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## Journal



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# THE APPLIED COMPUTATIONAL ELECTROMAGNETICS SOCIETY 

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# Comparative Study of Analytical and Numerical Techniques in Modeling Electromagnetic Scattering from Single and Double Knife-Edge in 2D Ground Wave Propagation Problems 

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#### Abstract

This paper presents a comparative study of some analytical and numerical techniques in the solution of a classical problem of electromagnetic scattering from single and double knife edge above ground. The results of the analytical exact and asymptotic techniques (such as uniform theory of diffraction, parabolic equation diffraction method) are compared with the two-way split-step parabolic equation method (SSPE), through several numerical simulations. Salutary discussions on their capabilities and limitations are presented. Codes used in the simulations are provided in the end.


Index Terms - Diffraction, electromagnetic scattering, geometric optics (GO), ground wave propagation, two-way split-step parabolic equation method, uniform theory of diffraction (UTD).

## I. INTRODUCTION

Electromagnetic ground wave propagation over the Earth's surface is affected by several scattering phenomena, such as reflection, refraction, and diffraction. Developing a rigorous method for understanding the effects of varying conditions on ground wave propagation is usually a challenging task because of the vast variability of the medium parameters and also the complexity of the surfaces and obstacles that re-direct the propagating energy. Initially, the prediction of the ground wave propagation problem has been
achieved by means of some analytical exact or analytical asymptotic techniques [1-17], which require the geometry to be represented as a member of a set of some canonical geometries. The exact techniques express the solution in the form of infinite series or integral and may serve as reference but might be poorly convergent at high frequencies when the dimensions of the objects are much larger than the wavelength. Asymptotic techniques have been widely used for decades to solve problems at high (usually optical) frequencies. These high-frequency asymptotic (HFA) approaches employ simplified models of electromagnetic wave reflection, refraction and diffraction such as geometric optics (GO), physical optics (PO), geometric theory of diffraction (GTD) [8-9], uniform theory of diffraction (UTD) [10-11], physical theory of diffraction (PTD) [12-13]. The GO model describes only the incident, reflected and refracted waves on the illuminated side of the scatterers. The GTD model is complementary to the GO model, in the sense that it overcomes some limitations of GO by including the diffraction mechanism. However, the GTD model exhibits singularities along the incident and reflection shadow boundaries (ISB and RSB). The uniform theory of diffraction (UTD) model achieves smooth wave behavior along these boundaries. This family (GO, GTD, UTD) is also known as ray tracing techniques in the literature. The PO
and PTD models, on the other hand, are based on induced currents on the surfaces of the scatterers. The PTD model supplements PO by introducing non-uniform ("fringe") edge currents to incorporate the effect of diffraction into the solution. In spite of simplicity, the asymptotic techniques suffer from two major drawbacks: (i) Solutions are available only for certain canonical objects, such as wedge (or knife-edge), cylinder. (ii) (Unless ray-shooting is not used) verticallyvarying refractivity profiles in the troposphere cannot be handled.

With the advances in computers, some numerical techniques have been devised to solve the ground wave propagation problem involving complex geometries and inhomogeneous environments. Split-step parabolic equation (SSPE) method [18-22] has been widely used in propagation modeling because of its capability in modeling both horizontally- and vertically-varying atmospheric refraction (especially ducting) effects. It solves an initial-value problem starting from a source (or an antenna), and marching out in range by computing the field along the vertical direction at each range by means of step-by-step Fourier transformations. The classical SSPE method deals only with forward propagating waves, and thus, it is a one-way model that is valid in the paraxial region. Recently, a two-way SSPE algorithm has been introduced [23, 24] to incorporate the backward propagating waves into the one-way SSPE by utilizing an iterative forward-backward scheme for modeling multipath effects. This algorithm has been served for the public's use via a novel, MATLAB ${ }^{\circledR}$ based software tool (PETOOL) [25]. In spite of several advantages, the SSPE method takes the diffraction effects into account within the paraxial approximation, degrading the accuracy of the approach in deep-shadow regions where the diffracted fields dominate.

The organization of this paper is as follows: In Sec. II, analytical exact and asymptotic methods are briefly formulated. In Sec. III, fundamentals of the two-way SSPE are outlined. In Sec. IV, the techniques are compared through several numerical simulations. In the Appendix, the codes used in the simulations are presented.

Except Sec. II C, the time dependence of the form $\exp (-i \omega t)$ is assumed.

## II. ANALYTICAL EXACT AND ASYMPTOTIC MODELS

This section discusses various analytical approaches, belonging to the family of ray-based techniques. The problem of interest is the 2D electromagnetic ground wave propagation where single or multiple knife-edges are located on a flat conducting surface and illuminated by a linesource (see Fig. 1). The problem of single or multiple knife-edge (or wedge in general) suspended in a homogeneous medium is one of the classical, canonical problems, and plays a fundamental role in the construction of the highfrequency asymptotic techniques. If the edges are located on a conducting surface, the image theory can be employed to account for the multiple reflections from the ground or edges. The total field is then obtained by the sum of the direct ray, the reflected rays emanating from image sources, and the diffracted rays from the tip of the edges, by also checking the line-of-sight (LOS) conditions between the source and the observation point. The beauty of this model is that it yields physical insight, and contributions of every ray item can be observed separately.


Fig. 1: Geometry of the problem, illustrating some possible ray contributions for different observation points (A, B, C). [Red: direct, Green: diffracted, Blue: reflected or diffracted/reflected].

The subsections below formulate various techniques to modeling of the diffracted field from the tip of a single wedge in the absence of ground (see Fig. 2). The more general problem in Fig. 1 involving multiple bouncing of the rays can be handled by a successive implementation of GO and diffraction algorithms depending on different LOS conditions (this is indeed just a coding issue). The wedge in Fig. 2 is illuminated by a cylindrically diverging line-source. The tip-tosource distance and the source angle are denoted
by $r_{0}$ and $\varphi_{0}$, respectively; whereas the tip-toobserver distance and the observation angle are represented by $r$ and $\varphi$, respectively. The wedge exterior angle $\alpha$ is set to $2 \pi$ in the simulations to model the knife-edge. On the other hand, more realistic, hill-type obstacles may be modeled easily by setting $\alpha \neq 2 \pi$. The wedge surface is assumed to be perfect electric conductor. As conventional in the diffraction theory, the polarization will be referred to as soft or hard polarizations, which are equivalent to horizontal or vertical polarizations, respectively.


Fig. 2. Problem of scattering from a wedge.

## A. Exact series model

Analytical exact solution of the problem in Fig. 2 can be obtained by using the separation of variables method, which reduces the 2D wave equation into two 1 D wave equations; one in the angular domain, the other in the radial domain. Depending on the boundary conditions, the angular domain solutions are expressed in terms of sine or cosine functions, whereas the radial domain solutions are constructed by means of Bessel or Hankel functions. Bessel and Hankel functions are appropriate for $r \leq r_{0}$ and for $r>r_{0}$, respectively.

Assuming a line source of the form of $I_{0} \delta\left(r-r_{0}, \varphi-\varphi_{0}\right)$, the exact total field for soft and hard boundary conditions, respectively, are expressed in series as follows: [15]

$$
u_{s}^{t}= \begin{cases}\frac{4 \pi}{\alpha} \sum_{m=1}^{\infty} J_{v_{m}}(k r) H_{v_{m}}^{(1)}\left(k r_{0}\right) f_{s}\left(\varphi, \varphi_{0}\right) & \text { if } r \leq r_{0}  \tag{1}\\ \frac{4 \pi}{\alpha} \sum_{m=1}^{\infty} J_{v_{m}}\left(k r_{0}\right) H_{v_{m}}^{(1)}(k r) f_{s}\left(\varphi, \varphi_{0}\right) & \text { if } r \geq r_{0}\end{cases}
$$

$$
u_{h}^{t}= \begin{cases}\frac{4 \pi}{\alpha} \sum_{m=0}^{\infty} \varepsilon_{m} J_{v_{m}}(k r) H_{v_{m}}^{(1)}\left(k r_{0}\right) f_{h}\left(\varphi, \varphi_{0}\right) & \text { if } r \leq r_{0}  \tag{2}\\ \frac{4 \pi}{\alpha} \sum_{m=0}^{\infty} \varepsilon_{m} J_{v_{m}}\left(k r_{0}\right) H_{v_{m}}^{(1)}(k r) f_{h}\left(\varphi, \varphi_{0}\right) & \text { if } r \geq r_{0}\end{cases}
$$

Here, $k$ is the wavenumber, $v_{m}=m \pi / \alpha, \varepsilon_{m}=1 / 2$ if $m=0$ and $\varepsilon_{m}=1$ otherwise; and $f_{s}=\sin \left(v_{m} \varphi_{0}\right) \sin \left(v_{m} \varphi\right), f_{h}=\cos \left(v_{m} \varphi_{0}\right) \cos \left(v_{m} \varphi\right)$.

Since the total field is the sum of the diffracted field and the GO field (i.e., $u_{s, h}^{t}=u_{s, h}^{d}+u_{s, h}^{G O}$ ), the diffracted field $u_{s, h}^{d}$ can be obtained by subtracting the GO field from the total field. The GO field is the sum of the incident (direct) field and the reflected field emanating from the image source (i.e., $u_{s, h}^{G O}=u_{s, h}^{i n c}+u_{s, h}^{r}$ where $u_{s, h}^{r}=u_{s, h}^{i n c, \text { image }}$ ). The incident field is expressed in terms of Hankel function, i.e., $u_{\mathrm{s}}^{i n c}=H_{0}^{(1)}(k R)$, where $R$ is the distance between the source (actual or image) and observation points (i.e., $R_{\text {actual }}$ or $R_{\text {image }}$ ). Hence, the GO field can be expressed with respect to the LOS conditions, as follows:

$$
\begin{equation*}
u_{s, h}^{G O}=\varepsilon_{1} H_{0}^{(1)}\left(k R_{\text {actual }}\right) \mp \varepsilon_{2} H_{0}^{(1)}\left(k R_{\text {image }}\right), \tag{3}
\end{equation*}
$$

where $(-)$ and $(+)$ are for soft and hard polarizations, respectively. Moreover,

$$
\begin{align*}
& \varepsilon_{1}=\left\{\begin{array}{l}
1 \text { for } \varphi \leq \pi+\varphi_{0} \\
0 \text { for } \varphi>\pi+\varphi_{0}
\end{array},\right.  \tag{4}\\
& \varepsilon_{2}=\left\{\begin{array}{l}
1 \text { for } \varphi \leq \pi-\varphi_{0} \\
0 \text { for } \varphi>\pi-\varphi_{0}
\end{array} \text { if } 0 \leq \varphi_{0} \leq \alpha-\pi\right),  \tag{5}\\
& \varepsilon_{2}=\left\{\begin{array}{l}
1, \text { for } 0<\varphi<\pi-\varphi_{0} \\
0, \text { for } \pi-\varphi_{0}<\varphi<2 \alpha-\pi-\varphi_{0} \quad\left(\text { if } \alpha-\pi \leq \varphi_{0} \leq \pi\right) \\
1, \text { for } 2 \alpha-\pi-\varphi_{0}<\varphi<\alpha
\end{array}\right. \tag{6}
\end{align*}
$$

Note that the case $0 \leq \varphi_{0} \leq \alpha-\pi$ in (5) and $\alpha-\pi \leq \varphi_{0} \leq \pi$ in (6) refer to Single-Side (only one face) and Double-Side illuminations, respectively.

Under the conditions where $k r_{0} \gg 1$ and $k r \gg 1$, asymptotic forms of Hankel functions can be used. Hence, the diffracted field can be cast into the following form:

$$
\begin{equation*}
u_{s, h}^{d}=u_{0} d_{s, h} \frac{\mathrm{e}^{i k r}}{\sqrt{r}} \tag{7}
\end{equation*}
$$

where $u_{0}=H_{0}^{(1)}\left(k r_{0}\right)$ is the incident field at the tip of the wedge, and $d_{s, h}$ is the diffraction coefficient.

Thus, (7) allows us to compute the diffraction coefficient approximately after the exact diffracted field is computed, i.e., $d_{s, h}=\sqrt{r}\left(u_{s, h}^{d} / u_{0}\right) \mathrm{e}^{-i k r}$.

It is useful to note that the number of terms in the series expressions increases drastically as the frequency increases and/or the observation point moves away from the tip of the wedge. This might obviously place a bottleneck on the computation time during the numerical implementation.

## B. Exact integral model

Analytical exact solution of the wedge diffraction problem can also be obtained by an integral representation presented by Bowman \& Senior in a handbook [6]. The diffracted fields can be expressed as allows:

$$
\begin{align*}
u_{s, h}^{d} & =\left[V_{d}\left(-\pi-\varphi+\varphi_{0}\right)-V_{d}\left(\pi-\varphi+\varphi_{0}\right)\right] \\
& \mp\left[V_{d}\left(-\pi-\varphi-\varphi_{0}\right)-V_{d}\left(\pi-\varphi-\varphi_{0}\right)\right], \tag{8}
\end{align*}
$$

where (-) and $(+)$ are for soft and hard polarizations, respectively. Here,

$$
\begin{equation*}
V_{d}(\beta)=\frac{1}{2 \pi n} \int_{0}^{\infty} H_{0}^{(1)}[k R(i \kappa)] \frac{\sin (\beta / n)}{\cosh (\kappa / n)-\cos (\beta / n)} d \kappa \tag{9}
\end{equation*}
$$

where $n=\alpha / \pi$ and $R(\eta)=\sqrt{r^{2}+r_{0}^{2}+2 r r_{0} \cos (\eta)}$.
Note that the total fields can be determined by adding the GO fields to the diffracted fields in (8). Similarly, the diffracted field can be expressed in the form of (7), from which the diffraction coefficient is calculated. Away from the shadow boundaries, asymptotic form of the diffraction coefficient can be derived as follows:

$$
\begin{align*}
d_{s, h}^{a s y m p}= & \frac{\sin (\pi / n)}{n}\left[\left(\cos \frac{\pi}{n}-\cos \frac{\varphi-\varphi_{0}}{n}\right)^{-1}\right. \\
& \left.\mp\left(\cos \frac{\pi}{n}-\cos \frac{\varphi+\varphi_{0}}{n}\right)^{-1}\right] \frac{\mathrm{e}^{i \pi / 4}}{\sqrt{2 \pi k}}, \tag{10}
\end{align*}
$$

where (-) and (+) are for soft and hard polarizations, respectively. Here, the complex exponential term $\mathrm{e}^{\mathrm{i} / 4}$ indicates that the wedge tip is the caustic of edge-diffracted rays. Note that the critical part of this representation is the numerical integration of the $V_{d}(\beta)$ integral in (9), which contains singularities on the complex $\beta$ plane because of Hankel function and the denominator [26]. Since the direct numerical integration along positive real axis is very time consuming, a deformed contour can be used to accelerate the computations.

## C. Uniform theory of diffraction (UTD) model

In this model, the diffracted field is also expressed in the same form in (7), except that $u_{0}=H_{0}^{(2)}\left(k r_{0}\right)$ and $\mathrm{e}^{i \pi / 4}$ is replaced by $\mathrm{e}^{-j \pi / 4}$ due to the time dependence convention in UTD. The UTD diffraction coefficients are given as [10]:

$$
\begin{align*}
& d_{s, h}=\frac{-e^{-j \pi / 4}}{2 n \sqrt{2 \pi k}}\left\{\left[\cot \left(\frac{\pi-\xi^{-}}{2 n}\right) F\left(k L g^{+} \xi^{-}\right)\right.\right. \\
&\left.+\cot \left(\frac{\pi+\xi^{-}}{2 n}\right) F\left(k L g^{-} \xi^{-}\right)\right]  \tag{11}\\
& \mp\left[\cot \left(\frac{\pi-\xi^{+}}{2 n}\right) F\left(k L g^{+} \xi^{+}\right)\right. \\
&\left.\left.+\cot \left(\frac{\pi+\xi^{+}}{2 n}\right) F\left(k L g^{-} \xi^{+}\right)\right]\right\}
\end{align*}
$$

where (-) and (+) are for soft and hard polarizations, respectively. Furthermore, $\xi^{+}=\varphi+\varphi_{0}, \quad \xi^{-}=\varphi-\varphi_{0}$ and $F(X)$ is the Fresnel function given as follows:

$$
\begin{equation*}
F(X)=2 j \sqrt{X} e^{j X} \int_{\sqrt{X}}^{\infty} e^{-j \tau^{2}} d \tau \tag{12}
\end{equation*}
$$

and $L, g^{ \pm}$are computed as follows:

$$
\begin{equation*}
L=\frac{r r_{0}}{r+r_{0}}, g^{ \pm}(\xi)=2 \cos ^{2}\left(\frac{2 n \pi N^{ \pm}-\xi}{2}\right), \tag{13}
\end{equation*}
$$

where $N^{ \pm}=( \pm \pi+\xi) / 2 n \pi$ are the integers that most closely satisfy this expression.

Note that the cotangent functions in (11) possess singularities at the shadow boundaries, and hence, can be replaced by the following:

$$
\begin{align*}
& \cot \left(\frac{\pi \pm \beta}{2 n}\right) F\left(k L g^{ \pm} \xi\right)  \tag{14}\\
& \simeq n\left[\sqrt{2 \pi k L} \operatorname{sgn}(\varepsilon)-2 k L \varepsilon e^{-j \pi / 4}\right] e^{-j \pi / 4}
\end{align*}
$$

for small $\varepsilon \rightarrow 0$. Similarly, the total fields are obtained by adding the GO and the diffracted fields, according to the LOS conditions. Finally, note that the UTD diffraction coefficients are equivalent to (10) away from the shadow boundaries, by replacing $\mathrm{e}^{\mathrm{i} \mathrm{\pi /4}}$ with $\mathrm{e}^{-\mathrm{j} \pi / 4}$.

## C. Parabolic equation (PE) model

The parabolic equation (PE) diffraction method provides a correct first-order approximation to the diffracted field in the case when $k r \gg 1$ and $k r_{0} \gg 1$ [13]. Note that, in spite of the similarity of
name, this model is different from the split-step parabolic equation (SSPE) method, which is a numerical marching type algorithm described in Sec. III. The PE-based diffracted field can be determined as follows:
$u_{s, h}^{d}=u_{0} \sqrt{\frac{r_{0}}{r_{0}+r}} W_{s, h}\left(k \rho, \varphi, \varphi_{0}\right) \mathrm{e}^{i k r}, \rho=\frac{r r_{0}}{r+r_{0}}$,
where

$$
\begin{equation*}
W_{s, h}\left(k \rho, \varphi, \varphi_{0}\right)=w\left(k \rho, \varphi-\varphi_{0}\right) \mp w\left(k \rho, \varphi+\varphi_{0}\right), \tag{16}
\end{equation*}
$$

where (-) and (+) are for soft and hard polarizations, respectively. Here,

$$
\begin{equation*}
w(k \rho, \psi) \mathrm{e}^{i k \rho}=\frac{1}{\pi \sqrt{2}} \mathrm{e}^{i(k \rho+\pi / 4)} \int_{-\infty}^{\infty} \mathrm{e}^{-k \rho s^{2}} \frac{\frac{1}{n} \sin \frac{\pi}{n}}{\cos \frac{\pi}{n}-\cos \frac{\eta(s)+\psi}{n}} d s, \tag{17}
\end{equation*}
$$

which is equivalent to the following form:

$$
\begin{align*}
w(k \rho, \psi) \mathrm{e}^{i k \rho}= & \frac{1}{\pi \sqrt{2}} e^{i\left(k \rho+\frac{\pi}{4}\right.} \int_{-\infty}^{\infty} e^{-k \rho s^{2}}\left[\frac{1}{2 n} \cot \frac{\eta(s)+\psi-\pi}{2 n}\right. \\
& -\frac{1}{2} \csc \frac{\zeta(s)+\psi-\pi}{2} \sec \frac{\zeta(s)}{2}  \tag{18}\\
& \left.-\frac{1}{2 n} \cot \frac{\eta(s)+\psi+\pi}{2 n}\right] d s \\
& +e^{-i k \cos \mu} \frac{e^{-i \pi / 4} \sqrt{\sqrt{x}}}{\sqrt{\sin } \frac{\psi-\pi}{2}} \int_{\sqrt{2 k \rho} \sin \frac{\mu-\pi}{2}}^{e^{i k^{2}}} d x
\end{align*}
$$

where $\eta=\sqrt{2} s \mathrm{e}^{-\mathrm{i} \pi / 4}, \quad s=\sqrt{2} \mathrm{e}^{-\mathrm{i} \pi / 4} \sin \zeta / 2$ and $\zeta(s)=-i \ln \left(1+i s^{2}+i s \sqrt{s^{2}-2 i}\right)$. This form is convenient for numeric calculation and asymptotic evaluation [26]. In the vicinity of the saddle point ( $|s| \ll 1$ ), the following approximations can be made:

$$
\begin{align*}
& \zeta(s)=s \sqrt{2} \mathrm{e}^{-i \pi / 4}+O\left(s^{2}\right)=\eta(s)+O\left(s^{2}\right),  \tag{19a}\\
& \text { sec } \frac{\zeta(s)}{2}=1+O\left(s^{2}\right) . \tag{19b}
\end{align*}
$$

Thus, the two first terms in the square brackets of (18), which are singular at the saddle point when $\psi=\varphi \pm \varphi_{0}=\pi$, completely cancel each other and the standard saddle point technique can be applied to the integral over the variable $s$.

Finally, the diffraction coefficients are found as follows:

$$
\begin{equation*}
d_{s, h}=W\left(k \rho, \varphi, \varphi_{0}\right) \sqrt{\frac{r r_{0}}{r+r_{0}}}, \tag{20}
\end{equation*}
$$

which reduces to (10) away from the shadow boundaries.

## III. TWO-WAY SPLIT-STEP PARABOLIC EQUATION MODEL

The parabolic wave equation (PWE) is derived from the 2D Helmholtz wave equation by separating the rapidly varying phase term to obtain a reduced function varying slowly in range for propagating angles close to the paraxial (horizontal) direction. The PWE can be converted to an initial value problem and can be solved by a marchingtype numerical algorithm. The Fourier split-step parabolic equation (SSPE) is a powerful algorithm which accepts the initial field at a reference range (e.g., at an antenna) and then advances in range by computing the field along longitudinal direction at each range step. The classical SSPE is known as one-way approach because it handles only the forward-propagating waves, and cannot account for the backscattered ones. The classical one-way SSPE determines the longitudinal field at range $x+\Delta x$ as follows [20]:

$$
\begin{align*}
u(x+\Delta x, z)= & \exp \left[i k\left(n^{2}-1\right) \frac{\Delta x}{2}\right] \times \\
& F^{-1}\left\{\exp \left[-i p^{2} \frac{\Delta x}{2 k}\right] F\{u(x, z)\}\right\} \tag{21}
\end{align*}
$$

where $F$ denotes the Fourier Transform, $p=k_{z}$ is the transform variable (i.e., transverse wavenumber $p=k \sin \theta$ where $\theta$ is the propagation angle from the horizontal), and $n$ is the refractive index. Eqn. (21) is valid for propagation angles up to $10^{\circ}-15^{\circ}$, and hence, it is known as narrow-angle SSPE. Long-range propagation can be accurately modeled with the narrow-angle SSPE because propagation angles encountered in such problems are usually less than a few degrees. However, short range propagation problems, as well as the problems involving multiple reflections and diffractions because of hills and valleys with steep slopes, can be modeled more accurately with the wide-angle SSPE, which is effective for propagation angles up to $40^{\circ}-45^{\circ}$ [22, 25]. The wide-angle SSPE obtains the field at each range as follows:

$$
\begin{align*}
& u(x+\Delta x, z)=\exp [i k(n-1) \Delta x] \times \\
& F^{-1}\left\{\exp \left[i k \Delta x\left(\sqrt{1-\frac{p^{2}}{k^{2}}}-1\right)\right] F\{u(x, z)\}\right\} \tag{22}
\end{align*}
$$

Although the one-way SSPE model is quite effective in modeling electromagnetic propagation above the Earth's irregular surface through inhomogeneous atmosphere, it suffers from the disability of handling backward-propagating waves. The forward waves might be sufficient for longrange propagation scenarios. However, the backward waves become significant in the presence of obstacles that re-direct the incoming wave, which renders a necessity of estimating the multipath effects accurately. Recently, a two-way SSPE algorithm was implemented in [23] to incorporate the backward-propagating waves into the solution, through a recursive forward-backward scheme to model the electromagnetic propagation over a staircase-approximated terrain (i.e., the terrain is modeled like a train of knife-edges). If the wave meets the terrain (or knife-edge), it is partially-reflected by imposing the appropriate boundary conditions on the terrain facet, and is marched out in the backward direction by reversing the paraxial direction in the PWE formulation. Note that the same form of (22) is used during the backward propagation. At each step where the wave hits the terrain, the wave is split into two components (forward and backward). Each wave component continues to march out in its own paraxial direction. The convergence of the algorithm is checked against a certain threshold criterion comparing the total fields at each iteration. The two-way SSPE algorithm was implemented in MATLAB and presented as a software tool (called PETOOL), which can be downloaded from [25].

## IV. NUMERICAL SIMULATIONS

This section demonstrates the test results of the analytical and numerical techniques over different propagation scenarios. The first scenario involves a single knife-edge of height 150 m located at 3 km . The line-source is at 50 m height; the frequency is 300 MHz ; and the polarization is soft. The 3D maps of the propagation factor ${ }^{1}$ within the entire domain are plotted in Fig. 3. Note that the exact models are not included in this figure because they are extremely timeconsuming.

A good agreement is observed among the results of the analytical techniques. When the two-

[^0]way SSPE is compared with the analytical techniques, a good conformity is obtained in the region before 3 km that is in the interference region, but a slight discrepancy is observed in the region after 3 km that is in the deep-shadow / diffraction region. This discrepancy might be due to ignoring some of the less contributing ray components, and/or due to the limitation of SSPE within the paraxial regions. It is also observed that artificial effects around the shadow boundaries are clearly apparent on the maps of the analytical methods. However, SSPE model provides smoother wave behavior around the transition regions.


Fig. 3. 3D PF maps for a single knife edge above ground at 3 km range illuminated by a line-source at 50 m height (soft polarization): (a) Two-way SSPE, (b) GO+UTD, (c) GO+PE [f = 300 MHz ].

All techniques are also compared in Fig. 4, which shows the polar plot of the total field along a circle of radius 50 m centered at the tip of the single knife edge. In addition, in Figs. 5(a) and 5(b), the PF values are plotted at two different values of range ( 2.8 km and 4 km , which are in interference and shadow regions, respectively). In Figs. 5(c) and (d), similarly, the values are plotted at different height values ( 140 m and 160 m ). It is concluded from the results that all analytical methods are in good harmony with each other. The results of the two-way SSPE are slightly different in the shadow region, as discussed above. However, it is remarkable to emphasize that the efficiency of the two-way SSPE, in terms of the computation time, is quite superior to the analytical techniques. Moreover, the two-way SSPE can easily be applied to any type of terrain or any number of knife-edges with little effort. If the analytical techniques are sorted according to the computation time, UTD is the fastest, and then the PE, exact integral and exact series methods come in order. But, especially in the exact methods, the computation time is highly dependent on the frequency and the distance between the observation point and the edge. Note that analytical exact models serve as reference if numerically computed accurately.


Fig. 4. Polar plot of the total field along a circle of radius 50 m centered at the tip of the single knife edge shown in Fig. 3. [s: series, i: integral].


Fig. 5. 2D PF plots for the single knife edge problem shown in Fig. 3: (a) PF vs. height at 2.8 km range, (b) PF vs. height at 4 km range, (c) PF vs. range at 140 m height, (d) PF vs. range at 160 m height. [s: series, i: integral].

The second scenario is the same as the first one, except that the polarization is hard and the line-source is at 100 m height. The results are plotted in Figs. 6, 7, and 8. Again, a good agreement is observed among different models.


Fig. 6. 3D propagation factor maps for a single knife edge above ground at 3 km range illuminated by a line-source at 100 m height (hard polarization): (a) Two-way SSPE, (b) GO+UTD, (c) GO + PE. $[\mathrm{f}=300 \mathrm{MHz}]$.

The last scenario considers double knife-edge of heights 150 m and 170 m located at 3 km and 5 km , respectively. The line-source is at 25 m height; the frequency is 300 MHz ; and the polarization is soft. The results are plotted in Figs. 9,10 , and 11. In comparing the two-way SSPE
with the analytical methods, the contributions of the waves hitting the walls up to 3 times are superposed. To achieve fair comparisons up to the third degree of reflections, the analytical methods account for 35 types of rays bouncing from the walls and the ground. The multiple bouncing of the diffracted fields from the walls and the ground is ignored due their negligible effects compared to strong reflections.


Fig. 7. Polar plot of the total field along a circle of radius 50 m centered at the tip of the single knife edge shown in Fig. 6. [s: series, i: integral].

Again, a good agreement is observed especially in the regions where strong multipath effects are observed, but some discrepancies exist between SSPE and the diffraction algorithms in the deep shadow region.

Simulations with double knife-edges and with other frequencies are repeated for the other polarization (i.e., hard boundary condition) and similar agreement among the models are obtained.



Fig. 8. 2D PF plots for the single knife edge problem shown in Fig. 6: (a) PF vs. height at 2.8 km range, (b) PF vs. height at 4 km range, (c) PF vs. range at 140 m height, (d) PF vs. range at 160 m height. [s: series, i: integral].


Fig. 9. 3D propagation factor maps for double knife edge above ground at 3 km and 5 km ranges illuminated by a line-source at 25 m height (soft polarization): (a) Two-way SSPE, (b) GO+UTD, (c) GO+PE. [f = 300 MHz .

## V. CONCLUSION

Analytical (exact and asymptotic) techniques and the two-way SSPE were discussed and compared through several numerical simulations in the context of the problem of scattering from single and double edge above ground. It was concluded that the results are in good agreement in general, however, some differences might be observed in the deep shadow region or around the shadow boundaries. The disparity of the two-way SSPE in the shadow region, which indeed exhibits "small" field values, might be tolerated in favor of
its computational efficiency. Finally, MATLABbased codes were provided in Appendix.

(a)

(b)

Fig. 10. Polar plot of the total field along a circle of radius 50 m centered at the tip of each knife edge shown in Fig. 9: (a) Edge at 3 km range, (b) Edge at 5 km range. [s: series, i: integral].




Fig. 11. 2D propagation factor (PF) plots for the double knife edge problem shown in Fig. 9: (a) PF vs. height at 2.5 km range, (b) PF vs. height at 4 km range, (c) PF vs. range at 100 m height, (d) PF vs. range at 160 m height. [s: series, i integral].

## APPENDIX MATLAB Codes

Sample codes for the diffraction algorithms in Sec. II are given in Tables 1-5 below. The codes computing the 3D field maps of the single or double edge problem (with respect to various LOS conditions) can be provided upon request from the authors. Note that the two-way SSPE algorithm (PETOOL) can be downloaded from [25].

Table 1: Input parameters common to the codes clear all; format long;
freq $=300$; \%input('frequency $[\mathrm{MHz}]=$ ');
alfamax = 360; \%input('wedge angle [Deg] = ');
$\mathrm{I} 0=1 ; \%$ input('strength of line source $=$ ');
$\mathrm{r}=50$; \%input('distance of the observer to wedge $=$ ');
$\mathrm{r} 0=212$; \%input('distance of the source to wedge = ');
ang0 $=45$; \%input('incident angle [Deg] = ');
ang = 150; \%input('observation angle [Deg] = ');
c = 3e8; \% speed of light
alfa $=$ alfamax*pi/180; \% change wedge angle degree to radians ang $0=$ ang $0 * \mathrm{pi} / 180$; \% change incident angle degree to radians ang $=$ ang*pi/180; \% change observation angle degree to radians error $=1 \mathrm{e}-12$;
freq $=$ freq*1e6; rlam $=r *$ freq/c; $k=2 *{ }^{*} i^{*}$ freq/c; $k r=k * r ; k r 0=k^{*} r 0 ;$
Table 2: Code of the exact by series method
Us=0; Uh=0; n=0; vn=0; err1 = 1e6; err2 = 1e6; coeff1 = pi*I0/(1i*alfa); $\mathrm{U} 0=\mathrm{I} 0 * \mathrm{besselh}(0,1, \mathrm{kr} 0) /(4 * 1 \mathrm{i})$;
\% define N for Soft BSc
while (err1 > error)
$\mathrm{vn} 1=n * \mathrm{pi} /$ alfa; vn2 $=(\mathrm{n}+1)^{*} \mathrm{pi} /$ alfa; $\mathrm{vn} 3=(\mathrm{n}+2)^{*} \mathrm{pi} /$ alfa;
$\mathrm{vn} 4=(\mathrm{n}+3) * \mathrm{pi} /$ alfa; vn5 $=(\mathrm{n}+4)^{*} \mathrm{pi} /$ alfa;
if $\mathrm{r}<=\mathrm{r} 0$
$\mathrm{a} 1=$ besselj(vn1,kr)*besselh(vn1,1,kr0)*sin(vn1*ang0)*sin(vn1*ang); a2 $=\operatorname{besselj}(\mathrm{vn} 2, \mathrm{kr}) *$ besselh(vn2,1,kr0)*sin(vn2*ang0)*sin(vn2*ang); a3 $=$ besselj $(\mathrm{vn} 3, \mathrm{kr}) *$ besselh(vn3,1,kr0) ${ }^{*} \sin (\mathrm{vn} 3 *$ ang 0$) * \sin (\mathrm{vn} 3 * \mathrm{ang})$; $\mathrm{a} 4=\operatorname{besselj}(\mathrm{vn} 4, \mathrm{kr}) *$ besselh(vn4,1,kr0)* $\sin (\mathrm{vn} 4 * a n g 0) * \sin (\mathrm{vn} 4 * \mathrm{ang})$; a5 = besselj(vn5,kr)*besselh(vn5,1,kr0)*sin(vn5*ang0)*sin(vn5*ang); Xs = a1+a2+a3+a4+a5; err1 = abs(Xs); else
$\mathrm{a} 1=\operatorname{besselj}(\mathrm{vn} 1, \mathrm{kr} 0) *$ besselh(vn1,1,kr)$* \sin (\mathrm{vn} 1 * a n g 0) * \sin (\mathrm{vn} 1 * a n g) ;$ a2 $=$ besselj(vn2,kr0)*besselh(vn2,1,kr)*sin(vn2*ang0)*sin(vn2*ang); a3 $=$ besselj $(\mathrm{vn} 3, \mathrm{kr} 0) *$ besselh $(\mathrm{vn} 3,1, \mathrm{kr})^{*} \sin (\mathrm{vn} 3 *$ ang 0$) * \sin (\mathrm{vn} 3 * \mathrm{ang})$; $\mathrm{a} 4=\operatorname{besselj}(\mathrm{vn} 4, \mathrm{kr} 0) *$ besselh(vn4,1,kr)* $\sin (\mathrm{vn} 4 * a n g 0) * \sin (\mathrm{vn} 4 * \mathrm{ang})$; a5 = besselj(vn5,kr0)*besselh(vn5,1,kr)*sin(vn5*ang0)*sin(vn5*ang); Xs = a1+a2+a3+a4+a5; err1 = abs(Xs); end

## $\mathrm{n}=\mathrm{n}+1$;

end
$\mathrm{N} 1=\mathrm{n}+10$;
\% define N for Hard BSc
$\mathrm{n}=0$; $\mathrm{vn}=0$;
while (err2 > error)
vn1 $=n *$ pi/alfa; vn2 $=(n+1)^{*}$ pi/alfa; vn3 $=(n+2) * p i / a l f a ;$
$\mathrm{vn} 4=(\mathrm{n}+3)^{*} \mathrm{pi} /$ alfa; vn5 $=(\mathrm{n}+4)^{*} \mathrm{pi} /$ alfa;
if ( $n==0$ ) eps $=0.5$; else eps $=1$; end
if $\mathrm{r}<=\mathrm{r} 0$
$\mathrm{b} 1=$ eps*besselj $(\mathrm{vn} 1, \mathrm{kr}) *$ besselh(vn1,1,kr0) ${ }^{*} \cos (\mathrm{vn} 1 * \operatorname{ang} 0){ }^{*} \cos (\mathrm{vn} 1 * \mathrm{ang}) ;$
b2 = eps*besselj(vn2,kr)*besselh(vn2,1,kr0)*cos(vn2*ang0)*cos(vn2*ang);
$\mathrm{b} 3=$ eps*besselj $(\mathrm{vn} 3, \mathrm{kr})^{*}$ besselh $(\mathrm{vn} 3,1, \mathrm{kr} 0)^{*} \cos \left(\mathrm{vn} 3^{*} \mathrm{ang} 0\right)^{*} \cos (\mathrm{vn} 3 * \mathrm{ang})$;
$\mathrm{b} 4=$ eps*besselj $(\mathrm{vn} 4, \mathrm{kr}) *$ besselh $(\mathrm{vn} 4,1, \mathrm{kr} 0) * \cos (\mathrm{vn} 4 * \mathrm{ang} 0) * \cos (\mathrm{vn} 4 * \mathrm{ang}) ;$
b5 = eps*besselj(vn5,kr)*besselh(vn5,1,kr0)*cos(vn5*ang0)*cos(vn5*ang);
Xh = b1+b2+b3+b4+b5; err2 = abs(Xh);
else
b1 = eps*besselj(vn1,kr0)*besselh(vn1,1,kr)* $\cos (\mathrm{vn} 1 * a n g 0) * \cos (\mathrm{vn} 1 * a n g)$;
b2 = eps*besselj(vn2,kr0)*besselh(vn2,1,kr)*cos(vn2*ang0)*cos(vn2*ang);
b3 $=$ eps ${ }^{*}$ besselj $(\mathrm{vn} 3, \mathrm{kr} 0){ }^{*}$ besselh $(\mathrm{vn} 3,1, \mathrm{kr})^{*} \cos (\mathrm{vn} 3 * \text { ang } 0)^{*} \cos (\mathrm{vn} 3 * \mathrm{ang})$;
$\mathrm{b} 4=$ eps*besselj(vn4,kr0)*besselh(vn4, $1, \mathrm{kr})^{*} \cos (\mathrm{vn} 4 * \mathrm{ang} 0) * \cos (\mathrm{vn} 4 * \mathrm{ang})$;
b5 = eps*besselj(vn5,kr0)*besselh(vn5,1,kr)* $\cos \left(\mathrm{vn5}{ }^{*}\right.$ ang0) ${ }^{*} \cos \left(\mathrm{vn5}{ }^{*} \mathrm{ang}\right)$;
Xh = b1+b2+b3+b4+b5; err2 = abs(Xh);
end

[^1]Table 3: Code of the exact by integral method
sgm $=\left(2^{*}\right.$ pi-alfa $) / 2 ;$ phi0 $=$ ang $0+$ sgm; $B=$ ang + sgm;
$\mathrm{nu}=\left(2^{*} \mathrm{pi}-2^{*} \mathrm{sgm}\right) / \mathrm{pi} ; \mathrm{phi}=\mathrm{B} ; \mathrm{m}=0 ; \mathrm{U} 0=\operatorname{besselh}(0,1, \mathrm{kr} 0)$;
V1 = Vd_Int(-pi-phi + phi0,r,r0,nu,k); V2 = Vd_Int(pi-phi+phi0,r,r0,nu,k);
V3 = Vd_Int(-pi-phi-phi0+2*sgm,r,r0,nu,k);
V4 = Vd_Int(pi-phi-phi0 0 2*sgm,r,r0,nu,k);
UsnDIFF $=(\mathrm{V} 1-\mathrm{V} 2-\mathrm{V} 3+\mathrm{V} 4) / \mathrm{U} 0 ; \%$ exact normalised diffracted fields
UhnDIFF $=(V 1-V 2+V 3-V 4) / U 0 ; \%$ exact normalised diffracted fields
BWsnDIFF $=$ UsnDIFF ${ }^{*}$ sqrt( $\left.2{ }^{*} \mathrm{pi}{ }^{*} \mathrm{kr}\right) / \exp \left(1 \mathrm{i}^{*} \mathrm{kr}\right) ; \%$ diff coeff for soft BCs
BWhnDIFF $=$ UhnDIFF*sqrt(2*pi*kr)/exp( $1 i^{*}$ kr); \% diff coeff for hard BCs
BWsnDIFF $=\operatorname{conj}\left(B W s n D I F F / s q r t\left(2 *{ }^{*}{ }^{*} \mathrm{k}\right)\right)$
BWhnDIFF $=\operatorname{conj}($ BWhnDIFF/sqrt(2*pi*k))
\% Function : Vd_Int.m (calculate Vd integral on deformed contour )
function result = Vd_Int(beta,ro,ro0,nu,k)
eps $=1 \mathrm{e}-12 ; \quad \mathrm{d}=1 \mathrm{e}-3 ; \%$ step size of the integration
$\operatorname{Mmax}=5000 ; y 0=\min (\mathrm{pi}, \mathrm{abs}($ beta) $) ;$ fun = 1e6; $\mathrm{x}=\mathrm{y} 0-0.01$; while ( abs(fun)>eps ) \% controlling maximum x value
 fun $=$ besselh $\left(0,1, \mathrm{k} * \mathrm{R} \_\mathrm{it}\right) . /(\cosh (\mathrm{z} / \mathrm{nu})-\cos (\mathrm{beta} / \mathrm{nu})) ; \mathrm{x}=\mathrm{x}+0.2$; end
xmax $=x ;$ tan_teta $=(y 0-0.01) / x m a x ; \%$ slope of the deformed contour $\mathrm{x}=0$ :d:xmax; $\mathrm{z} 1=\mathrm{x}+1 \mathrm{i}^{*} \mathrm{x} *$ tan_teta; \% deformed contour
R_it_1 $=\operatorname{sqrt}\left(\mathrm{ro}{ }^{*} \mathrm{ro}+\mathrm{ro} 0{ }^{*} \mathrm{ro} 0+2 *{ }^{*}\right.$ ro ${ }^{*} \mathrm{ro} 0 * \cos \left(1 \mathrm{i}^{*} \mathrm{z} 1\right)$ );
fun1 = $\sin ($ beta/nu)*besselh( $0,1, \mathrm{k} *$ R_it_1)./(cosh(z1/nu)-cos(beta/nu));
$\mathrm{y} 1=\operatorname{trapz}(\mathrm{z} 1$, fun 1$) ; \quad$ result $=(\mathrm{y} 1) /\left(2^{*} \mathrm{pi}{ }^{*} \mathrm{nu}\right)$;
Table 4: Code of the UTD method
phi0 = -ang0; n = (alfa)/pi; B = -ang; phi = B;
ksi_p = phi+phi0; ksi_m = phi-phi0; \% ksi+ and ksi-

Nmp $=(-$ pi+ksi_p $) /\left(2 *{ }^{*} i^{*} n\right) ;$ Nmm $=(-$ pi + ksi_m) $/(2 * p i * n) ; \% N-$
$\mathrm{m}=0$; U0 $=$ besselh $(0,1, \mathrm{kr} 0)$; coeff $=\exp \left(-1 i^{*} \mathrm{pi} / 4\right) /\left(2^{*} \mathrm{n}^{*} \mathrm{sqrt}\left(2 * \mathrm{pi}{ }^{*} \mathrm{k}\right)\right.$ );
if $(\mathrm{Npp}<1) \|(\mathrm{Npm}<1), \mathrm{Npp}=0 ; \mathrm{Npm}=0$;

```
elseif (Npp>=1) | (Npm>=1), Npp = 1; Npm = 1;
end
if (Nmp<0)|(Nmm<0), Nmp = -1; Nmm = -1;
elseif (0<=Nmp<1)|(0<=Nmm<1), Nmp = 0; Nmm = 0;
elseif (Nmp>=1) || (Nmm>=1), Nmp = 1; Nmm = 1;
end
gpp = 1+\operatorname{cos(ksi_p-2*n*pi*Npp); gpm = 1+cos(ksi_m-2*n*pi*Npm);}
gmp = 1+cos(ksi_p-2*n*pi*Nmp); gmm = 1+cos(ksi_m-2*n*pi*Nmm);
x1 = k*r**r0*gpm/(r+r0); x2 = k*r*r0*gmm/(r+r0);
x3 = k* *}\mp@subsup{}{}{*}\mp@subsup{\textrm{r}}{}{2}0*gpp/(r+r0); x4 = k*r* * 0*gmp/(r+r0);
F1 = Fresnel_Int(x1); F2 = Fresnel_Int(x2); F3 = Fresnel_Int(x3);
F4 = Fresnel_Int(x4);
UsDIFF_UTD = -coeff*(cot((pi+ksi_m)/(2*n))*F1+cot((pi-
ksi_m)/(2*n))*F2..
    -cot((pi+ksi_p)/(2*n))*F3-cot((pi-ksi_p)/(2*n))*F4); % coeff for soft BCs
UhDIFF_UTD = -coeff*(cot((pi+ksi_m)/(2*n))*F1+cot((pi-
ksi_m)/(2*n))*F2..
    +cot((pi+ksi_p)/(2*n))*F3+cot((pi-ksi_p)/(2*n))*F4); % coeff for hard BCs
Us_UTD=UsDIFF_UTD*sqrt(2*pi*k);
Uh_UTD=UhDIFF_UTD*sqrt(2*pi*k);
% ------------------------- Total Field for UTD 
if (ang >= 0) && (ang < pi-ang0) % --------- Region I -------
    R1=sqrt(r*r+r0*r0-2*r*r0*
    R2=sqrt(r**r+r0*r0-2*r*r0*
    Uxs=besselh(0,1,k*R1)-besselh(0,1,k*R2);
    Uxh=besselh(0,1,k*R1)+besselh(0,1,k*R2);
    UsTOTAL_UTD = UsDIFF_UTD+Uxs/U0;
    UhTOTAL_UTD = UhDIFF_UTD+Uxh/U0;
elseif (ang >= pi-ang0) && (ang < pi+ang0) % -------- Region II -------
    R1=sqrt(r*r+r0*r0-2*r*r0*cos(phi-phi0)); Ux=besselh(0,1,k*R1);
    UsTOTAL_UTD = UsDIFF_UTD+Ux/U0;
    UhTOTAL_UTD = UhDIFF_UTD+Ux/U0;
elseif (ang >= pi+ang0) && (ang <= alfa) % -------- Region III ------
    UsTOTAL_UTD = UsDIFF_UTD; UhTOTAL_UTD = UhDIFF_UTD;
end
Us_UTD = Us_UTD/sqrt(2*pi*k)
Uh_UTD = Uh_UTD/sqrt(2*pi*k)
% Function : Fresnel_Int.m (Fresnel integral)
function result = Fresnel_Int(x)
Ml = 0.3; Mu=5.5; coeff = 2* 1i**qrt(x)*exp(1i*x);
if (x<Ml)
result = exp(1i*(pi/4+x))*(sqrt(pi*x)-2*x*exp(1i*pi/4)-(2/3)*x*x*exp(-
1i*pi/4));
elseif (x > Mu)
    result = 1+1i/(2*x)-3/(4*x^2)-15*1i/(8*x^3)+75/(16*x^4);
else
    eps = 1e-12; Mmax = 5000; d = 1e-3; y_old = 1e6; ii = 0;
    for M=sqrt(x)+1:Mmax
    ii = ii+1; t1 = sqrt(x):d:M; t2=0:d:M-sqrt(x); t = t1-1i*t2;
    fun = exp(-1i**.*t); y(ii) = trapz(t,fun); error = abs(y(ii)-y_old);
    y_old = y(ii);
    if (error<eps), result = y(ii)*coeff; break;
    else continue
    end
    end
    result = y(ii)*coeff;
end
```

Table 5: Code of the PE diffraction method

[^2]$y(M)=\operatorname{trapz}(s$, fun $) ;$ error $=\operatorname{abs}\left(y(M)-y_{-}\right.$old $) ; y_{-}$old $=y(M) ;$
if (error<eps) , result $=y(M) ; N=M$; break;
else, continue
end
end

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# Numerical Prepackaging with PMC Lid - Efficient and Simple Design Procedure for Microstrip Circuits Including the Packaging 

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#### Abstract

The paper presents an efficient method for the design of printed microstrip circuit with packaging in mind, referred to as numerical prepackaging with a perfectly magnetic conductive (PMC) lid. The method comprises making the design including the packaging from the start by using a PMC lid, rather than first designing the open-aired circuit; and thereafter, considering the packaging effect and the often required retuning of the circuits themselves. The advantage is that no parallel plate modes can propagate between the perfect electric conductor (PEC) ground plane and the PMC lid plate if the spacing is smaller than an effective quarter of wavelength. This provides a limited computational volume so that the computation time is significantly reduced in the case of the finite element method (FEM) or the finite difference time domain method (FDTD). By using numerical packaging with PMC lid, the ideal PMC lid has to be realized afterwards e.g. by using a lid of nails, which is a minor task as compared to existing approaches.


Index terms- Cavity resonance, FDTD, FEM, PMC, PEC, shielded microstrip line.

## I. INTRODUCTION

In the design of printed microwave circuits using numerical methods such as the finite difference time domain method (FDTD) [1], finite element method (FEM) [2], finite integral method [3] or any volume type formulations, it is necessary to limit the computational space using virtual boundaries such as radiation boundaries [4]
or perfect matching boundaries [5] in terms of different implementations as described in [6-8]. In order to use such boundaries some restriction should be enforced to make valid computations. For example, it is recommended that the distance between these boundaries and the actual circuits to be in order of a quarter wavelengths at the low frequency to have reliable results. Some of the perfect matching boundary conditions require several layers [1] that in some cases would be eight layers. Such requirement regarding the distance of the virtual boundaries increases the overhead of computational time and storage. After reaching a final circuit design in such a way, one will always need to package the circuit. In order to have a compact package circuit it is important that the package cavity be very close to the circuit. However, such packaging should not affect the circuit performance. Some engineers might use resistive loading inside the packaging cavity to suppress resonances of the cavity appearing within the operating frequency band. Such a technique increases the losses and reduces the circuit efficiency. It would be better if the packaging is not affecting the circuit efficiency and at the same time be compact and nearly lossless. However, as the packaging gets closer to the circuit, packaging might interact with the circuit and affect its performance and the designer might find that some tuning of his circuit is necessary to achieve the desired performance. Such a tuning may be quite tedious to perform and requires experience in designing circuits.

Here, we propose an efficient procedure to design printed circuits with packaging in mind.

Packaging is needed for mechanical protection, but also to shield the circuits from outside strong fields and interference, and to prevent any possible radiation emissions from the circuit. In some cases resistive loading inside the package cavity can be used to suppress high order modes. Here, we propose, during the initial design to use perfect magnetic conductor (PMC) lid to shield the circuit and to consider it as an ideal packaging approach, which also will confine the computational domain volume. From the circuit point of view, one might consider the PMC as an open circuit surface dual to the perfect electric conductor (PEC) that is a short circuit surface. Note that a metal surface to a very good approximation can be considered a PEC in electromagnetic (EM) field analysis. PMC and PEC surfaces can be used as boundaries that limit the computational domain. Since PMC could be considered as an open circuit, its influence on the printed circuit is much smaller than a PEC (i.e. a metal surface). Possible parallel plate waveguide modes will be suppressed as far as the effective height of the parallel plates between the ground PEC and the PMC lid is less than an effective quarter wavelength [6]. As resonances are suppressed from the cavity the convergence of the FDTD method should improve as well. One might object on using PMC as it does not exist in nature, but nowadays we can artificially realize PMC or high impedance surfaces or what is referred to as an artificial magnetic conductor (AMC) using periodic surfaces. There are many ways to realize such surfaces, but it is important to realize it in a way that will not increase the losses of the system. Actually, the PMC lid packaging relates more to realizing a parallelplate cut-off (or stopband) between PEC and AMC, and different ways of realizing this are studied numerically in [10]. The simplest realization is in terms of a lid of nails or pins, for which the practical demonstration of usefulness to packaging is described in [11].

The PMC prepackaging is a result of research on soft and hard surfaces originating from a generalization of the corrugated surfaces used in corrugated horn antennas. This surface concept was first defined in 1988, and improved in 1990 [12]. The hard surface was already in 1996 applied to realize what today is known as cloaking [13]. The original soft surface is a transverse PEC/PMC
strip grid acting as an anisotropic electromagnetic bandgap (EBG) surface, as explained in [14] and [16]. Other marvels of EBG surfaces are described in [15]. The ideal PEC surface, PMC and PEC/PMC strip grids are in [16] referred to as canonical surfaces. The use of simplified canonical representations of advanced periodic surfaces with many details is very important in conceptual and numerical work; in conceptual work they improve physical understanding and creativity, and in numerical work they are fast and convenient to use in initial studies. The canonical surface concept has e.g. resulted in the present PMC prepackaging approach. Note that canonical representation of the EBG surface is proposed in [17], although not yet being so generally applicable as the PEC, PMC, and PEC/PMC strip grid. The EBG surface can also be used to create parallel-plate cut-off [10]. The concepts of soft and hard surfaces and canonical surfaces are also the background of the gap waveguide technology, as introduced in [6], verified by measurements in [11-18], and studied by plane wave spectral domain solutions in [19] and by classical subdomain plane wave expansions in [20]. The latter theoretical work is extended to more analytic expressions in [20]. There exists three types of gap waveguides; ridge gap waveguide, groove gap waveguide, and microstrip gap waveguide [22]. The PMC packaged microstrip line of the present paper is a kind of microstrip gap waveguide.

In this paper, we perform a parametric study of microstrip transmission lines in the presence of PMC cover. We look at what is the proper separation distance between the microstrip lines and the PMC that does not affect the original characteristic of the microstrip lines as ideal transmission line. We also study the effect of the PMC shielding on microstrip lines that violates the radiation condition as provided in [23-24]. The study shows how the PMC shielding removes such radiation condition constraints. Also, we look at some discontinuities that cause radiation losses from the circuit and how the PMC shielding also suppress such radiation losses. After that we introduce a periodic structure that is designed to realize the PMC surface and compare the ideal PMC shielding with the realized artificial magnetic conductor. We believe that such a procedure will eliminate or at least reduce the
needs for any tuning for the printed circuit after packaging. In addition, the design can be performed in one combined process, which certainly will be much faster than what is currently performed (as the circuit is designed in one process and the packaging in a second process). For the proposed design to be implemented, the software has to have the ability to model and include PMC surfaces. There are several commercial software codes that have such capabilities. Here, we considered a commercial software CST microwave studio [25], which is based on the volume integral equation in frequency and time domain.

## II. COMPARISON OF RESONANCE FREQUENCIES OF CAVITIES WITH PEC OR PMC LIDS

Usually, printed circuits are packaged in a conducting cavity, i.e. the circuit is located in a metal box so that it is surrounded by six conducting walls (ground plane, four conducting sidewalls, and a conducting lid). This creates an inhomogeneous cavity as shown in Fig. 1, i.e. being filled with a dielectric substrate layer located on the ground plane wall and with metal traces on the other side of it. The height h of the cavity is suggested to be higher than the substrate surface by 5 or 4 times substrate thickness in order not to affect the characteristic impedances of the microstrip lines, and not to interact with the circuit, and in turn not to affect the designed circuit performance. Since the total height is usually small, a transverse magnetic $\mathrm{TM}_{\mathrm{z}}$ mode with mainly z-directed electric field is supported. This makes the first resonance of the cavity determined by the dimensions of the ground-plane of the cavity. If the cavity modes are within the operating frequency band of the circuit, the cavity will interact strongly with the circuit and distort its characteristics. The effect is in particular severe when packaging active components such as mixers, oscillators, and amplifiers. The cavity may be loaded with resistive sheets or absorbers in order to suppress the resonances.

However, if the cavity has a PMC lid, the $\mathrm{TM}_{\mathrm{z}}$ mode will be in cut-off, as well as $\mathrm{TE}_{\mathrm{z}}$ modes, as long as the cavity height is smaller than a quarter of the wavelength, effectively. Therefore, the first resonance will appear at a much higher frequency
than for the PEC lid case. The PMC concept is rather new, so it is not possible to find fundamental results like resonance frequencies of conducting cavities with PMC lids. Therefore, we will here present the formulas together with those of a conducting cavity with PEC lid. To that end, we consider an ideal rectangular cavity with PEC floor and sidewalls, and either a PEC or PMC lid at a height h above the floor, and the cavity has a dielectric layer of thickness $t$ at the floor, as shown in Fig. 1. The cavity planar dimensions are $a \times b$, and $\varepsilon_{1}$ is the relative permittivity of the dielectric substrate.


Fig. 1. Partially filled cavity with PEC base and sidewalls, and (a) PEC lid and (b) PMC lid. The sidewalls are not shown.

The resonance frequencies can for the PEC lid case be obtained by solving the transcendental dispersion equations, in [26] for $k_{z}$, i.e.
for $\mathrm{TM}_{\mathrm{x}}$ mode:
$\frac{\mathrm{k}_{\mathrm{z} 1}}{\varepsilon_{1}} \tan \mathrm{k}_{\mathrm{z} 1} t=-\frac{\mathrm{k}_{\mathrm{z2}}}{\varepsilon_{2}} \tan \mathrm{k}_{\mathrm{z} 2}(h-t)$
for $\mathrm{TE}_{\mathrm{x}}$ mode:
$\mathrm{k}_{\mathrm{z} 1} \cot \mathrm{k}_{\mathrm{z} 1} t=-\mathrm{k}_{\mathrm{z} 2} \cot \mathrm{k}_{\mathrm{x} 2}(h-t)$
Similar equations can be derived for the PMC lid case, taking the form:
for $\mathrm{TM}_{\mathrm{x}}$ modes:
$\frac{\mathrm{k}_{\mathrm{z1}}}{\varepsilon_{1}} \tan \mathrm{k}_{\mathrm{z} 1} t=\frac{\mathrm{k}_{\mathrm{zz}}}{\varepsilon_{2}} \cot \mathrm{k}_{\mathrm{z2}}(h-t)$
for $\mathrm{TE}_{\mathrm{x}}$ modes:
$\mathrm{k}_{\mathrm{z} 1} \cot \mathrm{k}_{\mathrm{z} 1} t=\mathrm{k}_{\mathrm{z} 2} \tan \mathrm{k}_{\mathrm{z} 2}(h-t)$
where in both cases

$$
\begin{align*}
& k_{z 1}=\left[\omega^{2} \varepsilon_{1} \mu-\left(\frac{n \pi}{a}\right)^{2}+\left(\frac{m \pi}{b}\right)^{2}\right]^{1 / 2}  \tag{3.a}\\
& k_{z 2}=\left[\omega^{2} \varepsilon_{2} \mu-\left(\frac{n \pi}{a}\right)^{2}+\left(\frac{m \pi}{b}\right)^{2}\right]^{1 / 2} \tag{3.b}
\end{align*}
$$

and where $n$ and $m$ are integer numbers.
For a homogeneous cavity there exists closed form expressions for the resonant frequencies, which for the $\mathrm{TM}_{\mathrm{z}}$ case is given as:

$$
\begin{align*}
& f(G H z)_{m n l}^{P E C}=\sqrt{\left[\left(\frac{l \pi}{d}\right)^{2}+\left(\frac{n \pi}{a}\right)^{2}+\left(\frac{m \pi}{b}\right)^{2}\right]}, l=0,1,2 \cdots, m=1,2 \cdots, n=1,2 \cdots  \tag{4.a}\\
& f(G H z)_{m l l}^{P M C}=\sqrt{\left[\left(\frac{(l+1) \pi}{2 d}\right)^{2}+\left(\frac{n \pi}{a}\right)^{2}+\left(\frac{m \pi}{b}\right)^{2}\right]}, l=0,1,2 \cdots, m=1,2 \cdots, n=1,2 \cdots \tag{4.b}
\end{align*}
$$

Table 1 shows the first resonance in a cavity with different dimensions and different dielectric materials, computed from the above formulas. For a homogenized cavity, the base dimensions of the cavity determine the first resonant frequency when the cover is PEC, but the height determines the first resonance frequency when the cover is PMC. In both cases, the height must be much smaller than the cavity base dimensions and smaller than a quarter of a wave length. It is in our interest to show results when the cavity is partially filled with dielectric substrate and the separation between the dielectric surface and the top is small. The results are shown in Table 1 for different separation
distances ranging from $2 t$ (i.e. $h=3 \mathrm{~mm}$ ) to $4 t$ (i.e. $h=5 \mathrm{~mm})$. We see that the size of the cavity base and the dielectric loading determines the first resonance for the PEC case, and again the cavity height and the permittivity determine the first resonance for the PMC case. It can also be seen that the resonance frequency decrease when the permittivity increases and when the separation between the substrate and the lid increases. The resonances for the PMC lid case are 3 to 5 times higher than those of the PEC case.

## III. PMC PACKAGING EXAMPLES FOR MICROSTRIP CIRCUITS

First, we consider a simple $50 \Omega$ microstrip line as a two port device, and we use waveguide ports in CST in order to have perfect match to the microstrip line ports. We need this perfect match in order to get good accuracy in determining the reflection coefficient and associated transmission losses. The computational domain is bounded by the radiation box as suggested by the software developer. Thereafter, the PMC is added with different separations between the substrate surface and the PMC. Notice that the upper bound of the computational domain now is bounded by the PMC, and there is no need for any upper radiation boundary. However, the side walls of the computational domains are still bounded by what is suggested by the software developer. The computational domain is now limited between the ground plane and the PMC. This reduces the computational domain a lot. To make simple design guide, the separation distance will be related to the physical substrate thickness and transmission line width, but we have to keep in mind that this distance must be smaller than the quarter of a wavelength of the upper frequency to

Table 1: First resonance frequency in GHz of a cavity with PEC lid and PMC lid, with a substrate thickness $t=1 \mathrm{~mm}$

| cavity base <br> $a \times b(\mathrm{~mm})$ | $\varepsilon_{1}$ | $\mathrm{~h}=5 \mathrm{~mm}$ |  | $\mathrm{~h}=4 \mathrm{~mm}$ |  | $\mathrm{~h}=3 \mathrm{~mm}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | PEC lid | PMC lid | PEC lid | PMC lid | PEC lid | PMC lid |
| $50 \times 50$ | 1.0 | 4.243 | 15.588 | 4.243 | 19.224 | 4.243 | 25.357 |
|  | 2.3 | 3.994 | 15.329 | 3.930 | 18.781 | 3.820 | 24.281 |
|  | 10.2 | 3.832 | 14.274 | 3.724 | 16.525 | 3.538 | 18.982 |
|  | 1.0 | 3.905 | 15.588 | 3.905 | 19.224 | 3.905 | 25.357 |
|  | 2.3 | 3.677 | 15.260 | 3.618 | 18.729 | 3.517 | 24.246 |
|  | 10.2 | 3.528 | 14.224 | 3.429 | 16.497 | 3.258 | 18.970 |
| $60 \times 60$ | 1.0 | 3.536 | 15.588 | 3.536 | 19.224 | 3.536 | 25.357 |
|  | 2.3 | 3.329 | 12.277 | 3.275 | 18.677 | 3.184 | 24.211 |
|  | 10.2 | 3.195 | 14.182 | 3.106 | 16.469 | 2.951 | 18.956 |

make sure that the high order modes of the cavity will not be excited. Several parameters are considered such as the permittivity and thickness of the substrate.
a) Insertion losses: Here, we consider an example of a 20 mm long $50 \Omega$ microstrip line with a substrate thickness $t=0.635 \mathrm{~mm}$ and two different relative permittivities of 10.2 and 6.15 and loss tangent 0.0027 . The conductor is considered to be copper. The transmission coefficient $S_{21}$ is shown versus frequency in Fig. 2 for three cases; open, with PEC at $4 t$ from the substrate, and PMC. It can be seen that packaging eliminates the radiation losses which is large for the open case above 18 GHz . However, the PMC packaging provides smaller insertion losses than both the open and PEC cases. It can also be seen that higher permittivity cause larger insertion loss. It should be mentioned that based on [6], the microstrip line will have radiation losses if:

$$
\begin{equation*}
f\{G H z\} \times t\{m m\}>2.14 \sqrt{\varepsilon_{r}} \tag{5}
\end{equation*}
$$

which corresponds to 10.8 GHz when the relative permittivity is 10.2 and 8.4 GHz when it is 6.15 .


Fig. 2. Insertion losses of a straight microstrip line for different packaging cases, and for two different relative permittivities.
b) Characteristic Impedance: Figure 3 shows the variation of the characteristic impedance with frequency for the PMC lid located at different heights above the substrate. As expected, the characteristic impedance increases as the PMC lid gets closer to the line, because then the electric fields becomes more confined to the line so that its effective width is reduced. As such, the characteristic impedance increases since the characteristic impedance value is inversely proportional to the effective width of the microstrip line. The results indicate that a $3 t$ to $4 t$ air gap is sufficient to keep the characteristic impedance the same as the open case (no lid).


Fig. 3. Effect of the PMC lid on the characteristicimpedance of the microstrip line, $g=h-t$.
c) Bended microstrip line: A microstrip line of 20 mm length and having a right angle bend is shown in Fig. 4a. The bend is mitered at the corner to reduce the effect of the discontinuity at the bend. It should be mentioned that physical microstrip line discontinuities can be designed for minimum reflections, but radiation losses are always inevitable. Here, two cases are considered. One with a thin substrate ( $\mathrm{t}=.25 \mathrm{~mm} ; \varepsilon_{\mathrm{r}}=2.2, \mathrm{w}$ $=.8 \mathrm{~mm})$ and the other with a thicker substrate $(\mathrm{t}=$ $0.815 \mathrm{~mm} ; \varepsilon_{\mathrm{r}}=3.38, \mathrm{w}=1.85 \mathrm{~mm}$ ). Fig. 4b shows the comparison between the thin substrate case and the thick substrate case under open condtion and also the thick substrate covered with PMC at 3h. It can be seen that the PMC cover removed any kind of radiation loss and provides an overall better insertion loss, even for the open thin case.


Fig.4(a). Bended microstrip line.


Fig. 4(b). Comparison between bended microstrip line on thin or thick substrates, open and with PMC lid 3t above substrate.

## III. Realization of PMC Packaging

PMC is not a natural material so it has to be realized artificially. One method to realize the PMC lid is using a lid of periodic conducting nails [27] as shown in Fig. 5a. The artificially realized AMC surfaces have limited bandwidth, unlike the PEC that is a good approximation of conducting metal surfaces at all microwave frequencies. The lid of nails consists of nails connected to a solid ground plane which then is turned up-side-down. The magnetic surface appears at the open end of the nails, due to a transformation of the waves from the shorted end of the nails to the open end. The waves will in the nail layer propagate mainly along the nails, independent of the angle of arrival on the nails surface.

The pin structure is periodic, and therefore one cell can be analyzed by using the eigenmode solver of CST. The dispersion diagram of such an infinite periodic nail surface over a smooth metal surface is shown in Fig. 5b. We see that the structure is providing a stop band of parallel-plate modes (i.e. it is acting as an AMC surface) over a frequency band of $1: 2$. The $S_{21}$ computed for a
right angle bended microstrip line covered by the lid of nails is shown in Fig. 5c. We see that the bandwidth is similar to the bandwidth obtained from the dispersion diagram. We, also, see that the losses are very close to the ideal PMC case. A parameter study of the stopband can be found in [7].


Fig. 5 (a). Geometry of the bended microstrip line in a cavity with metallic sidewalls and lid of nails.


Fig. 5 (b). Dispersion diagram obtained for unit cell of periodic pin with $\mathrm{a}=3 \mathrm{~mm}$ and $\mathrm{p}=6.75 \mathrm{~mm}$.


Fig. 5 (c). Simulated $S_{21}$ of the right angle microstrip line with different $a$ and the same $a / p$.

We now consider an example with an inhomogeneous nonrectangular cavity with the same right angle bended microstrip line, as shown in Fig. 6a. However, here the microstrip line is printed on a GaAs substrate with permittivity 12.9 and loss tangent 0.006 at 10 GHz , and thickness t $=50 \mu \mathrm{~m}$. The $50 \Omega$ microstrip line has a width of $30 \mu \mathrm{~m}$. The dispersion diagram from one cell is shown in Fig. 6b. $\mathrm{S}_{21}$ versus frequency is shown in Fig. 6c. Figure 6c shows the performance of the three different cases; open, PEC lid, and PMC lid. We see that the PMC lid gives the smallest insertion loss, and the PEC lid curve shows some resonances within the frequency band.


Fig. 6 (a). Geometry of the bended microstrip line in an inhomogenously filled nonrectangular cavity with metal.


Fig. 6(b). Dispersion diagram obtained from the for one cell with inverted pyramidal-shaped pins that provide the wider bandwidth than the rectangular pins.


Fig. 6(c). $\mathrm{S}_{21}$ of the right angle microstrip line for the open case, PEC lid, PMC lid, and inverted pyramidalshaped pin lid.

## IV. Computation Summary

All the computations mentioned so far were done with CST Microwave Studio. Table 2 shows the number of mesh cells used for the different cases. For the PEC lid, PMC lid, and open case, we made a comparison based on the required number of cells. However, the open case needs matching layers, and the number of cells in this layer is not included in the number. CST does not provide the cell numbers in the matching layers that is actually five layers in the present case. Still, from the CPU time shown in Table 3, we see that the open case needs almost twice the CPU time of the PMC lid case when the PMC is at a height of 2 t .

Table 2: Comparison of mesh cell numbers for different simulation cases

| Simulation case <br> (Bended microstrip line) | Total <br> Mesh <br> cells |
| :---: | :---: |
| Open case (without matching layers) | 258763 |
| PMC lid (at a distance of t ) | 251550 |
| PMC lid (at a distance of $2 \times \mathrm{t}$ ) | 301860 |
| PEC lid (at a distance of $3 \times \mathrm{t}$ ) | 473616 |
| Pin lid case (a $=3 \mathrm{~mm}, \mathrm{p}=6.75 \mathrm{~mm}$ ) | 773616 |

Table 3: Comparison of CPU time for different simulation cases

| Simulation case <br> (bended micosrtrip line) | Simulation <br> time |
| :---: | :---: |
| PMC lid (at a distance of 2t) | 2 min 42 sec |
| PEC lid (at a distance of 3t) | 3 min 10 sec |
| Open (with 5 matching layers) | 4 min 48 sec |
| Pin Lid (a $=3 \mathrm{~mm}, \mathrm{p}=6.75$ <br> $\mathrm{~mm})$ | 7 min 12 sec |

## V. PACKAGING OF PRACTICAL MICROSTRIP FILTER

As a proof of previously mentioned PMC and pin lid packaging concept, a $3^{\text {rd }}$ order microstrip coupled line bandpass filter is manufactured as shown in Fig. 7a. Three cases are considered: open unpackaged case, smooth metal package case, and pin lid packaging case. Figure 7b shows the measurement results compared with the simulated ideal PMC case.


Fig. 7(a). Photo of the microstrip coupled line filter with the pin lid cover on the side.


Fig. 7(b). The microstrip filter measurement response in three cases compared with the simulated PMC packaging case.

As seen from the measurement results of Fig. 7, ideal PMC and the pin lid packaging provide a
good filter response in the pass band and the rejection bands. On the other hand, for the open case and smooth metal lid case, the response of the filter is distorted within the pass band, which of course is not desired. The details of this work can be found in [24].

## VI. CONCLUSION

An efficient method for the design of printed microstrip circuit with packaging in mind was presented. The results showed that the presented PMC prepackaging concept is more appropriate than PEC packaging regardless of the computational efficiency because of the wider bandwidth that can be provided by the PMC, and in particularly for packaging of large printed circuit boards. Engineers can use shorter time during the design process and optimize the circuit already from the start with the packaging in mind. After having designed with ideal PMC, the PMC can be realized by e.g. the bed of nails. This can be designed for a large stopband using simple design rules or the design curves in [10] to operate within the frequency band of interest. Larger stopbandwidths can be achieved by using inverted pyramidal-shaped pins, or mushrooms (patches with via holes).

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# Generalized Formulation for the Scattering from a Ferromagnetic Microwire 

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#### Abstract

In this paper, we present a full wave analysis for the scattering of an obliquely incident plane wave of an arbitrary polarization angle due to an infinite magnetized ferromagnetic microwire. The analysis is based on expanding the incident and scattered waves as infinite sets of cylindrical waves and matching the corresponding modes on the surface of the microwire to obtain the unknown amplitudes of the scattered fields. The gyromagnetic properties of the ferromagnetic material introduce a coupling between the electric and magnetic field components inside the microwire. This coupling is the reason of cross polarization in the scattered field.


Index Terms: Analytical techniques, ferrites, ferromagnetic microwires, scattering.

## I. INTRODUCTION

Ferromagnetic microwires based on amorphous ferromagnetic alloy compositions have gained a significant interest in RF and microwave applications due to their unique electromagnetic properties and the feasibility of simple fabrication and manufacturing process [1-2]. These ferromagnetic materials are characterized by high electrical conductivity, ferromagnetic resonance, gyromagnetic properties, and tunable characteristics. These properties make ferromagnetic microwires to be good candidates for different applications in microwave range including synthesizing artificial double negative metamaterials, shielding materials, and absorbing materials [3-5]. Another similar configuration for synthesizing double negative metamaterial is based on an array of conducting wires embedded
inside a ferromagnetic host [6]. Tuning properties of ferromagnetic microwires make them suitable to obtain tunable synthetic materials [7-11]. This can also be used to design microwave sensing tools like stress sensors [10]. In addition, it may be also expected to obtain polarization rotation from these ferromagnetic materials due to their gyromagnetic properties. This polarization rotation can introduce an additional feature which may be useful to other applications like electromagnetic polarizer or radar cross section reduction.

These unique properties of ferromagnetic microwires were the motivation for different authors to introduce different techniques for modeling the electromagnetic wave interaction with these microwires. Effective medium properties and magneto-impedance are used to obtain simple analytical expressions for the equivalent effective parameters of ferromagnetic microwire composites [11]. Other numerical techniques like TLM and FDTD can also be used to present a numerical analysis for the scattering from general gyrotropic media [12-13]. However, the small diameter of the ferro-magnetic microwire represents a significant computational challenge in these numerical methods. More recently, Liberal et al [14-15] introduced a full wave analysis for the scattering of an infinite ferromagnetic microwire and an infinite array of these microwires. Their analysis is based on expanding the incident and scattered waves as superpositions of cylindrical waves. The unknown amplitudes of the scattered fields are obtained by applying the boundary conditions on the surface of the microwire. They also extended their analysis to study the scattering of a ferromagnetic
microwire inside a rectangular waveguide and verified their analysis with experimental results. The advantage of this analysis is that it is full wave analysis which includes all the parameters of the ferromagnetic microwire without introducing approximation. Liberal et al [14-15] showed that this analysis cannot be obtained by other numerical techniques like method of moments or finite difference time domain due to the complicated properties of the ferromagnetic materials combined with the very small radius of the microwire.


Fig. 1. Geometry of the problem.
However, the analysis of Liberal et al [14-15] was limited only for the case of normal incidence on the microwire with an electric field polarization parallel to its axis. For the mentioned applications of ferromagnetic microwires, it may be required to have a general insight of the electromagnetic wave interaction with ferromagnetic microwire at different angles of incidence and different polarization. This was the motivation here to extend their analysis for the case of the scattering from a ferromagnetic microwire due to an obliquely incident plane wave of an arbitrary polarization as shown in Fig. 1. The main difference in this case is coupling between the electric and magnetic field components in the ferromagnetic material due to its gyromagnetic properties. This coupling vanishes completely for normal incidence. Thus, this property is not included in the analysis of Liberal et al [14-15]. The resulting wave equation inside the
ferromagnetic microwire in this case is represented as a fourth order differential equation as shown in the analysis in the following section. This is similar to the problem of electromagnetic wave interaction with ferrite material in closed and open waveguides which was studied by different authors [16-19].

In the following section, we present the analysis of the problem. In Section 3, sample results for the scattering of two different ferromagnetic microwires are presented for different angle of incidence and different polarization angles. Then concluding remarks are discussed.

## II. THEORY AND ANALYSIS

In the region $\rho \geq a_{0}$ outside the microwire, the incident plane wave and scattered fields along the longitudinal direction of the wire can be represented as a combination of inward and outward cylindrical functions as follows:

$$
\begin{aligned}
& E_{z}^{\text {inc }}=\left(E_{0} \sin \theta_{\text {inc }} \cos \alpha_{\text {inc }}\right) \sum_{n=-\infty}^{\infty} j^{n} J_{n}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi}, \\
& H_{z}^{\text {inc }}=\left(\frac{E_{0}}{Z_{0}} \sin \theta_{\text {inc }} \sin \alpha_{\text {inc }}\right) \sum_{n=-\infty}^{\infty} j^{n} J_{n}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi}, \\
& E_{z}^{s}=\left(E_{0} \sin \theta_{\text {inc }}\right) \sum_{n=-\infty}^{\infty} C_{n} H_{n}^{(2)}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z_{z}} z} e^{-j n \phi},(2-\mathrm{b}) \\
& H_{z}^{s}=\left(\frac{E_{0}}{Z_{0}} \sin \theta_{\text {inc }}\right) \sum_{n=-\infty}^{\infty} D_{n} H_{n}^{(2)}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi},(2-\mathrm{b})
\end{aligned}
$$

where $\quad \beta_{z}=k_{0} \cos \theta_{\text {inc }}$ is the longitudinal propagation constant and $\beta_{\rho 0}=k_{0} \sin \theta_{\text {inc }}$ is the radial propagation constant. The amplitude of the incident field is assumed to be unity such that $E_{0}=1 \mathrm{~V} / \mathrm{m}$. It should be noted that the polarization angle of the scattered is included in the unknown amplitudes of the scattered electric and magnetic fields, $C_{n}$ and $D_{n}$, respectively.

The transverse field components can be obtained in terms of the longitudinal field components. For the present case, we are interested in the $\phi$ components of the electric and magnetic fields to be applied in the tangential boundary conditions on the microwire. These $\phi$ components can be presented in terms of the longitudinal field components as follows:

$$
\begin{align*}
& E_{\phi}=\left(-\frac{j \beta_{z}}{\beta_{\rho 0}^{2}} \frac{1}{\rho} \frac{\partial E_{z}}{\partial \phi}+\frac{j \omega \mu_{0}}{\beta_{\rho 0}^{2}} \frac{\partial H_{z}}{\partial \rho}\right),  \tag{3-a}\\
& H_{\phi}=\left(-\frac{j \omega \varepsilon_{0}}{\beta_{\rho 0}^{2}} \frac{\partial E_{z}}{\partial \rho}-\frac{j \beta_{z}}{\beta_{\rho 0}^{2}} \frac{1}{\rho} \frac{\partial H_{z}}{\partial \phi}\right) . \tag{3-b}
\end{align*}
$$

By applying (3) into (1) and (2), the incident and scattered $\phi$ field components are obtained as:

$$
\begin{align*}
& E_{\phi}^{\text {inc }}=-E_{0} \frac{n \cos \theta_{\text {inc }} \cos \alpha_{i n c}}{k_{0} \rho \sin \theta_{\text {inc }}} \sum_{n=-\infty}^{\infty} j^{n} J_{n}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{2} z} e^{-j n \phi} \\
& +j E_{0} \sin \alpha_{i n c} \sum_{n=-\infty}^{\infty} j^{n} J_{n}^{\prime}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{2} z} e^{-j n \phi}, \\
& H_{\phi}^{\text {inc }}=-j \frac{E_{0}}{Z_{0}} \cos \alpha_{\text {inc }} \sum_{n=-\infty}^{\infty} j^{n} J_{n}^{\prime}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{2} z} e^{-j n \phi} \\
& -\frac{E_{0}}{Z_{0}} \frac{n \cos \theta_{\text {inc }} \sin \alpha_{\text {inc }}}{k_{0} \rho \sin \theta_{\text {inc }}} \sum_{n=-\infty}^{\infty} j^{n} J_{n}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z_{2} z}} e^{-j n \phi} \text {, } \\
& E_{\phi}^{s}=-E_{0} \frac{n \cos \theta_{\text {inc }}}{k_{0} \rho \sin \theta_{\text {inc }}} \sum_{n=-\infty}^{\infty} C_{n} H_{n}^{(2)}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi} \\
& +j E_{0} \sum_{n=-\infty}^{\infty} D_{n} H_{n}^{\prime(2)}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi}, \\
& H_{\phi}^{s}=-j \frac{E_{0}}{Z_{0}} \sum_{n=-\infty}^{\infty} C_{n} H_{n}^{\prime(2)}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi} \\
& -\frac{E_{0}}{Z_{0}} \frac{n \cos \theta_{\text {inc }}}{k_{0} \rho \sin \theta_{\text {inc }}} \sum_{n=-\infty}^{\infty} D_{n} H_{n}^{(2)}\left(\beta_{\rho 0} \rho\right) e^{-j \beta_{z} z} e^{-j n \phi} .
\end{align*}
$$

On the other hand, the fields inside the ferromagnetic microwire biased by a dc magnetic field along its axis at $\rho \leq a_{0}$ are related as:

$$
\begin{align*}
& \frac{1}{\rho} \frac{\partial E_{z}}{\partial \phi}-\frac{\partial E_{\phi}}{\partial z}=-j \omega \mu H_{\rho}+\omega \kappa H_{\phi},  \tag{5-a}\\
& \frac{\partial E_{\rho}}{\partial z}-\frac{\partial E_{z}}{\partial \rho}=-j \omega \mu H_{\phi}-\omega \kappa H_{\rho}  \tag{5-b}\\
& \frac{1}{\rho} \frac{\partial\left(\rho E_{\phi}\right)}{\partial \rho}-\frac{1}{\rho} \frac{\partial E_{\rho}}{\partial \phi}=-j \omega \mu_{0} H_{z},  \tag{5-c}\\
& \frac{1}{\rho} \frac{\partial H_{z}}{\partial \phi}-\frac{\partial H_{\phi}}{\partial z}=j \omega\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right) E_{\rho},  \tag{5-d}\\
& \frac{\partial H_{\rho}}{\partial z}-\frac{\partial H_{z}}{\partial \rho}=j \omega\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right) E_{\phi},  \tag{5-е}\\
& \frac{1}{\rho} \frac{\partial\left(\rho H_{\phi}\right)}{\partial \rho}-\frac{1}{\rho} \frac{\partial H_{\rho}}{\partial \phi}=j \omega\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right) E_{z}, \tag{5-f}
\end{align*}
$$

where $\mu$ and $k$ are the elements of the permeability tensor of the ferromagnetic material and $\sigma$ is its electrical conductivity. The parameters of ferromagnetic materials are given by [14-15]:
$\mu=\mu_{0}$
$+\mu_{0}\left(\frac{\omega_{0} \omega_{m}\left(\omega_{0}^{2}-\omega^{2}\left(1-\alpha^{2}\right)\right)-j \alpha \omega \omega_{m}\left(\omega_{0}^{2}+\omega^{2}\left(1+\alpha^{2}\right)\right)}{\left[\omega_{0}^{2}-\omega^{2}\left(1+\alpha^{2}\right)\right]^{2}+4 \omega_{0}^{2} \omega^{2} \alpha^{2}}\right)$
$k=\mu_{0}\left(\frac{\omega \omega_{m}\left(\omega_{0}^{2}-\omega^{2}\left(1+\alpha^{2}\right)\right)-j 2 \omega_{0} \omega_{m} \omega^{2} \alpha}{\left[\omega_{0}^{2}-\omega^{2}\left(1+\alpha^{2}\right)\right]^{2}+4 \omega_{0}^{2} \omega^{2} \alpha^{2}}\right)$,
where $f_{0}=\omega_{0} / 2 \pi$ is the Larmor frequency, $f_{m}=\omega_{m} / 2 \pi$ is the resonance frequency at the saturation limit and $\alpha$ is a dimensionless damping factor.

It should be noted that the longitudinal dependence of the field components inside the microwire is the same as outside the microwire to satisfy the tangential propagation boundary condition. Thus, the field dependence inside the microwire is also $e^{-j \beta_{2} z}$ where $\beta_{z}=k_{0} \cos \theta_{\text {inc }}$ is the same as in the free space surrounding the microwire.

The transverse field components inside the ferromagnetic microwire can be obtained in terms of the longitudinal field components as follows:

$$
\begin{align*}
& E_{\rho}=\frac{1}{D}\left(a \frac{\partial E_{z}}{\partial \rho}+b \frac{1}{\rho} \frac{\partial E_{z}}{\partial \phi}-e \frac{\partial H_{z}}{\partial \rho}-d \frac{1}{\rho} \frac{\partial H_{z}}{\partial \phi}\right),(7-\mathrm{a})  \tag{7-a}\\
& E_{\phi}=\frac{1}{D}\left(-b \frac{\partial E_{z}}{\partial \rho}+a \frac{1}{\rho} \frac{\partial E_{z}}{\partial \phi}+d \frac{\partial H_{z}}{\partial \rho}-e \frac{1}{\rho} \frac{\partial H_{z}}{\partial \phi}\right),(7-\mathrm{b})  \tag{7-b}\\
& H_{\rho}=\frac{1}{D}\left(b \frac{\omega \varepsilon}{\beta_{z}} \frac{\partial E_{z}}{\partial \rho}-a \frac{\omega \varepsilon}{\beta_{z}} \frac{1}{\rho} \frac{\partial E_{z}}{\partial \phi}+a \frac{\partial H_{z}}{\partial \rho}+b \frac{1}{\rho} \frac{\partial H_{z}}{\partial \phi}\right)  \tag{7-c}\\
& H_{\phi}=\frac{1}{D}\left(a \frac{\omega \varepsilon}{\beta_{z}} \frac{\partial E_{z}}{\partial \rho}+b \frac{\omega \varepsilon}{\beta_{z}} \frac{1}{\rho} \frac{\partial E_{z}}{\partial \phi}-b \frac{\partial H_{z}}{\partial \rho}+a \frac{1}{\rho} \frac{\partial H_{z}}{\partial \phi}\right) \tag{7-d}
\end{align*}
$$

where

$$
\begin{gather*}
a=j \beta_{z} \beta_{\rho}^{2},  \tag{8-a}\\
b=\omega^{2} \kappa \beta_{z}\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right),  \tag{8-b}\\
d=-j \omega \mu\left(\beta_{\rho}^{2}-\frac{\omega^{2} \kappa^{2}}{\mu}\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right)\right),  \tag{8-c}\\
e=\omega \kappa \beta_{z}^{2},  \tag{8-d}\\
D=\left(\omega^{2} \kappa \varepsilon\right)^{2}-\beta_{\rho}^{4},  \tag{8-е}\\
\beta_{\rho}=\sqrt{\omega^{2} \mu\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right)-\beta_{z}^{2}} . \tag{8-f}
\end{gather*}
$$

By inserting Eq. (7) with the definitions of Eq. (8) into Eq. (5), one can obtain two coupled wave
equations of the longitudinal field components as follows [16-17]:

$$
\begin{align*}
& \nabla_{t}^{2} E_{z}+c_{1} E_{z}-j d_{1} H_{z}=0,  \tag{9-a}\\
& \nabla_{t}^{2} H_{z}+f_{1} H_{z}+j g_{1} E_{z}=0, \tag{9-b}
\end{align*}
$$

where

$$
\begin{gather*}
c_{1}=\left(\beta_{\rho 1}^{2}-\frac{\omega^{2} \kappa^{2}}{\mu}\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right)\right),  \tag{10-a}\\
d_{1}=\frac{\mu_{0} \omega \kappa \beta_{z}}{\mu},  \tag{10-b}\\
f_{1}=\frac{\mu_{0} \beta_{\rho}^{2}}{\mu},  \tag{10-c}\\
g_{1}=\frac{\omega \kappa \beta_{z}}{\mu}\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right) . \tag{10-d}
\end{gather*}
$$

It should be noted that the coupling between the longitudinal electric and magnetic field components vanishes for the case of normal incidence where $\beta_{z}=0$. These two coupled wave equations can be represented as two decoupled fourth order differential equation as follows:

$$
\begin{equation*}
\left(\nabla_{t}^{2} \nabla_{t}^{2}+\left(f_{1}+c_{1}\right) \nabla_{t}^{2}+f_{1} c_{1}-d_{1} g_{1}\right) \Psi_{z}=0, \tag{11}
\end{equation*}
$$

where $\Psi_{z}$ is either $E_{z}$ or $H_{z}$. This fourth order differential equation can be represented as a multiplication of two wave equations of different propagation constants as follows:

$$
\begin{equation*}
\left(\nabla_{t}^{2}+\gamma_{\rho 1}^{2}\right)\left(\nabla_{t}^{2}+\gamma_{\rho 2}^{2}\right) \Psi_{z}=0, \tag{12-a}
\end{equation*}
$$

where the two propagation constants are:

$$
\begin{equation*}
\gamma_{\rho 1 / 2}=\sqrt{\frac{1}{2}\left(\left(f_{1}+c_{1}\right) \pm \sqrt{\left(f_{1}-c_{1}\right)^{2}+4 d_{1} g_{1}}\right)} . \tag{12-b}
\end{equation*}
$$

The general solution of Eq. (12) for the longitudinal electric field inside the microwire in terms of cylindrical waves is [16-17]:

$$
\begin{equation*}
E_{z}^{t}=\sum_{n=-\infty}^{\infty}\left(A_{n} J_{n}\left(\gamma_{\rho 1} \rho\right)+B_{n} J_{n}\left(\gamma_{\rho 2} \rho\right)\right) e^{-j n \phi} e^{-j \beta_{2} z} . \tag{13-a}
\end{equation*}
$$

A similar solution can be obtained for the longitudinal magnetic field. The unknown amplitudes of the longitudinal magnetic field can be related to the corresponding ones of the longitudinal electric field by inserting Eq. (13-a) into Eq. (9-b). Thus, the general solution of the longitudinal magnetic field component in terms of the unknown amplitudes of the longitudinal electric field component is given by:
$H_{z}^{t}=\sum_{n=-\infty}^{\infty}\left(\eta_{1} A_{n} J_{n}\left(\gamma_{\rho 1} \rho\right)+\eta_{2} B_{n} J_{n}\left(\gamma_{\rho 2} \rho\right)\right) e^{-j n \phi} e^{-j \beta_{z} z}$,
(13-b)
where the coupling coefficients between the longitudinal electric and magnetic field components are given by:

$$
\begin{equation*}
\eta_{1 / 2}=\frac{j g_{1}}{\gamma_{\rho 1 / 2}^{2}-f_{1}} . \tag{14}
\end{equation*}
$$

By inserting Eq. (13) into Eqs. (7-b) and (7-d), one can obtain the $\phi$ components of the electric and magnetic fields inside the microwire as:

$$
\begin{align*}
& E_{\phi}^{t}=\sum_{n=-\infty}^{\infty}\left[A_{n} \mathrm{X}_{1 n}(\rho)+B_{n} \mathrm{X}_{2 n}(\rho)\right] e^{-j n \phi} e^{-j \beta_{2} z},  \tag{15-a}\\
& H_{\phi}^{t}=\sum_{n=-\infty}^{\infty}\left[A_{n} \Lambda_{1 n}(\rho)+B_{n} \Lambda_{2 n}(\rho)\right] e^{-j n \phi} e^{-j \beta_{2} z}, \tag{15-b}
\end{align*}
$$

where

$$
\begin{align*}
\mathrm{X}_{\text {in }}(\rho) & =\frac{1}{D}\left(d \eta_{i} \gamma_{\rho i}-b \gamma_{\rho i}\right) J_{n}^{\prime}\left(\gamma_{\rho i} \rho\right) \\
& +\frac{1}{D} \frac{j n\left(e \eta_{i}-a\right)}{\rho} J_{n}\left(\gamma_{\rho i} \rho\right),  \tag{16-a}\\
\Lambda_{i n}(\rho) & =\frac{1}{D}\left(a \gamma_{\rho i} \frac{\omega}{\beta_{z}}\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right)-b \gamma_{\rho i} \eta_{i}\right) J_{n}^{\prime}\left(\gamma_{\rho i} \rho\right) \\
-j & \frac{1}{D} \frac{n}{\rho}\left(a \eta_{i}+b \frac{\omega}{\beta_{z}}\left(\varepsilon_{0}-j \frac{\sigma}{\omega}\right)\right) J_{n}\left(\gamma_{\rho i} \rho\right) . \tag{16-b}
\end{align*}
$$

"i" here stands for either 1 or 2 . By applying the boundary conditions of the tangential field components at the boundary of the microwire one can obtain a linear system of equations for each cylindrical wave mode " $n$ " to obtain the corresponding unknown field amplitudes as shown in Eq. 17 below.

By solving the above system of equation one can obtain the unknown amplitudes of the scattered and penetrated field components. For a very thin wire compared with the free space wave length as in the present case of the microwire, the zero-order cylindrical wave is the dominant mode. The amplitudes of the other modes are much smaller such that they can be ignored compared with the amplitudes of the zero-order mode [14].

$$
\begin{align*}
& {\left[\begin{array}{cccc}
J_{n}\left(\gamma_{\rho 1} a_{0}\right) & J_{n}\left(\gamma_{\rho 2} a_{0}\right) & -\sin \theta_{0} H_{n}^{(2)}\left(\beta_{\rho 0} a_{0}\right) & 0 \\
Z_{0} \eta_{1} J_{n}\left(\gamma_{\rho 1} a_{0}\right) & Z_{0} \eta_{2} J_{n}\left(\gamma_{\rho 1} a_{0}\right) & 0 & -\sin \theta_{0} H_{n}^{(2)}\left(\beta_{\rho 0} a_{0}\right) \\
k_{0} a_{0} \sin \theta_{0} X_{1 n}\left(a_{0}\right) & k_{0} a_{0} \sin \theta_{0} X_{2 n}\left(a_{0}\right) & n \cos \theta_{0} H_{n}^{(2)}\left(\beta_{\rho 0} a_{0}\right) & \left.-j k_{0} a_{0} \sin \theta_{0} H_{n}^{\prime(2)}\left(\beta_{\rho 0}\right) a_{0}\right) \\
Z_{0} k_{0} a_{0} \sin \theta_{0} \Lambda_{1 n}\left(a_{0}\right) & Z_{0} k_{0} a_{0} \sin \theta_{0} \Lambda_{2 n}\left(a_{0}\right) & j k_{0} a_{0} \sin \theta_{0} H_{n}^{\prime(2)}\left(\beta_{\rho 0} a_{0}\right) & n \cos \theta_{0} H_{n}^{(2)}\left(\beta_{\rho 0} a_{0}\right)
\end{array}\right]\left[\begin{array}{l}
A_{n} \\
B_{n} \\
C_{n} \\
D_{n}
\end{array}\right]} \\
& \quad=\left[\begin{array}{c}
j^{n} \sin \theta_{\text {inc }} \cos \alpha_{\text {inc }} J_{n}\left(\beta_{\rho 0} a_{0}\right) \\
j^{n} \sin \theta_{\text {inc }} \sin \alpha_{\text {inc }} J_{n}\left(\beta_{\rho 0} a_{0}\right) \\
-n \cos \theta_{\text {inc }} \cos \alpha_{\text {inc }} j^{n} J_{n}\left(\beta_{\rho 0} a_{0}\right)+k_{0} a_{0} \sin \theta_{\text {inc }} \sin \alpha_{\text {inc }} j^{n+1} J_{n}^{\prime}\left(\beta_{\rho 0} a_{0}\right) \\
-j^{n+1} k_{0} a_{0} \sin \theta_{\text {inc }} \cos \alpha_{\text {inc }} J_{n}^{\prime}\left(\beta_{\rho 0} a_{0}\right)-j^{n} n \cos \theta_{\text {inc }} \sin \alpha_{\text {inc }} J_{n}\left(\beta_{\rho 0} \rho\right)
\end{array}\right] . \tag{17}
\end{align*}
$$

Thus, we are concentrating only on $C_{0}$ and $D_{0}$ in the results shown in the following section.

## III. RESULTS AND DISCUSSIONS

In this section, we present sample results for the scattering of two ferromagnetic microwires at different angles of incidence and different polarization angles. The ferromagnetic material is assumed to be $\left(\mathrm{Co}_{0.94} \mathrm{Fe}_{0.06}\right)_{75} \mathrm{Si}_{12.5} \mathrm{~B}_{12.5}$. The parameters of this ferromagnetic material are [1415] $f_{0}=\omega_{0} / 2 \pi=5.666 \mathrm{GHz}, \quad \alpha=0.02 \quad$ and $f_{m}=\omega_{m} / 2 \pi=11.642 \mathrm{GHz}$.

For ferromagnetic materials, there are two main mechanisms for electromagnetic wave propagation, namely along the dc magnetization biasing or perpendicular to it [20]. For the former mechanism, the plane wave can be presented a superposition of two oppositely circular polarized waves. One of them has an equivalent permeability $\mu+k$ while the other has an equivalent permeability $\mu-k$. The first circular polarization has a resonance at Larmor frequency while the second circular polarization does not have any resonance effect. On the other hand, for the case of electromagnetic wave propagation which is perpendicular to the dc magnetic biasing while its electric field is parallel to the dc magnetic biasing, the equivalent permeability is $\mu_{e}=\left(\mu^{2}-k^{2}\right) / \mu$. This case is known as an extra ordinary wave [20]. The equivalent permeability in this case has a resonance frequency at 9.9 GHz for the present ferromagnetic material. The final case, where both the propagation direction and the electric field polarization are perpendicular to the dc magnetic biasing, the equivalent permeability is the permeability of free space. Thus, for the present material we have two resonance
frequencies, Larmor resonance frequency at 5.666 GHz and extra ordinary wave resonance at 9.9 GHz.

The following results shown in Figures 2 to 5 present the amplitudes of the zero-order modes of the scattered fields as functions of frequency $f$ and angle of incidence $\theta_{\text {inc }}$ for different polarization angles $\alpha_{\text {inc }}$. The radius of the microwire in Figs. 2 and 3 is $45 \mu \mathrm{~m}$ while the corresponding one in Figs. 4 and 5 is $4 \mu \mathrm{~m}$. It can be noted that only the resonance effect of extra ordinary wave is detected in the amplitudes of the scattered fields. For the case of $\alpha_{\text {inc }}=0^{\circ}$, the incident field is pure TM wave. Thus, $D_{0}$ in this case corresponds to the amplitude of a cross polarized component of the scattered field. The resonance behavior in this case appears as a minimum in the amplitude of the co-polarized scattered field and as a maximum in the amplitude of the cross-polarized field component. It can be noted that the cross polarized component vanishes completely for normal incidence while it has a peak at $\theta_{\text {inc }}$ in the range from $40^{\circ}$ to $60^{\circ}$ depending on the frequency and the radius of the microwire.

For the case of a polarization angle $\alpha_{\text {inc }}=45^{\circ}$, the main characteristics of the reflected $E_{z}$ component is the same as in the case of the pure TM wave. However, the amplitude of $C_{0}$ is decreased in this case by nearly a factor of $\cos \alpha_{i n c}$. On the other hand, the amplitude of $D_{0}$ is monotonically increased with frequency and angle of incidence for the case of the thick microwire while it is nearly not affected for the thin microwire.

Finally, for the case of a polarization angle $\alpha_{\text {inc }}=90^{\circ}$ where the incident wave represents pure TE wave, the amplitude of $D_{0}$ is the co-polarized component of the scattered field while $C_{0}$ is the cross-polarized component. It can be noted that in this case the co-polarized component is not characterized by any resonance behavior. However, for the thick microwire, the results show a resonance behavior at a cross-polarized component around the resonance frequency of extra ordinary wave. This resonance behavior is nearly negligible in the case of the thin microwire. It can also be noted that the cross polarized component vanishes completely in normal incidence as in the case of TM incident wave.

From these results, it can be concluded that the scattering of magnetized ferromagnetic microwire introduces a cross polarized component for oblique incidence and the frequency response of the scattered field is mainly characterized by a resonance effect around the resonance frequency of extra ordinary waves in most cases for both copolarized and cross-polarized components.

## IV. CONCLUSION

In this paper, we presented a full wave analysis for the scattering of an obliquely incident plane wave of an arbitrary polarization angle due to an infinite magnetized ferromagnetic microwire. The present analysis represents a generalization for a previously published analysis based on only normal incidence TM wave. The main difference in the present case lies in the coupling between the electric and magnetic field components due to the gyromagnetic properties of the ferromagnetic microwire. This field coupling introduces cross polarized component in the scattered field. This cross polarized component vanishes completely for the case of normal incidence. The scattered field components are mainly characterized by resonance effects close to the resonance of extra ordinary wave in most cases for both co-polarized and cross-polarized components.

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(a) $\alpha_{\text {inc }}=0^{\circ}$.

(b) $\alpha_{\text {inc }}=45^{\circ}$.

(c) $\alpha_{\text {inc }}=90^{\circ}$.

Fig. 2. $C_{0}$ scattering coefficient for a ferromagnetic microwire of a radius $r_{0}=45 \mu \mathrm{~m}$ as functions of frequency and angle of incidence $\left(\theta_{\text {inc }}\right)$ at different polarization angles $\left(\alpha_{\text {inc }}\right)$.

(a) $\alpha_{i n c}=0^{\circ}$.


(c) $\alpha_{i n c}=90^{\circ}$.

Fig. 3. $D_{0}$ scattering coefficient for a ferromagnetic microwire of a radius $r_{0}=45 \mu \mathrm{~m}$ as functions of frequency and angle of incidence $\left(\theta_{\text {inc }}\right)$ at different polarization angles $\left(\alpha_{i n c}\right)$.

(a) $\alpha_{\text {inc }}=0^{\circ}$

(b) $\alpha_{\text {inc }}=45^{\circ}$

(c) $\alpha_{\text {inc }}=90^{\circ}$

Fig. 4. $C_{0}$ scattering coefficient for a ferromagnetic microwire of a radius $r_{0}=4 \mu \mathrm{~m}$ as functions of frequency and angle of incidence $\left(\theta_{\text {inc }}\right)$ at different polarization angles $\left(\alpha_{\text {inc }}\right)$.

(c) $\alpha_{\text {inc }}=90^{\circ}$

Fig. 5. $D_{0}$ scattering coefficient for a ferromagnetic microwire of a radius $r_{0}=4 \mu \mathrm{~m}$ as functions of frequency and angle of incidence $\left(\theta_{\text {inc }}\right)$ at different polarization angles $\left(\alpha_{\text {inc }}\right)$.
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# An Equivalent Dipole-Moment Method Based Multilevel Fast Multipole Algorithm for Dielectric Objects 

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#### Abstract

An equivalent dipole-moment method (EDM) based multilevel fast multipole algorithm (MLFMA), namely EDM-MLFMA, is proposed for the electromagnetic scattering from threedimensional (3D) dielectric objects. In this scheme, the MLFMA is employed to accelerate the interactions of equivalent dipoles in the far regions by expanding the 3D dyadic Green's function into a multipole expression. The memory requirement and CPU time of the conventional EDM are reduced from $O\left(N^{2}\right)$ to $O(N \log N)$, where $N$ is the number of unknowns. Numerical results are presented to validate the efficiency and accuracy of this method.


Index Terms - Equivalent dipole-moment method (EDM), multilevel fast multipole algorithm (MLFMA), volume integral equation (VIE).

## I. INTRODUCTION

The scattering of electromagnetic (EM) waves from dielectric materials is an important research area because of the wide applications in dielectric radomes, anisotropic substrates, absorbing materials, etc. [1-2]. In the method of moments (MoM), the volume integral equation (VIE) is generally appropriate for dielectric objects with inhomogeneity and anisotropy. However, the conventional MoM converts the VIE to a dense matrix equation, which is very expensive and formidable especially for large scale EM targets. The $O\left(N^{2}\right)$ memory and computational complexity for iterative solvers is required for both computing the MoM impedance matrix elements and solving this dense matrix equation, where $N$ is the number of unknowns.

More recently, the equivalent dipole-moment method (EDM) [3-4] has been developed to efficiently generate the impedance matrix elements for surface integral equation (SIE). Later, the EDM was extended to deal with the isotropic media [5] and anisotropic media [6-7]. The EDM is based on the commonly used Rao-WiltonGlisson (RWG) [8] and Schaubert-Wilton-Glisson (SWG) [9] basis functions. In the EDM, each RWG triangle pair or SWG tetrahedron pair is viewed as a dipole model with an equivalent dipole moment. The main advantage of the EDM is that the impedance matrix element can be expressed in an extremely simplified form. However, the memory requirement and the matrixsolve time do not change, and the complexities are still $O\left(N^{2}\right)$.

In this article, the EDM is speeded up by the multilevel fast multipole algorithm (MLFMA) [10-16] for solving the EM scattering from threedimensional (3D) dielectric materials in freespace. All the SWG basis functions are modeled as equivalent dipole models and divided into multilevel cubical groups. Through expanding the 3D dyadic Green's function [12] in the formulation of the EDM using the addition theorem [17-18], the interactions between the source and field equivalent dipoles in nonnearby groups are transformed into aggregation, translation and disaggregation operators. Benefiting from the octree-structured grouping, the interpolation and anterpolation, only $O(N)$ impedance elements at the finest level should be calculated and stored, and the computation complexity as well as the memory requirement of the conventional EDM are reduced from $O\left(N^{2}\right)$ to $O(N \log N)$.

The remainder of the paper is organized as follows. In Section II, the EDM is briefly presented for the VIE. Then we describe how the MLFMA is used to accelerate the EDM in detail. In Section III, some numerical results about the bistatic radar cross section (RCS) are given to verify the efficiency and accuracy of the method. Finally, conclusions are drawn in Section IV.

## II. FORMULATIONS

## A. Basic principles of the EDM

Considering an arbitrarily shaped 3D scattering target, which consists of electric anisotropic material characterized by permittivity tensor $\overline{\bar{\varepsilon}}(\mathbf{r})$.

The VIE can be constructed by

$$
\begin{equation*}
\mathbf{E}^{i}(\mathbf{r})=\overline{\bar{\varepsilon}}^{-1}(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r})+j \omega \mathbf{A}(\mathbf{r})+\nabla \phi(\mathbf{r}), \tag{1}
\end{equation*}
$$

where

$$
\begin{gather*}
\mathbf{A}(\mathbf{r})=\mu_{0} \int_{v} \mathbf{J}\left(\mathbf{r}^{\prime}\right) G\left(\mathbf{r}, \mathbf{r}^{\prime}\right) d v^{\prime},  \tag{2}\\
\phi(\mathbf{r})=-\frac{1}{j \omega \varepsilon_{0}} \int_{v} \nabla^{\prime} \cdot \mathbf{J}\left(\mathbf{r}^{\prime}\right) G\left(\mathbf{r}, \mathbf{r}^{\prime}\right) d v^{\prime} \tag{3}
\end{gather*}
$$

$\mathbf{E}^{i}(\mathbf{r})$ denotes the incident electric field. $\mathbf{D}(\mathbf{r})$ denotes the total electric flux density. $\mathbf{J}(\mathbf{r})$ is the equivalent volume current. $\varepsilon_{0}$ and $\mu_{0}$ are the freespace permittivity and permeability, respectively. $G\left(\mathbf{r}, \mathbf{r}^{\prime}\right)=e^{-j k \mathbf{r}^{\prime} \mathbf{r}^{\prime}} /\left(4 \pi\left|\mathbf{r}-\mathbf{r}^{\prime}\right|\right)$ stands for the freespace scalar Green's function.

The unknown equivalent volume current $\mathbf{J}(\mathbf{r})$ can be expanded by a set of SWG basis functions as

$$
\begin{equation*}
\mathbf{J}(\mathbf{r})=j \omega \overline{\bar{\kappa}}(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r})=\sum_{n=1}^{N} I_{n} \overline{\bar{\kappa}}_{n} \cdot \mathbf{f}_{n}(\mathbf{r}), \tag{4}
\end{equation*}
$$

where $\mathbf{f}_{n}(\mathbf{r})$ represents the $n$th SWG basis function which is defined on two adjoining tetrahedron elements $T_{n}^{ \pm}$sharing the $n$th face $S_{n}$ in the volume mesh. $I_{n}$ is the unknown expansion coefficient. $N$ is the number of the faces in the volume mesh. $\overline{\bar{\kappa}}(\mathbf{r})=\overline{\mathbf{I}}-\overline{\bar{\varepsilon}}_{r}^{-1}(\mathbf{r})$ is the contrast ratio [19]. $\overline{\mathbf{I}}$ is the unit tensor.

Substituting Eq. (2) - Eq. (4) into Eq. (1) and using the Galerkin's method, finally the matrix equation of the VIE can be obtained by

$$
\begin{equation*}
\sum_{n=1}^{N} Z_{m n} I_{n}=V_{m}, m=1 \sim N, \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
Z_{m n}= & j \omega\left\langle\mathbf{f}_{m}(\mathbf{r}), \mathbf{A}_{n}(\mathbf{r})\right\rangle+\left\langle\mathbf{f}_{m}(\mathbf{r}), \nabla \phi_{n}(\mathbf{r})\right\rangle \\
& +\frac{1}{j \omega \varepsilon_{0}}\left\langle\mathbf{f}_{m}(\mathbf{r}), \overline{\bar{\varepsilon}}_{r}^{-1} \cdot \mathbf{f}_{n}(\mathbf{r})\right\rangle, \tag{6}
\end{align*},
$$

are the impedance matrix elements.

$$
\begin{equation*}
V_{m}=\left\langle\mathbf{f}_{m}(\mathbf{r}), \mathbf{E}^{i}(\mathbf{r})\right\rangle, \tag{7}
\end{equation*}
$$

are the right-hand side vector elements.
In the conventional MoM, the multipoint Gaussian quadrature is usually used to calculate Eq. (6). However, this process can be simplified in the EDM. The basic idea of the EDM is that the fields radiated by the current in a SWG element are approximated as the fields due to an infinitely small dipole with an equivalent moment [5]. Based on this assumption, the interaction of two basis functions can be replaced by the interaction of two infinitely small dipoles, except they are very close to each other.

The equivalent dipole moment in the $n$th SWG element can be obtained by the integration of the volume current over the tetrahedron pair $T_{n}^{ \pm}[6]$.

$$
\begin{align*}
\mathbf{m}_{m}= & \int_{T_{n}^{ \pm}} \overline{\bar{\kappa}}_{n} \cdot \mathbf{f}_{n}\left(\mathbf{r}^{\prime}\right) d v^{\prime} \approx a_{n} \overline{\bar{\kappa}}_{n}^{+} \cdot\left(\mathbf{r}_{n s}^{c}-\mathbf{r}_{n}^{c+}\right),  \tag{8}\\
& +a_{n} \overline{\bar{K}}_{n}^{-} \cdot\left(\mathbf{r}_{n}^{c-}-\mathbf{r}_{n s}^{c}\right)
\end{align*}
$$

where $\mathbf{r}_{n}^{c \pm}$ and $\mathbf{r}_{n s}^{c}$ are the position vectors of the centroid of $T_{n}^{ \pm}$and the centroid of the common face $S_{n}$, respectively.

The impedance elements $Z_{m n}$ can be expressed in a very simple form [5-6]

$$
\begin{align*}
Z_{m n}= & \frac{\eta e^{-j k R}}{4 \pi}\left[\mathbf{m}_{m}^{\prime} \cdot \mathbf{m}_{n}\left(\frac{j k}{R}+C\right)\right. \\
& \left.-\left(\mathbf{m}_{m}^{\prime} \cdot \hat{\mathbf{R}}\right)\left(\hat{\mathbf{R}} \cdot \mathbf{m}_{n}\right)\left(\frac{j k}{R}+3 C\right)\right], \tag{9}
\end{align*}
$$

where $\mathbf{R}=\mathbf{r}_{m n}=\mathbf{r}_{m}-\mathbf{r}_{n}$ is the vector from the center point $\mathbf{r}_{n}$ of the $n$th equivalent dipole to the center point $\mathbf{r}_{m}$ of the $m$ th equivalent dipole. $R=|\mathbf{R}|$, $\hat{\mathbf{R}}=\mathbf{R} / R . \eta$ is the intrinsic impedance of medium in free-space, and $C=[1+1 /(j k R)] / R^{2}$.

$$
\begin{equation*}
\mathbf{m}_{m}^{\prime}=a_{m}\left(\mathbf{r}_{m}^{c-}-\mathbf{r}_{m}^{c+}\right), \tag{10}
\end{equation*}
$$

is the moment of the SWG basis function [9].
It should be mentioned that the matrix elements are computed by the EDM directly when the distance of two SWG elements is greater than the critical distance $0.2 \lambda_{0}$ in this paper.

## B. Acceleration of the EDM using the MLFMA

Although the mutual impedance elements can be calculated by the EDM efficiently, the computational complexity for performing a matrix vector product (MVP) is still $O\left(N^{2}\right)$ for iterative methods and all the impedance elements must be stored. So the EDM is limited to solve electrically small targets. In this work, the MLFMA is employed to accelerate the MVP and reduce the memory requirement.

To employ the MLFMA, the entire target needs multilevel grouping first, then nonempty cubes are recorded using octree-structured data at all levels. We consider two dipoles $m$ and $n$, which belong to two far groups $j$ and $i$, respectively. Equation (9) of the impedance element $Z_{m n}$ can be rewritten as

$$
\begin{equation*}
Z_{m n}=j k \eta \mathbf{m}_{m}^{\prime} \cdot \overline{\mathbf{G}}\left(\mathbf{r}_{m}, \mathbf{r}_{n}\right) \cdot \mathbf{m}_{n}, \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
\overline{\mathbf{G}}\left(\mathbf{r}_{m}, \mathbf{r}_{n}\right)=[\overline{\mathbf{I}}(1+C)-\hat{\mathbf{R}} \hat{\mathbf{R}}(1+3 C)] G\left(\mathbf{r}_{m}, \mathbf{r}_{n}\right), \tag{12}
\end{equation*}
$$

is the dyadic Green's function in free-space, in which

$$
\begin{equation*}
C=\frac{1}{j k R}+\frac{1}{(j k R)^{2}} . \tag{13}
\end{equation*}
$$

In addition, the vector $\mathbf{R}=\mathbf{r}_{m n}$ can be rewritten as $\mathbf{R}=\mathbf{r}_{j i}+\mathbf{r}_{m j}-\mathbf{r}_{n i}$, in which $\mathbf{r}_{j i}=\mathbf{r}_{O_{j}}-\mathbf{r}_{O_{i}}$, $\mathbf{r}_{m j}=\mathbf{r}_{m}-\mathbf{r}_{O_{j}}, \mathbf{r}_{n i}=\mathbf{r}_{n}-\mathbf{r}_{O_{i}}, \mathbf{r}_{O_{i}}$ and $\mathbf{r}_{O_{j}}$ are the center positions of group $i$ and $j$, respectively. Since the two groups are the far group pair, the addition theorem can be used to expand the dyadic Green's function as [12]

$$
\begin{equation*}
\overline{\mathbf{G}}\left(\mathbf{r}_{m}, \mathbf{r}_{n}\right) \approx \int[\overline{\mathbf{I}}-\hat{\mathbf{k}} \hat{\mathbf{k}}] e^{-j \mathbf{k} \cdot\left(\mathbf{r}_{m} \mid \mathbf{r}_{n j}\right)} T_{L}\left(\mathbf{k}, \mathbf{r}_{j i}\right) d^{2} \hat{\mathbf{k}}, \tag{14}
\end{equation*}
$$

where
$T_{L}\left(\mathbf{k}, \mathbf{r}_{j i}\right)=\frac{k}{16 \pi^{2}} \sum_{l=0}^{L}(-j)^{l+1}(2 l+1) h_{l}^{(2)}\left(k r_{j i}\right) P_{l}\left(\hat{\mathbf{k}} \cdot \mathbf{r}_{j i}\right)$
.(15)
$h_{l}^{(2)}(\cdot)$ is the spherical Hankel function of the second kind, and $P_{l}(\cdot)$ is the Legendre function of order $l$.
Substituting Eq. (14) into Eq. (11), and using the identity of $\quad(\overline{\mathbf{I}}-\hat{\mathbf{k}} \mathbf{\mathbf { k }})=(\overline{\mathbf{I}}-\hat{\mathbf{k}} \mathbf{\mathbf { k }}) \cdot(\overline{\mathbf{I}}-\hat{\mathbf{k}} \hat{\mathbf{k}})$, the impedance element $Z_{m n}$ can be represented by

$$
\begin{equation*}
Z_{m n}=j k \eta \int \mathbf{R}_{m}(\hat{\mathbf{k}}) \cdot T_{L}\left(\mathbf{k}, \mathbf{r}_{j i}\right) \mathbf{F}_{n}(\hat{\mathbf{k}}) d \hat{\mathbf{k}}, \tag{16}
\end{equation*}
$$

in which the vectors $\mathbf{F}_{n}(\hat{\mathbf{k}})$ and $\mathbf{R}_{m}(\hat{\mathbf{k}})$ are radiation function and receive function, respectively and are given by

$$
\begin{align*}
& \mathbf{F}_{n}(\hat{\mathbf{k}})=\mathbf{m}_{n} \cdot(\overline{\mathbf{I}}-\hat{\mathbf{k}} \hat{\mathbf{k}}) e^{j \mathbf{k} \cdot \mathbf{r}_{i n}},  \tag{17}\\
& \mathbf{R}_{m}(\hat{\mathbf{k}})=\mathbf{m}_{m}^{\prime} \cdot(\overline{\mathbf{I}}-\hat{\mathbf{k}} \hat{\mathbf{k}}) e^{-j \mathbf{k} \mathbf{r}_{\mathrm{m}_{j}}} . \tag{18}
\end{align*}
$$

Physical interpretation of this expansion is that a spherical wave from a dipole in free space is expanded by the sum of an infinite number of plane waves. Using Eq. (16), then the MVP can be computed fast through an aggregation-translationdisaggregation form. The complexities of computation and memory requirement can achieve $O(N \log N)$ profiting from the interpolation, anterpolation, and the grid-tree data structure [1011]. At the finest level, only $O(N)$ impedance elements should be calculated and stored. Further more, part of them can be efficiently calculated by the EDM.

## III. NUMERICAL RESULTS

In this section, we present some numerical results to validate the efficiency and accuracy of the new method. In the following examples, all the simulations are performed on a personal computer with the Pentium(R) Dual CPU E5500 with 2.80 GHz (only one core is used) and 2.0 GB RAM. The GMRES iterative solver is employed to obtain an identical residual error $\leq 0.001$ and the block diagonal preconditioner is used.


Fig. 1. Bistatic RCSs in $\theta \theta$ polarization of a spherical dielectric shell illuminated by a uniform plane wave with the incident direction of $(\theta, \phi)=\left(0^{\circ}, 0^{\circ}\right)$.

First, we consider the scattering problem of a spherical dielectric shell. The shell's inner and outer radii are 0.5 m and 0.55 m and the relative permittivity of the shell is $\overline{\bar{\varepsilon}}_{r}=1.5 \overline{\mathbf{I}}$, as shown in the inset of Fig. 1. The shell is discretized into 5309 tetrahedrons, and the total number of unknowns is 12213 . The size of the finest group is set to 0.15 m and a 3-level EDM-MLFMA is used. The bistatic RCS for $\theta \theta$ polarization calculated by the EDM-MLFMA agrees well with the conventional EDM and the Mie series solution shown in Fig. 1. At $180^{\circ}$, the RCS value of the Mie series is -0.97987 dBsm , and the results obtained by the EDM-MLFMA and the conventional EDM are -0.97359 and -0.97464 dBsm , respectively. The root mean square (rms) errors of the EDM-MLFMA and the conventional EDM are 0.448 and 0.441 dBsm , respectively.


Fig. 2. Bistatic RCSs in $\theta \theta$ polarization of a dielectric cylinder illuminated by a uniform plane wave with the incident direction of $(\theta, \phi)=\left(0^{\circ}, 0^{\circ}\right)$.

Then the bistatic RCS of a dielectric cylinder is considered. The radii and height of the cylinder are 0.25 m and 2.0 m . The relative permittivity of the cylinder is $\overline{\bar{\varepsilon}}_{r}=1.5 \overline{\mathbf{I}}$. The target is discretized into 5668 tetrahedrons, and the total number of unknowns is 11897. A 4-level EDM-MLFMA is used and the size of the group at the finest level is 0.15 m . The bistatic RCS in $\theta \theta$ polarization obtained by the EDM-MLFMA agrees well with the conventional EDM shown in Fig. 2.

Finally the bistatic RCS of an anisotropic dielectric target is considered. The target is constituted by 5 small slabs with the same size 1.0 $\mathrm{m} \times 4.0 \mathrm{~m} \times 0.05 \mathrm{~m}$, but the 5 small slabs have
different relative permittivities (see Fig. 3, $\overline{\bar{\varepsilon}}_{r 1}$ $=[1.5, \mathrm{j}, 0 ;-\mathrm{j}, 1.5,0 ; 0,0,2.0], \overline{\bar{\varepsilon}}_{\mathrm{r} 2}=(1.75-0.5 \mathrm{j}) \overline{\mathbf{I}}$ and $\left.\overline{\bar{\varepsilon}}_{\mathrm{r} 3}=[2.0, \mathrm{j}, 0 ;-\mathrm{j}, 2.0,0 ; 0,0,1.5]\right)$. The target is discretized into 49470 tetrahedrons, and the total number of unknowns is 115620. A 6-level EDMMLFMA is used and the group at the finest level is also with the size of 0.15 m . The bistatic RCS in $\theta \theta$ polarization obtained by the EDM-MLFMA is shown in Fig. 3. This target could not be calculated by the conventional EDM, because the memory is not enough. The conventional EDM is estimated to require about 100 GB memory and 4 h CPU time.


Fig. 3. Bistatic RCSs in $\theta \theta$ polarization of an anisotropic dielectric target illuminated by a uniform plane wave with the incident direction of $(\theta, \phi)=\left(0^{\circ}, 0^{\circ}\right)$.

Table 1 summarizes the CPU time and memory requirment of the above simulations. It can be seen that the EDM-MLFMA saves much CPU time and memory than the conventional EDM.

Table 1: Comparison of CPU time and memory requirement of the conventional EDM and EDMMLFMA

| Method | EDM |  | EDM-MLFMA |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Time | Memory | Time | Memory |
| Problem 1 | 132 s | 1188 MB | 51 s | 145 MB |
| Problem 2 | 138 s | 1128 MB | 58 s | 171 MB |
| Problem 3 | - | - | 520 s | 1390 MB |

## IV. CONCLUSION

In this article, the EDM based MLFMA is introduced and applied to solve the VIE in the electromagnetic scattering of dielectric targets.

The new method can reduce both the CPU time and memory requirement of the conventional EDM to $O(N \log N)$. In the future, this method will be extended and applied to solve surface integral equation (SIE) and volume-surface integral equation (VSIE).

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# Improved Weakly Conditionally Stable Finite-Difference TimeDomain Method 

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#### Abstract

To circumvent the inaccuracy in the implementation of the perfect-electric-conductor (PEC) condition in the weakly conditionally stable finite-difference time-domain (WCS-FDTD) method, an improved weakly conditionally stable (IWCS) FDTD method is presented in this paper. In this method, the solving of the tridiagonal matrix for the magnetic field component is replaced by the solving of the tridiagonal matrix for the electric field components; thus, the perfect-electric-conductor (PEC) condition for the electric field components is implemented accurately. The formulations of the IWCS-FDTD method are given, and the stability condition of the IWCSFDTD scheme is presented analytically. Compared with the WCS-FDTD method, this new method has higher accuracy in the implantation of the PEC condition, which is demonstrated through numerical examples.


Index Terms - FDTD method, perfect-electricconductor (PEC) condition, weakly conditionally stable FDTD method.

## I. INTRODUCTION

To overcome the Courant limit on the time step size of the FDTD method [1,2], unconditionally stable methods such as the alternating-direction implicit FDTD (ADI-FDTD) scheme [3-12] and a weakly conditionally stable finite-difference time-domain (WCS-FDTD) method has been developed recently [13,14]. In the WCS-FDTD method, the CFL condition is not removed totally, but being weaker than that of the conventional FDTD method. The time step in this scheme is only determined by one space discretization, which is extremely useful for problems where a very fine mesh is needed in one
or two directions. The WCS-FDTD method has better accuracy and higher computation efficiency than the ADI-FDTD method, especially for larger field variation.

However, in the WCS-FDTD method, updating of $H_{y}$ component needs the unknown $E_{X}$ and $E_{Z}$ components at the same time step, thus, the $H_{y}$ component has to be updated implicitly by solving tridiagonal matrix [10], which results in a large inaccuracy in the implementation of the perfect-electric-conductor (PEC) condition for the $E_{X}$ and $E_{z}$ components. To circumvent this problem, an improved weakly conditionally stable finite-difference time-domain (IWCS-FDTD) is presented in this paper. In this method, the solving of the tridiagonal matrix for the $H_{y}$ component is replaced by the solving the tridiagonal matrix of the $E_{X}$ and $E_{Z}$ components; thus, the perfect-electric-conductor (PEC) condition for the $E_{X}$ and $E_{z}$ components is implemented easily. The formulations of the IWCS-FDTD method are presented, and the final updating equations are given. The stability condition of the IWCS-FDTD scheme is presented analytically. Compared with the WCS-FDTD method, this new method has higher accuracy in the implantation of the PEC condition, which is demonstrated through numerical examples.

## II. FORMULATION FOR THE IWCSFDTD METHOD

In a linear, non-dispersive, and lossless medium, the 3D WCS-FDTD scheme in reference [10] can be written as:
$E_{x}^{n+1}=E_{x}^{n}+2 a D_{y} H_{z}^{n+1 / 2}-a D_{z}\left(H_{y}^{n+1}+H_{y}^{n}\right)$

$$
\begin{gather*}
E_{z}^{n+1}=E_{z}^{n}-2 a D_{y} H_{x}^{n+1 / 2}+a D_{x}\left(H_{y}^{n+1}+H_{y}^{n}\right)  \tag{2}\\
H_{y}^{n+1}=H_{y}^{n}+b D_{x}\left(E_{z}^{n+1}+E_{z}^{n}\right) \\
-b D_{z}\left(E_{x}^{n+1}+E_{x}^{n}\right)  \tag{3}\\
H_{x}^{n+3 / 2}= \\
H_{x}^{n+1 / 2}-2 b D_{y} E_{z}^{n+1}  \tag{4}\\
+b D_{z}\left(E_{y}^{n+3 / 2}+E_{y}^{n+1 / 2}\right)  \tag{5}\\
H_{z}^{n+3 / 2}= \\
H_{z}^{n+1 / 2}+2 b D_{y} E_{x}^{n+1}  \tag{6}\\
\\
\quad b D_{x}\left(E_{y}^{n+3 / 2}+E_{y}^{n+1 / 2}\right) \\
E_{y}^{n+3 / 2}= \\
E_{y}^{n+1 / 2}-a D_{x}\left(H_{z}^{n+3 / 2}+H_{z}^{n+1 / 2}\right) \\
\\
+a D_{z}\left(H_{x}^{n+3 / 2}+H_{x}^{n+1 / 2}\right)
\end{gather*}
$$

here, $\quad a=\Delta t / 2 \varepsilon \quad, \quad b=\Delta t / 2 \mu, \quad D_{w}=\partial / \partial w$ ( $w=x, y$ ) represents the first derivative operator with respect to $w . \varepsilon$ is the permittivity and $\mu$ is the permeability of the medium, $n$ and $\Delta t$ are the index and size of time-step.

Obviously, updating of $H_{y}$ component, as shown in eq. (3), needs the unknown $E_{X}$ and $E_{Z}$ components at the same time step. In the WCSFDTD scheme, the $H_{y}$ component is updated by substituting eqs. (1) and (2) into eq. (3) [10] directly. This results in a broadly band matrix equation which is updated by solving two tridiagonal matrix equation.

Here, we apply a new technique to solve eqs. (1)-(3). For simplicity, we write eqs. (1)-(3) in a new form as,

$$
\begin{gather*}
E_{x}^{n+1}=E_{x}^{*}-a D_{z} H_{y}^{n+1}  \tag{7}\\
E_{z}^{n+1}=E_{z}^{*}+a D_{x} H_{y}^{n+1}  \tag{8}\\
H_{y}^{n+1}=H_{y}^{*}+b D_{x} E_{z}^{n+1}-b D_{z} E_{x}^{n+1} \tag{9}
\end{gather*}
$$

where,

$$
\begin{gather*}
E_{x}^{*}=E_{x}^{n}+2 a D_{y} H_{z}^{n+1 / 2}-a D_{z} H_{y}^{n}  \tag{10}\\
E_{z}^{*}=E_{z}^{n}-2 a D_{y} H_{x}^{n+1 / 2}+a D_{x} H_{y}^{n}  \tag{11}\\
H_{y}^{*}=H_{y}^{n}+b D_{x} E_{z}^{n}-b D_{z} E_{x}^{n} \tag{12}
\end{gather*}
$$

Obviously, updating of $E_{X}$ and $E_{Z}$ components, as shown in eqs. (7) and (8), needs the unknown $H_{y}$ component at the same
time step. By substituting eq. (9) into eqs. (7) and (8) we can obtain,

$$
\begin{align*}
& \left(1-a b D_{2 z}\right) E_{x}^{n+1}=E_{x}^{*}-a D_{z} H_{y}^{*}-a b D_{z} D_{x} E_{z}^{n+1} \\
& \left(1-a b D_{2 x}\right) E_{z}^{n+1}=E_{z}^{*}+a D_{x} H_{y}^{*}-a b D_{z} D_{x} E_{x}^{n+1} \tag{13}
\end{align*}
$$

Multiplying eq.(13) by factor $\left(1-a b D_{2 x}\right)$, and subtracting $\left(a b D_{z} D_{x}\right) \times$ eq.(14), we have,

$$
\begin{align*}
& \left(1-a b D_{2 x}-a b D_{2 z}\right) E_{x}^{n+1} \\
& =\left(1-a b D_{2 x}\right) E_{x}^{*}-a D_{z} H_{y}^{*}-a b D_{z} D_{x} E_{z}^{*} \tag{15}
\end{align*}
$$

The left side of eq. (15) is a broadly-banded matrix equation which is solved expensively. To improve the computation efficiency, new terms are added at the both side of eq.(15),

$$
\begin{align*}
& \left(1-a b D_{2 x}-a b D_{2 z}+a^{2} b^{2} D_{2 x} D_{2 z}\right) E_{x}^{n+1} \\
& =\left(1-a b D_{2 x}\right) E_{x}^{*}-a D_{z} H_{y}^{*}-a b D_{z} D_{x} E_{z}^{*} \\
& \quad+a^{2} b^{2} D_{2 x} D_{2 z} E_{x}^{n} \tag{16}
\end{align*}
$$

It is equivalent to the following,

$$
\begin{align*}
& \left(1-a b D_{2 x}\right)\left(1-a b D_{2 z}\right) E_{x}^{n+1} \\
& =\left(1-a b D_{2 x}\right) E_{x}^{*}-a D_{z} H_{y}^{*}-a b D_{z} D_{x} E_{z}^{*} \\
& \quad+a^{2} b^{2} D_{2 x} D_{2 z} E_{x}^{n} \tag{17}
\end{align*}
$$

Dividing eq. (17) into two sub-steps, we have,

$$
\begin{array}{r}
\left(1-a b D_{2 x}\right) e_{x}=a^{2} b^{2} D_{2 x} D_{2 z} E_{x}^{n}-a D_{z} H_{y}^{*}-a b D_{z} D_{x} E_{z}^{*} \\
\left(1-a b D_{2 z}\right) E_{x}^{n+1}=e_{x}+E_{x}^{*} \tag{18.a}
\end{array}
$$

With this manipulation, the updating of the $E_{x}$ component requires the solution of two tridiagonal matrices (18.a) and (18.b).

In a similar way, updating of $E_{Z}$ component can be written as,

$$
\begin{align*}
& \left(1-a b D_{2 z}\right) e_{z} \\
& =a^{2} b^{2} D_{2 x} D_{2 z} E_{z}^{n}+a D_{x} H_{y}^{*}-a b D_{z} D_{x} E_{x}^{*} \tag{19.a}
\end{align*}
$$

$$
\begin{equation*}
\left(1-a b D_{2 x}\right) E_{z}^{n+1}=e_{z}+E_{z}^{*} \tag{19.b}
\end{equation*}
$$

After the solving of the $E_{x}^{n+1}$ and $E_{z}^{n+1}$ components, the $H_{y}$ component can be updated using equation (3) explicitly, thus, the solving of the tridiagonal matrix for the $H_{y}$ component in the WCS-FDTD method is replaced by the solving of the tridiagonal matrix for the $E_{X}$ and $E_{Z}$ components here.

Approximating each derivative in space by centered second-order finite differences, we can obtain the final updating equations for $E_{X}, E_{Z}$ and $H_{y}$ components. Such as, the updating equations for $E_{X}$ component are as follows,

$$
\left(1+\frac{2 a b}{\Delta x^{2}}\right) e_{x}\left(i+\frac{1}{2}, j, k\right)
$$

$-\frac{a b}{\Delta x^{2}} e_{x}\left(i+\frac{3}{2}, j, k\right)-\frac{a b}{\Delta x^{2}} e_{x}\left(i-\frac{1}{2}, j, k\right)$
$=\frac{a^{2} b^{2}}{\Delta z^{2} \Delta x^{2}}\left[\begin{array}{l}E_{x}^{n}\left(i+\frac{3}{2}, j, k+1\right)-E_{x}^{n}\left(i+\frac{1}{2}, j, k+1\right) \\ -E_{x}^{n}\left(i+\frac{1}{2}, j, k+1\right)+E_{x}^{n}\left(i-\frac{1}{2}, j, k+1\right)\end{array}\right]$
$-\frac{2 a^{2} b^{2}}{\Delta z^{2} \Delta x^{2}}\left[\begin{array}{c}E_{x}^{n}\left(i+\frac{3}{2}, j, k\right)-E_{x}^{n}\left(i+\frac{1}{2}, j, k\right) \\ -E_{x}^{n}\left(i+\frac{1}{2}, j, k\right)+E_{x}^{n}\left(i-\frac{1}{2}, j, k\right)\end{array}\right]$
$+\frac{a^{2} b^{2}}{\Delta z^{2} \Delta x^{2}}\left[\begin{array}{c}E_{x}^{n}\left(i+\frac{3}{2}, j, k-1\right)-E_{x}^{n}\left(i+\frac{1}{2}, j, k-1\right) \\ -E_{x}^{n}\left(i+\frac{1}{2}, j, k-1\right)+E_{x}^{n}\left(i-\frac{1}{2}, j, k-1\right)\end{array}\right]$
$-a \frac{H_{y}^{*}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right)-H_{y}^{*}\left(i+\frac{1}{2}, j, k-\frac{1}{2}\right)}{\Delta z}$
$-\frac{a b}{\Delta z \Delta x}\left[\begin{array}{l}E_{z}^{*}\left(i+1, j, k+\frac{1}{2}\right)-E_{z}^{*}\left(i, j, k+\frac{1}{2}\right) \\ -E_{z}^{*}\left(i+1, j, k-\frac{1}{2}\right)+E_{z}^{*}\left(i, j, k-\frac{1}{2}\right)\end{array}\right]$

$$
\begin{align*}
& \left(1+\frac{2 a b}{\Delta z^{2}}\right) E_{x}^{n+1}\left(i+\frac{1}{2}, j, k\right) \\
& -\frac{a b}{\Delta z^{2}} E_{x}^{n+1}\left(i+\frac{1}{2}, j, k+1\right) \\
& -\frac{a b}{\Delta z^{2}} E_{x}^{n+1}\left(i+\frac{1}{2}, j, k-1\right) \\
& =e_{x}\left(i+\frac{1}{2}, j, k\right)+E_{x}^{*}\left(i+\frac{1}{2}, j, k\right) \tag{20.b}
\end{align*}
$$

where,

$$
\begin{align*}
& H_{y}^{*}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right)=H_{y}^{n}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right) \\
& +b\left(\frac{E_{z}^{n}\left(i+1, j, k+\frac{1}{2}\right)-E_{z}^{n}\left(i, j, k+\frac{1}{2}\right)}{\Delta x}\right] \\
& \left.-\frac{E_{x}^{n}\left(i+\frac{1}{2}, j, k+1\right)-E_{x}^{n}\left(i+\frac{1}{2}, j, k\right)}{\Delta z}\right]  \tag{21}\\
& E_{z}^{*}\left(i+1, j, k+\frac{1}{2}\right)=E_{z}^{n}\left(i+1, j, k+\frac{1}{2}\right) \\
& +a \frac{H_{y}^{n}\left(i+\frac{3}{2}, j, k+\frac{1}{2}\right)-H_{y}^{n}\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right)}{\Delta x} \\
& -\frac{2 a}{\Delta y}\left[\begin{array}{l}
\left.H_{x}^{n+1 / 2}\left(i+1, j+\frac{1}{2}, k+\frac{1}{2}\right)\right] \\
\left.E_{x}^{*}\left(i+\frac{1}{2}, j, k\right)=E_{x}^{n+1 / 2}\left(i+1, j-\frac{1}{2}, k+\frac{1}{2}\right)\right] \\
\left.-a-\frac{1}{2}, j, k\right) \\
-a\left(i+\frac{1}{2}, j, k+\frac{1}{2}\right)-H_{y}^{n}\left(i+\frac{1}{2}, j, k-\frac{1}{2}\right) \\
\Delta z
\end{array}\right.  \tag{22}\\
& +\frac{2 a}{\Delta y}\left[-H_{z}^{n+1 / 2}\left(i+\frac{1}{2}, j-\frac{1}{2}, k\right)\right]
\end{align*}
$$

(23)

The solving of the $E_{y}$ component, same as that in the WCS-FDTD method, is updated implicitly by substituting eqs. (4) and (5) into eq. (6),

$$
\begin{align*}
& \left(1-a b D_{2 x}-a b D_{2 z}\right) E_{y}^{n+3 / 2} \\
& =\left(1+a b D_{2 x}+a b D_{2 z}\right) E_{y}^{n+1 / 2} \\
& -2 a D_{x} H_{z}^{n+1 / 2}+2 a D_{z} H_{x}^{n+1 / 2}  \tag{24}\\
& -2 a b D_{x} D_{y} E_{x}^{n+1}-2 a b D_{y} D_{z} E_{z}^{n+1}
\end{align*}
$$

The left side of eq. (24) is also a broadlybanded matrix equation. Adding new terms at the both side of eq.(24) and dividing it into two substeps, we can obtain the updating of the $E_{y}$ component as follows,

$$
\begin{align*}
& \left(1-a b D_{2 x}\right) e_{y} \\
& =\left(1+a b D_{2 x}\right)\left(1+a b D_{2 z}\right) E_{y}^{n+1 / 2} \\
& -2 a D_{x} H_{z}^{n+1 / 2}+2 a D_{z} H_{x}^{n+1 / 2}  \tag{25.a}\\
& -2 a b D_{x} D_{y} E_{x}^{n+1}-2 a b D_{y} D_{z} E_{z}^{n+1} \\
& \quad\left(1-a b D_{2 z}\right) E_{y}^{n+3 / 2}=e_{y} . \tag{25.b}
\end{align*}
$$

Thus, at each time step the IWCS-FDTD method requires the solution of six tridiagonal matrices and two explicit equations.

## III. WEAKLY CONDITIONAL STABILITY OF THE IWCS-FDTD METHOD

The relations between the field components of the IWCS-FDTD method can be represented in matrix forms,

$$
\begin{equation*}
[A][\Lambda]^{n+1}=[B][\Lambda]^{*}+[C][\Lambda]^{n} \tag{26}
\end{equation*}
$$

where,

$$
[A]=\left[\begin{array}{cccccc}
S & 0 & 0 & 0 & 0 & 0 \\
2 a b D_{x} D_{y} & S & 2 a b D_{z} D_{y} & 0 & 0 & 0 \\
0 & 0 & S & 0 & 0 & 0 \\
0 & -b D_{z} & 2 b D_{y} & 1 & 0 & 0 \\
b D_{z} & 0 & -b D_{x} & 0 & 1 & 0 \\
-2 b D_{y} & b D_{x} & 0 & 0 & 0 & 1
\end{array}\right]
$$

$$
\left.\left.\begin{array}{l}
{[B]=\left[\begin{array}{cccccc}
1-a b D_{2 x} & 0 & -a b D_{z} D_{x} & 0 & -a D_{z} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-a b D_{z} D_{x} & 0 & 1-a b D_{2 z} & 0 & a D_{x} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]} \\
{[C]=\left[\begin{array}{cccccc}
a^{2} b^{2} D_{2 x} D_{2 z} & 0 & 0 & 0 & 0 & 0 \\
0 & S_{2} & 0 & 2 a D_{z} & 0 & -2 a D_{x} \\
0 & 0 & a^{2} b^{2} D_{2 x} D_{2 z} & 0 & 0 & 0 \\
0 & b D_{z} & 0 & 1 & 0 & 0 \\
-b D_{z} & 0 & b D_{x} & 0 & 1 & 0 \\
0 & -b D_{x} & 0 & 0 & 0 & 1
\end{array}\right]} \\
{[\Lambda]^{n+1}=\left[E_{x}^{n+1}\right.} \\
E_{y}^{n+3 / 2} \\
E_{z}^{n+1} \mathrm{H}_{x}^{n+3 / 2}
\end{array} \mathrm{H}_{y}^{n+1} \mathrm{H}_{z}^{n+3 / 2}\right]^{\prime}\right] .
$$

According eqs. (10)-(12), we have,

$$
\begin{equation*}
[\Lambda]^{*}=[D][\Lambda]^{n} \tag{27}
\end{equation*}
$$

with,

$$
[D]=\left[\begin{array}{cccccc}
1 & 0 & 0 & 0 & -a D_{z} & 2 a D_{y} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -2 a D_{y} & a D_{x} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-b D_{z} & 0 & b D_{x} & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

Substituting eq. (27) into eq. (26), we obtain,

$$
\begin{equation*}
[\Gamma][\Lambda]^{n}=0 \tag{28}
\end{equation*}
$$

here,

$$
\begin{aligned}
& {[\Gamma]=[A] \zeta-[B][D]-[C]} \\
& =\left[\begin{array}{cccccc}
\left(S \zeta-S_{1}\right) & 0 & 2 a b D_{z} D_{x} & -2 a^{2} b f & 2 a D_{z} & f_{x} \\
2 a b D_{x} D_{y} \zeta\left(S \zeta-S_{2}\right) & 2 a b D_{z} D_{y} \zeta-2 a D_{z} & 0 & 2 a D_{x} \\
2 a b b_{2} D_{x} & 0 & \left(S \zeta \zeta-S_{3}\right) & f_{z} & -2 a D_{x} 2 a^{2} b f \\
0 & -L_{z} & 2 b D_{y} \zeta & \zeta-1 & 0 & 0 \\
L_{z} & 0 & -L_{x} & 0 & \zeta-1 & 0 \\
-2 b D_{y} \zeta & L_{x} & 0 & 0 & 0 & \zeta-1
\end{array}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
& S_{1}=1-a b D_{2 x}+a b D_{2 z}+a^{2} b^{2} D_{2 x} D_{2 z} \\
& S_{3}=1+a b D_{2 x}-a b D_{2 z}+a^{2} b^{2} D_{2 x} D_{2 z} ; \\
& f=D_{x} D_{y} D_{z} ; \\
& f_{z}=2 a D_{y}\left(1-a b D_{2 z}\right) ; \\
& f_{x}=-2 a D_{y}\left(1-a b D_{2 x}\right) ; \\
& L_{w}=b D_{w}(\xi+1), w=x, z .
\end{aligned}
$$

For a nontrivial solution of eq. (28), the determinant of the coefficient matrix in eq. (28) should be zero,
$|\Gamma|=0$
By solving eq. (29), we have,

$$
\begin{equation*}
(\zeta-1)^{2}\left(M^{2} \zeta^{2}-2 N \zeta+M^{2}\right)\left(M \zeta^{2}-2 P \zeta+M\right)=0 \tag{30}
\end{equation*}
$$

here,

$$
\begin{aligned}
& M=\left(1-a b D_{2 x}\right)\left(1-a b D_{2 z}\right) \\
& N=\left(1+a b D_{2 x}\right)\left(1+a b D_{2 z}\right)\left(1-a b D_{2 x}\right)\left(1-a b D_{2 z}\right) \\
& +2 a b D_{2 y}\left(1-a b D_{2 x}-a b D_{2 z}\right)\left(1+a^{2} b^{2} D_{2 x} D_{2 z}\right) \\
& P=\left(1+a b D_{2 x}\right)\left(1+a b D_{2 z}\right)+2 a b D_{2 y}
\end{aligned}
$$

The growth factor $\zeta$ is obtained,

$$
\begin{align*}
& \zeta_{1,2}=1  \tag{31}\\
& \zeta_{3,4}=\left(N \pm \sqrt{N^{2}-M^{4}}\right) / M^{2}  \tag{32}\\
& \zeta_{5,6}=\left(P \pm \sqrt{P^{2}-M^{2}}\right) / M \tag{33}
\end{align*}
$$

To satisfy the stability condition during the field advancement, the module of growth factor $\zeta$ can't be larger than 1. It is evident that the module of $\zeta_{1,2}$ is unity.

For the values of $\zeta_{3,4}$ and $\zeta_{5,6}$, when the conditions $M^{4} \geq N^{2}$ and $M^{2} \geq P^{2}$ are satisfied, $\left|\zeta_{3,4}\right|=\left|\zeta_{5,6}\right|=1$ can be obtained. Approximating each derivative in space by centered second-order finite differences, we can obtain the limitation for time-step size in the IWCS-FDTD method as follows,

$$
\begin{equation*}
\Delta t \leq \Delta y / c \tag{34}
\end{equation*}
$$

where $c=1 / \sqrt{\varepsilon \mu}$ is the speed of light in the medium.

The stable condition of the IWCS-FDTD method is same as that of WCS-FDTD method. The maximum time-step size for IWCS-FDTD method is only determined by one spatial increment $\Delta y$. This is due to that the explicit difference is only used in the $y$ direction.

## IV. NUMERICAL VALIDATION

To demonstrate the accuracy and efficiency of the proposed theory, a numerical example is presented here. A metal plate with dimension $60 \mathrm{~mm} \times 60 \mathrm{~mm}$ is shown in Fig.1. Twenty five apertures of 2 mm length and 2 mm width are cut on the plate. All the distances between the apertures are 10 mm . A uniform plane wave polarized along the $z$-direction, is normally incident on the aperture, and the time dependence of the excitation function is as follows,

$$
\begin{equation*}
E_{z}(t)=\exp \left[-4 \pi\left(t-t_{0}\right)^{2} / T\right] \tag{35}
\end{equation*}
$$

where $T$ and $t_{0}$ are constants, and both equal to $2 \times 10^{-9}$. In such a case, the highest frequency of interest is 1 GHz . The observation point is set at the back of the plate and is 50 mm far from the plate.

Applying the FDTD method to compute the time domain electric field component $E_{Z}$ at the observation point, to simulate the apertures precisely, the cell size around the aperture must be small. We choose $\Delta x=\Delta z=0.5 \mathrm{~mm}$ around the apertures. The cell size $\Delta y$ is set to be 25 mm . To satisfy the stability condition of the FDTD algorithm, the time-step size for the conventional FDTD is $\Delta t \leq 1.17$ ps. For the WCS-DTD and IWCS-FDTD scheme, the maximum time increment is only related to the space increments $\Delta y$, that is, $\Delta t \leq 83.33 \mathrm{ps}$. Five-cellthick CPML layers are used to terminate the grid, and are placed five cells from the metal plate on all sides. The implementation of the plane wave is same as that of conventional FDTD method. The metal plate is viewed as a perfect electronic conductor and the tangential electric field values at the metal plate should to be zeros.

In the WCS-FDTD method, the $E_{x}$ and $E_{Z}$ components at the metal plate are set zeros directly after they are updated by using eqs. (2) and (3); while in the IWCS-FDTD method, the

PEC boundary condition for the $E_{X}$ and $E_{Z}$ components are implemented following the strategy descried in reference [8], by incorporating the PEC condition into the solving of the tridiagonal matrices.


Fig. 1. Geometric configuration of the metal plate.
We perform the numerical simulation for an 83 ns time history by using the IWCS-FDTD method under time step size 83.33ps. The result is shown in Fig. 2. The total time steps are almost 1,000. It can be seen from Fig. 1 that no instability problem is observed, which numerically validates the stability condition of eq. (34).

To demonstrate the high computational efficiency and accuracy of the IWCS-FDTD method, we perform the numerical simulations for a 5 ns time history by using the conventional FDTD, WCS-FDTD, and IWCS-FDTD methods, and compare the computation times and accuracy of these methods. In the conventional FDTD method, the time-step size keeps a constant of 1.17 ps, while in the WCS-FDTD and IWCS-FDTD methods, we use time-step size 83.33 ps.


Fig. 2. Numerical result using IWCS-FDTD method with time step size 83.33ps.

Figure 3 shows the electric field component $E_{Z}$ at observation point calculated by using the conventional FDTD, WCS-FDTD, and IWCSFDTD methods. It can be seen from this figure that only the result calculated by the IWCS-FDTD method agrees well with the result calculated by the conventional FDTD method. The result calculated by the WCS-FDTD method deviates from that of the conventional FDTD method significantly.

It is apparent that the IWCS-FDTD method has higher accuracy than the WCS-FDTD method in the implementation of the PEC condition. The reason for the inaccuracy of WCS-FDTD method is that, in the WCS-FDTD method, updating of $H_{y}$ component needs the unknown $E_{X}$ and $E_{Z}$ components at the same time step, thus, implementation of the PEC condition for the $E_{X}$ and $E_{Z}$ components must be incorporated into the solving of the $H_{y}$ component. The WCS-FDTD method neglects this and results in serious error in the implementation of the PEC condition.

To complete this simulation, the computation times for the conventional FDTD method, WCSFDTD method and IWCS-FDTD method are $41.18,5.05$, and 5.37 minutes, respectively. Due to large time step size applied, the CPU time for the WCS-FDTD and IWCS-FDTD methods are almost $1 / 8$ of that for the conventional FDTD method.


Fig. 3. The comparison of the results calculated by using the conventional FDTD, WCS-FDTD, and IWCS-FDTD methods

## V. CONCLUSION

An improved weakly conditionally stable FDTD method is presented in this paper to circumvent the inaccuracy in the implementation of the perfect-electric-conductor condition in the WCS-FDTD method. The stability condition of the IWCS-FDTD scheme is presented analytically and the numerical performance of the proposed method over the WCS-FDTD method is demonstrated through numerical example.

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# Dual Band-Notch Small Square Monopole Antenna with Enhanced Bandwidth Characterestics for UWB Applications 

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#### Abstract

This article proposes a novel printed monopole antenna for ultra wideband applications with dual band-notch function. The antenna consists of a square radiating patch with four rectangular slits and a ground plane with a pair of T-shaped parasitic structures, which provides a wide usable fractional bandwidth of more than $165 \%$ (2.79-14.83 GHz). In order to generate single band-notch characteristics, we use two rectangular slits in the corners of square radiating patch. By adding two rectangular slits in the center of the modified radiating patch, a dual band notch function is achieved and also by inserting a pair of T-shaped parasitic structures in the ground plane, additional resonances are excited and hence much wider impedance bandwidth can be produced, especially at the higher band. The measured results reveal that the presented dual band-notch monopole antenna offers a very wide bandwidth with two notched bands, covering all the $5.2 / 5.8 \mathrm{GHz}$ WLAN, $3.5 / 5.5 \mathrm{GHz}$ WiMAX and 4 GHz C bands. The designed antenna has a small size of $12 \times 18 \mathrm{~mm}^{2}$.

Index Terms - Dual band notch function, square monopole antenna, ultra-wideband (UWB) applications.


## I. INTRODUCTION

In UWB communication systems, one of the key issues is the design of a compact antenna while providing wideband characteristic over the whole operating band. Consequently, a number of microstrip antennas with different geometries have been experimentally characterized. Moreover, other strategies to improve the impedance bandwidth which do not involve a modification of the geometry of the planar antenna have been investigated [1-4].

The frequency range for UWB systems between 3.1 to 10.6 GHz will cause interference to the existing wireless communication systems, such as, the wireless local area network (WLAN) for IEEE 802.11a operating in $5.15-5.35 \mathrm{GHz}$ and $5.725-$ 5.825 GHz bands, WiMAX (3.3-3.6 GHz and Cband (3.7-4.2 GHz), so the UWB antenna with a single and dual band-stop performance is required. Recently to generate the frequency band-notch function, modified planar monopoles have been recently proposed [5-10]. In [5] and [6], different shapes of the slits (i.e., W-shaped and folded trapezoid) are used to obtain the desired band notched characteristics. Multiple [7] halfwavelength U-shaped slits are embedded in the radiation patch to generate the single and multiple
band-notched functions, respectively. In [8], bandnotch function is achieved by using a T-shaped coupled-parasitic element in the ground plane.

In this paper, a new dual band-notched printed monopole antenna with multi resonance performance is presented. In the proposed structure, single band-notch function is provided by cutting two rectangular slits in the corners of the square radiating patch and dual band-notch characteristic is obtained by cutting four rectangular slits in the radiating patch. Finally by inserting a pair of T-shaped parasitic structures on the ground plane, additional resonances are excited and the bandwidth is improved that achieves a fractional bandwidth with multi resonance performance of more than $165 \%$.

## II. ANTENNA DESIGN

As shown in Fig. 1, the proposed monopole antenna is fed by a microstrip line and it is printed on a FR4 substrate of thickness 1.6 mm . As shown in Fig. 1, the proposed antenna consists of a square radiating patch with four rectangular slits and a ground plane which is partially modified with two T-shaped parasitic structures. The basic antenna structure consists of a square patch, a feedline, and a ground plane. The square patch has a width $W$. As can be seen in Fig. 1, the width and length of the feed line, which is connected to the patch are $W_{f}$ and $L_{f}$, respectively. On the other side of the substrate, a conducting ground plane of width $W_{\text {sub }}$ and length $L_{\text {gnd }}$ is placed. The width $W_{f}$ of the microstrip feedline is fixed at 2 mm . The proposed antenna is connected to a $50 \Omega$ SMA connector for signal transmission.
In this study, four modified rectangular slits with variable dimensions are used in order to generate the dual frequency band-stop performance, as displayed in Fig. 1. These slits can create additional surface current paths in the antenna therefore perturb the resonant response [7]. At the notch frequencies, the current flows are more dominant around these slits, and they are oppositely directed between the slits edges [10]. Also T-shaped parasitic structures play an important role in the broadband characteristics of this antenna because they can achieve additional resonances and improve the bandwidth [10]. In other words, the impedance bandwidth is effectively improved at the upper frequency that
can be regarded as a parasitic resonator electrically to the square monopole.
The optimal dimensions of the designed antenna

$$
\begin{aligned}
& \text { are as follows: } W_{\text {sub }}=12 \mathrm{~mm}, \\
& L_{\text {sub }}=18 \mathrm{~mm}, h_{\text {sub }}=1.6 \mathrm{~mm}, W=10 \mathrm{~mm}, \\
& W_{f}=2 \mathrm{~mm}, L_{f}=7 \mathrm{~mm}, W_{S 3}=0.5 \mathrm{~mm}, \\
& L_{S}=8.75 \mathrm{~mm}, W_{S 1}=2 \mathrm{~mm}, L_{S 1}=9 \mathrm{~mm}, L_{X}=0.5 \mathrm{~mm}, \\
& W_{X}=1 \mathrm{~mm}, W_{P}=1.2 \mathrm{~mm}, \quad L_{P}=8 \mathrm{~mm}, \\
& W_{P 1}=2.75 \mathrm{~mm}, L_{P 1}=3.5 \mathrm{~mm}, W_{P 2}=0.5 \mathrm{~mm} \text { and } \\
& L_{\text {gnd }}=3.5 \mathrm{~mm} .
\end{aligned}
$$



(b)
(a)

(c)

Fig. 1. Geometry of proposed antenna with four rectangular slits and a pair of T-shaped parasitic structures, (a) side view, (b) top view, (c) bottom view.

## III. RESULTS AND DISCUSSIONS

The microstrip-fed monopole antenna was constructed and studied to demonstrate the effect of the proposed dual band-notch function and bandwidth enhancement technique. The numerical and experimental results of the input impedance and radiation characteristics are presented and discussed. The parameters of this proposed antenna are studied by changing one or two parameters at a time and while others are fixed.

The simulated results are obtained using the Ansoft simulation software high-frequency structure simulator (HFSS) [12].
Figure 2 shows the structures of various antennas used for simulation studies. Return loss characteristics for ordinary square patch antennas (Fig. 2(a)), with a pair of rectangular slits (Fig. 2(b)), and with four rectangular slits (Fig. 2(c)) are compared in Fig 3. As shown in Fig. 3, in the proposed antenna configuration, the ordinary square monopole can provide the fundamental and next higher resonant radiation band at 4.3 and 8.2 GHz , respectively. In order to generate single band-notch characteristics, two rectangular slits are used at the corners of square radiating patch. By adding two rectangular slits in the center of modified radiating patch a dual band-notch function is achieved that covering all the $5.2 / 5.8 \mathrm{GHz}$ WLAN, $3.5 / 5.5 \mathrm{GHz}$ WiMAX and $4-\mathrm{GHz} \mathrm{C}$ bands. Also the input impedance of the proposed antenna structure and the various monopole antenna structures that are shown in Fig. 2, are shown in Fig. 4.

To understand the phenomenon behind this dual band-notch performance, the simulated current distribution on the modified radiating patch for the proposed antenna without T-shaped parasitic structures at the notch frequencies of 3.8 GHz and 5.5 GHz is presented in Fig. 5(a) and 5(b), respectively. As it can be observed in Fig. 5(a), $5(\mathrm{~b})$, the current is concentrated on the edges of the interior and exterior of the rectangular slits at 3.8 GHz and 5.5 GHz . Therefore, the antenna impedance changes at these frequencies due to the band-notch properties of the rectangular slits.


Fig. 2. (a) The ordinary square antenna, (b) the square antenna with two rectangular slit in corners, (c) the square antenna with four rectangular slits.


Fig. 3. Simulated return loss characteristics for antennas shown in Figure 2.


Fig. 4. Simulated VSWR characteristics for the proposed antenna structure and antennas shown in Figure 2.


Fig. 5. Simulated surface current distributions on the radiating patch for (a) the proposed antenna without T-shaped parasitic structures at first bandnotch central frequency ( 3.8 GHz ), (b) the proposed antenna without T -shaped parasitic structures at second band-notch central frequency ( 5.5 GHz ).

In order to increase the upper frequency bandwidth, two T-shaped parasitic structures are inserted in the ground plane of the proposed antenna, as displayed in Fig. 1. Three such Tshaped parasitic structures with different sizes are specified in Table I as cases 1,2 and 3.

Table 1: Three cases of proposed antenna with different values of T-shaped parasitic structures

| Case | $\mathrm{L}_{\mathrm{Pl}^{(\mathrm{mmm})}}$ | $\mathrm{L}_{\mathrm{P}}{ }^{(\mathrm{mmm})}$ | $\mathrm{W}_{\mathrm{PI}^{(\mathrm{mmm})}}$ | $\mathrm{W}_{\mathrm{P}}{ }^{(\mathrm{mmm})}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3 | 9 | 3.5 | 1 |
| 2 | 3.5 | 8 | 2.75 | 1.2 |
| 3 | 5 | 7 | 1 | 1.5 |


(b)

Fig. 6. Simulated return loss and VSWR characteristics for the proposed antenna without Tshaped parasitic structures and three cases 1,2 and 3 with it as shown in Table 1.

Figure 6 shows the effects of T-shaped parasitic structures with different values on the impedance matching in comparison with the same antenna without them. It is found that by inserting the T-
shaped parasitic structures with suitable dimensions in the ground plane additional resonances are excited and hence much wider impedance bandwidth with multi-resonance characteristics can be produced, especially at the higher frequencies.
The simulated VSWR curves with different values of $L_{S}$ are plotted in Fig. 7. As shown in Fig. 6, when the height of the slits increases from 5.25 to 9.25 mm , the center of lower notch frequency is decreased from 4.8 to 3.45 GHz and also the center of higher notch frequency is decreased from 6.5 to 4.9 GHz . From these results, we can conclude that the notch frequency is controllable by changing the interior height of the corner slit.


Fig. 7. Simulated VSWR characteristics for various values of $L_{S}$.


Fig. 8. Simulated VSWR characteristics for various values of $W_{\text {S3 }}$.

Figure 8 illustrates the simulated VSWR characteristics with various values of $W_{S 3}$. As the
width $W_{S 3}$ increases from 0.2 to 1.5 mm , the filter bandwidth is varied from 0.6 to 1.3 GHz for lower notch frequency and also varied from 0.8 to 1.2 GHz for higher notch frequency.


Fig. 9. Photograph of the realized antenna, (a) top view, (b) bottom view.


Fig. 10. Measured and simulated VSWR for the proposed antenna.

A prototype of the proposed monopole antenna, as shown in Fig. 9, was fabricated and tested in the Antenna Laboratory at Microwave Technology Company (MWT), and the VSWR were measured using a HP 8720ES network analyzer in an anechoic chamber. Figure 10 shows the measured and simulated VSWR characteristics of the proposed antenna. The fabricated antenna has the frequency band of 2.79 to over 14.83 GHz with two rejection bands around 3.45-4.23 and 5.075.89 GHz . As shown in Fig. 10, there exists a discrepancy between measured data and the simulated results which could be due to the effect of the SMA port. In order to confirm the accurate return loss characteristics for the designed antenna, it is recommended that the manufacturing and measurement processes need to be performed carefully.

The difference in the measured and simulated results in Fig. 10 can also be due to the use of FR4. The designed and fabricated FR4 substrate might not have the same dielectric constant value. As in reality the dielectric constant of an FR4 substrate usually can be anything from 4.1-4.9 GHz.

(a)

(b)

(c)


| - co-pol | (measured) |
| :---: | :---: |
| - co-pol | (simulated) |
| --- cross-pol (measured) |  |
| --- cross-pol (simulated) |  |

Fig. 11. Measured and simulated radiation patterns of the proposed antenna. (a) 4.8 GHz , (b) 7.5 GHz, and (c) 10 GHz .

The radiation patterns have been measured inside an anechoic chamber. Figure 11 shows the measured and simulated radiation patterns including the co-polarization and crosspolarization in the $H$-plane ( $x$-z plane) and $E$-plane ( $y-z$ plane). The main purpose of the radiation patterns is to demonstrate that the antenna actually
radiates over a wide frequency band. It can be seen that the radiation patterns in $x$-z plane are nearly omnidirectional for the three frequencies. As shown in Fig. 11, the measured radiation patterns agree very well with the simulated results at the three frequencies.

Figure 12 shows the effects of the rectangular slits, and T-shaped parasitic structures on the maximum gain in comparison with the same antenna without them. As shown in Fig. 12, the basic antenna has a gain that is low at 2 GHz and increases with frequency. It is found that the gain of the square antenna is decreased with the use of the rectangular slits, and T-shaped parasitic structures. It can be observed in Fig. 12 that by using a square radiating patch with a four rectangular slits and two T-shaped parasitic structures, two sharp decrease of maximum gain in the notched frequencies band at 3.9 and 5.5 GHz are shown. For other frequencies outside the notched frequencies band, the antenna gain with the filter is similar to those without it. A twoantenna technique is used to measure the radiation gain. It can be seen that for the frequency notch band the gain is negative, which is expected based on the return loss and VSWR measurements.


Fig. 12. Measured maximum gain comparisons for the ordinary square antenna and the proposed antenna.

## V. CONCLUSION

In this paper, a novel compact wideband planar monopole antenna (PMA) with variable single and dual band-notched characteristics has been proposed for various UWB applications. The
fabricated antenna has the frequency band of 2.79 to over 14.83 GHz with two rejection bands around $3.3-4.23$ and $5.07-5.89 \mathrm{GHz}$. By cutting four rectangular slits with variable dimensions on the square radiating patch, dual band-notch characteristics generated and also by inserting two inverted T-shaped conductor-backed plane, additional resonances are excited and hence much wider impedance bandwidth can be produced, especially at the higher band. The proposed antenna has a simple configuration and is easy to fabricate. Experimental results show that the proposed antenna could be a good candidate for UWB applications.

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# Optimal Design of an Ultra-Wideband Antenna with the Irregular Shape on Radiator using Particle Swarm Optimization 

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#### Abstract

An ultra-wideband antenna with an irregular shape radiator for ultra-wideband applications has been designed by particle swarm optimization along with the simulator, HFSS. The proposed antenna design process is automated. Also, the better results can be obtained efficiently. The proposed antenna is fed by a coplanar waveguide line. The optimized antenna has a compact substrate size of 34.0 mm by 33.35 mm . Results show that the optimized irregular shape of the proposed antenna outperforms the antennas with uniform rectangular shapes in impedance matching. The optimized antenna can cover the spectrum of ultra-wideband ( $3.1 \mathrm{GHz}-10.6 \mathrm{GHz}$ ). The current distributions are investigated for describing the antenna characteristics. Moreover, the proposed UWB antenna has good characteristics of radiation, transmission, and impedance bandwidth.


Index Terms - Optimization, planar antennas, PSO, ultra-wideband.

## I. INTRODUCTION

Ultra-wideband (UWB) [1] antennas have been received much attention from antenna designers and researchers. Many UWB antennas were designed using planar structure due to advantages of low profile, low cost, light weight, and easy fabrication. The dimensions and geometry of the radiator largely affect the impedance matching. To achieve the design goal, some of planar UWB antennas adopted different shapes such as rectangle [2, 3], circle [4], triangle [5], and octagon [6] as the radiators. Other studies used the staircase-shape on the edges of the
radiators [7, 8]. However, those shapes have uniform or smooth edges on the radiator and the dimensions of the radiator were determined by experimental efforts.

This study uses the particle swarm optimization (PSO) method, combined with the finite element method based simulator, HFSS, to design and optimize the proposed UWB antenna. The radiator of the proposed UWB antenna consists of five rectangular patches. The dimension of each rectangular patch can be arbitrarily adjusted by the PSO method to achieve the design goal. Hence, the shape of the optimized radiator would be various. The developed PSO method can control the HFSS and can change the antenna dimensions without any manual adjustments during the optimization process. In this way, optimized results are obtained automatically. The main advantage of the proposed approach is that it reduces the time required to solve an electromagnetic problem. The better results can be obtained efficiently. Compared with the studies shown in [2-8], the irregular shape radiator obtained by the proposed approach can be considered as novel.

The PSO is an effective algorithm which was proposed in 1995 by James Kennedy and Russell Eberhart jointly [9]. Many electromagnetic problems [10-12] have been designed using PSO. The basic concept of PSO is from the study on animals' group behaviors, such as a swarm of birds randomly searching for food. The birds (particles) distribute randomly and fly with random direction and velocity in a finite area initially. If one of the birds finds a location close to the food (the optimum), the bird changes its flying direction and velocity toward the food.

Simultaneously, the bird tells other birds to fly around the bird. The other birds would change their direction and velocity to get closer to the food. The birds search for the entire space until one of them finds the food. The PSO algorithm is inspired from this model. For the sake of brevity, other detailed concepts of PSO are excluded here and they can be found in [13].

Optimized dimensions of the proposed planar UWB antenna is fabricated on a cheap FR4 substrate. The antenna radiator is fed by a coplanar waveguide (CPW) line [14]. A good impedance matching is obtained in the UWB spectrum. Moreover, a reasonable agreement between simulated reflection coefficient and measured one is observed. Simulated results show that near omnidirectional radiation patterns are achieved. A parametric study was performed by simulation. This result shows that the optimized irregular shape of the proposed antenna outperforms the antennas with uniform rectangular shapes on the radiators in impedance matching. Good frequencydomain characteristics of maximum gain, group delay and phase of S21 were measured. The results verified that the proposed UWB antenna can apply to any short- or long-range communication systems.

Table 1: The dimensions and optimization ranges of the initial antenna (Unit: mm)

| Parameter | Size | Optimization range |
| :--- | :--- | :--- |
| L0 | 5.0 | $0.5-7.0$ |
| Lg | 15.0 | $9.0-17.0$ |
| Wg | 15.0 | $16.0-26.0$ |
| L1 | 5.0 | $1.0-7.0$ |
| W1 | 4.4 | $0.5-8.0$ |
| L2 | 5.0 | $1.0-7.0$ |
| W2 | 4.6 | $0.5-8.0$ |
| L3 | 5.0 | $1.0-7.0$ |
| W3 | 4.8 | $0.5-8.0$ |
| L4 | 5.0 | $1.0-7.0$ |
| W4 | 5.0 | $0.5-8.0$ |
| L5 | 5.0 | $1.0-7.0$ |
| W5 | 5.2 | $0.5-8.0$ |

## II. PSO AND ANTENNA DESIGN

Figure 1 shows the geometry of the initial antenna whose radiator consisting of five different rectangular patches. Before the optimization, the initial antenna was created in the full-wave
electromagnetic simulator, HFSS. In the PSO optimization, the optimizer has more flexibility when there are more numbers of parameters to be optimized. At the same time, more numbers of EM simulations is required in iteration. To compromise the order of complexity and flexibility, five rectangular patches of the radiator are adopted in this study. Table 1 lists the dimensions of the initial antenna. Five rectangular patches are denoted L1 to L5 and W1 to W5. The sizes of them are used for creating the initial antenna only. The initial values of five rectangular patches do not affect the optimization result since the PSO is a global optimizer. It is not required using any starting points in the optimization process of the PSO. The gap between the radiator and ground is L0. The length and width of the ground plane are Lg and 2 Wg , respectively. The W0 $(4.2 \mathrm{~mm})$ and $\mathrm{G}(0.3 \mathrm{~mm})$ are the width and the gap of the $50 \Omega \mathrm{CPW}$, respectively. The W0 and $G$ are fixed during optimization. The proposed antenna is to be fabricated on an FR4 substrate with thickness of 0.8 mm , relative permittivity of 4.4, and loss tangent of 0.02 . In this optimization, thirteen parameters, L0, Lg, Wg, L1 to L5, and W1 to W5 are to be optimized within their optimization range to achieve the UWB specification. If the designer wants the antenna to


Fig. 1. The geometry of the initial antenna.


Fig. 2. The best fitness versus iteration.

Table 2: The dimensions of the optimized antennas (Unit: mm)

| Parameter | Size | Parameter | Size |
| :--- | :--- | :--- | :--- |
| W0 | 4.2 | L0 | 1.35 |
| Wg | 14.28 | Lg | 15.87 |
| W1 | 13.97 | L1 | 2.67 |
| W2 | 12.77 | L2 | 2.66 |
| W3 | 15.49 | L3 | 3.0 |
| W4 | 16.0 | L4 | 3.79 |
| W5 | 11.29 | L5 | 4.66 |
| be compact, the designer may limit the |  |  |  | optimization ranges of parameters. The reflection coefficient $\left|S_{11}\right|$ of the optimized antenna should be below -10 dB in the UWB spectrum (frequency between 3.1 GHz to 10.6 GHz ).

To optimize the proposed antenna using PSO and HFSS, the first step is to create a script of HFSS. The HFSS script is recorded while creating the geometry of the initial antenna. The PSO serves as an external optimizer to control the HFSS. The next step is to modify the value of each parameter in the script. The PSO code can identify and modify the dimension of each parameter in the script. The third step is to execute the modified script. The HFSS simulates the antenna using the modified geometry to obtain the simulated results. The fitness is then used to evaluate the obtained results for PSO optimization process. In this study, the obtained $\left|S_{11}\right|$ is used to calculate the fitness using (1).


Fig. 3. The fabricated antenna on an FR4 substrate with the antenna size of 34.0 mm by 33.35 mm .


Fig. 4. The measured and simulated reflection coefficients $\left|S_{11}\right|$ of the proposed antenna.

$$
\begin{gather*}
\text { Fitness }=\sum\left(\left|S_{11}(f)\right|-\mid S_{11, d}(f)\right)\left[\frac{1+\operatorname{sgn}\left(\left|S_{11}(f)\right|-\left|S_{11, d}(f)\right|\right)}{2}\right] \Delta f, \\
\left|S_{11, d}(f)\right|=-12 \mathrm{~dB}, \text { for } 2.8 \mathrm{GHz} \leq f \leq 11.0 \mathrm{GHz} . \tag{1}
\end{gather*}
$$

Where, the $\Delta f$ is frequency interval set to 0.01 GHz . The $\left|S_{11, d}\right|$ is the desired reflection coefficient, which is stricter than that of the UWB


Fig. 5. The geometry of four different widths on the radiator of the UWB antennas.
specification. The fitness is the different area between $\left|S_{11}\right|$ and $\left|S_{11, d}\right|$ that should be as small as possible during optimization. The smaller the fitness is, the better optimization result obtained. The PSO optimization process is terminated when the maximum number of iterations is reached or when the fitness goes to zero. In the PSO setting, the maximum number iteration is set to 100 . The reflecting boundary condition [13] and 27 particles and are used.

## III. RESULTS AND DISCUSSIONS

The PSO optimization process is ended after the maximum number of iterations, 100, is reached. Figure 2 shows the curve of the best fitness versus iteration. The fitness value is dropped significantly during the first ten iterations. Then, the fitness slowly decreases as the number of iteration increases. The optimized dimensions of the global best are used to fabricate the proposed UWB antenna. Table 2 lists the optimized dimensions of the proposed antenna. Figure 3 reveals the picture of the antenna fabricated on an FR4 substrate with the antenna size of 34.0 mm by 33.35 mm . The measured
$\left|S_{11}\right|$ is obtained using an Agilent N5230A vector network analyzer (VNA). Figure 4 shows the curves of measured and simulated $\left|S_{11}\right|$. A reasonable agreement between them is observed. The impedance bandwidth of the simulated $\left|S_{11}\right|$ is $8.04 \mathrm{GHz}(2.94-10.98 \mathrm{GHz}$ below -10 dB ).

To compare the performance of the proposed antenna with that of the UWB antenna with a uniform shape of the radiator, a parametric study was performed in this study. The proposed antenna has five different widths, W1 to W 5 , on its radiator as shown in Table 2. The proposed antenna is denoted A0 as shown in Fig. 5(a). The minimum value of W1 to W5 is W5 ( 11.29 mm ). The widths of the radiator are uniformed to W5, and the antenna is denoted as A1, which can be seen in Fig. 5(b). The lengths, L1 to L5, are fixed as those of A0. The other two antennas, A2 and A3, can be done by the similar way. The antenna, A2, has a uniform width W 4 ( 16.0 mm , the maximum value of W 1 to W 5 ) of its radiator as shown in Fig. 5(c). The antenna, A3, has a uniform width ( 13.91 mm , the mean of W1 to W5) of its radiator as shown in Fig. 5(d).
Figure 6 shows the simulated reflection coefficients $\left|S_{11}\right|$ of A0 to A3. Poor $\left|S_{11}\right|$ of A1 and A2 can be found. After comparing A0 to A3, the results show that the two curves are almost the same below 4.0 GHz ; whereas, the $\left|S_{11}\right|$ of A0 is better than that of A3 in the frequencies between 4.0 GHz and 10.5 GHz . However, the A3 has a higher cut-off frequency (below -10 dB ) at 11.28 GHz.


Fig. 6. The comparison of the antennas, A0 to A3, on simulated reflection coefficients $\left|S_{11}\right|$.


Fig. 7. The current distributions of the proposed UWB antennas at (a) 3.1 GHz , (b) 5.5 GHz , (c) 8.0 GHz , and (d) 10.6 GHz . The figures on the left-hand side and the right-hand side are magnitudes and directions, respectively.

Figure 7 shows the magnitudes and directions of current distributions of the proposed UWB antennas at $3.1,5.5,8.0$, and 10.6 GHz , respectively. The current distributions are kept symmetrical about the $z$-axis at all frequencies. The current densities are much stronger near the edges of the ground plane and the radiator. The currents on the irregular edges of the radiator increase the path length of the currents. It makes the antenna size reduction. Moreover, the irregular

(a) 3.1 GHz

(b) 5.5 GHz

(c) 8.0 GHz

Fig. 8. Simulated radiation patterns (normalized) of the proposed antenna at (a) 3.1 GHz , (b) 5.5 GHz , and (c) 8.0 GHz .
edges affect the impedance matching especially at higher frequencies as the results also can be confirmed and shown in Fig. 6. Figure 8 shows normalized $E_{\theta}$ and $E_{\phi}$ in the x-y and x-z planes at frequencies of $3.1 \mathrm{GHz}, 5.5 \mathrm{GHz}$, and 8.0 GHz . The $E_{\theta}$ is near omni- irectional in the x-y plane at these three frequencies. The cross-polar component $E_{\phi}$ is very small (less than -40 dB ) in the $x-z$ plane. The results indicate that the proposed antenna is a good candidate for UWB applications. Figure 9 shows the measured maximum gain of the proposed antenna. The maximum gain increases as the frequency increases. The gain is 1.82 dBi at 3.0 GHz ; whereas, the gain is 4.92 dBi at 11.0 GHz .


Fig. 9. The measured maximum gain of the proposed antenna.

To measure the dispersion and linearity of the proposed antenna, two identical proposed UWB antennas are placed in a face to face orientation (co-polarized direction) as shown in Fig. 10 (a). The two antennas are connected to port 1 and port 2 of the VNA (Agilent N5230A), respectively and fed by the low power 0 dBm coming from the two VNA ports. The two antennas are separated at a distance of 50 mm from each other. This short distance ensures a pure transmission relationship between two transmitted and received antennas. Other factors interfering with the transmission can be viewed as minor. Under this placement, the group delay was measured for the property of dispersion and measured the phase of the $\left|S_{21}\right|$ for the property of linearity. Figure 10 (a) shows the measured group delay of the proposed antenna. The variation of the group delay is less than 1 ns in the UWB spectrum. The result shows that a pulse distortion does not occur and a good linearity can be ensured in far-field region [7]. Figure 10 (b) shows the measured phase of $\left|S_{21}\right|$. A good linearity of the $\left|S_{21}\right|$ phase is observed between 3.0 GHz and 8.5 GHz . If the frequencies are larger than 8.5 GHz , the variation of the $\left|S_{21}\right|$ phase becomes large. The loss becomes larger in the higher frequencies due to using a high loss FR4 substrate in this study such that the linearity of the $\left|S_{21}\right|$ phase becomes a little poor. However, the proposed UWB antenna still has good properties of dispersion and linearity in the band of interest.


Fig. 10. Measured results of the proposed antenna, (a) group delay and (b) $\left|S_{21}\right|$ phase.

## VI. CONCLUSION

This study presents a planar ultra-wideband antenna with an irregular shape on its radiator for UWB applications. The particle swarm optimization combined with the finite element method based simulator, HFSS to design and optimize the proposed antenna. The antenna design process is automated and better results can be obtained efficiently. The proposed approach successfully obtains the desired goal. The antenna has a compact substrate size of 34.0 mm by 33.35 mm . Optimization results indicate that the proposed antenna has good characteristics of impedance bandwidth and radiation pattern in the band of interest. The parametric study of the proposed antenna shows that the optimized irregular shape of the proposed antenna
outperforms the antennas with uniform rectangular shapes in impedance matching in the UWB spectrum. The current distributions have been used to interpret the antenna characteristics. Measured results of the maximum gain, group delay, and phase of $\left|S_{21}\right|$ demonstrate good transmission properties of the proposed antenna. To sum up, the proposed antenna is suitable for UWB applications.

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# A Novel Harmonic Suppression Antenna with Both Compact Size and Wide Bandwidth 

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#### Abstract

A novel CPW-fed harmonic suppression antenna with both compact size and wide bandwidth is proposed and investigated in this paper. The second and third harmonic frequencies are suppressed effectively with square-shaped lattices inserted in the ground plane. Furthermore, by exploiting the Vivaldishaped structure and two simple rectangular slots, wide bandwidth ( 1.86 GHz ) ranging from 1.9 GHz to 3.76 GHz is also obtained with rather compact size of $10 \mathrm{~mm} \times 10 \mathrm{~mm}$, which is much smaller than that have ever been reported before. Eventually, the measured results confirmed the simulated dates.


Index Terms - Compact size, harmonic suppression, PBG, Vivaldi shaped structure, wide bandwidth.

## I. INTRODUCTION

The harmonic suppressed antenna (HSA), due to its great advantages such as low cost, small size, and easy integration, has become more and more attractive for various applications, such as wireless communications and microwave power transmission (MPT) [1-4]. Usually, a harmonic suppression filter will be required in those systems to avoid the harmonic influence on the overall system performance. However, it is bulky, expensive and hard to be integrated in monolithic microwave integrated (MMIC) devices. To overcome these defects, HSAs have been widely investigated these years: shorting pin [5], tuning stub [6], defected ground plane structure (DGS) [7], photonic band gap (PBG) [8], and the electromagnetic band gap structure (EBG) [9] can
all suppress the harmonic effectively. Besides, considerable research efforts have been made towards the miniaturization and broadband of HSAs. In [10], by inserting symmetrical slots in the ground plane of an open-ended CPW-fed transmission line, fairly compact dimensions, $26 \mathrm{~mm} \times 15 \mathrm{~mm}$ in physical size , and $0.3 \lambda_{g} \times$ $0.17 \lambda_{g}\left(\lambda_{g}\right.$ is the wavelength in the substrate at the resonant frequency) in electrical size was obtained. In [11], a broadband HSA, with a $10-\mathrm{dB}$ impedance bandwidth spanning from 1.56 GHz to $2.88 \mathrm{GHz}(1.32 \mathrm{GHz})$, was achieved by exploiting PBG structures with cross-shaped lattices. However, the compact HSA suffers a quite narrow bandwidth of 200 MHz , and the broadband HSA is very limited by its rather big size of about $50 \mathrm{~mm} \times 60 \mathrm{~mm}$.

In this paper, a novel harmonic suppression antenna with both compact size and wide bandwidth is presented. To suppress harmonics, PBG structures with square-shaped lattices were exploited. On the other hand, by employing the Vivaldi-shaped structure and two rectangular slots, a rather compact and small dimensions, $10 \mathrm{~mm} \times$ 10 mm in physical size, and $0.1 \lambda_{g} \times 0.1 \lambda_{g}$ in electrical size, is achieved with a wide bandwidth of 1.86 GHz spanning from 1.9 GHz to 3.76 GHz . Eventually, the experiment results agree well with the simulated dates.

## II. ANTENNA DESIGN

The existing planar microstrip antennas are mostly in symmetrical structures, especially the printed monopoles antennas [12-13]. For those symmetrical structures their half formats have all the necessary dimension features for the resonant


Fig. 1. Geometry of the proposed HSA.
frequencies. Therefore, it is expected that the miniaturized half structures of these symmetrical printed monopole antennas can achieve similar impedance features as the un-miniaturized full structures. Particularly, by simply exploiting its structural symmetry, $40 \%$ reduction in size was realized in [14]. Accordingly, similar but modified technology is applied to reduce the total size of the proposed harmonic suppression antenna at the beginning design, and then the PBG structure is used to suppress the harmonic frequencies.

Figure 1 shows the geometry of the proposed compact HSA with wide bandwidth. This antenna is printed on the RO4003C substrate with size of $10 \mathrm{~mm} \times 10 \mathrm{~mm}$, thickness of 0.5 mm , and dielectric constant of 3.38 . The width of the coplanar waveguide feeding line $w_{f}$ is fixed at 1.6 mm and the gap $d_{0}$ is fixed at 0.1 mm to achieve $50 \Omega$ characteristic impedance.

Half structures of a Vivaldi-shaped printed monopole antenna, which is formed by the index gradient curve, are exploited at first. In order to further reduce the antenna size, a rectangular ground plane with a rectangular slot was added to the left side of the coplanar waveguide feeding line. Moreover, another rectangular sot was etched in the right ground plane. On the other hand, to suppress the harmonic frequencies, PBG structures with square-shaped lattices were inserted only in the left ground plane. Particularly,
the two rectangular slots, which can control the strength of coupling and guide the current distribution, are critical to the design and need to be carefully adjusted. At last, a fairly compact size and harmonic suppression as well as wide bandwidth are obtained at the same time.

## III. RESULATS AND DISCUSSION

The proposed harmonic suppression antenna with both compact size and wide bandwidth are constructed, and the numerical and experimental results are presented and discussed. The parameters of this proposed antenna are studied by changing one parameter at a time and fixing the others. To fully understand the behavior of the antenna's structure and to determine the optimum parameters, the antenna was analyzed using the business software CST. And the prototype of the proposed antenna with optimal design, i.e. $a=2 \mathrm{~mm}, \quad s=0.3 \mathrm{~mm}, \quad w_{s}=0.5 \mathrm{~mm}, \quad d=5 \mathrm{~mm}$, $d_{1}=4.5 \mathrm{~mm}, \quad L_{\mathrm{s}}=6 \mathrm{~mm}, \quad b=1 \mathrm{~mm}, \quad r_{0}=0.1 \mathrm{~mm}$, $r=0.4 \mathrm{~mm}$, is shown in Fig. 2.


Fig. 2. Prototype of the proposed HSA.
In order to minimize the physical size of the proposed antenna, a rectangular slot is introduced into the right ground plane to alter the input impedance characteristics. As illustrated in Fig. 3, the slot can effectively lengthen the equivalent electrical length and the resonant frequency goes lower when the length of the slot $L_{s}$ increases gradually.

Figure 4 exhibits the effects of the PBG on the impedance matching in comparison to the same antenna without PBG and three cases with PBG. It is found that the PBG structure successfully
suppressed the second and third harmonic frequencies. Furthermore, an extra resonant frequency is excited, thus a much wider impedance bandwidth is obtained. Also, from the results shown in Fig. 4, it can be observed that the parameter $b$ have only impact on the extra resonant frequency. As the parameter $b$ decreases from 1.1 mm to 0.9 mm gradually, the second resonant frequency increases correspondingly, while the first resonant frequency almost remains still.


Fig. 3. Simulated S11 curves for the proposed antenna with different length of $L s$.


Fig. 4. Simulated S11 curves for the proposed antenna without PBG and with different PBG dimension of $b$.

Similarly, the S11 curves for various distance $d_{1}$ are plotted in Fig. 5. It can be observed that the parameter $d_{1}$ also mainly affects the second resonant frequency. And as d1 increases from 3.5 mm to 4.5 mm , the second resonant frequency
shifts left. However, as shown in Fig. 6, the width $w_{s}$ not only has impact on the second frequency but also the first one. The first resonant frequency decreases when the width $w_{s}$ increases, while the second resonant frequency increases.


Fig. 5. Simulated S11 curves for various distance $d_{1}$.


Fig. 6. Simulated S11 curves for various width $w_{s}$.

Eventually, the simulated and measured S11 of the proposed harmonic suppression antenna with optimized design are demonstrated in Fig. 7. Because FEKO is mostly used for large electrical size problems and the proposed antenna is very small, only CST, HFSS as well as IE3D were used to validate the results. Compared with the HFSS solver which is based on frequency domain, the transient solver of CST is based on time domain; thus, it is more suitable for wideband simulation cases. Therefore, the simulated results by the CST are quite similar to the measured results. Besides, the IE3D is based on the MOM algorithm, and it's
more suitable for planar structures. From the results in Fig. 7, it can be seen that the three results are similar and the -10 dB impedance bandwidth of the proposed HSA is ranging from 1.9 GHz to 3.76 GHz , which is almost the maximum bandwidth since the lowest second harmonic frequency is 3.8 GHz . The experiment results validate the design with only a slight shift. Furthermore, it is also noticed that the antenna is only $10 \mathrm{~mm} \times 10 \mathrm{~mm}$ in physical size and $0.11 \lambda \mathrm{~g}$ $\times 0.11 \lambda_{g}$ in electrical size, which is much smaller than that have ever been reported before. Compared with the above mentioned compact HSA and the broadband HSA, this compact broadband harmonic suppression antenna proposed in this paper offers an even smaller size and broader bandwidth, e.g. about $76 \%$ and $96 \%$ reduction in size, $830 \%$ and $41 \%$ enhancement in bandwidth, respectively.


Fig. 7. The simulated and measured S11 for the proposed HSA with PBG structure.

Furthermore, as a very important aspect of a practical antenna, the efficiency of the proposed antenna is also investigated. The efficiency of the proposed HSA with PBG structure is shown in Fig. 8. And the efficiency of the reference antenna without the PBG structure is also shown for comparison. It can be observed that the reference antenna without the PBG structure is below $50 \%$ in the bandwidth since the total antenna size is rather small. Moreover, the suppression of the harmonics affects the antenna efficiency a lot. In the working band from 1.9 GHz to 3.76 GHz , the antenna efficiency almost remains the same. However, compared with the antenna without harmonics suppression, the antenna efficiency
with harmonics suppression goes much lower in the harmonics bands.


Fig. 8. The efficiency of the proposed HSA with PBG structure and the reference antenna without PBG structure.

Additionally, the radiation patterns of the proposed HSA with PBG structure and the reference antenna without PBG structure at 3GHz, 6 GHz , and 9 GHz are shown in Fig. 9 (a), (b), and (c), respectively. It illustrates that compared with the reference antenna, the proposed HSA could suppress the second and the third harmonics without deteriorating the fundamental radiation patterns.

## IV. CONCLUSION

In this paper, a novel CPW-fed antenna with harmonic suppression is proposed and investigated. Both compact size and wide bandwidth are achieved by employing both the Vivaldi-shaped and PBG structures along with two simple rectangular slots. Particularly, it exhibits a smallest dimension up to now, $10 \mathrm{~mm} \times 10 \mathrm{~mm}$ in physical size, and $0.11 \lambda_{g} \times 0.11 \lambda_{g}$ in electrical size, with a rather wide bandwidth spanning from 1.9 GHz to 3.76 GHz . With all these promising features, the proposed HSA is very attractive for wireless communication and microwave power transmission applications.

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Fig. 9. The radiation patterns of the proposed HSA with PBG structure and the reference antenna without PBG structure at a) 3 GHz , b) 6 GHz , and c) 9GHz.

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# A Novel Circular Disc Monopole Antenna for Dual-Band WLAN Applications 

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#### Abstract

This paper presents a novel and simple circular disc monopole antenna with a tunable H shaped ground plane which provides two separate resonant modes to achieve 2.4 - and $5.5-\mathrm{GHz}$ dualband operation for wireless local area network (WLAN) applications. The proposed antenna is printed on a low cost substrate FR4 ( $\varepsilon_{r}=4.4$ ) and excited using a $50 \Omega$ microstrip transmission line. Lower resonant mode of the antenna has an impedance bandwidth ( $\left|\mathrm{S}_{11}\right|<-10 \mathrm{~dB}$ ) of 700 MHz ( $2250-2950 \mathrm{MHz}$ ), and the upper resonant mode has a bandwidth of $1100 \mathrm{MHz}(4900-6000 \mathrm{MHz})$. For validation purposes, an antenna prototype was fabricated and tested. The measured and simulated results show relatively good agreement and confirm satisfactory radiation characteristics. Effects of varying the widths of the arms of the Hshaped ground plane on antenna performance have also been studied.


Index Terms - Dual-band, H-shaped ground plane, monopole antenna, WLAN.

## I. INTRODUCTION

During the recent years, there are rapid developments in wireless local area network (WLAN) applications. In order to satisfy the 2.45 $\mathrm{GHz}(2400-2483 \mathrm{MHz})$ band of IEEE $802.11 \mathrm{~b} / \mathrm{g}$, $5.2 \mathrm{GHz}(5150-5350 \mathrm{MHz}$ ), and 5.8 GHz ( $5725-$ 5825 MHz ) band of 802.11a WLAN standard, antennas with enhanced dual- or multi-frequency capabilities to meet the IEEE 802.11 WLAN standards in the $2.45 / 5.2 / 5.8 \mathrm{GHz}$ operating bands, are in demand for practical WLAN applications. Printed monopole antennas are good candidates owing to their attractive features, e.g. simple
structures, low cost, low profile, light weight, omni-direction radiation patterns, easy realization, and convenience for integrating with microwave monolithic integrated circuit (MMIC) technologies.

Various kinds of printed monopole antennas, such as a microstrip fed printed double-T monopole antenna [1] and antennas with double Lslits [2], G-shaped radiating strip [3], semicircular and rectangular patch with H-shaped ground [4], and other antennas [5-9] have been reported to achieve dual band characteristics, which cover the $2.4-$ and $5.2-\mathrm{GHz}$ WLAN bands. For the available designs, the antennas reported in [1, 4] can not achieve a dual-band response with sufficiently large bandwidth to cover the $2.45 / 5.5 \mathrm{GHz}$ (24002483/ $5150-5825 \mathrm{MHz}$ ) bands. The antenna reported in [2] does not satisfy low-profile configurations. Other antennas mentioned in [3-9] are either large in antenna size or complex in antenna structure. Furthermore, the antennas in [10-12] require a shorting pin for the ground connection inside the printed monopole antenna element. This increases the design complexity as well as the fabrication cost. The antennas mentioned in [13, 14] provide a method to obtain broad bandwidth with a printed circular disc patch, but they are not suitable for WLAN applications. It noted that the longer strip controls the lower band of the antenna, while the shorter strip and the parasitic strip together generate the wide operating band for the upper resonance, so a popular method to obtain broad bandwidth is to use parasitic patches and short the parasitic or the monopole patches. As known, the antenna resonant frequencies depend on more parameters like slot
width, length etc, and thus the complexities of simulation, optimization, and fabrication for antenna increase by adding the parasitic patches.

Based on the background of the researches above, this paper proposes a novel microstrip-fed dual-band monopole antenna with simple structure, small size, and tuneable central frequency ratios. The proposed antenna is comprised of a circular disc patch and an Hshaped ground, it can be easily excited by a $50 \Omega$ microstrip line. By tuning the widths of the arms of the ground plane, the central frequency ratios of the two operating frequencies can be adjusted. This means that the H -shaped ground plane can be a universal structure to design the dual-band printed monopole antenna which can satisfy different demands. Good impedance matching can be obtained for operating frequencies in the $2.45 / 5.2 / 5.8 \mathrm{GHz}$ bands. The three-dimensional (3D) EM simulator HFSS (based on the finite element method) is used. In this paper, $50 \Omega$ wave port, ideal radiation boundary, solution frequency of 2.45 and 5.5 GHz is adopted) is used for design simulation. Design details of the proposed antenna are described, and results of impedance bandwidth, the radiation characteristics and gains are given and discussed in this paper.

This paper is organized as follows. The proposed antenna structure is briefly described in Section II. The measured and simulated results are discussed in Section III. Finally, the implications and significance of this work are briefly discussed in Section IV.

## II. THE PROPOSED ANTENNA STRUCTURE AND DESIGN

Shown in Fig. 1 (a) is the schematic of the proposed antenna. The antenna with a radius of $R$ and a $50 \Omega$ microstrip feed line are printed on the same side of the dielectric substrate (in this paper, the FR4 substrate of thickness 1.6 mm and relative permittivity 4.4 was used as a lossless dielectric model). $L$ and $W$ denote the length and the width of the dielectric substrate, respectively. The width of the microstrip feed line is fixed at $W_{1}=3 \mathrm{~mm}$ to achieve $50 \Omega$ impedance. $d$ is the distance of the feed gap between the feed point and the ground plane. On the other side of the substrate, the conducting H-shaped ground plane only covers the section of the microstrip feed line. Shown in Fig. 1
(b) is the return loss of antennas with different patches, the same H-shaped ground plane and the same dielectric substrate. The bandwidth of antenna with circular disc patch is larger and better than that with rectangular patch. The antenna still produces dual-band with the different patches. As a result, a common method to obtain broad bandwidth is to use circular disc patch, and the major effect of the H -shaped ground plane is to make current paths of the two resonant frequencies different, and thus dual resonant modes are excited.

(a)

(b)

Fig. 1. (a) The configuration of the proposed monopole antenna. (b) The return loss of antennas with rectangular patch and circular disc patch.

In order to obtain the optimal size, the main parameters affecting the antenna bandwidth can be discussed in Fig. 2. The simulated results of three different lengths of $d=0.8,1.1$ and $1.4 \mathrm{~mm}, L_{2}=2$, 3 and $4 \mathrm{~mm}, R_{1}=8,10$ and 12 mm are presented in Fig. 2 (a), Fig. 2 (b), Fig. 2 (c), respectively, and
other dimensions of antenna are not changed. For the three cases in Fig. 2 (a), the upper band is narrow and not good enough when a larger or smaller $d$ is selected, and the lower band is narrow when the smaller $d$ is chosen. In Fig. 2 (b) and Fig. 2 (c) show that as the length of $L_{2}$ and $R_{1}$ increase, the upper band becomes larger and the lower band becomes smaller. As a result, the best size of antenna is presented in Table1.

(a)

(b)

(c)

Fig. 2. Simulated reflection coefficient for antennas with various (a) d, (b) $L_{2}$, (c) $R_{1}$.

Table 1: Nominal design parameters of the proposed antenna

| Design parameters | Value (mm) |
| :---: | :---: |
| $R$ | 10 |
| $L$ | 40 |
| $l_{1}$ | 8 |
| $l_{2}$ | 3 |
| $I_{3}$ | 3 |
| $d$ | 1.1 |
| $W$ | 40 |
| $W_{1}$ | 3 |
| $W_{2}$ | 12 |
| $W_{3}$ | 26 |

## II. RESULTS AND ANALYSIS

## A. Radiation characteristic

To investigate the electromagnetic characteristics of the proposed antenna, the excited surface current distributions of the H-shaped ground at the two central resonant frequency 2.45 and 5.5 GHz are simulated, and the sketch is drawn in Fig. 3. It is shown that when the antenna is excited with the same phases and magnitudes, the excited surface current of the lower resonant mode of the antenna is strong in the larger arm of the H -shaped ground plane and weak in the upper arm of the H -shaped ground plane. On the contrary, the excited surface current of the higher resonant mode of the antenna is strong in the upper arm of the H -shaped ground plane and weak in the larger arm of the H -shaped ground plane, and thus the width $\left(W_{3}\right)$ of the upper arm of the H-shaped ground plane controls the second or upper operating band, the width ( $W$ ) of the larger arm of the H -shaped ground plane controls the first or lower operating band. WLAN bands can be achieved by adjusting the widths of the arms of the H -shaped ground.

To further validate the basic principle of the proposed antenna, a practical example illustrated in Fig. 4 is presented and examined using both simulations and prototype measurements. The dimensions of the antenna are given in Table 1.

The simulated and measured reflection coefficient (dB) of the dual-band antenna is shown in Fig. 5. It is clearly seen that two wide operating bandwidths are obtained, low-frequency bandwidth of the measurement is larger than that of the simulation and high-frequency bandwidth of
the measurement is equal to that of the simulation. The measured lower band achieves a -10 dB impedance bandwidth of $26.9 \%$, ranging from 2.25 GHz to 2.95 GHz , with respecting to the central frequency at 2.6 GHz , and the measured bandwidth for the upper mode reaches 1.1 GHz ( $4.9-6.0 \mathrm{GHz}$ ), or about $20.2 \%$, referred to the central frequency at 5.45 GHz . Obviously, the antenna can operate over the bands which cover the required bandwidths of the IEEE 802.11 WLAN standards in the band at 2.45 GHz (24002484 MHz ), $5.2 \mathrm{GHz}(5150-5350 \mathrm{MHz}$ ), and 5.8 GHz ( $5725-5825 \mathrm{MHz}$ ). Although there are some discrepancies between the simulated and measured results, which is probably due to the accuracy issues in making the prototype antenna as well as simulation or measurement errors and the stability of dielectric material, the agreement between the measured data and the simulated results is fairly good.

The simulated radiation patterns of the antenna at the central frequencies of the two WLAN bands are shown in Fig. 6 and Fig. 7, it is seen that the obtained radiation patterns are not as good as those of a conventional ideal monopole antenna, which has a good omni-directional pattern, they are more directional. For the E-plane (xz-plane), the radiation patterns of the proposed antenna are similar at $2.45 / 5.25 / 5.75 \mathrm{GHz}$. For the H -plane (yz-plane), the radiation pattern at 2.45 GHz can also be seen to be more directional than those at $5.25 / 5.75 \mathrm{GHz}$, the radiation pattern at $5.25 / 5.75 \mathrm{GHz}$ has a good omni-direction.

Table 2 shows the antenna gains at lower operating band. In the 2.4 GHz band, the peak gain is 4.8 dBi at $2.8 \mathrm{GHz}, 3.5 \mathrm{dBi}$ at 2.4 GHz , and 1.4 dB of gain variation observed. Table 3 shows the antenna gain at upper operating band. In the 5.5 GHz band, the peak gain reaches 4.1 dBi at 6 GHz , 2.9 dBi at 5.2 GHz , and 4.0 dBi at 5.8 GHz , gain variation is less than 1.5 dB . Gain tends to be a constant in the whole band.

Table 2: Realized gain of lower operating band

| Frequency (GHz) | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Gain $(\mathrm{dBi})$ | 3.4 | 3.5 | 4.3 | 4.8 | 4.6 |

Table 3: Realized gain of upper operating band

| Frequency (GHz) | 4.9 | 5.2 | 5.5 | 5.8 | 6.0 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Gain $(\mathrm{dBi})$ | 3.1 | 2.9 | 2.8 | 4.0 | 4.1 |



Fig. 3. Simulated surface current distributions of the H -shaped ground plane: (a) at 2.45 GHz , (b) at 5.5 GHz.


Fig. 4. The photo and configuration of the proposed monopole antenna.

Shown in Fig. 8 is the radiation efficiency for antenna, it can be seen that the radiation efficiency reaches more than $70 \%$ at lower operating band, and it is above $75 \%$ at upper operating band. In the non-operating band, the radiation efficiency for antenna is below $60 \%$. In the whole band, the antenna obtains a good radiation characteristic.


Fig. 5. Reflection coefficient of the antenna.

(a)

(b)

(c)

Fig. 6. E-plane radiation patterns of the proposed antenna. (a) 2.45 GHz , (b) 5.25 GHz , (c) 5.75 GHz.

(a)

(b)


Fig. 7. H-plane radiation patterns of the proposed antenna. (a) 2.45 GHz , (b) 5.25 GHz , (c) 5.75 GHz.


Fig. 8. Simulated radiation efficiency for antenna.

## B. Effects of varying the widths of the arms of the ground plane

The parameters effect on the antenna's resonant frequency and impedance bandwidth are studied.

Figure 9 shows that the central frequency of the higher resonant mode decreases with the increase of the width of the upper arm of the ground plane, while that of the lower resonant mode is nearly unchanged, and both of the impedance bandwidths are almost unchanged. Figure 10 shows that the central frequency of the lower resonant mode decreases with the increase of the width of the larger arm of the ground plane, while that of the higher resonant mode is nearly unchanged, and both of the impedance bandwidths are almost unchanged. This means that by varying the widths of the arms of the H -shaped ground plane, the central frequency ratio of the proposed
antenna can be adjusted. Making a compromise between size and bandwidths of the antenna, the final parameters are shown in Table 1.


Fig. 9. Simulated reflection coefficient for antennas with various $W_{3}$.


Fig. 10. Simulated reflection coefficient for antennas with various $W$.

As a result, one of the advantages of the proposed antenna is that it can meet the different bands for communication equipments by changing the widths of the arms of the H -shaped ground plane without changing other parameters.

## IV. CONCLUSION

A novel dual-band monopole antenna excited by microstrip has been demonstrated in this paper. The proposed antenna has a simple structure. Moreover, by adjusting the widths of the arms of the H -shaped ground plane, the central frequency ratio of the antenna can be tuned so that the H shaped ground plane can be used for designing the dual-band printed monopole antennas in order to satisfy different communication needs. After
optimizing the geometric parameters of the antenna, the proposed antenna can provide two discrete operating bands, which cover both 2.45 $\mathrm{GHz}(2.4-2.484 \mathrm{GHz}$ ) and $5 \mathrm{GHz}(5.15-5.825$ GHz ) bands. The final measured results show satisfactory performance. Because the measured and simulated results are found to be in good agreement, the proposed antenna has been successfully designed for use in dual-band WLAN applications.

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# Electric Field Distribution in a Biological Cell for Various Electrode Configurations-A Simulation Study 

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#### Abstract

Electroporation or electropermeabilization (EP) is a non-viral physical process of inducing and/or increasing permeability of biological membranes by the application of high intensity and short duration electrical pulses. The resulting high field strength in the membrane can lead to the formation of regions of increased permeability, often called pores that allow transmembrane transport of macromolecules, such as DNA and chemo drugs. The efficacy of EP depends on a number of parameters, such as electric field strength, duration, number of pulses, size of target cell, and type of drug or DNA to be fed, to mention a few parameters. While electric field strength, duration, and number of pulses are the dominant parameters for the electroporation effect, the electric field distribution also depends on the shape, the size, and the material of electrodes used. The various electrodes used in practice are the parallel plate electrodes, the needle electrodes, the needle array electrodes, and the caliper electrodes. Both six needle array and parallel plate electrodes were used in the skin cancer trials. The electric field distribution varies with the type of electrode. The choice of electrodes depends upon the type of application. For example, parallel plate electrodes produce good results in human clinical trials and the distance between the electrodes can be easily varied. Needle electrodes are the simplest, but due to their configuration, the field distribution is highly nonlinear; there is higher field strength around the needle tip. That is one of the reasons for using multiple needles or needle arrays to make the field distribution as uniform as possible. It is of practical interest to study the electric field distribution of various electrodes to make a more informed choice of which electrodes to use. This


paper presents the results of a study of the electric field distribution in biological tissues for various electrode configurations. Magwel was used in this study. For the same voltage applied, the triangular and arc electrode configuration developed the highest electric field strength. Specifically, we develop simulation results that enhance or enable the acquisition of information from cells.

Index Terms - Cell culture, dielectrophoresis, electrodes, HeLa cell.

## I. INTRODUCTION

Electroporation is the physical process of inducing transient permeability of biological membranes by short pulses of electric fields [1-3]. The resulting high field strength in the membrane can lead to the formation of areas of increased permeability, often called pores, which allow transmembrane transport of molecules. This effect has been used in the laboratory for more than a decade as a research tool to facilitate cellular uptake of genetic material in vitro. More recently, electroporation has also been found effective for the intracellular delivery of molecules in living tissues, which led to a variety of medical applications. Some of these applications have already proceeded to clinical trials. The effectiveness of this technique is evidenced by the Phase I and II skin cancer clinical trials [3]. The most important parameters for effective electroporation are the electrical field strength and the length of time the field applied (pulse length) [4, 5]. A number of other parameters can affect the efficacy of electroporation, such as the shape of the electrical pulse, polarity, number of and interval between pulses, size of target cells, and thermal conditions during and after the pulse, as
well as other cellular and environmental factors [4-7]. Electric field in the tissue is generated by a potential difference (voltage) between electrodes surrounding, or adjacent to, the tissue. A wide variety of electrode configurations have been developed, appropriate for each specific therapeutic goal [8, 9].

## II. NEED FOR DIFFERENT CONFIGURATIONS OF ELECTRODES

The role of the electrodes is to act as a conduit to transform the voltage pulse from a pulse generator into local electric fields in tissues. The most commonly used electrodes are parallel plate electrodes [3, 8, and 9] and needle electrodeseither a six needle array [3, 8-10] or two needles [11] are used [12].
(a) Parallel plate electrodes. It's the simplest configuration and it produces good results in human clinical trials with tumors close to the surface due to the uniform electric field it develops [8]. With parallel plate electrodes, it is very easy to adjust the distance between them to obtain the desired field strength. However, plate electrodes are less efficacious for deeper tumors, where needle electrodes are more useful [8].
(b) Needle Electrodes. A pair of needles is also a very simple system [8, 11]. However, due to the needle configuration, the field of single-pairneedle electrodes is highly divergent, with high field strength near the needle tip. That is one of the reasons why multiple needles or needle arrays are needed to develop a more even electric field distribution [8]. Apparently, different cross sections of the electrodes will affect the electric field distribution. In this paper, the electric field distributions due to various electrode configurations are studied using Magwel [13].

## III. ELECTRODES

To manipulate cells, one must create an electric field of a certain shape and strength. The two approaches to doing this are to either integrate electrodes within the microsystem or have them be external [14].

## A. Internal electrodes

Internal electrodes, uncommon in EP systems, are the predominant approach to generating electric fields for dielectrophoresis (DEP), mainly
because of the superior control they offer for locally shaping the field. Electrodes act as equipotential surfaces that shape the field by controlling their location and voltage. Above, we saw that reducing the length of the electrodes favorably affected manipulation forces, and the simplest way to do this is to place electrodes close together. This necessitates electrodes that are internal to the system. The primary consideration that arises when using internal electrodes is that the electrodes can adversely interact with the electrolyte via either gas generation or corrosion. Because current is carried in metals by electrons and in electrolytes by ions, electrochemistry must occur at the electrode-electrolyte interface to transform the electron current into an ion current. The net result can be the production of gas $\left(\mathrm{H}_{2}\right.$, $\mathrm{O}_{2}$, etc.) or dissolution of the electrode, both of which can disrupt operation of the device. This is a serious concern at dc, explaining the relative absence of EP systems that use internal electrodes. For DEP systems, electrochemical effects are typically avoided by operating at $>10$ 's kHz in saline (lower frequencies can be tolerated in liquids of lower ionic strength). Gas generation is also voltage dependent, so higher voltages are more likely to lead to deleterious effects. Given that one usually wishes to operate at a high frequency ( $>100$ 's kHz ) when using DEP with cells, electrochemistry does not pose any practical limitations.

## B. External electrodes

The alternate approach is to use external electrodes. Very common in EP systems, external electrodes are rare in DEP systems. One advantage of using external electrodes is that they do not have to be fabricated with the device, resulting in a simpler fabrication process. The electrodes themselves often consist of platinum wires that are inserted into the port holes of the device. The electric field in systems that use external electrodes is shaped by using a straight channel (for EP) or inserting low polarizability obstructions (such as glass or polymer posts [15, 16]) in the channel (for DEP). There is no theoretical limit to the electric fields and forces that can be generated using external electrodes, although there are practical limits. Higher overall fields (such as for EP) are created by increasing the voltage or decreasing the spacing between the
obstructions, limited by the fact that openings cannot be made any smaller than the cells. Additionally, the fields exist throughout the volume of the electrolyte, which can lead to significant heating. Thus, published DEP devices for use with cells have typically used very-low conductivity water to minimize heating [15, 16]. Finally, fairly large voltages (approx. $0.1 \sim 1 \mathrm{kV}$ ) are needed to generate the required fields, and thus, these devices are limited to dc or lowfrequency operation. This is fine for EP, which operates at dc, but restricts DEP manipulations to only the low frequency region of the ClausiusMossotti (CM) factor. The need to minimize solution conductivity, operate at or near dc, and minimize chamber volume make external electrodes best suited for use with robust cellssuch as bacteria-that can tolerate being placed in low-conductivity buffers or as endpoint analysis on cells that are not needed for downstream use.

## C. Effects of electric field on cell

Because electrical cell manipulation exposes cells to strong electric fields, one needs to know how these electric fields might affect cell physiology. Ideally, one would like to determine operating conditions that will not affect the cells and use those conditions to constrain the design. Of course, cells are poorly understood complex systems and thus it is impossible to know for certain that one is not perturbing the cell. However, all biological manipulations-cell culture, microscopy, flow cytometry, etc.-alter cell physiology. What is most important is to minimize known influences on cell phenotype and then use controls to account for unknown influences. The known influences of electric fields on cells can be split into the effects due to current flow, which causes heating, and direct interactions of the fields with the cell. We consider each of these in turn.

## IV. SIMULATION MODEL

We chose the multiphysics model (conductive and dielectric media dc \& electrostatics model) when using Magwel. The general model cell culture system with the HeLa cell is shown in Fig. 1. Tables 1 and 2 give the dimensions and other electrical data used in this simulation, respectively.


Fig. 1. 3D view of the simulation domain for the cell culture system.

Modeling of cell and medium: Figure 1 shows the schematic view of the cell culture system in which the Hela cell is considered as a monolayer of cells located at the bottom of the culture dish. The cell is modeled as a homogeneous sphere with a diameter of 35 mm and a thickness of $3 \mu \mathrm{~m}$. On the other hand, the culture medium is modeled as DMEM with 5 mm of height. Both the cell and the medium have their corresponding electrical parameters at different frequencies as listed in Table 1. The copper electrodes are used to apply electric field into the culture dish and are modeled as internal electrodes. The dimension and shape of the electrodes influence the generation of the E field in the vicinity of the electrodes which in turn induces field in the cell.

Table 1: Simulation parameters

| Simulation domain | $\begin{gathered} 50 \mathrm{~mm} \mathrm{X} \\ \mathrm{~mm} \end{gathered}$ |
| :---: | :---: |
| Culture dish | circular type flux, diameter= 35 mm , height= 9.5 mm |
| Cell culture medium | Dulbecco’s Modified Eagle Medium (DMEM), height=5mm |
| Cell | Circular, monolayer, diameter $=35 \mathrm{~mm}$, height $=3 \mu \mathrm{~m}$ |
| Electrodes (internal) | Copper, height $=15 \mathrm{~mm}$ ( with different shape) |
| Spacing between electrodes | 1 mm |
| Applied voltage | $\begin{gathered} 10 \mathrm{~V} \text { at } \mathrm{f}=100 \mathrm{kHz} \\ (\text { small signal }) \\ \hline \end{gathered}$ |

Table 2: Dielectric properties of the HeLa cell and culture medium [17, 18]

| Freque <br> ncy | Cell <br> E | Cell <br> $\sigma$ | Mediu <br> m |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 Hz | Medium <br> $\sigma .0 \mathrm{e}$ <br> 4 | 0.0001 | 80 | 1.6 |
| 100 Hz | 5.5 e <br> 4 | 0.0007 | 78 | 1.55 |
| 100 kH <br> z | 5 e 4 | 0.001 | 76 | 1.5 |
| 1 MHz | 1.5 e <br> 4 | 0.06 | 74 | 1.5 |

## A. Angular plate electrodes configuration

The copper electrodes are placed inclined (Vshaped) in the layer of cell and medium as shown in Fig. 2(a). The contact is facilitating the input bias to the electrodes. Here the left electrode has a 10 V dc while the right one is at ground state (O V ). A small signal is also considered to the analysis with ac with frequency of 100 kHz . The small gap ( 1 mm ) is maintained between the lower parts making angular orientation. Figures 2(b), 2(c), and 2(d) show the results with potential distribution, E field distribution, and magnetic vector potential, respectively. The higher electric
field intensity region is clearly observed at the smallest gap between the electrodes.

(a)

(c)

(b)

(d)


Fig. 2. Simulation model for the HeLa cell using inclined electrodes ( $\mathrm{x}-\mathrm{y}$ plane, at $\mathrm{y}=21553 \mu \mathrm{~m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$, at the position of cell layer marked with red arrow) (a) Cell layer view, (b) potential distribution, (c) spatial E field distribution, and (d) magnetic vector potential.

We have plotted the E field intensity in the x direction at $\mathrm{y}=21553 \mu \mathrm{~m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ (cell layer) as shown in Fig. 3. The sharp peak electric field strength at the gap of the electrodes is 64.5 $\mathrm{V} / \mathrm{m}$ which is the actual field strength obtained in the center of the electrodes. The nominal field strength (voltage divided by the distance between the two electrodes) is $10 \mathrm{kV} / \mathrm{m}$.

So, we can tell the actual field strength is much lower than the nominal value. This is true because field will be reduced inside the dielectric bodies as the cell having greater value of permittivity than the outside liquid medium. Two
other minor peaks are also observed at the both side of the main peak. Those are the interfacial field due to the conductor-dielectric interface.


Fig. 3. E field variation along x axis at $\mathrm{y}=21553$ $\mu \mathrm{m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ at $\mathrm{f}=100 \mathrm{kHz}$. $\mathrm{E}_{\text {max }}=64.5 \mathrm{~V} / \mathrm{m}$ using inclined electrodes as shown in model of Fig 2(a).

## B. Triangular electrodes configuration

This model is simulated using two triangular shape electrodes placed at the same gap in between the electrodes. The results of E field distribution are plotted with the function of x coordinates (micrometer). Figure 5 shows the E field variation along the x axis at $\mathrm{y}=26608 \mu \mathrm{~m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ at $\mathrm{f}=100 \mathrm{kHz}$. The maximum electric field is found to be $105 \mathrm{~V} / \mathrm{m}$ right at the gap of the electrodes. The sharp and pointed peak is found due to the symmetry in the discretization of electrodes.



Fig. 4. Simulation model for the HeLa cell using triangular electrodes (a) cell layer view, (b) potential distribution, (c) spatial E field distribution, and (d) magnetic vector potential.


Fig. 5. E field variation along x axis at $\mathrm{y}=26608$ $\mu \mathrm{m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ at $\mathrm{f}=100 \mathrm{kHz}$. $\mathrm{E}_{\max }=105 \mathrm{~V} / \mathrm{m}$ using inclined electrodes as shown in the model of Fig 4(a).

## C. Needle electrodes configuration

The electric field distribution is not smooth near to the gap of electrodes as in Fig 6(a). Because the field is highly non-uniform due to the geometry of electrodes and no linear discretization effect at the electrodes edges that can be seen from Figs. 6(b), 6(c), and 6(d).


Fig. 6. Simulation model for the HeLa cell using needle electrodes (a) cell layer view, (b) potential distribution, (c) spatial E field distribution, and (d) magnetic vector potential.

Figure 7 shows the variation of E field along the x axis at $\mathrm{y}=26045 \mu \mathrm{~m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ (cell layer) at $\mathrm{f}=100 \mathrm{kHz}$. The maximum E field is observed to be $96 \mathrm{~V} / \mathrm{m}$. The neighboring effect of the conductive electrodes and conductor-dielectric interface would produce such an effect which can be noticed from a small rise-up at both sides of the main peak.

## D. Parallel plate electrodes configuration

In this configuration model shown in Fig. 8(a) and the simulation results Figs 8(b), 8(c) and 8(d), the electric field distribution is found quite smooth and comparatively even at the entire gap of electrodes. Because the field is uniform due to the geometry of electrodes and also linear discretization is possible at the electrode edges. The high level electric field strength occurs between the cell and electrodes plates.
Figure 9 shows the variation of E field along x axis at $\mathrm{y}=20400 \mu \mathrm{~m}$ and $\mathrm{z}=1503 \mu \mathrm{~m}$ (cell layer) at $\mathrm{f}=100 \mathrm{kHz}$. The maximum E field is observed to
be $96 \mathrm{~V} / \mathrm{m}$. The neighboring effect of the conductive electrodes and conductor-dielectric interface produce small rise-up at the both side of the main peak.


Fig. 7. E field variation along x axis at $\mathrm{y}=26045$ $\mu \mathrm{m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ at $\mathrm{f}=100 \mathrm{kHz} . \mathrm{E}_{\max }=96 \mathrm{~V} / \mathrm{m}$ using needle electrodes as shown in model of Fig. 6(a).


Fig. 8. Simulation model for the HeLa cell using parallel electrodes (a) Cell layer view, (b) potential distribution, (c) spatial E field distribution, and (d) magnetic vector potential.


Fig. 9. E field variation along x axis at $\mathrm{y}=20400$ $\mu \mathrm{m}$ and $\mathrm{z}=1503 \mu \mathrm{~m}$ at $\mathrm{f}=100 \mathrm{kHz}$. $\mathrm{E}_{\max }=96 \mathrm{~V} / \mathrm{m}$ using parallel electrodes as shown in model of Fig 8(a).

## E. Arc electrodes configuration

The non-smooth electric field distribution is found at the entire gap of electrodes shown in model and their results in Figs. 10(a) and 10(b). Because the field is non-uniform due to the geometry of electrodes and linear discretization is difficult to achieve at the electrodes edges. Figure 11 shows the variation of E field along x axis at $\mathrm{y}=$ $26181 \mu \mathrm{~m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ (cell layer) at $\mathrm{f}=100$ kHz . The maximum E field is observed to be 134 V/m.


Fig. 10. Simulation model for the HeLa cell using inclined electrodes (a) cell layer view, (b) spatial E field distribution.


Fig. 11. E field variation along x axis at $\mathrm{y}=26181$ $\mu \mathrm{m}$ and $\mathrm{z}=1501 \mu \mathrm{~m}$ at $\mathrm{f}=100 \mathrm{kHz}$. $\mathrm{E}_{\max }=134 \mathrm{~V} / \mathrm{m}$.

Table 3 summarizes the results of the simulation of the HeLa cell model using different type and shape of the electrodes. The shape of the electrodes characterizes the linearity of the field distribution at the edges. The highest electric field strength due to highly non-uniformity occurs at the area closest to the sharp point. Domain discretization at the structure of interest is also important for the accurate results. The peak electric field exists only at the spacing (gap) of the electrodes. Hence, the location, shape, and orientation of the electrodes influence the overall contribution of the E field to the cell and medium. For the monolayer of the HeLa cell, the localized electric field as obtained from the above field distribution does not affect the entire layer of the cell. Thus, the average electric field should be more pronounced and effective for the polarization of the dielectric properties of the cell. In that case, the array of electrode could be a good choice where the uniform distribution is important. For the manipulation of small dielectric objects (cell), different types of array like interdigitated or polynomial electrodes array can also perform better than single electrode.

Table 3: Comparison of E fields with electrode's type and their orientations

| Electrode's <br> types and <br> orientation | Maximum E <br> field (V/m) | Uniformity |
| :---: | :---: | :---: |
| Flat and <br> angular | 64.5 | Non uniform |
| Triangular | 105 | Non uniform |
| Rod (pin) | 96 | Non uniform |
| Flat and <br> parallel | 96 | Uniform |
| Arc | 134 | Non uniform |

## V. CONCLUSION

The simulation of a monolayer of HeLa cell is presented here with the different types, shape, and orientation of electrodes. The electric field distribution for electroporation applications strongly depends on the electrode configurations. Parallel plate electrodes and needle electrodes of various dimensions are most commonly used for preclinical and clinical applications [3, 8-10]. Parallel plate electrodes give the most uniform electric field distribution with the simplest configuration as seen in this simulation and this correlates well with the published data [8]. Needle electrodes need to be used for deeply-seated tumors where parallel plate electrodes cannot be used. Due to the sharp needle point, needle electrodes give non-uniform field distribution with the highest field strength near the tip of the needle as the field homogeneity depends on the shape of the electrodes [19-21]. This point has to be taken into consideration during electroporation applications since the electric field in the tissue will be different for a given area and hence the efficacy of the technique will vary.

The cell and the medium are equivalently characterized by their electrical constants at certain frequency. As the medium is considered to be dispersive, the results are shown to be at certain frequency namely 100 kHz . It was found that up to 100 MHz , the results does not differ significantly as the size of the cell is much less than the wavelength (low frequency problem). The work also demonstrates the proper choice of the electrodes would be significant for the cell manipulation and cell culture.

The presented ideas facilitate the dielectrophoresis as a building block for lab-on-achip devices which can be easily fabricated using the existing microelectrode photolithography techniques. The design of the electrodes, the choice of the suspending medium, the applied peak voltage and frequency can be pre-determined to optimize the operation of the device.

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# APPLIED COMPUTATIONAL ELECTROMAGNETICS SOCIETY JOURNAL <br> http://aces.ee.olemiss.edu 

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Each paper is required to manifest some relation to applied computational electromagnetics. Papers may address general issues in applied computational electromagnetics, or they may focus on specific applications, techniques, codes, or computational issues. While the following list is not exhaustive, each paper will generally relate to at least one of these areas:

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[^0]:    ${ }^{1}$ The propagation factor (PF) is the field strength relative to its free-space value in dB .

[^1]:    $\mathrm{n}=\mathrm{n}+1$;
    end
    $\mathrm{N} 2=\mathrm{n}+10 ; \mathrm{n}=0$;
    \% Calculating Diffractted and Total Field
    while $\mathrm{N} 1>\mathrm{n}$
    $\mathrm{vn}=\mathrm{n}$ *i/alfa;
    if $n=0$ eps $=0.5$; else eps $=1$; end
    if $\mathrm{r}<=\mathrm{r} 0, \mathrm{Us}=\mathrm{Us}+\mathrm{besselj}(\mathrm{vn}, \mathrm{kr}) * \operatorname{besselh}(\mathrm{vn}, 1, \mathrm{kr} 0) * \sin (\mathrm{vn} * \mathrm{ang} 0) * \sin \left(\mathrm{vn}{ }^{*} \mathrm{ang}\right)$;
    else, Us=Us+eps*besselj(vn,kr0)*besselh(vn, $1, \mathrm{kr}$ )*sin(vn*ang0)*sin(vn*ang);
    end
    $\mathrm{n}=\mathrm{n}+1$;
    end
    $\mathrm{n}=0$;
    while $\mathrm{N} 2>\mathrm{n}$
    $\mathrm{vn}=\mathrm{n}^{*} \mathrm{pi} /$ alfa;
    if $\mathrm{n}==0$ eps $=0.5$; else eps $=1$; end
    if $\mathrm{r}<=\mathrm{r} 0$
    Uh=Uh+eps*besselj(vn,kr)*besselh(vn,1,kr0)* $\cos (\mathrm{vn} * a n g 0) * \cos (\mathrm{vn} * \mathrm{ang})$;
    else
    Uh=Uh+eps*besselj(vn,kr0)*besselh(vn, $1, \mathrm{kr})^{*} \cos (\mathrm{vn} * \mathrm{ang} 0){ }^{*} \cos (\mathrm{vn} * \mathrm{ang})$;
    end
    $\mathrm{n}=\mathrm{n}+1$;
    end
    Us=Us*coeff1; Uh=Uh*coeff1;
    if (ang >= 0 ) \&\& (ang < pi-ang0) \% -------- Region I -------
    R1=sqrt(r*r $+\mathrm{r} 0 *{ }^{*} 0-2 *{ }^{*}{ }^{*} \mathrm{r} 0 * \cos ($ ang-ang 0$)$ );
    R2 $=\mathrm{sqrt}\left(\mathrm{r}{ }^{*} \mathrm{r}+\mathrm{r} 0 *_{\mathrm{r}} 0-2 *_{\mathrm{r}} *_{\mathrm{r}} 0 * \cos (\right.$ ang + ang 0$)$ );
    Uxs $=10 *$ besselh $(0,1, \mathrm{k} * \mathrm{R} 1) /(4 * 1 \mathrm{i})-\mathrm{I} 0 *$ besselh $(0,1, \mathrm{k} * \mathrm{R} 2) /(4 * 1 \mathrm{i})$;
    Uxh=I0*besselh( $0,1, \mathrm{k} * \mathrm{R} 1) /(4 * 1 \mathrm{i})+\mathrm{I} 0 * \operatorname{besselh}(0,1, \mathrm{k} * \mathrm{R} 2) /(4 * 1 \mathrm{i})$;
    Usdiff = Us-Uxs; Uhdiff = Uh-Uxh;
    elseif (ang >= pi-ang0) \&\& (ang < pi+ang0) \% -------- Region II -------
    R1=sqrt(r*r+r0*r0-2*r*r0*cos(ang-ang0));
    Ux=I0*besselh( $\left.0,1, \mathrm{k}^{*} \mathrm{R} 1\right) /\left(4^{*} 1 \mathrm{i}\right) ;$ Usdiff $=$ Us-Ux; Uhdiff = Uh-Ux;
    elseif (ang >= pi+ang0) \&\& (ang <= alfa) \% -------- Region III -------
    Usdiff = Us; Uhdiff = Uh;
    end
    UhnTOTAL_SERIES= (Uh/U0); \% exact normalised Total fields hard
    UhnDIFF_SERIES=(Uhdiff/U0)*sqrt(2*pi*kr)/exp(1i*kr); \% exactdiffco hard
    UsnTOTAL_SERIES= (Us/U0); \% exact normalised Total fields Soft
    UsnDIFF_SERIES=(Usdiff/U0)*sqrt(2*pi*kr)/exp(1i*kr); \% exactdiffco soft
    UsnDIFF_SERIES $=$ conj(UsnDIFF_SERIES/sqrt(2*pi*k))
    UhnDIFF_SERIES $=$ conj(UhnDIFF_SERIES/sqrt(2*pi*k))

[^2]:    $\mathrm{n}=$ alfa/pi; rho= ${ }^{*}$ r0/(r+r0); m=0; krho=k*rho; psi = ang-ang0; w1 = Int_calcPE2(alfa,krho,psi)*exp(1i*(krho+pi/4))/(pi*sqrt(2)); psi $=$ ang + ang 0 ;
    w2 = Int_calcPE2(alfa,krho,psi)*exp(1i*(krho+pi/4))/(pi*sqrt(2));
    Ws=(w1-w2)*exp(-1i*krho); Wh=(w1+w2)*exp(-1i*krho);
    Us_PE $=$ Ws ${ }^{*}$ sqrt(r0/(r0+r))*exp(1i*kr); \% normalized PE diff field -soft Uh_PE $=\mathrm{Wh}^{*} \mathrm{sqrt}(\mathrm{r} 0 /(\mathrm{r} 0+\mathrm{r}))^{*} \exp \left(1 \mathrm{i}^{*} \mathrm{kr}\right) ; \%$ normalized PE diff field -hard PEs_DiffCoeff = Us_PE*sqrt(2*pi*kr)/exp(1i*kr);
    PEh_DiffCoeff = Uh_PE*sqrt(2*pi*kr)/exp(1i*kr);
    PEs_DiffCoeff $=$ conj(PEs_DiffCoeff)/sqrt $(2 * \mathrm{pi} * \mathrm{k})$
    PEh_DiffCoeff = conj(PEh_DiffCoeff)/sqrt(2*pi*k)
    \% Function : Int_calcPE2.m
    function result = Int_calcPE2(alfa,kr,psi)
    eps $=1 \mathrm{e}-12 ; \mathrm{n}=$ alfa/pi; y_old $=1 \mathrm{e} 6$; result $=0$;
    for $\mathrm{M}=1: 100$
    $\mathrm{s}=-\mathrm{M}: 0.0001: \mathrm{M} ;$ eta $=\mathrm{sqrt}(2) .{ }^{*} \mathrm{~s} . * \exp \left(-1 \mathrm{i}^{*} \mathrm{pi} / 4\right)$;
    $\mathrm{F} 2=(1 / \mathrm{n}) . * \sin (\mathrm{pi} / \mathrm{n}) . /(\cos (\mathrm{pi} / \mathrm{n})-\cos ((\mathrm{eta}+\mathrm{psi}) / \mathrm{n}))$; fun $=\exp (-k r * s . * s) . * F 2$;

