A New Left-Handed Metamaterial Structure Based on Split-Triangle Resonators (STRs)

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Abstract — In this paper, small compact split triangle-resonators (STRs) with left-handed metamaterial (LHM) properties is proposed. The parameters of this new resonator are found by using the transfer matrix method and Nicholson-Ross-Weir method. Our results indicate that this structure could be used to realize the refractive index medium. Using the x-direction of the electromagnetic propagation wave, a simultaneous negative permeability and permittivity is obtained. The origin of the negative refractive index is a resonance due to the internal inductance and capacitance of the structure. In addition to simultaneous permittivity and permeability, the copper resonator has the advantage of being positioned on only one side of the FR4 substrate.

Index Terms — Metamaterials, split triangle resonators (STRs).

I. INTRODUCTION

The development of artificial materials (metamaterials) with negative refraction index or left-handed materials (LHM) has been a subject of growing interest in recent years. Apart from its exotic electrodynamics properties (such as the reversal of Snell's law, Doppler Effect and Cherenkov radiation), key to this interest is the potential applicability of these metamaterials to the fabrication of RF and microwave components based on left handedness. Due to negative values of effective permittivity and permeability, LHM are negative refractive index (NRI) media with antiparallel phase and group velocities. Namely, the wavevector k forms a left handed triplet with the vectors E and H (the electric and magnetic field intensity) and the wave fronts for propagating EM waves travel towards the source, i.e. opposite to the direction of energy flow [1, 2].

The renewed interest in the subject is due to the rediscovery by Pendry [10] of an old idea by Veselago [11] that materials with simultaneously negative permittivity and permeability can be regarded as negative refractive index materials. Then, a left-handed metamaterial was first implemented in a two dimensional periodic array of split ring resonators and long wire strips by Smith [8].

Recently, there has been growing interest in both the theoretical and experimental study of metamaterials. Many properties and potential applications of left-handed metamaterials (LHM) have been explored and analyzed theoretically. It has been proposed that LHM could be used to build a perfect lens with sub-wavelength resolution [3], and studies have been done on backward waves propagation [4-5], waveguides [21-26], antennas [22, 23, 24] Cerenkov radiation [6], and resonators [7]. The logical approach was to excite the split ring resonators and wire strips in order to force the structure to behave like magnetic and electric dipoles, respectively [9]. Since then, there have been large numbers of experimental investigations on the observation of this phenomenon. SRR/wire-type LHM opens a new field of electromagnetic response with matter. However, there are still some drawbacks such as high losses and limited bandwidth and anisotropic property preventing its further development. These issues prompted researchers to explore new designs such as the Omega pattern [12-13-14], S-Type [27], fishnet [15], and so on.

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In this paper, we present a new LHM resonator with simultaneous negative permittivity and permeability which can be used for conception of microwave antennas and filters design. The structure is composed by two coupled split triangle resonators (STRs) printed on only one side of the FR4 substrate. Therefore, it doesn't need other elements on the opposite side unlike most resonators proposed to date. We study the unit cell of the proposed resonator using two different approaches based on S-parameters that are the standard retrieval method and the Nicholson-Ross-Weir (NRW) approach.

The paper is organized as follows. In Section 2, the design for the constitutive elements of the LHM screen is described. In Section 3, we present the description of the two methods (retrieval and NRW) used. In Section 4, compared results are presented for x direction of propagation. Finally, in Section 5, conclusions are summarized.

II. RESONATOR DESIGN

The STR is formed by two coupled conducting triangles printed on a dielectric slab. Assuming a particle size much smaller than the free space wavelength, the STR's essentially behaves as a quasistatic R*LC* circuit fed by the external magnetic flux linked by the particle.

Figure 1a shows the cubic unit cell of the proposed structure, composed by a 0.5 mm thick substrate of FR4 ($\varepsilon_r = 4.4$, loss tangent of 0.02) and a copper STR positioned on the top side of the substrate. The cubic cell dimension is a=7mm. Figure 1b presents the planar view of the top side of the unit.

III. NUMERICAL METHODS DESCRIPTIONS

S-parameters were determined via full-wave simulations. Effective medium parameters (ε, μ) were determined using two methods: the inversion of *S* parameters for the experimental characterization of unknown materials presented by the Nicholson-Ross-Weir approach [16-17] and the standard transfer matrix method [18-19].

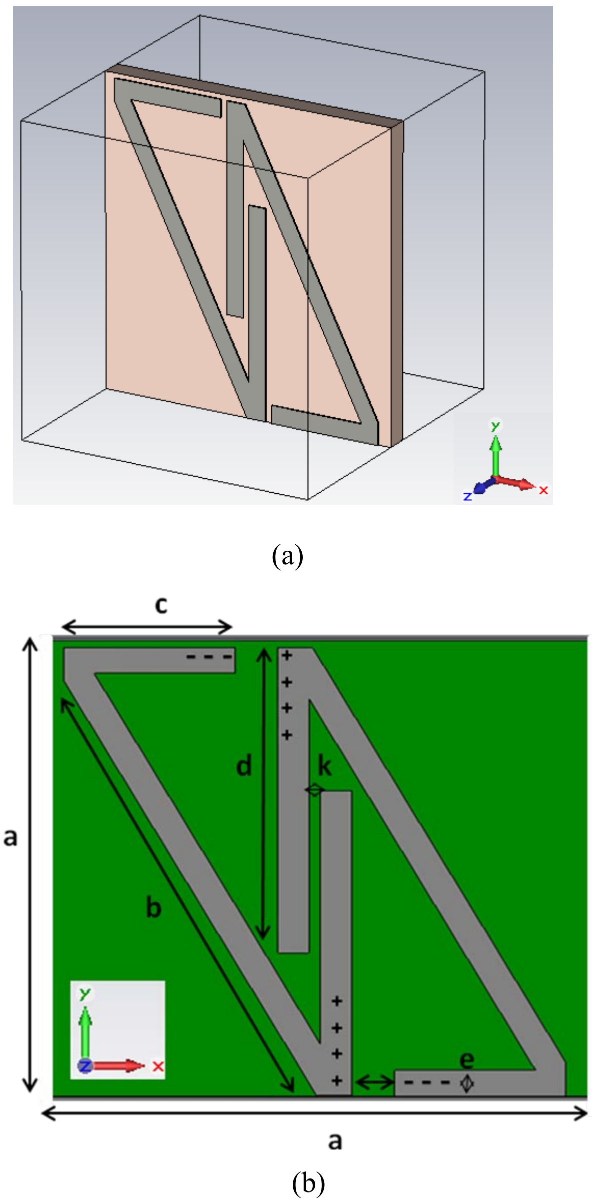


Fig. 1. Split triangle resonators (STRs): (a) perspective view of the unit cell, (b) planar view of the unit cell, a=7mm; b=7.89mm; c=2.4mm; d=4.3mm; e=0.4mm; P=0.2mm; k=0.2mm.

A: Standard retrieval method

Assuming a homogeneous medium, knowing the refractive index *n* and wave impedance *z* allows us to find μ and ε. The transfer matrix can be defined from:

$$F' = TF, \qquad (1)$$

$$\text{with: } F = \begin{pmatrix} E \\ H_{red} \end{pmatrix}. \tag{2}$$

E and H_{red} are the complex electric and magnetic field amplitudes. Here, the magnetic field assumed throughout is a reduced magnetic field [28] having the normalization $H_{red} = (+i\omega\mu_0 H)$. The transfer matrix for a homogeneous 1D slab has the analytic form

$$\text{And: } T = \begin{pmatrix} \cos(nkd) & -\frac{z}{k}\sin(nkd) \\ \frac{k}{z}\sin(nkd) & \cos(nkd) \end{pmatrix}. \tag{3}$$

The elements of the S matrix can be found from the elements of the T matrix [20].

For a case of homogeneous material, such as the parallelepiped-shape proposed, T11=T22=Ts and det(**T**)=1. We obtain a symmetric S matrix and finally analytic expressions on index and impedance given by:

$$n = \frac{1}{kd}\cos^{-1}\left[\frac{1}{2S_{21}}\left(1 - S_{11}^2 + S_{21}^2\right)\right], \tag{4}$$

$$Z = \sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \tag{5}$$

with:

$$\varepsilon = n/z, \tag{6}$$

$$\mu = nz. \tag{7}$$

B: Nicholson-Ross-Weir (NRW) approach

The Nicolson-Ross-Weir (NRW) equations enable the calculation of the complex permeability and permittivity of an unknown material sample entirely filling the cross-section of a reflectionless airline from the measured S-parameters. The relation between measured S-parameters and material properties is derived by considering the multiple reflections of a unit amplitude wave incident upon the air-sample interfaces within the waveguide.

The NRW method begins the expression of the transmission term, T from equation:

$$T = \frac{V_1 - \Gamma}{1 - \Gamma V_1}, \tag{8}$$

with:

$$\Gamma = \frac{T - V_2}{1 - \Gamma V_2}. \tag{9}$$

Γ= reflection coefficient

and:

$$V_1 = S_{21} + S_{11}, \tag{10}$$

$$V_2 = S_{21} - S_{11}, \tag{11}$$

and:

We obtain from (8) and (9) the equation:

$$1 - T = \frac{(1+\Gamma)(1-V_1)}{1 - \Gamma V_1}. \tag{12}$$

Assuming that the electrical thickness of the LHM slab is not large $(i.e., k_{real}d \le 1)$ and aware that the wave number

$$K = \frac{\omega\sqrt{\varepsilon_r \mu_r}}{c} = k_0\sqrt{\varepsilon_r \mu_r}. \tag{13}$$

The transmission term can be written as $T \approx 1 - jkd$ to obtain the approximate results of permittivity and permeability:

$$\varepsilon_r \approx \frac{2}{jk_0 d}\frac{1 - V_1}{1 + V_1}, \tag{14}$$

$$\mu_r \approx \frac{2}{jk_0 d}\frac{1 - V_2}{1 + V_2}, \tag{15}$$

where:

$$V_1 = S_{21} + S_{11}, \tag{16}$$

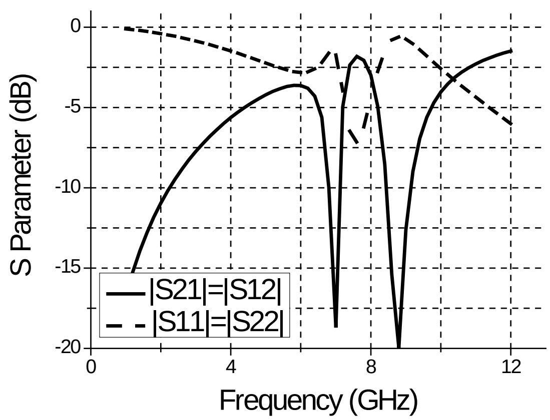
$$V_2 = S_{21} - S_{11}, \tag{17}$$

$$k_0 = \omega / c, \tag{18}$$

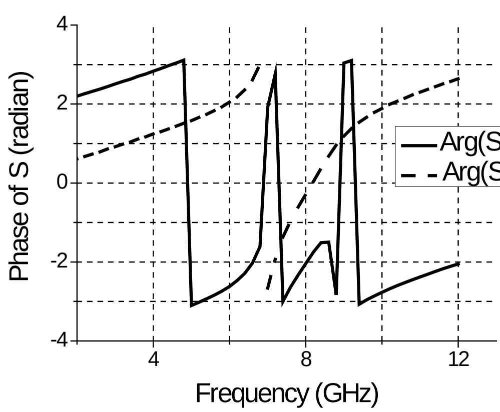
$$\omega = \textit{radian frequency}.$$

IV. RESULTS AND DISCUSSION

We study the particle along the X propagation. Figures 2a and 2b, show the amplitude and phase information of the calculated S parameters for the metamaterial structure, it can be seen that $S11$ is equal to $S22$, and $S12$ is equal to $S21$, since the structure is symmetric in the x-direction. Accordingly, using the standard retrieval method [18] and the Nicholson-Weir-Ross approach, the results for an impedance, effective refractive index, effective permittivity, and permeability are presented. The impedance shown in Fig. 3a, shows that the structure is indeed roughly matched at 7GHz. Referring to Fig. 3(c) and Fig. 3(d), the range of the simultaneous negative permittivity and permeability starts from 6.9 GHz to 7.6 GHz. Also, Fig. 3(b) confirms the negative index using the two methods in the same band. The deviation between the results obtained from the two extraction methods in Fig. 3b and Fig. 3c is due to the approximation of equations of ε and μ in NRW extraction.

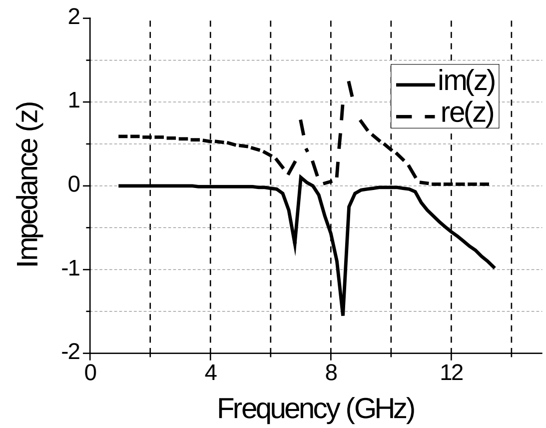


(a)



(b)

Fig. 2. (a) Magnitude and (b) phase of the simulated S parameters for the unit cell in Fig. 1a.



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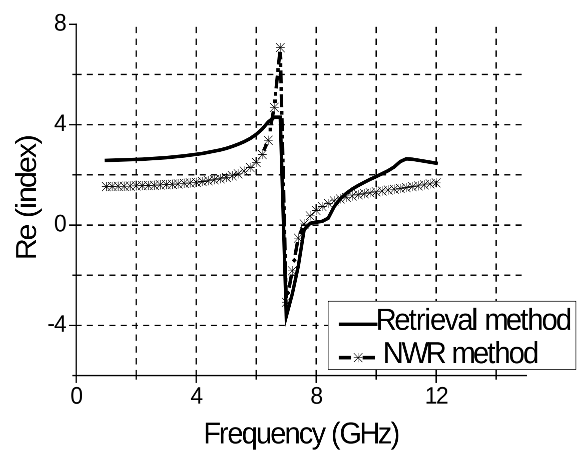

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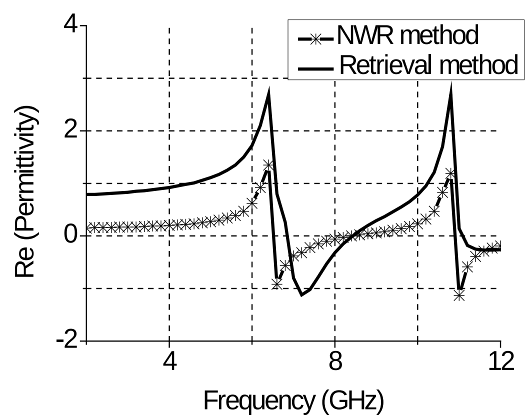
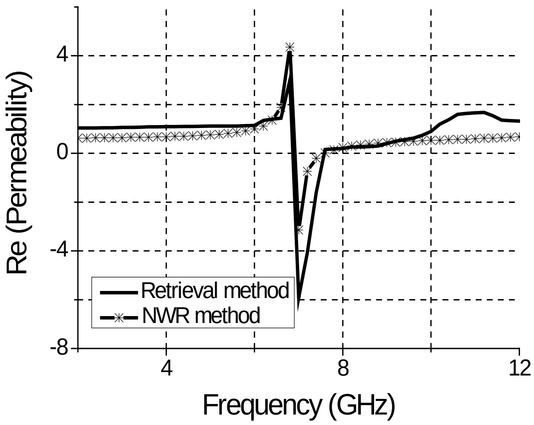


(c)



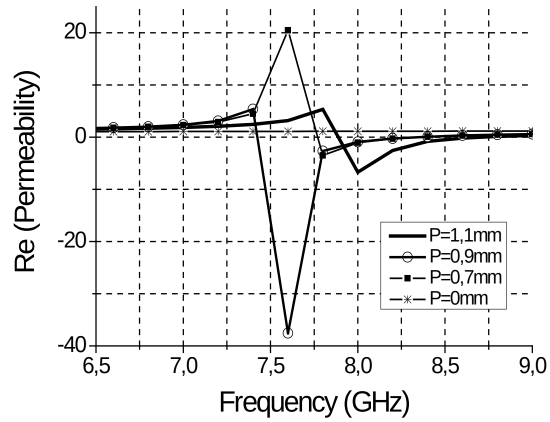
(d)

Fig. 3. (a) Retrieved impedance, (b) retrieved and NRW index, (c) retrieved and NRW real part of permittivity, (d) retrieved and NRW real part of permeability.

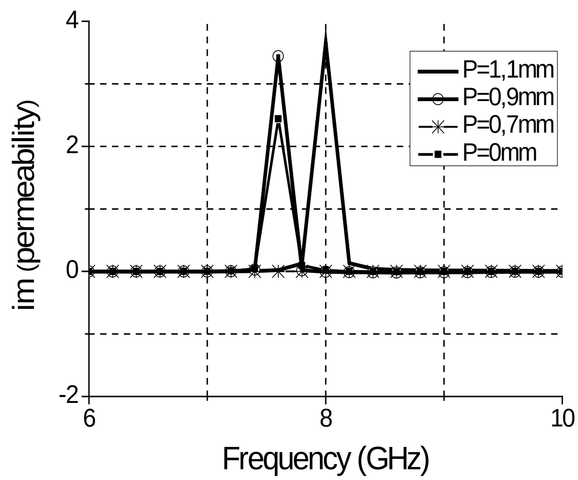
Figures 4 and 5 show the real and imaginary parts of the permeability and permittivity for various values of P, respectively. We observe that the general variation of the P-parameter has no impact on the signs of the permittivity (Fig. 5a-b). There is just a variation of the resonance frequency due to the variation of the total length of the radiating element. On the other hand, the parameter has an impact on the permeability. For P = 0, our resonator has a constant permeability (Fig. 4a), moreover, the real part of the refractive index (Fig. 6a) remains negative because observing the Fig. 4b, the imaginary permeability is positive for P=0, and having the real part of permittivity is negative (Fig. 5a), we obtain the necessary condition for a negative refractive index [25]:

$$\varepsilon'\mu'' + \varepsilon''\mu' \prec 0. \qquad (19)$$

Figure 6b shows the imaginary part of refractive index.

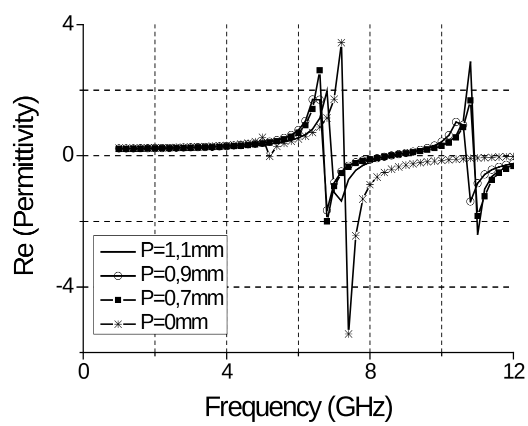


(a)

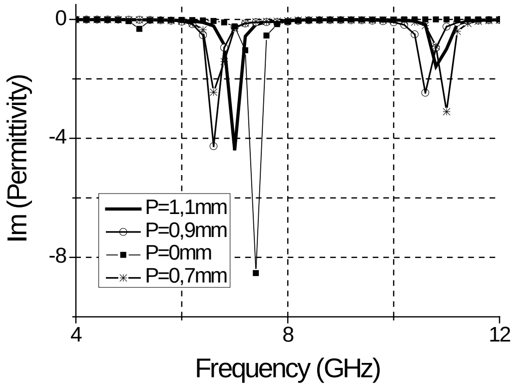


(b)

Fig. 4. (a) Standard retrieved real part of permeability for various P, (b) standard retrieved imaginary part of permeability for various P.

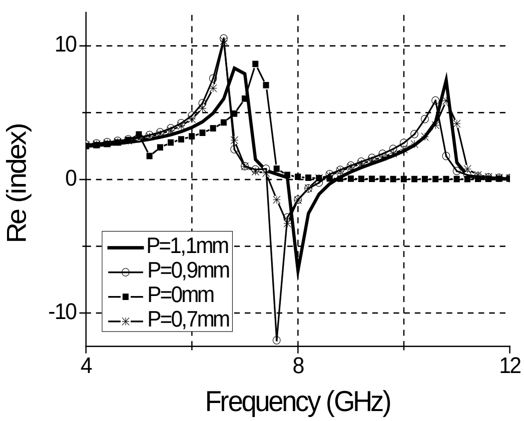


(a)

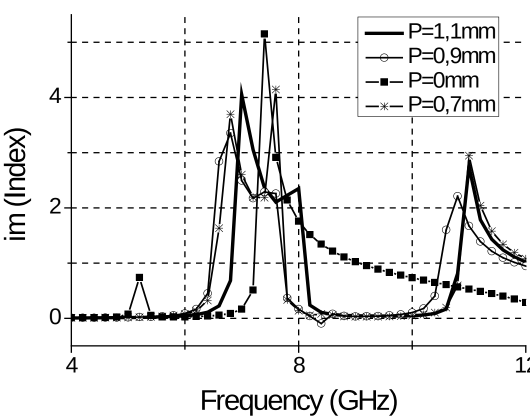


(b)

Fig. 5. (a) Standard retrieved real part of permittivity for various P, (b) standard retrieved imaginary part of permittivity for various P.



(a)



(b)

Fig. 6. (a) Standard retrieved real part of index for various P, (b) standard retrieved imaginary part of index for various P.

V. CONCLUSION

The design and study of a new LHM resonator based on two split triangles has been shown at microwave frequencies. The good agreement between NRW and standard retrieval method for two different directions of propagation is observed. The results confirm the existence of the LHM with simultaneous negatives permittivity and permeability. The effect of "P" parameter on the magnetic resonance of STR was observed. However, STRs present the important advantage of double negative ε and μ printing metallic elements just in one side (top side) of the substrate.

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