

Analysis of Microstrip Antennas using The Volume Surface Integral Equation Formulation and the Pre-Corrected Fast Fourier Transform Method

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Abstract — A rigorous and effective analysis based on the volume-surface integral equation (VSIE) formulation and pre-corrected-fast Fourier transform method (P-FFT) is presented for the problems of finite microstrip antennas which are modeled by combined conducting and dielectric materials. Several typical microstrip antennas and conformal microstrip antenna arrays are reconsidered; the comparisons of results from calculation and measurement validate the algorithm. Different feed methods are also considered to excite the antennas and conformal arrays. All the problems could be solved on a small computer with high efficiency and good precision.

Index Terms — Conformal antenna, microstrip antenna, precorrected-fast Fourier transform method, volume-surface integral equation.

I. INTRODUCTION

Microstrip patch antennas have been widely used in satellite communications, aircrafts, radars, biomedical applications, and reflector feeds, due to their advantages of low profile, simple structure, low cost and compatibility with integrated circuits. It is important to develop an efficient electromagnetic numerical method to analyze antennas with good precision.

There are always two approaches to analyze microstrip antennas, one approach is to model the antenna with infinite ground and substrate, derive the Green's function to the special multilayer structure of the antenna [1]; then, the unknown surface currents of the patch can be solved by full-

wave method. This approach does not account for the edge effects of a finite antenna, and it will be very difficult to get the related Green's function when the microstrip antenna is conformed onto a host with sophisticated and irregular shapes [2]. Another approach is to deal with the finite antenna directly by full-wave method, one of the popular numerical methods for this problem is based on the surface integral equation (SIE) formulation [3] or the hybrid volume-surface integral equation (VSIE) [4,5] formulation, for thin dielectric sheet problem, the volume integral approach seems a better alternative than the surface integral approach to the thin sheet problem [6]. In this paper, the VSIE formulation is applied to solve problems consisting of arbitrarily composite conducting-dielectric objects.

But this approach is considerably difficult for electrical large antenna arrays because of the necessity of solving a large matrix equation, which requires a large computer memory and CPU time. Domain decomposition method (DDM) [7], characteristic basis function method (CBFM) [8], and parallel computation techniques [9] could be applied to decompose the large problems to many smaller problems and alleviate the problem by assigning the memory requirements and CPU time. Fast solvers could also be used to reduce the storage memory and CPU time to some reasonable extent, such as conjugate gradient fast Fourier transform method (CG-FFT) [10], fast multipole algorithm (FMM), or multilevel fast multipole algorithm (MLFMA) [11,12], adaptive integral method (AIM) [13] and pre-corrected-fast Fourier transform method (p-FFT) [14,15].

Among these types of fast algorithms, FFT-form algorithms have relatively simple implementation on personal computers as compared to the FMM. CG-FFT method requires the integral equation to be discretized on rectangular grids which has limited its usage to conformal antenna objects, and as compared in [16], p-FFT can use larger grid spacing than the AIM, so the p-FFT algorithm is applied in this paper.

Several typical microstrip antennas or arrays including wideband microstrip antenna, microstrip patch antenna, and microstrip conformal arrays are characterized, and their respective numerical results are presented to demonstrate good accuracy and efficiency of the present design. Two feeding models are used in the examples including probe feed model [15,17] and microstrip line feed model [18], in which, the probe feed model use the uniform prism as demonstrated in [17], so RWG basis can be used for probe junction modeling.

II. FORMULATIONS AND EQUATIONS

Formulations for VSIE [5, 19, 20] and the main progress of p-FFT method are discussed here. The method in [21] is used to solve the singularity problems of potential integrations. Incomplete LU factorization with threshold (ILUT) pre-conditioner [22] is applied to improve the condition number of the impedance matrix, and the generalized minimum residual method (GMRES) is employed to solve the matrix equation for a faster convergence [22].

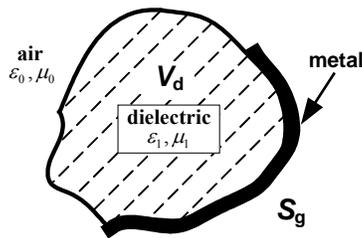


Fig. 1. Composite dielectric and metal model.

A. Coupled volume-surface integral equation

The mixed conduction-dielectric problem is considered here as shown in Fig. 1. To the dielectric obstacles, the volume equivalence theorem is used [23], then the scattered fields are produced by the equivalent volume polarization currents \mathbf{J}_d . To consider the scattering of perfectly conducting surfaces, the physical equivalent theorem can be

used [23], and the fields are best determined using surface equivalent current densities \mathbf{J}_c . For all these currents are situated in free space, so the free-space dyadic Green's function could be utilized in the computation of the scattered fields $\mathbf{E}^s(\mathbf{r})$, which could be expressed as

$$\mathbf{E}_\alpha^s = i\omega\mu_b \int_\alpha \overline{\overline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_\alpha d\mathbf{r}', \quad \alpha = S \text{ or } V, \quad (1)$$

where $\overline{\overline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}')$ is the dyadic Green's function, expressed as

$$\overline{\overline{\mathbf{G}}}(\mathbf{r}, \mathbf{r}') = \left(\overline{\overline{\mathbf{I}}} + \nabla\nabla/k_b^2 \right) \frac{e^{ik_b|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}. \quad (2)$$

k_b is the wave number for the background media.

Using boundary condition, the field integral equations are given by

$$\frac{\mathbf{D}(\mathbf{r})}{\epsilon_1(\mathbf{r})} = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}^s(\mathbf{r}) \quad \text{in } V_d, \quad (3)$$

$$\mathbf{E}_{\tan}^i = -\mathbf{E}_{\tan}^s \quad \text{on } S_g, \quad (4)$$

where V_d is the volume of the dielectric substrate, S_g is the surfaces of the metal. $\mathbf{D}(\mathbf{r})$ is the electric flux density in the dielectric region. \mathbf{E}^i is the field due to the impressed source. As expressed in (1), \mathbf{E}^s is the scattered field due to the currents of the conductors \mathbf{J}_c and the equivalent volume currents \mathbf{J}_d in the dielectric region, which are expanded as follows

$$\mathbf{J}_c(\mathbf{r}) = \sum_{n=1}^{N_s} I_n^S f_n^S(\mathbf{r}), \quad \mathbf{r} \text{ on } S_g, \quad (5)$$

$$\mathbf{J}_d(\mathbf{r}) = j\omega \sum_{n=1}^{N_v} I_n^V \kappa(\mathbf{r}) f_n^V(\mathbf{r}), \quad \mathbf{r} \text{ in } V_d. \quad (6)$$

RWG basis functions [19] and SWG basis functions [20] are used here to discretize the models. Galerkin's method [24] is used for MoM, then the matrix equation is built for the solution of the unknown surface and volume currents. To solve the problem directly, the memory requirement is of the order $O(N^2)$, and the CPU is of the order $O(N^3)$. For electrically large problems, it is inefficient to use MoM directly, while p-FFT method could be used to alleviate the problem to some extent.

The antenna in Fig. 4 is modeled by 2299 tetrahedral elements and 962 triangular patches, with 6795 unknowns to the whole problem. At a frequency of 7.78 GHz, the p-FFT accelerated method needs about 412 MB memory with the near-zone threshold distance set to 0.27λ , number of near field interactions is 12,935,915. Time for per iteration is 0.147s, a tolerance of 0.1% for GMRES can be achieved in only 21 iterations, total CPU time is 9.25 minutes for the solution. The generalized pencil-of-function method (GPOF) is used to retrieve the input impedance of the antenna [29].

The calculated return loss results are compared with the measured results in Fig. 5. A positive offset of about 180 MHz in the resonant frequency can be observed in Fig. 5. Except for the reasons of meshes density and fabrication error, the error relates to the nature of SWG and RWG or other low-order basis functions. SWG or other low-order dielectric basis functions are unable to exactly satisfy the boundary condition of the vanishing tangential E-field component on the metal-dielectric surface [17]. And for RWG or other low-order surface basis functions, it is difficult to express the fringing currents of conducting patches, according to the fringing effects of patch antenna, the inductive currents change very quickly near the fringe of a patch, and the fringing currents focus to the patch edge which will cause a much more higher current density near the fringe, but RWG basis function can not render the fringe effecting phenomenon very well, that's why the resonant frequency calculated by MoM always has a positive offset to the actual resonant frequency. Another reason is that the feed model used in Fig. 4 has introduced a self-inductance and caused frequency shift due to the strip shorted to the ground. The losses of dielectric and metal are also not considered in the numerical method.

In Fig. 6, the radiation patterns of the calculated results using the method in the paper are compared to the measured results and numerical results using the FDTD [26] and PMCHWT [30] methods, as shown in the diagram, very good agreements are obtained by comparing the calculated results using the method of the paper to the measured results. While in some angles, the results from FDTD method in [26] and PMCHWT method in [30] have a lot of

differences from the measured results, the reason for the lack of agreement is that, the finite ground plane is not modeled in the FDTD and PMCHWT solutions [26, 30].

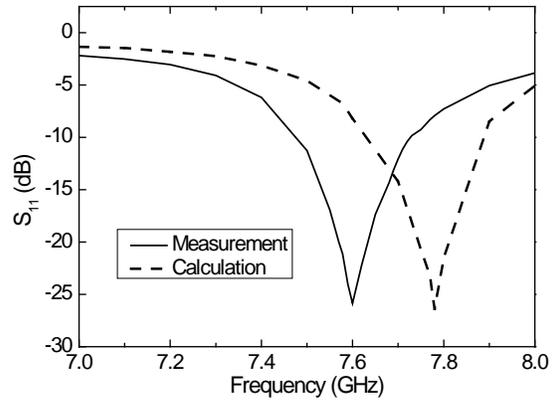


Fig. 5. S_{11} (dB) results comparison.

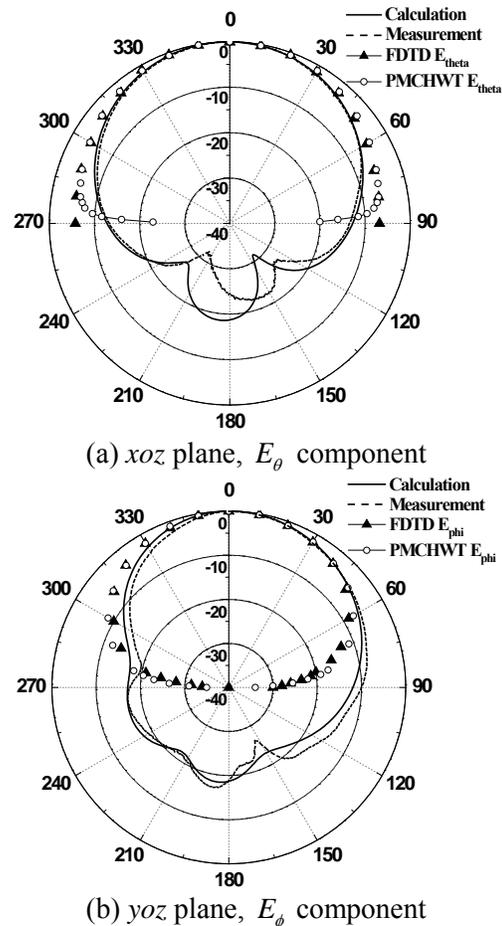


Fig. 6. Radiation patterns comparison at the resonant frequency, the calculated results are compared with measured results, calculated results based on FDTD in [26] and the numerical results

based on the PMCHWT method in [30]. E_θ component in xoz plane and E_ϕ component in $yo z$ plane are calculated and compared with the measurement.

C. Cylindrical microstrip arrays

Conformal microstrip antenna arrays are usually utilized in reality on surfaces of aircrafts, missiles, or many other land vehicles, which conform to prescribed shapes such as cylinders, cones, spheres, or some other complex geometries. Design of conformal array is a challenging problem for it is much more flexible than designing a planar array.

In [27], two examples of conformal cylindrical arrays were analyzed, assuming that the cylindrical ground plane and substrates have an infinite extent in the axial directions, and the patch surface currents orthogonal to the excitation polarization in each element were neglected, the mutual couplings between array elements were also ignored. In this paper, radiation characteristics of conformal array with different radii (chosen as 50mm, 76mm, and 106mm, respectively) are analyzed and compared.

As shown in Fig. 7, a 4×4 patch antenna array is conformed to a PEC cylinder with finite boundary. The geometry and dimensions of the cylindrical microstrip array calculated are shown in Fig. 7a. The interelement spacing is selected to be the same in the ϕ - and z - directions. As shown in Fig. 7a, the length and width of the microstrip feed line are 4 mm and 1 mm, respectively, the feed point locates with a distance of 0.5 mm to the end of the microstrip feed line. The probe feed model is shown in Fig. 7b [17]. For the limitation of our computation condition, in order to reduce the unknowns and memory required, the cylinder is cut by two planes as shown in Fig. 7a. The dielectric substrate of array has a thickness of h with relative permittivity $\epsilon_r = 2.94$. The calculated E-plane (xoz plane) and H-plane (xoy plane) patterns of the three cases ($a=106$ mm, $a=76$ mm, and $a=50$ mm) are compared as shown in Fig. 8, along with the, measured results provided in [27].

It is observed that the three radiation patterns are almost the same in the E-plane, the relative level of side-lobe will increase as the curvature increases. For the microstrip feeding method, the

calculated directive gains of the three cases are 20.36, 19.74, and 17.76, respectively, which will decrease as the curvature increases, Memory requirements and CPU time for p-FFT fast solver used to calculate the examples $a=106$ mm and $a=76$ mm are listed in Table 1. From Table 1, less than 1 GB memories are used to solve the problems. It should be pointed out that, for the limitation of our computation resource, less than 1.2 GB memories could be used, and preconditioner is not used in the conformal cases, which causes a lot of time for computation convergence. For all these cases, the tolerance for GMRES is set to 0.01. The convergence rates of probe fed cases are much lower than the microstrip fed cases, as more dense meshes should be applied to model the probe, which resulted in a larger condition number of the matrix.

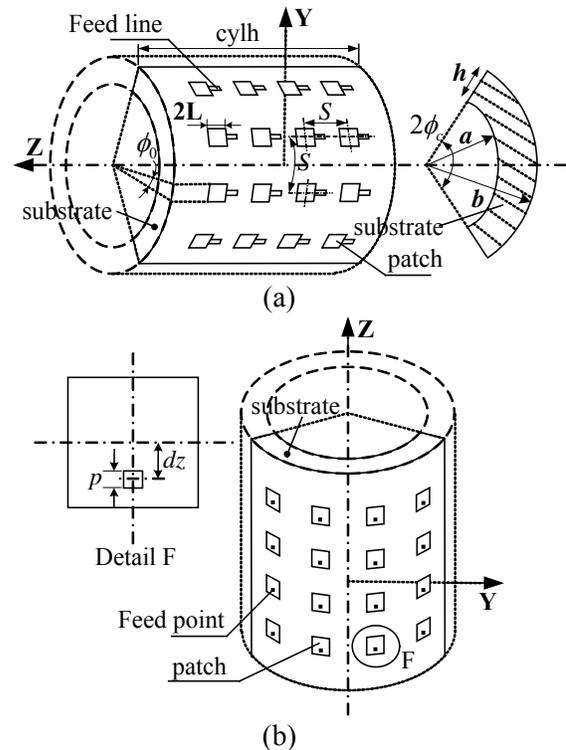


Fig. 7. Geometry of the cylindrical microstrip antenna array, frequency is at 16.2 GHz; $a=76$ mm, $h=0.254$ mm, $b=a+h$, $S=15$ mm, $2L=7.2$ mm, $2b\phi_0 = 5$ mm, $cyllh=66$ mm, $2\phi_c = 46^\circ$, $dz=1$ mm, $p=0.2$ mm. (a) Microstrip-line fed, (b) probe fed.

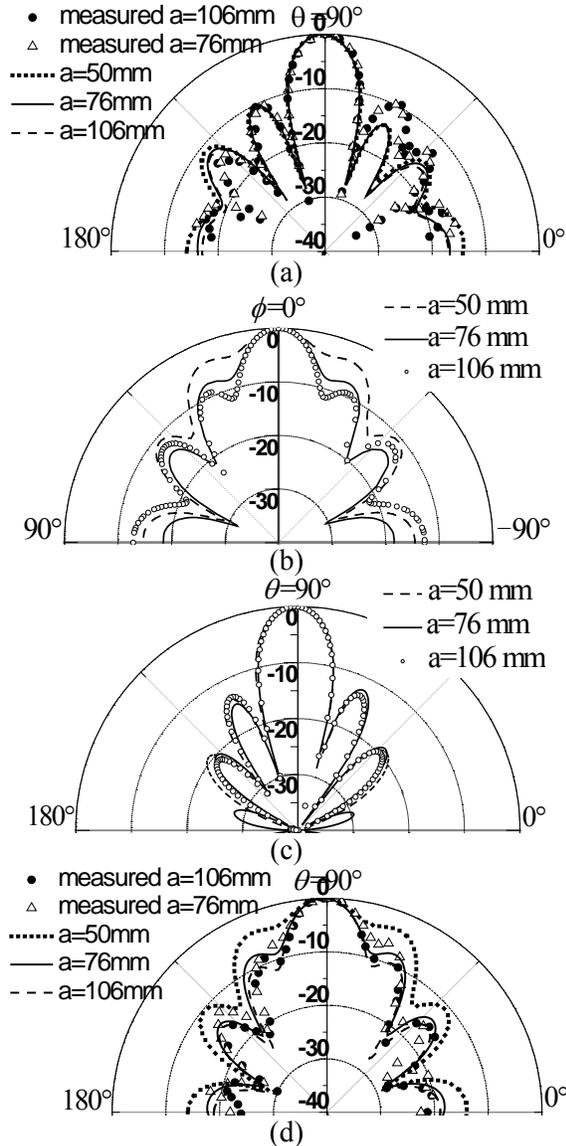


Fig. 8. Calculated and measured patterns [27].
 (a) probe feed, E-plane, E_θ component.
 (b) probe feed, H-plane, E_θ component.
 (c) microstrip-line feed, E-plane, E_θ component.
 (d) microstrip line feed, H-plane, E_θ component.

Table 1: Memory required and CPU time for the calculation of cylindrical arrays

	Cylindrical array				
	$a=76$ mm		$a=106$ mm		
	Probe	strip	probe	strip	
Unknowns	45939	51224	46544	47158	
Near-zone threshold (λ_0)	0.23	0.22	0.245	0.26	
Memory (MB)	P-FFT	645	799	737	834
	MoM	55648	60059	49586	50903
Per iteration time (s)	0.82	1.26	1.12	0.87	
Total CPU time (h)	12.83	0.87	15.97	1.75	

IV. CONCLUSION

The numerical method based on the volume-surface integral equation (VSIE) formulation and pre-corrected-fast Fourier transform method (p-FFT) fast solver is used to analyze the radiation problems of microstrip antennas, probe feed and microstrip line feed methods are both applied in the paper to excite the antennas or arrays. Numerical examples demonstrate that the p-FFT method yields an effective reduction of memory requirement and computational cost for large problems. At the same time, good agreements between calculated results and measured results are obtained. The reasons for the resonant frequency offset problem are analyzed in the paper and the problem remains to be solved.

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