Improved Weakly Conditionally Stable Finite-Difference Time-Domain Method

Juan Chen

School of Electronic and Information Engineering, Xi'an Jiaotong University, Xi'an 710049, China
chenjuan0306@yahoo.com.cn

Abstract **─** To circumvent the inaccuracy in the implementation of the perfect-electric-conductor (PEC) condition in the weakly conditionally stable finite-difference time-domain (WCS-FDTD) method, an improved weakly conditionally stable (IWCS) FDTD method is presented in this paper. In this method, the solving of the tridiagonal matrix for the magnetic field component is replaced by the solving of the tridiagonal matrix for the electric field components; thus, the perfect-electric-conductor (PEC) condition for the electric field components is implemented accurately. The formulations of the IWCS-FDTD method are given, and the stability condition of the IWCS-FDTD scheme is presented analytically. Compared with the WCS-FDTD method, this new method has higher accuracy in the implantation of the PEC condition, which is demonstrated through numerical examples.

Index Terms **─** FDTD method, perfect-electric-conductor (PEC) condition, weakly conditionally stable FDTD method.

I. INTRODUCTION

To overcome the Courant limit on the time step size of the FDTD method [1,2], unconditionally stable methods such as the alternating-direction implicit FDTD (ADI-FDTD) scheme [3-12] and a weakly conditionally stable finite-difference time-domain (WCS-FDTD) method has been developed recently [13,14]. In the WCS-FDTD method, the CFL condition is not removed totally, but being weaker than that of the conventional FDTD method. The time step in this scheme is only determined by one space discretization, which is extremely useful for problems where a very fine mesh is needed in one or two directions. The WCS-FDTD method has better accuracy and higher computation efficiency than the ADI-FDTD method, especially for larger field variation.

However, in the WCS-FDTD method, updating of H_y component needs the unknown E_x and E_z components at the same time step, thus, the H_y component has to be updated implicitly by solving tridiagonal matrix [10], which results in a large inaccuracy in the implementation of the perfect-electric-conductor (PEC) condition for the E_x and E_z components. To circumvent this problem, an improved weakly conditionally stable finite-difference time-domain (IWCS-FDTD) is presented in this paper. In this method, the solving of the tridiagonal matrix for the H_y component is replaced by the solving the tridiagonal matrix of the E_x and E_z components; thus, the perfect-electric-conductor (PEC) condition for the E_x and E_z components is implemented easily. The formulations of the IWCS-FDTD method are presented, and the final updating equations are given. The stability condition of the IWCS-FDTD scheme is presented analytically. Compared with the WCS-FDTD method, this new method has higher accuracy in the implantation of the PEC condition, which is demonstrated through numerical examples.

II. FORMULATION FOR THE IWCS-FDTD METHOD

In a linear, non-dispersive, and lossless medium, the 3D WCS-FDTD scheme in reference [10] can be written as:

$$E_x^{n+1} = E_x^n + 2aD_y H_z^{n+1/2} - aD_z \left(H_y^{n+1} + H_y^n \right) \quad (1)$$

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$$E_z^{n+1} = E_z^n - 2aD_y H_x^{n+1/2} + aD_x \left(H_y^{n+1} + H_y^n \right) \quad (2)$$

$$\begin{aligned} H_y^{n+1} = H_y^n + bD_x \left(E_z^{n+1} + E_z^n \right) \\ - bD_z \left(E_x^{n+1} + E_x^n \right) \end{aligned} \quad (3)$$

$$\begin{aligned} H_x^{n+3/2} = H_x^{n+1/2} - 2bD_y E_z^{n+1} \\ + bD_z \left(E_y^{n+3/2} + E_y^{n+1/2} \right) \end{aligned} \quad (4)$$

$$\begin{aligned} H_z^{n+3/2} = H_z^{n+1/2} + 2bD_y E_x^{n+1} \\ - bD_x \left(E_y^{n+3/2} + E_y^{n+1/2} \right) \end{aligned} \quad (5)$$

$$\begin{aligned} E_y^{n+3/2} = E_y^{n+1/2} - aD_x \left(H_z^{n+3/2} + H_z^{n+1/2} \right) \\ + aD_z \left(H_x^{n+3/2} + H_x^{n+1/2} \right) \end{aligned} \quad (6)$$

here, $a = \Delta t / 2\varepsilon$, $b = \Delta t / 2\mu$, $D_w = \partial / \partial w$ ($w = x, y$) represents the first derivative operator with respect to w . ε is the permittivity and μ is the permeability of the medium, n and Δt are the index and size of time-step.

Obviously, updating of H_y component, as shown in eq. (3), needs the unknown E_x and E_z components at the same time step. In the WCS-FDTD scheme, the H_y component is updated by substituting eqs. (1) and (2) into eq. (3) [10] directly. This results in a broadly band matrix equation which is updated by solving two tri-diagonal matrix equation.

Here, we apply a new technique to solve eqs. (1)-(3). For simplicity, we write eqs. (1)-(3) in a new form as,

$$E_x^{n+1} = E_x^* - aD_z H_y^{n+1} \quad (7)$$

$$E_z^{n+1} = E_z^* + aD_x H_y^{n+1} \quad (8)$$

$$H_y^{n+1} = H_y^* + bD_x E_z^{n+1} - bD_z E_x^{n+1} \quad (9)$$

where,

$$E_x^* = E_x^n + 2aD_y H_z^{n+1/2} - aD_z H_y^n \quad (10)$$

$$E_z^* = E_z^n - 2aD_y H_x^{n+1/2} + aD_x H_y^n \quad (11)$$

$$H_y^* = H_y^n + bD_x E_z^n - bD_z E_x^n \quad (12)$$

Obviously, updating of E_x and E_z components, as shown in eqs. (7) and (8), needs the unknown H_y component at the same time step. By substituting eq. (9) into eqs. (7) and (8) we can obtain,

$$\left(1 - abD_{2z}\right) E_x^{n+1} = E_x^* - aD_z H_y^* - abD_z D_x E_z^{n+1} \quad (13)$$

$$\left(1 - abD_{2x}\right) E_z^{n+1} = E_z^* + aD_x H_y^* - abD_z D_x E_x^{n+1} \quad (14)$$

Multiplying eq.(13) by factor $\left(1 - abD_{2x}\right)$, and subtracting $\left(abD_z D_x\right) \times$ eq.(14), we have,

$$\begin{aligned} &\left(1 - abD_{2x} - abD_{2z}\right) E_x^{n+1} \\ &= \left(1 - abD_{2x}\right) E_x^* - aD_z H_y^* - abD_z D_x E_z^* \end{aligned} \quad (15)$$

The left side of eq. (15) is a broadly-banded matrix equation which is solved expensively. To improve the computation efficiency, new terms are added at the both side of eq.(15),

$$\begin{aligned} &\left(1 - abD_{2x} - abD_{2z} + a^2 b^2 D_{2x} D_{2z}\right) E_x^{n+1} \\ &= \left(1 - abD_{2x}\right) E_x^* - aD_z H_y^* - abD_z D_x E_z^* \\ &\quad + a^2 b^2 D_{2x} D_{2z} E_x^n \end{aligned} \quad (16)$$

It is equivalent to the following,

$$\begin{aligned} &\left(1 - abD_{2x}\right)\left(1 - abD_{2z}\right) E_x^{n+1} \\ &= \left(1 - abD_{2x}\right) E_x^* - aD_z H_y^* - abD_z D_x E_z^* \\ &\quad + a^2 b^2 D_{2x} D_{2z} E_x^n \end{aligned} \quad (17)$$

Dividing eq. (17) into two sub-steps, we have,

$$\left(1 - abD_{2x}\right) e_x = a^2 b^2 D_{2x} D_{2z} E_x^n - aD_z H_y^* - abD_z D_x E_z^* \quad (18.a)$$

$$\left(1 - abD_{2z}\right) E_x^{n+1} = e_x + E_x^* \quad (18.b)$$

With this manipulation, the updating of the E_x component requires the solution of two tridiagonal matrices (18.a) and (18.b).

In a similar way, updating of E_z component can be written as,

$$\begin{aligned} &\left(1 - abD_{2z}\right) e_z \\ &= a^2 b^2 D_{2x} D_{2z} E_z^n + aD_x H_y^* - abD_z D_x E_x^* \end{aligned} \quad (19.a)$$

$$\left(1-abD_{2x}\right)E_z^{n+1}=e_z+E_z^{*} \qquad (19.b)$$

After the solving of the E_x^{n+1} and E_z^{n+1} components, the H_y component can be updated using equation (3) explicitly, thus, the solving of the tridiagonal matrix for the H_y component in the WCS-FDTD method is replaced by the solving of the tridiagonal matrix for the E_x and E_z components here.

Approximating each derivative in space by centered second-order finite differences, we can obtain the final updating equations for E_x , E_z and H_y components. Such as, the updating equations for E_x component are as follows,

$$\begin{aligned}
&\left(1+\frac{2ab}{\Delta x^2}\right)e_x\left(i+\frac{1}{2},j,k\right)\\
&-\frac{ab}{\Delta x^2}e_x\left(i+\frac{3}{2},j,k\right)-\frac{ab}{\Delta x^2}e_x\left(i-\frac{1}{2},j,k\right)\\
&=\frac{a^2b^2}{\Delta z^2\Delta x^2}\left[\begin{array}{l}E_x^n\left(i+\frac{3}{2},j,k+1\right)-E_x^n\left(i+\frac{1}{2},j,k+1\right)\\ -E_x^n\left(i+\frac{1}{2},j,k+1\right)+E_x^n\left(i-\frac{1}{2},j,k+1\right)\end{array}\right]\\
&-\frac{2a^2b^2}{\Delta z^2\Delta x^2}\left[\begin{array}{l}E_x^n\left(i+\frac{3}{2},j,k\right)-E_x^n\left(i+\frac{1}{2},j,k\right)\\ -E_x^n\left(i+\frac{1}{2},j,k\right)+E_x^n\left(i-\frac{1}{2},j,k\right)\end{array}\right]\\
&+\frac{a^2b^2}{\Delta z^2\Delta x^2}\left[\begin{array}{l}E_x^n\left(i+\frac{3}{2},j,k-1\right)-E_x^n\left(i+\frac{1}{2},j,k-1\right)\\ -E_x^n\left(i+\frac{1}{2},j,k-1\right)+E_x^n\left(i-\frac{1}{2},j,k-1\right)\end{array}\right]\\
&-a\frac{H_y^{*}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)-H_y^{*}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z}\\
&-\frac{ab}{\Delta z\Delta x}\left[\begin{array}{l}E_z^{*}\left(i+1,j,k+\frac{1}{2}\right)-E_z^{*}\left(i,j,k+\frac{1}{2}\right)\\ -E_z^{*}\left(i+1,j,k-\frac{1}{2}\right)+E_z^{*}\left(i,j,k-\frac{1}{2}\right)\end{array}\right]
\end{aligned} \qquad (20.a)$$

$$\begin{aligned}
&\left(1+\frac{2ab}{\Delta z^2}\right)E_x^{n+1}\left(i+\frac{1}{2},j,k\right)\\
&-\frac{ab}{\Delta z^2}E_x^{n+1}\left(i+\frac{1}{2},j,k+1\right)\\
&-\frac{ab}{\Delta z^2}E_x^{n+1}\left(i+\frac{1}{2},j,k-1\right)\\
&=e_x\left(i+\frac{1}{2},j,k\right)+E_x^{*}\left(i+\frac{1}{2},j,k\right)
\end{aligned} \qquad (20.b)$$

where,

$$\begin{aligned}
&H_y^{*}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)=H_y^{n}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)\\
&+b\left[\begin{array}{l}\dfrac{E_z^n\left(i+1,j,k+\frac{1}{2}\right)-E_z^n\left(i,j,k+\frac{1}{2}\right)}{\Delta x}\\ -\dfrac{E_x^n\left(i+\frac{1}{2},j,k+1\right)-E_x^n\left(i+\frac{1}{2},j,k\right)}{\Delta z}\end{array}\right]
\end{aligned} \qquad (21)$$

$$\begin{aligned}
&E_z^{*}\left(i+1,j,k+\frac{1}{2}\right)=E_z^{n}\left(i+1,j,k+\frac{1}{2}\right)\\
&+a\frac{H_y^n\left(i+\frac{3}{2},j,k+\frac{1}{2}\right)-H_y^n\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)}{\Delta x}\\
&-\frac{2a}{\Delta y}\left[\begin{array}{l}H_x^{n+1/2}\left(i+1,j+\frac{1}{2},k+\frac{1}{2}\right)\\ -H_x^{n+1/2}\left(i+1,j-\frac{1}{2},k+\frac{1}{2}\right)\end{array}\right]
\end{aligned} \qquad (22)$$

$$\begin{aligned}
&E_x^{*}\left(i+\frac{1}{2},j,k\right)=E_x^{n}\left(i+\frac{1}{2},j,k\right)\\
&-a\frac{H_y^n\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)-H_y^n\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z}\\
&+\frac{2a}{\Delta y}\left[\begin{array}{l}H_z^{n+1/2}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)\\ -H_z^{n+1/2}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)\end{array}\right]
\end{aligned} \qquad (23)$$

The solving of the E_y component, same as that in the WCS-FDTD method, is updated implicitly by substituting eqs. (4) and (5) into eq. (6),

$$\begin{aligned}&\left(1-abD_{2x}-abD_{2z}\right)E_y^{n+3/2}\\&=\left(1+abD_{2x}+abD_{2z}\right)E_y^{n+1/2}\\&-2aD_xH_z^{n+1/2}+2aD_zH_x^{n+1/2}\\&-2abD_xD_yE_x^{n+1}-2abD_yD_zE_z^{n+1}\end{aligned} \tag{24}$$

The left side of eq. (24) is also a broadly-banded matrix equation. Adding new terms at the both side of eq.(24) and dividing it into two sub-steps, we can obtain the updating of the E_y component as follows,

$$\begin{aligned}&\left(1-abD_{2x}\right)e_y\\&=\left(1+abD_{2x}\right)\left(1+abD_{2z}\right)E_y^{n+1/2}\\&-2aD_xH_z^{n+1/2}+2aD_zH_x^{n+1/2}\\&-2abD_xD_yE_x^{n+1}-2abD_yD_zE_z^{n+1}\end{aligned} \tag{25.a}$$

$$\left(1-abD_{2z}\right)E_y^{n+3/2}=e_y\ . \tag{25.b}$$

Thus, at each time step the IWCS-FDTD method requires the solution of six tridiagonal matrices and two explicit equations.

III. WEAKLY CONDITIONAL STABILITY OF THE IWCS-FDTD METHOD

The relations between the field components of the IWCS-FDTD method can be represented in matrix forms,

$$\left[A\right]\left[\Lambda\right]^{n+1}=\left[B\right]\left[\Lambda\right]^{*}+\left[C\right]\left[\Lambda\right]^{n} \tag{26}$$

where,

$$\left[A\right]=\begin{bmatrix}S&0&0&0&0&0\\2abD_xD_y&S&2abD_zD_y&0&0&0\\0&0&S&0&0&0\\0&-bD_z&2bD_y&1&0&0\\bD_z&0&-bD_x&0&1&0\\-2bD_y&bD_x&0&0&0&1\end{bmatrix}$$

$$\left[B\right]=\begin{bmatrix}1-abD_{2x}&0&-abD_zD_x&0&-aD_z&0\\0&0&0&0&0&0\\-abD_zD_x&0&1-abD_{2z}&0&aD_x&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}$$

$$\left[C\right]=\begin{bmatrix}a^2b^2D_{2x}D_{2z}&0&0&0&0&0\\0&S_2&0&2aD_z&0&-2aD_x\\0&0&a^2b^2D_{2x}D_{2z}&0&0&0\\0&bD_z&0&1&0&0\\-bD_z&0&bD_x&0&1&0\\0&-bD_x&0&0&0&1\end{bmatrix}$$

$$\left[\Lambda\right]^{n+1}=\left[E_x^{n+1}\ E_y^{n+3/2}\ E_z^{n+1}\ \mathrm{H}_x^{n+3/2}\ \mathrm{H}_y^{n+1}\ \mathrm{H}_z^{n+3/2}\right]'$$

$$\left[\Lambda\right]^{*}=\left[E_x^{*}\ E_y^{*}\ E_z^{*}\ \mathrm{H}_x^{*}\ \mathrm{H}_y^{*}\ \mathrm{H}_z^{*}\right]'$$

$$S=1-abD_{2x}-abD_{2z}+a^2b^2D_{2x}D_{2z}$$

$$S_2=1+abD_{2x}+abD_{2z}+a^2b^2D_{2x}D_{2z}$$

According eqs. (10)-(12), we have,

$$\left[\Lambda\right]^{*}=\left[D\right]\left[\Lambda\right]^{n} \tag{27}$$

with,

$$\left[D\right]=\begin{bmatrix}1&0&0&0&-aD_z&2aD_y\\0&0&0&0&0&0\\0&0&1&-2aD_y&aD_x&0\\0&0&0&0&0&0\\-bD_z&0&bD_x&0&1&0\\0&0&0&0&0&0\end{bmatrix}$$

Substituting eq. (27) into eq. (26), we obtain,

$$\left[\Gamma\right]\left[\Lambda\right]^{n}=0 \tag{28}$$

here,

$$\left[\Gamma\right]=\left[A\right]\zeta-\left[B\right]\left[D\right]-\left[C\right]$$

$$=\begin{bmatrix}\left(S\zeta-S_1\right)&0&2abD_zD_x&-2a^2bf&2aD_z&f_x\\2abD_xD_y\zeta&\left(S\zeta-S_2\right)&2abD_zD_y\zeta&-2aD_z&0&2aD_x\\2abD_zD_x&0&\left(S\zeta-S_3\right)&f_z&-2aD_x&2a^2bf\\0&-L_z&2bD_y\zeta&\zeta-1&0&0\\L_z&0&-L_x&0&\zeta-1&0\\-2bD_y\zeta&L_x&0&0&0&\zeta-1\end{bmatrix}$$

and

$$S_1 = 1 - abD_{2x} + abD_{2z} + a^2b^2D_{2x}D_{2z}$$
$$S_3 = 1 + abD_{2x} - abD_{2z} + a^2b^2D_{2x}D_{2z};$$
$$f = D_xD_yD_z;$$
$$f_z = 2a\,D_y\left(1 - a\,b\,D_{2z}\right);$$
$$f_x = -2a\,D_y\left(1 - a\,b\,D_{2x}\right);$$
$$L_w = b\,D_w\left(\xi + 1\right), w = x, z.$$

For a nontrivial solution of eq. (28), the determinant of the coefficient matrix in eq. (28) should be zero,

$$\left|\Gamma\right| = 0 \tag{29}$$

By solving eq. (29), we have,

$$\left(\zeta - 1\right)^2\left(M^2\zeta^2 - 2N\zeta + M^2\right)\left(M\zeta^2 - 2P\zeta + M\right) = 0 \tag{30}$$

here,

$$M = \left(1 - abD_{2x}\right)\left(1 - abD_{2z}\right)$$
$$N = \left(1 + abD_{2x}\right)\left(1 + abD_{2z}\right)\left(1 - abD_{2x}\right)\left(1 - abD_{2z}\right)$$
$$+2abD_{2y}\left(1 - abD_{2x} - abD_{2z}\right)\left(1 + a^2b^2D_{2x}D_{2z}\right)$$
$$P = \left(1 + abD_{2x}\right)\left(1 + abD_{2z}\right) + 2abD_{2y}$$

The growth factor ζ is obtained,

$$\zeta_{1,2} = 1 \tag{31}$$

$$\zeta_{3,4} = \left(N \pm \sqrt{N^2 - M^4}\right)\Big/M^2 \tag{32}$$

$$\zeta_{5,6} = \left(P \pm \sqrt{P^2 - M^2}\right)\Big/M \tag{33}$$

To satisfy the stability condition during the field advancement, the module of growth factor ζ can't be larger than 1. It is evident that the module of $\zeta_{1,2}$ is unity.

For the values of $\zeta_{3,4}$ and $\zeta_{5,6}$, when the conditions $M^4 \geq N^2$ and $M^2 \geq P^2$ are satisfied, $\left|\zeta_{3,4}\right| = \left|\zeta_{5,6}\right| = 1$ can be obtained. Approximating each derivative in space by centered second-order finite differences, we can obtain the limitation for time-step size in the IWCS-FDTD method as follows,

$$\Delta t \leq \Delta y / c \tag{34}$$

where $c = 1/\sqrt{\varepsilon\mu}$ is the speed of light in the medium.

The stable condition of the IWCS-FDTD method is same as that of WCS-FDTD method. The maximum time-step size for IWCS-FDTD method is only determined by one spatial increment Δy. This is due to that the explicit difference is only used in the *y* direction.

IV. NUMERICAL VALIDATION

To demonstrate the accuracy and efficiency of the proposed theory, a numerical example is presented here. A metal plate with dimension 60mm $\times$ 60mm is shown in Fig.1. Twenty five apertures of 2 mm length and 2 mm width are cut on the plate. All the distances between the apertures are 10 mm. A uniform plane wave polarized along the *z*-direction, is normally incident on the aperture, and the time dependence of the excitation function is as follows,

$$E_z(t) = \exp[-4\pi(t - t_0)^2 / T] \tag{35}$$

where T and t_0 are constants, and both equal to 2×10^{-9}. In such a case, the highest frequency of interest is 1 GHz. The observation point is set at the back of the plate and is 50mm far from the plate.

Applying the FDTD method to compute the time domain electric field component E_z at the observation point, to simulate the apertures precisely, the cell size around the aperture must be small. We choose $\Delta x = \Delta z = 0.5$ mm around the apertures. The cell size Δy is set to be 25mm. To satisfy the stability condition of the FDTD algorithm, the time-step size for the conventional FDTD is $\Delta t \leq 1.17$ps. For the WCS-DTD and IWCS-FDTD scheme, the maximum time increment is only related to the space increments Δy, that is, $\Delta t \leq 83.33$ ps. Five-cell-thick CPML layers are used to terminate the grid, and are placed five cells from the metal plate on all sides. The implementation of the plane wave is same as that of conventional FDTD method. The metal plate is viewed as a perfect electronic conductor and the tangential electric field values at the metal plate should to be zeros.

In the WCS-FDTD method, the E_x and E_z components at the metal plate are set zeros directly after they are updated by using eqs. (2) and (3); while in the IWCS-FDTD method, the

PEC boundary condition for the E_x and E_z components are implemented following the strategy descried in reference [8], by incorporating the PEC condition into the solving of the tri-diagonal matrices.

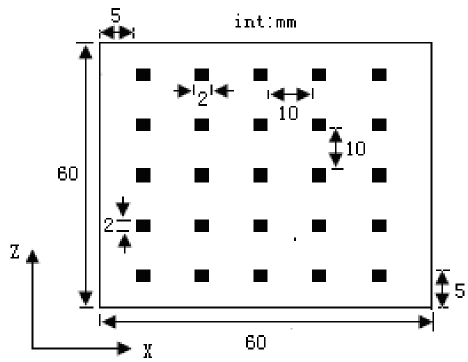

Fig. 1. Geometric configuration of the metal plate.

We perform the numerical simulation for an 83 ns time history by using the IWCS-FDTD method under time step size 83.33ps. The result is shown in Fig. 2. The total time steps are almost 1,000. It can be seen from Fig.1 that no instability problem is observed, which numerically validates the stability condition of eq. (34).

To demonstrate the high computational efficiency and accuracy of the IWCS-FDTD method, we perform the numerical simulations for a 5 ns time history by using the conventional FDTD, WCS-FDTD, and IWCS-FDTD methods, and compare the computation times and accuracy of these methods. In the conventional FDTD method, the time-step size keeps a constant of 1.17 ps, while in the WCS-FDTD and IWCS-FDTD methods, we use time-step size 83.33 ps.

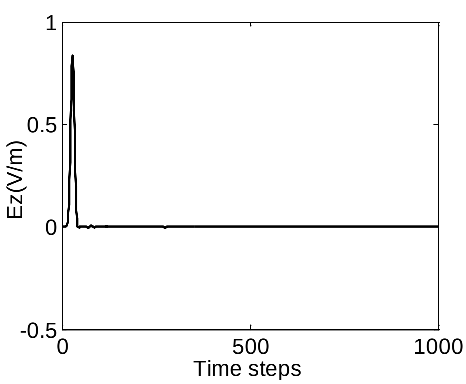

Fig. 2. Numerical result using IWCS-FDTD method with time step size 83.33ps.

Figure 3 shows the electric field component E_z at observation point calculated by using the conventional FDTD, WCS-FDTD, and IWCS-FDTD methods. It can be seen from this figure that only the result calculated by the IWCS-FDTD method agrees well with the result calculated by the conventional FDTD method. The result calculated by the WCS-FDTD method deviates from that of the conventional FDTD method significantly.

It is apparent that the IWCS-FDTD method has higher accuracy than the WCS-FDTD method in the implementation of the PEC condition. The reason for the inaccuracy of WCS-FDTD method is that, in the WCS-FDTD method, updating of H_y component needs the unknown E_x and E_z components at the same time step, thus, implementation of the PEC condition for the E_x and E_z components must be incorporated into the solving of the H_y component. The WCS-FDTD method neglects this and results in serious error in the implementation of the PEC condition.

To complete this simulation, the computation times for the conventional FDTD method, WCS-FDTD method and IWCS-FDTD method are 41.18, 5.05, and 5.37 minutes, respectively. Due to large time step size applied, the CPU time for the WCS-FDTD and IWCS-FDTD methods are almost 1/8 of that for the conventional FDTD method.

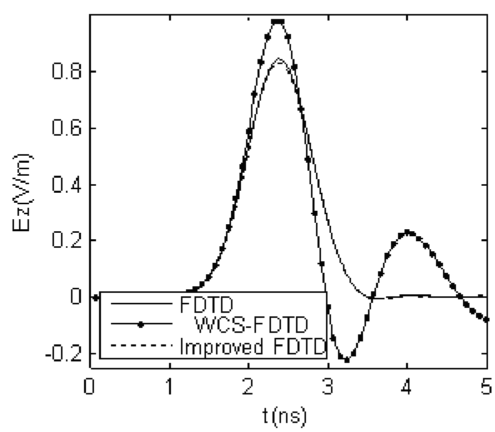

Fig. 3. The comparison of the results calculated by using the conventional FDTD, WCS-FDTD, and IWCS-FDTD methods

V. CONCLUSION

An improved weakly conditionally stable FDTD method is presented in this paper to circumvent the inaccuracy in the implementation of the perfect-electric-conductor condition in the WCS-FDTD method. The stability condition of the IWCS-FDTD scheme is presented analytically and the numerical performance of the proposed method over the WCS-FDTD method is demonstrated through numerical example.

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REFERENCES

[1] K. S. Yee, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," *IEEE Trans. Antennas Propagat.*, vol. 14, pp. 302-307, 1966.

[2] A. Taflove, *Computational Electrodynamics*, Norwood, MA: Artech House, 1995.

[3] F. Zheng, Z. Chen, and J. Zhang, "A Finite-Difference Time-Domain Method Without the Courant Stability Conditions," *IEEE Microwave Guided Wave Lett.*, vol. 9, pp. 441-443, 1999.

[4] T. Namiki, "3-D ADI-FDTD Method-Unconditionally Stable Time-Domain Algorithm for Solving Full Vector Maxwell's Equations," *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1743-1748, 2000.

[5] G. Sun and C. W. Trueman, "Some Fundamental Characteristics of the One-Dimensional Alternate-Direction-Implicit Finite-Difference Timedomain Method," *IEEE Trans. Microawave Theory Tech.*, vol. 52, pp. 46-52, 2004.

[6] M. Darms, R. Schuhmann, H. Spachmann, and T. Weiland, "Dispersion and Asymmetric Effects of ADI-FDTD," *IEEE Microwave Wireless Components Lett*, vol. 12, pp. 491-493, 2002.

[7] N. V. Kantartzis, "Multi-Frequency Higher-Order ADI-FDTD Solvers for Signal Integrity Predictions and Interference Modeling in General EMC Applications," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, pp. 1046-1060, 2010.

[8] J. Chen and J. Wang, "An Unconditionally Stable Subcell Model for Thin Wires in the ADI-FDTD Method," *Applied Computational Electromagnetic Society* (*ACES*) *Journal*, vol. 25, pp. 659-664, 2010.

[9] G. Sun and C. W. Trueman, "Accuracy of Three Unconditionally-Stable FDTD Schemes for Solving Maxwell's Equations," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 18, pp. 41-47, 2003.

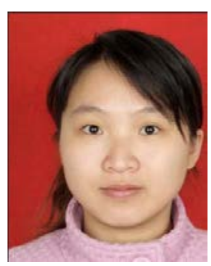
[10] G. Sun and C. W. Trueman, "An Unconditionally-Stable FDTD Method Based on the Crank–Nicolson Scheme for Solving the Three-Dimensional Maxwell's Equations," *Electron. Lett.*, vol. 40, pp. 589-590, 2004.

[11] J. Chen and J. Wang, "PEC Condition Implementation for the ADI-FDTD Method," *Microwave Opt. Technol. Lett.*, vol. 49, pp. 526-530, 2007.

[12] I. Ahmed and Z. Chen, "Error Reduced ADI-FDTD Methods," *IEEE Antennas Wireless Propag. Lett*, vol. 4, pp. 323-325, 2005.

[13] J. Chen and J. Wang, "Weakly Conditionally Stable Finite-Difference Time-Domain Method", *IET Microwaves, Antennas & Propagation*, vol. 4, pp. 1927-1936, 2010.

[14] J. Chen and J. Wang, "A Frequency-Dependent Weakly Conditionally Stable Finite-Difference Time-Domain Method for Dispersive Materials," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, pp. 665-671, 2010.



Juan Chen was born in Chongqing, China, in 1981. She received the Ph.D. degree in Electromagnetic Field and Microwave Techniques at the Xi'an Jiaotong University, Xi'an, China, in 2008. She now serves as an associate professor at Xi'an Jiaotong University. Her research interests are the numerical electromagnetic methods, antenna designs, and electromagnetic compatibility.