_{m=1}^{M} \hat{\lambda}_m \hat{\underline{v}}_m \hat{\underline{v}}_m^H \qquad (18)$$

where $\mathrm{V} = \left[\hat{\underline{v}}_1, \hat{\underline{v}}_2, \ldots, \hat{\underline{v}}_M\right]$ and $\mathrm{D} = diag\{\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_M\}$

Establishing of the signal and noise subspace is obtained using the following observations; Because it is assumed that $M > N$, the $M \times M$ eigenvalues matrix D and the $M \times M$ eigenvectors matrix V can divided into four parts as follows:

$$\mathrm{V}=\begin{bmatrix} \mathrm{V}_{11} & \mathrm{V}_{12} \\ \underbrace{\mathrm{V}_{21}}_{N} & \underbrace{\mathrm{V}_{22}}_{M-N} \end{bmatrix}\begin{matrix} \}\ N \\ \}\ M-N \end{matrix} \quad (19)$$

and

$$\mathrm{D}=\begin{bmatrix} \mathrm{D}_{1} & \mathrm{D}_{2} \\ \underbrace{\mathrm{D}_{3}}_{N} & \underbrace{\mathrm{D}_{4}}_{M-N} \end{bmatrix}\begin{matrix} \}\ N \\ \}\ M-N \end{matrix}. \quad (20)$$

The submatrices V_{11} and V_{21} define the signal subspace matrix V_S. The submatrices V_{12} and V_{22} define the noise subspace matrix V_B. V_S and V_B are the same signal and noise subspace defined by the MUSIC method. The two blocks D_2 and D_3 for the diagonal matrix D are two null matrices.

Assume first that there is no noise present. The spectral covariance matrix Γ can then be written

$$\begin{aligned}\Gamma=\Gamma_s=\mathrm{VEV}^{\mathrm{H}}&=[\mathrm{V_S V_B}]\begin{bmatrix}\mathrm{E} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathrm{V_S^H}\\ \mathrm{V_B^H}\end{bmatrix} \\ &=\mathrm{V_S E V_S^H}\end{aligned} \quad (21)$$

where E is the $N\times N$ diagonal matrix which contain the eigenvalues (in decreasing order) of the signal covariance matrix Γ_s.

Now assume that noise is included in the model. The spectral covariance matrix Γ can then be written

$$\begin{aligned}\Gamma&=\Gamma_s+\Gamma_b=\mathrm{V_S E V_S^H}+\sigma_b^2 I_M \\ &=\mathrm{V_S E V_S^H}+\mathrm{V_S}\left(\sigma_b^2 I_N\right)\mathrm{V_S^H}+\mathrm{V_B}\left(\sigma_b^2 I_{M-N}\right)\mathrm{V_B^H}. \\ &=[\mathrm{V_S V_B}]\,diag\{\mathrm{E}+\sigma_b^2 I_N,\sigma_b^2 I_{M-N}\}[\mathrm{V_S V_B}]^{\mathrm{H}}\end{aligned} \quad (22)$$

Thus, the $M-N$ dimensional subspace spanned by the $M-N$ noise eigenvectors may be justifiably referred to as the noise subspace. The N dimensional subspace spanned by the incident signal mode vectors may also be referred to as the signal subspace. Both subspaces are disjoint.

By including the partitioning of the matrix V and D, a new covariance matrix Γ is defined as

$$\begin{aligned}\Gamma&=\begin{bmatrix}\mathrm{V_{11}D_1V_{11}^H} & \mathrm{V_{11}D_1V_{21}^H}\\ \mathrm{V_{21}D_1V_{11}^H} & \mathrm{V_{21}D_1V_{21}^H}\end{bmatrix}+\begin{bmatrix}\mathrm{V_{12}D_4V_{12}^H} & \mathrm{V_{12}D_4V_{22}^H}\\ \mathrm{V_{22}D_4V_{12}^H} & \mathrm{V_{22}D_4V_{22}^H}\end{bmatrix} \\ &=[\mathrm{V_S V_B}]\,diag\{\mathrm{D_1},\mathrm{D_4}\}[\mathrm{V_S V_B}]^{\mathrm{H}}.\end{aligned} \quad (23)$$

Add and leave out the noise covariance matrix Γ_b, relation (23), can be written as

$$\Gamma=\mathrm{V_S}\left(\mathrm{D_1}-\sigma_b^2 I_N\right)\mathrm{V_S^H}+\mathrm{V_S}\left(\sigma_b^2 I_N\right)\mathrm{V_S^H}+\mathrm{V_B D_4 V_B^H} \quad (24)$$

By identification with (22), the signal covariance matrix can be written as:

$$\Gamma_s=\begin{bmatrix}\mathrm{V_{11}D_1^{'}V_{11}^H} & \mathrm{V_{11}D_1^{'}V_{21}^H}\\ \mathrm{V_{21}D_1^{'}V_{11}^H} & \mathrm{V_{21}D_1^{'}V_{21}^H}\end{bmatrix}=\begin{bmatrix}\mathrm{Q_{S1}} & \mathrm{Q_{S2}}\\ \mathrm{Q_{S3}} & \mathrm{Q_{S4}}\end{bmatrix}, \quad (25)$$

and the noise covariance matrix can be written as

$$\begin{aligned}\Gamma_b&=[\mathrm{V_S V_B}]\,diag\{\sigma_b^2 I_N,\mathrm{D_4}\}[\mathrm{V_S V_B}]^{\mathrm{H}} \\ &=\mathrm{V_S}\left(\sigma_b^2 I_N\right)\mathrm{V_S^H}+\Gamma_{bb}\end{aligned}, \quad (26)$$

where

$$\Gamma_{bb}=\begin{bmatrix}\mathrm{V_{12}D_4^{'}V_{12}^H} & \mathrm{V_{12}D_4^{'}V_{22}^H}\\ \mathrm{V_{22}D_4^{'}V_{12}^H} & \mathrm{V_{22}D_4^{'}V_{22}^H}\end{bmatrix}=\begin{bmatrix}\mathrm{Q_{B1}} & \mathrm{Q_{B2}}\\ \mathrm{Q_{B3}} & \mathrm{Q_{B4}}\end{bmatrix}, \quad (27)$$

$$\mathrm{D_1^{'}}=\mathrm{D_1}-\sigma_b^2 I_N=\mathrm{E} \quad \text{and} \quad \mathrm{D_4^{'}}=\mathrm{D_4}=\sigma_b^2 I_{M-N}.$$

The parameters Q_{S1}, Q_{S2}, Q_{S3} and Q_{S4} are, respectively, $N\times N$, $N\times(M-N)$, $(M-N)\times N$ and $(M-N)\times(M-N)$ dimensional submatrices. Q_{B1}, Q_{B2}, Q_{B3} and Q_{B4} are, respectively, $N\times N$, $N\times(M-N)$, $(M-N)\times N$ and $(M-N)\times(M-N)$ dimensional submatrices.

Lastly, the noise subspace R_B and the signal subspace R_S is obtained by a linear operation of the matrix formed from the noise and signal covariance matrix:

B. Iterative Subspace-Decomposition-Signal (ISDS) Method

From the signal covariance matrix (25), G_S is defined to be the $N\times(M-N)$ linear operator source. The rows of Q_{S3} can be expressed as a linear combination of linearly independents rows of Q_{S1}; equivalently, there is a $N\times(M-N)$ linear operator G_S between Q_{S1} and Q_{S3}

$$\mathrm{Q_{S3}}=\mathrm{G_S^H Q_{S1}}\Rightarrow \mathrm{V_{21}}=\mathrm{G_S^H V_{11}}. \quad (28)$$

Then it follows from (28), that

$$\mathrm{R_{Bs}^H}A(\theta)=0_{(M-N)\times N} \quad \text{or} \quad span\{\mathrm{R_{Bs}}\}\perp span\{A\}$$

and

$$span\{R_{Ss}\} = span\{A\}, \tag{29}$$

where $R_{Bs} = \begin{bmatrix} G_S \\ -I_{M-N} \end{bmatrix}$ and $R_{Ss} = \begin{bmatrix} I_N \\ G_S^H \end{bmatrix}$ are the noise and signal subspace matrices, respectively. Because the $M \times (M-N)$ matrix R_{Bs} has a full rank of $(M-N)$, the columns of R_{Bs} form the basis for the null space $\aleph(A^H(\theta))$ of $A^H(\theta)$, and clearly, the orthogonal projector onto this noise subspace is given by

$\Pi_{R_{Bs}} = R_{Bs}\left(R_{Bs}^H R_{Bs}\right)^{-1} R_{Bs}^H$, which implies that

$$\Pi_{R_{Bs}} \underline{a}(\theta) = 0_{M\times 1} \quad \text{for} \quad \theta = \theta_i \\ i = 1,\dots,N \tag{30}$$

where

$$\underline{a}(\theta) = \left[1, exp(-j2\pi f_0 \tau(\theta)),\dots,exp(-j2\pi f_0 (M-1)\tau(\theta))\right]^T$$

and $0_{M\times 1}$ is an $M \times 1$ null vector. Evidently, the directions can be estimated based on the orthogonal property (30).

C. Iterative Process

The common assumption used for the following iterations is that the noise subspace matrix R_B (i.e., R_{Bs} or R_{Bb}) has a full rank of $(M-N)$. Because it is assumed that $M > N$, the $M \times (M-N)$ noise subspace matrix R_{BI} can be divided, at the *I*th iteration, into two parts as follows:

$$R_{B_I} = \begin{bmatrix} W_I \\ Z_I \end{bmatrix} \begin{matrix} \}M-N \\ \}\quad N \end{matrix} \tag{31}$$

where R_{BI} and W_I are two submatrices with full rank, the row of Z_I can be expressed as a linear combination of linearly independent rows of W_I; equivalently there is a $N \times (M-N)$ linear operator G_{I+1} at the (I+1)th iteration, between W_I and Z_I

$$G_{I+1} W_I = Z_I. \tag{32}$$

Then it follows from (34), that

$$R_{B_{I+1}}^H A(\theta) = 0_{(M-N)\times N} \quad \text{or} \quad span\{R_{B_{I+1}}\} \perp span\{A\}$$

and

$$span\{R_{S_{I+1}}\} = span\{A\} \tag{33}$$

where

$R_{B_{I+1}} = \begin{bmatrix} G_{I+1} \\ I_{M-N} \end{bmatrix}$ and $R_{S_{I+1}} = \begin{bmatrix} -I_N \\ G_{I+1}^H \end{bmatrix}$ are the noise and signal subspace matrices at the (I+1)th iteration, respectively.

Thus, a continuation of iterative decomposition of the new noise subspace matrix enables us to reduce the contribution of the noise and consequently to increase the signal-to-noise ratio, by keeping the information useful of the signal to be detected. The ISD spectrum, at the *I*th iteration, can be expressed as:

$$F_{ISD}^I(\theta) = \frac{1}{\underline{a}^H(\theta) R_{B_I} R_{B_I}^H \underline{a}(\theta)} \tag{34}$$

where

$$R_{B_I} = \begin{bmatrix} G_I \\ I_{M-N} \end{bmatrix}. \tag{35}$$

The matrix $R_{B_I} R_{B_I}^H$ is a projection matrix onto the noise subspace. For steering vectors that are orthogonal to the noise subspace, the denominator of (34), will become very small, and thus the peaks will occur in $F_{ISD}^I(\theta)$ corresponding to the angle of arrival of the signal.

IV. SIMULATION RESULT

In this section, the estimation accuracies of the proposed method and the classical MUSIC technique are compared for the problem of DOA estimation. Some numerical examples are investigated to illustrate the performance of the proposed method more explicitly. The array herein is assumed to be a ULA composed of 5 isotropic sensors, whose spacing equal half-wavelength. The number of signals is assumed to be known *a priori*. The simulation results include probabilities rate of resolving the two sources and root mean squared errors (RMSE's) of estimated DOAs. In all the simulations of this paper successful simulations are those that show two distinct peaks.

A. RMSE of Estimated DOA

Suppose that the ULA receives two uncorrelated narrowband signals with equal power. We make 200 Monte Carlo (Realizations) runs for each experiment to compute the Root-Mean-Squared Errors (RMSE's) of estimated DOAs. The true DOAs are given by {88°, 93°}. The background noise is assumed to be a stationary Gaussian white random process with zero mean. The RMSE's of estimated DOAs versus SNR are shown in Fig. 2, where the number of sensors is 5 and the number of snapshots is 100. The Propagator method is also plotted for comparison.

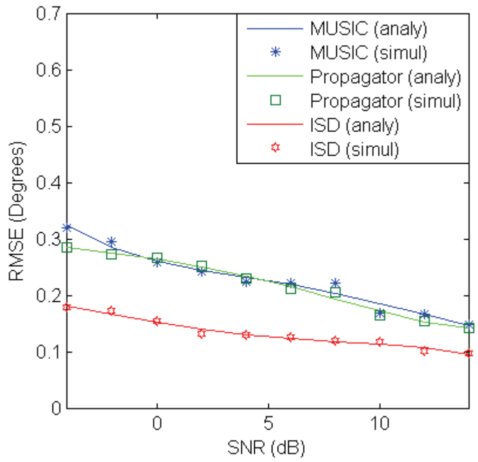


Fig. 2. RMSE's of estimated DOAs versus SNR. DOAs of signal 1 to 2 are 88° and 93°. The number of snapshots and the number of sensors are equal to 100 and 5, respectively.

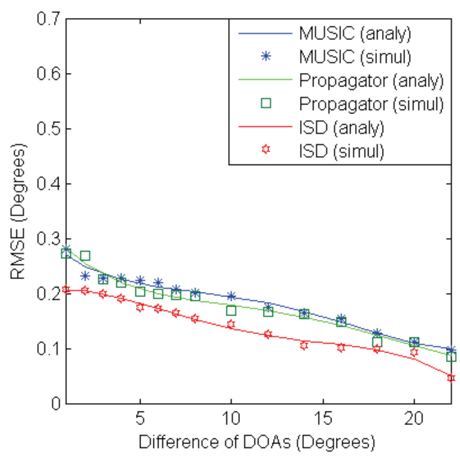


Fig. 3. RMSE's of estimated DOAs versus difference of DOAs between the two sources. The number of snapshots, SNR and the number of sensors are equal to 100, 12 dB and 5, respectively.

It is demonstrated in Fig. 2 that the ISD estimator provides the comparable estimation accuracy with the MUSIC and Propagator methods when SNR is greater than 10dB, and yields the more estimation accuracy than the latter as SNR $\le$ 10 dB. It should be noted that the RMSE for the proposed method is higher than the classical MUSIC and Propagator methods over the range of SNR that we simulated, especially in the case of low SNR (SNR < 5 dB), the proposed method surpasses the MUSIC estimator. The RMSE's of the two estimators (MUSIC and Propagator) approach to the ISD as SNR becomes high.

The RMSE of the DOA estimation error obtained by ISD, MUSIC and Propagator methods is plotted versus difference of DOAs between the two sources in Fig. 3. Another example is to demonstrate the superiority of the ISD method over the other method.

It is clearly seen that the ISD method has high-resolution capability.

B. Resolution Rate Capabilities

Let us assume that the number N of mobile users is 2 and each sample covariance matrix is estimated from 100 snapshots in the simulation.

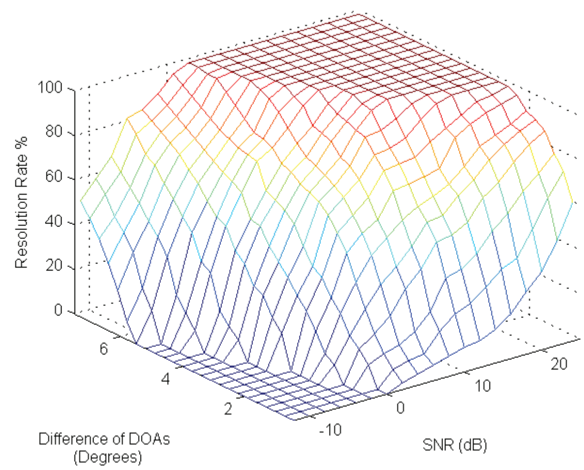
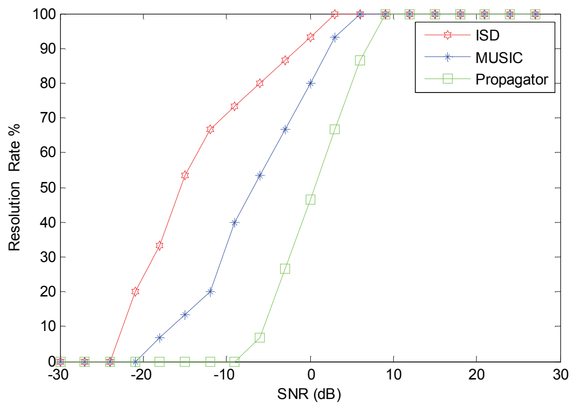


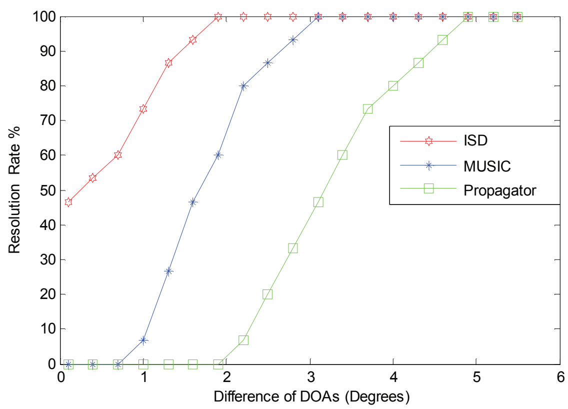
Fig. 4. Resolution rate capabilities versus SNR and difference of DOAs between the two sources. The number of snapshots and the number of sensors are equal to 100 and 5, respectively.

The two sources are considered resolved if the differences between the estimates and their respective true locations are both less than the separation between the sources, i.e., if $83° \le \hat{\theta}_1 < 93°$ and $88° \le \hat{\theta}_2 < 98°$. Throughout simulations, the powers of the sources are assumed equal.

The resolving rate capabilities obtained by the proposed ISD method versus SNR and difference of DOAs between the two sources are plotted in Fig. 4, where the number of snapshots and the number of sensors are equal to 100 and 5, respectively.



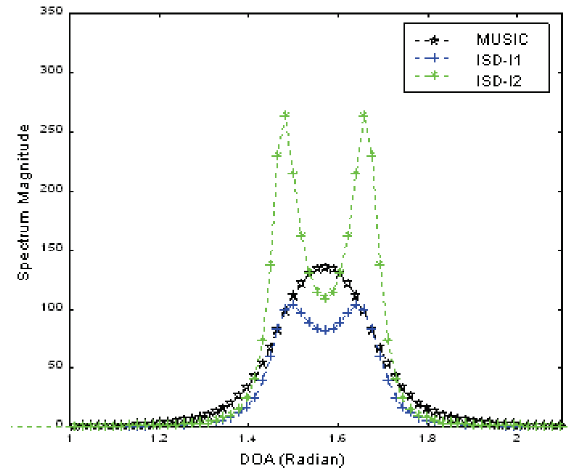
(a)



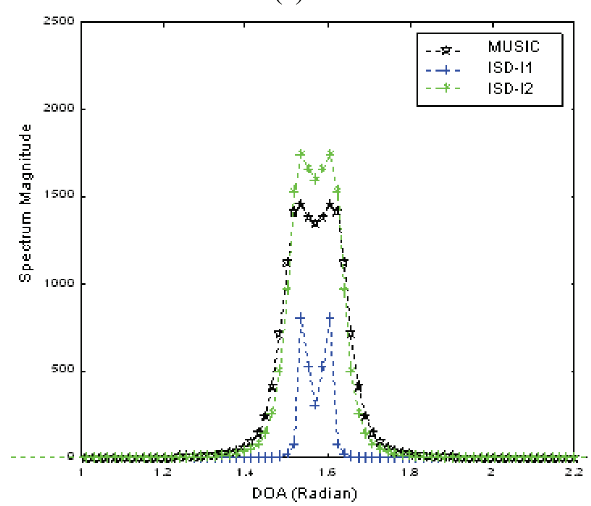
(b)

Fig. 5. Resolution rate capabilities, performance comparisons of the proposed and classical methods, (a) Resolution rate capabilities versus SNR. DOAs of signal 1 to 2 are 88° and 93°. The number of snapshots and the number of sensors are 100 and 5, respectively, (b) Resolution rate capabilities versus difference of DOAs between the two sources. The number of snapshots, SNR and the number of sensors are equal to 100, 12 dB and 5 respectively.

When the two sources are uncorrelated, the results are displayed in Fig. 5. Figure 5(a) shows that the resolution capabilities of the proposed and the classical MUSIC methods are practically the same. The resolution performance of the Propagator method is also included for comparison. It can be verified from Fig. 5(a) that to have a resolution rate more than 90 %, the SNR of ISD, MUSIC and Propagator method must be higher than -5 dB, 0 dB and 5 dB, respectively.



(a)



(b)

Fig. 6. Spectrum magnitude of proposed method and MUSIC versus DOA of sources, (a) Spectrum magnitude of proposed ISD method (for Iteration: I=1 and I=2) and MUSIC. DOAs of signal 1 to 2 are 88° and 93°. The number of snapshots, SNR and the number of sensors are equal to 100, -5 dB and 5, respectively, (b) Spectrum magnitude of proposed ISD method (for Iteration: I=1 and I=2) and MUSIC. DOAs of signal 1 to 2 are 88° and 93°, respectively. The number of snapshots, SNR and the number of sensors are equal to 100, 5 dB and 5 respectively.

As illustrated by Fig. 5(b) that though the classical MUSIC and Propagator are unable to resolve the sources when the difference of DOAs between the sources becomes less then 3° and 5°

respectively. The proposed ISD method can, however, estimate the DOAs of the sources more accurately when the difference of DOAs between the sources becomes less then 2°.

Figures 5 and 4 shows higher resolution capabilities of proposed method in resolving closely spaced sources with large power differences at low SNR condition compared to MUSIC and Propagator. The same as the first experiment in all successful simulations of MUSIC method the proposed algorithm has also been successful with a much distinct peaks.

To evaluate the dependence of the estimation accuracy of the proposed method on the number of iterations, the spectrum magnitude of estimated DOAs for the proposed ISD method for two different levels of iteration (I =1 and I = 2), is plotted in Fig. 6. The number of snapshots is 100, SNR changes at -5 dB in Fig. 6(a) and 5 dB in Fig. 6(b), and the number of sensor is 5. MUSIC method is also included for comparison.

The observation indicates that the proposed method can resolve the two DOAs at low SNR condition, unlike MUSIC method. This guarantees that the ISD approach can preserve high accuracy in the case of low-SNR.

V. CONCLUSION

The approach presented here for iterative subspace decomposition is very general and can be applied to all array configurations. The ISD method is interpretable in terms of the geometry of complex M spaces where in the eigen structure of the measured Γ matrix plays the central role. ISD method provides asymptotically unbiased estimates of a general set of signal parameters and its superiority over MUSIC accuracy bound. In geometric terms, ISD minimizes the distance from the steering vectors $\underline{a}(\theta)$ continuum to the signal subspace whereas maximum likelihood minimizes a weighted combination all component distances.

The effect of ISD on the performance of MUSIC is analyzed by numerically evaluating and comparing: (1) the RMSE in the spectral covariance estimates obtained using finite data, and (2) the resolution rate capabilities of the DOA estimates. It is shown that ISD leads to a significant improvement in the performance of the MUSIC estimator.

The array elements may be arranged in a regular or irregular pattern and may differ or be identical in directional characteristics (amplitude/phase) provided their polarization characteristics are all identical. The extension to include general polarizationally diverse antenna arrays will be more completely described in a future work.

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