Analytical and FEM Based Calculation of Electromagnetic Forces Exerted on Cylindrical Coils due to their own Current

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Abstract ─ Different parts of the cylindrical coils are exposed to electromagnetic forces due to electric current flowing through it. These forces can deform the coil in axial and radial directions in abnormal operating conditions. So, in design process of cylindrical coils in many magnetic devices, mechanical stresses exerted on different parts of these kinds of coils should be determined. In this paper, analytical expressions for the forces in axial direction are derived in order to calculate the forces exerted on different parts of the cylindrical coils. In order to evaluate the precision of the method, the finite element method (FEM) is used and the results obtained by FEM are compared with the results of the analytical equations. Results obtained by finite element analysis confirm the analytical method. Due to inherent difficulties in calculation of the forces in radial direction, distribution of the latter on the coil body is calculated by FEM. The results show that the outer turns of the coil in two axial ends are exposed to the largest axial tension, while radial stresses are largest in the middle parts of the coil. In this paper, the calculations focus on the cylindrical coils; however, the method can be used for the calculation of the magnetic force distribution on different parts of spiral coils, disc coils and any type of air-cored coils with different sizes.

Index Terms ─ Analytical method, cylindrical coil, electromagnetic force distribution, FEM.

I. INTRODUCTION

Due to the extensive application of cylindrical coils in industry such as linear tubular motors, magnetic launchers and casting industries [1-3], determining the distribution of the mechanical stresses on different parts of the former is necessary. The calculation of the magnetic force distribution on current-carrying coils is closely related to the calculation of the magnetic force between them. To calculate the force between these coils, a variety of methods have been proposed in literature. There are some empirical equations and tables to calculate the force between coils with different shapes [4]. Also, the variation of the mutual inductance between two coils is used to calculate the force between them. In the latter method, first the mutual inductance between two coils is calculated and then it is used to calculate the force [5-7]. There are many articles which discuss the calculation of the magnetic forces between magnetic coils. In references [8-10], magnetic force between spiral coils is calculated using a new and effective method. In other research, magnetic forces between cylindrical coils are calculated [11, 12]. In this paper, using the method developed in [10], the axial magnetic force distribution on different parts of the cylindrical coils is calculated. The results are validated using finite element method (FEM). Also, FEM is employed to calculate the distribution of the radial forces on the cylindrical coils.

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II. MAGNETIC FORCE BETWEEN TWO CONCENTRIC CIRCULAR ELEMENTS

Suppose a system of two current carrying rings as shown in Fig. 1. To calculate the force between them, the concept of vector magnetic potential is employed. The vector magnetic potential of ring 1 in any point P on ring 2 is equal to [13]:

$$\vec{A} = \frac{\mu_0 I_1}{4\pi} \oint_{C_1} \frac{d\vec{l}'}{R_1}, \tag{1}$$

where μ_0 is the vacuum permeability, I_1 is the current of ring 1, C_1 is the length of ring 1 and R_1 is the distance between the differential component of the source $d\vec{l}'$ at point p′ and the field point p (Fig. 2).

By obtaining the vector magnetic potential, the flux density is calculated using the following equation [13]:

$$\vec{B} = \vec{\nabla} \times \vec{A}. \tag{2}$$

To calculate the force of ring 1 exerted on ring 2, the following equation is employed [13]:

$$\vec{F}_{21} = I_2 \oint_{C_2} d\vec{l}_2 \times \vec{B}, \tag{3}$$

where I_2 is the current of ring 2. Using equations (1) and (2) in equation (3) and doing some mathematical calculations, the following equation for the force is obtained:

$$\vec{F}_{21} = \vec{a}_z \left(\frac{\mu_0 I_1 I_2 z k}{2\sqrt{ab}(1-k^2)} \right) [(1-k^2)K(k) - (1-(1/2)k^2)E(k)]. \tag{4}$$

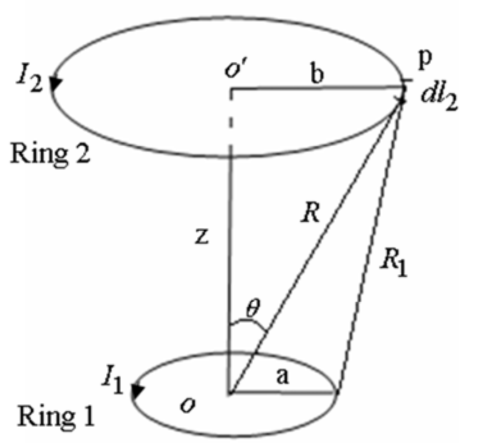

Fig. 1. Two current carrying concentric rings in z distance of each other.

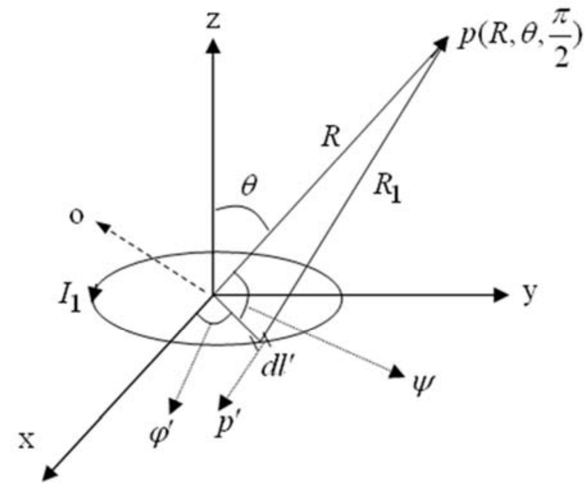

Fig. 2. Determination of the vector potential of a current carrying ring with radius a in any given point p.

In the aforementioned equation, a and b are radii of rings 1 and 2, respectively, z is axial distance of the two rings and k ($0 < k < 1$) is a constant coefficient and is equal to:

$$k = \sqrt{\frac{4ab}{(a+b)^2 + z^2}}, \tag{5}$$

and *K(k)* and *E(k)* are first and second order elliptic integrals respectively, which are defined as:

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1-k^2 \sin^2\theta)^{1/2}}. \tag{6}$$

$$E(k) = \int_0^{\frac{\pi}{2}} (1-k^2 \sin^2\theta)^{1/2} \, d\theta. \tag{7}$$

III. CALCULATION OF AXIAL DISTRIBUTION OF THE FORCE

In order to calculate the mechanical stresses exerted on different parts of the coil resulting from its current, the mesh-matrix method is employed. Consider *a* coil with the turn number of *N* shown in Fig. 3, where r_0 is the inner radius, *b* is the radial thickness and *a* is the height of the coil. As seen in this figure, the cross-section of the coil is divided into several segments. Here, the coil is divided into $n_r \times n_a$ cells. To calculate the distribution of the force on the different parts of the coil, the force between different filaments (in Fig. 3, each filament is specified with two cells in both sides) of the coil is calculated and then added together.

The force between filaments j and l is calculated using equation (4) as follows:

$$f_{jl} = \vec{a}_z\left(\frac{\mu_0 i^2 z_{jl} k'}{2\sqrt{r_j r_l}(1-k'^2)}\right)[(1-k'^2)K(k') - (1-(1/2)k'^2)E(k')]. \quad (8)$$

In the above equation, r_j and r_l are the radii of the filaments j and l, respectively, and z_{jl} is the center to center distance of the two filaments. The current of each filament is supposed to be concentrated on its center, and the current density of the entire coil is supposed to be uniform, and i which can be calculated using the following equation, is the current of each filament in the coil:

$$i = \frac{NI}{n_r \times n_a}. \quad (9)$$

In the above equation, I is the current of the coil. Parameter k' in equation (8) is a constant and is equal to:

$$k' = \sqrt{\frac{4r_j r_l}{(r_j + r_l)^2 + z_{jl}^2}}. \quad (10)$$

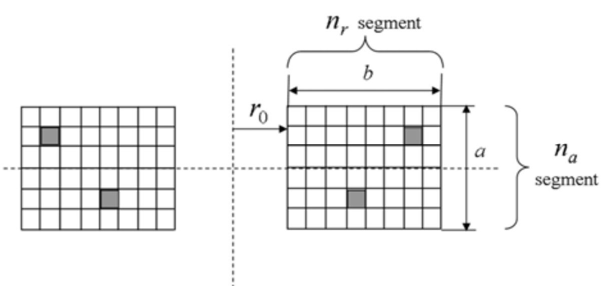

Fig. 3. Division of the coil into different meshes to calculate the force distribution.

IV. RESULTS

A. Analytical results

As mentioned before, the force between two filaments can be calculated using equation (8). In order to calculate the force exerted on specific part of the coil, one can use the effect of all filaments on that part and sum them up. The characteristics of the coil used for calculations are shown in Table 1. The cross-section of the coil has been divided into 71×35 segments, and the current of the coil is supposed to be 12 Amperes. Axially net magnetic force exerted on different parts of the coil is calculated. The force distribution on the outer part (turns located in outer part) of the coil in axial direction is illustrated in Fig. 4a. As seen in this figure, the magnitude of the force is symmetrical in respect to axial axis of the coil in such a way that there is no stress on the middle part of the coil. This is so because, the forces from both sides exerted on the middle part of the coil are equal in magnitude and opposite in direction and so cancel each other out. Figure 4b shows the distribution of the axial force on radial direction. According to this figure, moving across radial direction, the axial force exerted on related parts increases and after reaching a maximum value, it starts to decrease at the outer parts. In order to compare the force distribution on all parts of the coil body, the force profile among the axial and radial dimensions of the coil is illustrated in Fig. 5. It is clear from this figure that the outer parts of the coil at two axial axis ends are the most critical portions, because they are exposed to the largest tension. For instance, a force of about 0.6 N is exerted on the outer parts of the coil located at about 9.5cm in axial axis while the current following the coil is 12 amperes. The current can increase in fault conditions and give rise to deformation of the outer parts of the coil at two axial axis ends.

Table 1: Characteristics of the coil used in calculations

Number of Turns	**Inner radius, r_0 (cm)**	**Radial thickness, b (cm)**	**Heigh t, a (cm)**
2450	5	6	12

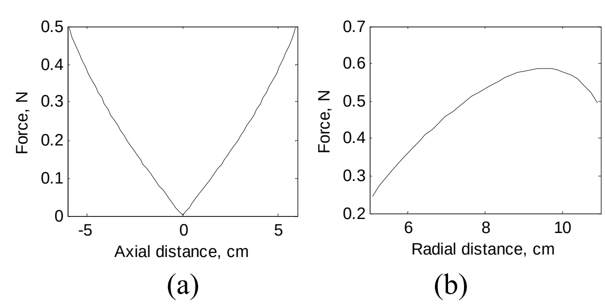

Fig. 4. (a) Axial force distribution on the outer part of the coil in axial direction. (b) Axial force distribution on lower part of the coil in radial direction.

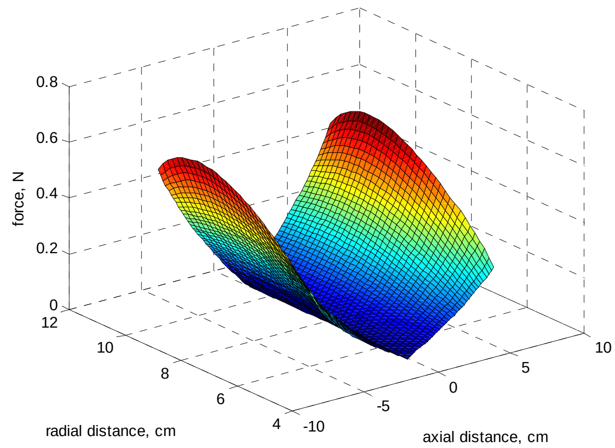

Fig. 5. The force distribution on all parts of the coil body.

B. FEM results

In order to evaluate the precision of the analytical method, in this section, a cylindrical coil is simulated using 2-D finite element method (FEM). The distribution of the flux density produced by the coil is illustrated in Fig. 6. Axial magnetic forces exerted on different parts of the coil due to current flowing through it, as well as radial forces are calculated using FEM. It should be mentioned that due to structure of the coil and inherent difficulties in calculations, only magnetic forces in axial direction is calculated using analytical method. Figure 7 illustrates the axial magnetic force distribution on different parts of the coil. It is clear from this figure that the force exerted on the outer parts of the axial end is the largest in comparison with other parts. In Fig. 7, half of the coil (lower half) is illustrated in order to clearly show the distribution of the force. Starting from the center of the coil, the axial force increases until it reaches the maximum value in axial end of the coil. In radial direction the axial forces increases and then reaches a maximum point and then decreases. So, the results of the Fig. 7 are in good agreement with those presented in Figs. 4 and 5.

The magnetic force is exerted on the coil body in radial and axial directions. In order to calculate the total force on different parts of the coil, the forces in two directions are summed up. Total force distribution on the coil body is illustrated in Fig. 8. As it is seen in this figure, the central parts of the coil are exposed to the largest force. This phenomenon is because of the exertion of the radial forces in these parts. It has been shown before that the axial force exerted on the middle parts of the coil from other parts cancel each other out; however, the radial forces exerted on the middle parts, arising from different parts of the coil are added to each other. Therefore, the middle parts experience largest radial force. Although the radial forces want to decompose the coil, their net effect on different parts is zero. For further clarity a part of the coil in Fig. 8 is magnified in Fig. 9.

The calculation of the distribution of the axial force on different parts of coil has been done using analytical method. The results are compared with FEM results. The coil has 1065 turns which is composed of 15 layers, each layer having 71 turns. The results are given in Table 2. Other specifications of the coil are given in Table 3. In Table 2, axial force distribution on the outer part of the coil in axial direction and on the lower part of the coil in radial direction is presented. In this table, distribution of the axial force is obtained by two methods in both axial and radial directions. As it is seen, the results obtained by FEM are in good accordance with analytical results confirming the proposed method. For better comparison, the results are plotted in Figs. 10 and 11. As the virtual work method gives accurate results in force calculations [14, 15], in this paper, the forces in FEM results are calculated by the former.

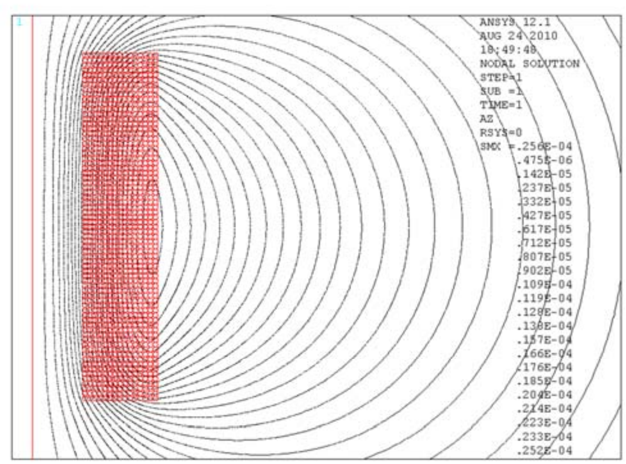

Fig. 6. Flux density distribution for the cylindrical coil.

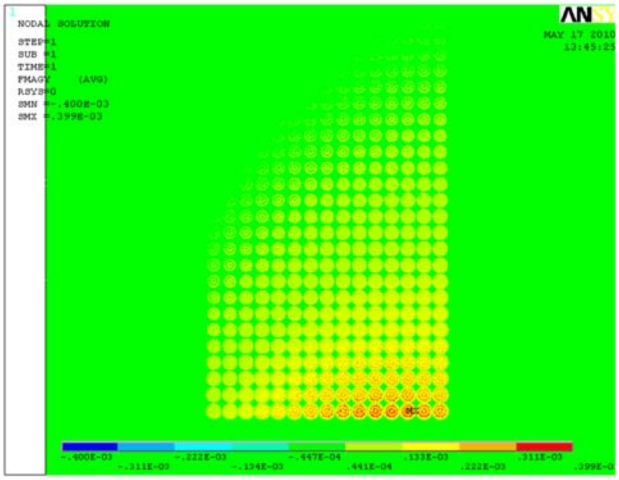

Fig. 7. Axial force distribution on different parts of the cylindrical coil.

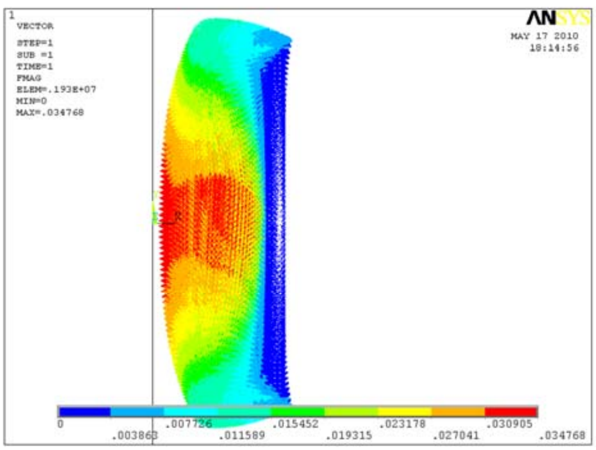

Fig. 8. Total force distribution on the coil's body.

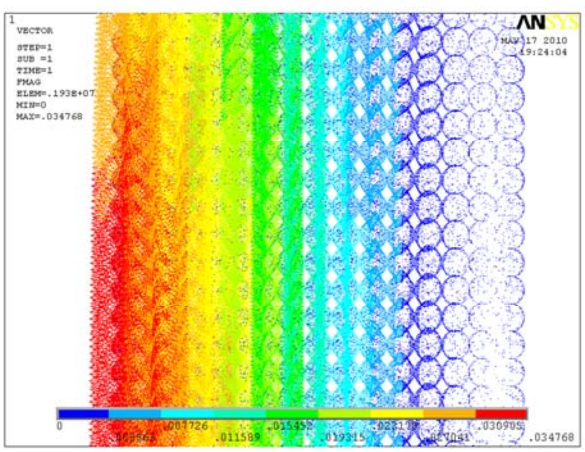

Fig. 9. A part of the coil in Fig. 8.

Table 2: comparison of the axial force distribution on coil's body by analytical method and FEM

	Distance(cm)	Force(N)	
		Analytical	**FEM**
radial direction	5.08	0.4904	0.5066
	5.40	0.5888	0.5911
	5.56	0.6265	0.6114
	5.88	0.6863	0.6837
	6.04	0.7090	0.7142
	6.52	0.7471	0.7511
	6.68	0.7484	0.7518
	7.00	0.7280	0.7306
	7.16	0.7019	0.6917
	7.32	0.6587	0.6500
axial direction	5.60	0.6587	0.6820
	5.12	0.5382	0.5380
	4.16	0.3730	0.3773
	3.36	0.2724	0.2574
	2.56	0.1923	0.1848
	1.92	0.1379	0.1450
	0.80	0.0548	0.0570
	0.16	0.0109	0.0121
	0	0	0
	-0.48	0.0327	0.0316
	-1.76	0.1252	0.1231
	-2.56	0.1923	0.1976
	-3.52	0.2906	0.3297
	-4.16	0.3730	0.3806
	-4.64	0.4479	0.4318
	-5.28	0.5732	0.6028
	-5.60	0.6587	0.6724

Table 3: Characteristics of the coil used in calculations of the analytical method and FEM

Number of Turns	1065
Inner radius, r_0 (cm)	5
Radial thickness, b (cm)	2.4
Height, a (cm)	11.36
Current of the coil (A)	20
Diameter of the wire used (mm)	1.6

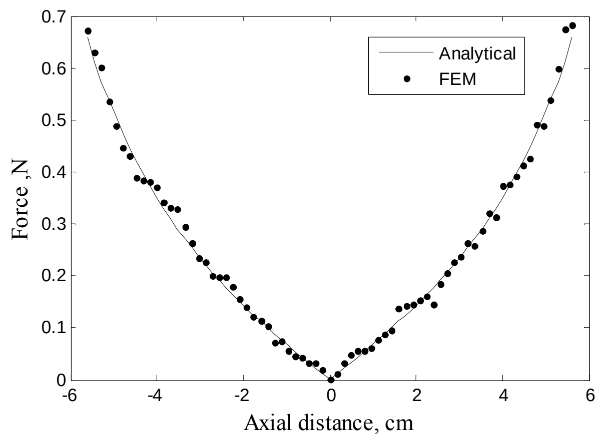

Fig. 10. Axial force distribution on the outer part of the coil in axial direction.

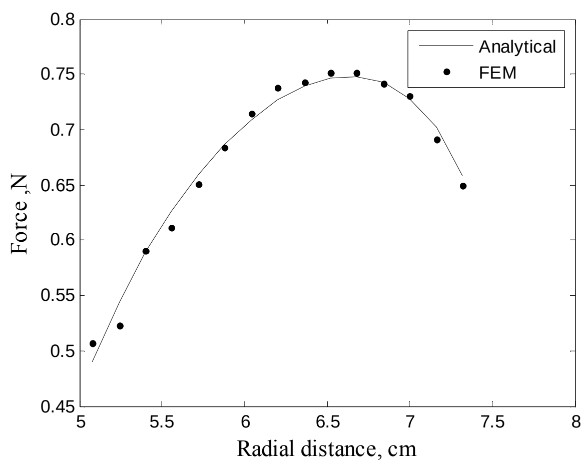

Fig. 11. Axial force distribution on lower part of the coil in radial direction.

V. CONCLUSION

In this paper, the axial and radial electromagnetic force distribution on cylindrical coil's body is calculated. Exact analytical equations are used to calculate the axial forces. According to the results obtained by MATLAB coding, the maximum axial stress is exerted on the outer parts of the coil at two axial axis ends. Thus, it is necessary to sufficiently support those parts of the coil in any application. On the other hand, considering the fact that the forces from both sides exerted on the middle parts of the coil cancel each other out, these parts of the coil are exposed to low tension. In order to verify the results of the analytical method, finite element method is employed. The results are confirmed the accuracy of the analytical method. Calculation of the forces in radial direction is also carried out by FEM. The results show that radial stresses are largest in the middle parts of the coil.

REFERENCES

[1] K. Fujisaki, "Magnetohydrodynamic Stability in Pulse Electromagnetic Casting", *IEEE Trans. Ind. Appl.*, vol. 39, no. 5, pp. 1442-1447, 2003.

[2] A. Meriched, M. Feliachi, and H. Mohellebi, "Electromagnetic Forming of Thin Metal Sheets", *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1808-1811, 2000.

[3] T. G. Engel, D. Surls, and W. C. Nunnally, "Prediction and Verification of Electromagnetic Forces in Helical Coil Launchers", *IEEE Trans. Magn.*, vol. 39, no. 1, pp. 112-115, 2003.

[4] F. W. Grover, *Inductance Calculations, Working Formulas and Tables*, Dover Publications, New York, 1962.

[5] S. Babic, C. Akyel, and. S. J Salon, "New Procedures for Calculating the Mutual Inductance of the System: Filamentary Circular Coil-Massive Circular Solenoid", *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1131-1134, 2003.

[6] K. B. Kim, E. Levi, Z. Zabar, and L. Birenbaum, "Mutual Inductance of Noncoaxial Circular Coils with Constant Current Density", *IEEE Trans. Magn.*, vol. 33, no. 5, pp. 4303-4309, 1997.

[7] C. Akyel, S. Babic, and S. Kinicic, "New and Fast Procedures for Calculating the Mutual Inductance of Coaxial Circular Coils (Circular Coil - Disk Coil)", *IEEE Trans. Magn.*, vol. 38, no. 5, pp. 2367-2369, 2002.

[8] A. Shiri, *Calculation of Magnetic Forces Between Current Carrying Coils in Two Plates*, M.Sc. Thesis, Iran University of Science &Technology, September 2006 (in Persian).

[9] A. Shiri and A. Shoulaie, "Calculation of Magnetic Forces Between Spiral Coils Using Mesh Approach", *24th International Power System Conference*, Tehran, Iran, November 2009 (in Persian).

[10] A. Shiri and A. Shoulaie, "A New Methodology for Magnetic Force Calculations Between Planar Spiral Coils", *Progress in Electromagnetics Research*, PIER 95, pp. 39-57, 2009.

[11] A. Shiri and A. Shoulaie, "New Results in Calculation of Magnetic Forces Between Cylindrical Coils", *23nd International Power System Conference*, Tehran, Iran, November 2008 (in Persian).

[12] A. Shiri, M. R. A. Pahlavani, and A. Shoulaie, "A New and Fast Procedure for Calculation of the Magnetic Forces Between Cylindrical Coils", *International Review of Electrical Engineering*, vol. 4, no. 5, pp. 1053-1060, 2009.

[13] D. K. Cheng, *Field and Wave Electromagnetics*, Second Edition, Prentice Hall, 1989.
[14] J. Mizia, K. Adamiak, A. R Eastham, and G. E. Dowson, "Finite Element Force Calculation: Comparison of Methods for Electric Machines", *IEEE Trans. Magn.*, vol. 24, no. 1, pp. 447-450, 1988.
[15] L. H. De Medeiros, G. Reyne, and G. Meunier, "Comparison of Global Force Calculations on Permanent Magnets", *IEEE Trans Magn.*, vol. 34, no. 5, pp. 3560-3563, 1998.

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