

# Method of Moments Analysis of Electromagnetic Transmission Through an Arbitrarily Shaped 3D Cavity in a Thick Conducting Plane

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**Abstract** — The method of moments (MOM) with surface equivalence principle is used to numerically solve the problem of electromagnetic scattering from and transmission through an arbitrarily shaped 3D cavity in a thick conducting plane is considered. The conducting walls of the cavity inside the ground plane are of arbitrary shape. The apertures at both ends of the cavity are also of arbitrary shape. An equivalent surface magnetic current placed on the top aperture produces the scattered field in the region where the impressed sources are. The total field inside the cavity is produced by two surface equivalent magnetic currents on the apertures and an equivalent surface electric current residing on the walls of the cavity as well as on both apertures. The transmitted field on the opposite side of the impressed sources is computed by an equivalent surface magnetic current residing on the bottom aperture. Computed results are compared with results in the literature obtained by using other methods. Very good agreement is observed.

**Index Terms** — Apertures, equivalence principle, moment methods.

## I. INTRODUCTION

The coupling of electromagnetic energy through apertures is an important problem in electromagnetic engineering. Bethe [1] offered solutions for coupling through a small circular aperture in a conducting plane wall of zero thickness, utilizing electric and magnetic

dipole moments. His solution, the so-called aperture polarizability method, has been used extensively as a basis for future research on aperture coupled systems. Arvas [2] computed polarizabilities of arbitrary shaped small apertures.

A major breakthrough in dealing with aperture problems came in 1976 when Harrington and Mautz [3] expressed aperture characteristics as admittance matrices, which depend only on the region being considered, being independent of the other region. The aperture coupling is then expressible as the sum of the two independent aperture admittance matrices. The numeric solution is carried out with the method of moments formulated by Harrington [4]. Auckland solved the problem of electromagnetic transmission through a slit in a thick conducting plane, when the cross-section of the slit is rectangular [5] and when it is arbitrarily shaped [6]. Park and Eom published a paper in which they use the aforementioned method to solve for the electromagnetic transmission through circular apertures in a thick conducting plane [7]. Imeci computed transmission through an arbitrary shaped aperture in a conducting plane separating air and a chiral medium [8]. A similar problem was solved by Jin and Volakis [9] using the finite element method. The purpose of this work is to solve this problem using MOM and surface equivalence principle. For homogeneous structures of arbitrary shape, surface meshing is usually simpler than volume meshing. In such cases the method of moments with surface equivalence principle can be simpler than finite

element method. To the best of our knowledge the present work is the first that solves the problem using the method of moments and surface equivalence principle.

## II. DEFINITION OF THE PROBLEM

The general problem considered here is shown in Fig. 1. The ground plane of thickness  $d$  is assumed to be a perfect electric conductor (PEC) of infinite size in  $x$  and  $y$  directions. Arbitrarily shaped apertures exist on each side of the thick ground plane. These apertures are connected with an arbitrarily shaped cavity. The regions above and below the ground plane as well as inside the cavity are linear, homogeneous, isotropic dielectric mediums. These regions are named regions  $a$ ,  $b$ , and  $c$  from the top to the bottom. In general, each region has a different electric permittivity ( $\epsilon$ ) and magnetic permeability ( $\mu$ ) than the others. The top region, region  $a$ , also contains impressed sources ( $\vec{J}^i, \vec{M}^i$ ) far away. These sources excite a time harmonic plane wave which illuminates the ground plane on the top side. The electric and magnetic fields in each region are unknown and they are going to be calculated by applying the equivalence principle, image theory and the method of moments.

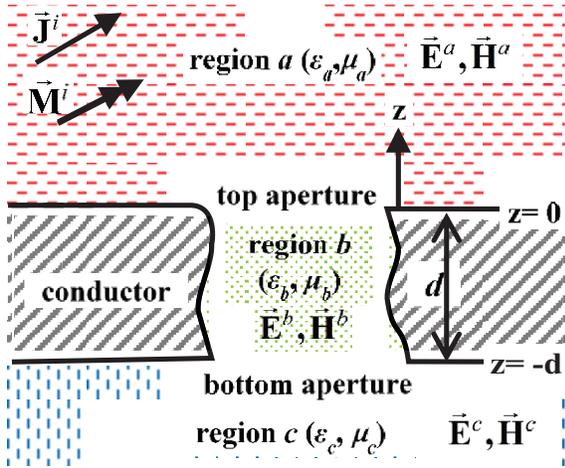


Fig. 1. Cross section view of the problem.

The equivalent problem for the top region is shown in Fig. 2. The impressed sources ( $\vec{J}^i, \vec{M}^i$ ) and the material ( $\epsilon_a, \mu_a$ ) are kept the same as those in the original problem. The top aperture surface of Fig. 1 is now covered by a patch of PEC in Fig. 2. Hence, the whole  $z=0$  plane in Fig. 2 is a PEC. Below this plane, the fields are set to be null fields. The tangential electric field in region  $a$  of Fig. 1 is zero just above the  $z=0$  plane except over the top aperture region. The electric field in the top aperture region is  $\vec{E}^a|_{z=0} = \vec{E}^a|_{z=0^+}$  in Fig. 1. By placing an equivalent magnetic surface current:

$$\vec{M}_1 = \vec{E}^a|_{z=0^+} \times \vec{n}_a = \vec{E}^a|_{z=0^+} \times \vec{z}, \quad (1)$$

over this newly placed patch of PEC in Fig. 2, we guarantee that the tangential electric field just above this current in Fig. 2 is the same as the tangential electric field at the same points of Fig. 1. In (1),  $\vec{z}$  is the unit vector in the  $z$ -direction and  $z=0^+$  indicates the limit as  $z$  approaches zero from the above. Then the fields in Fig. 2, produced by the impressed sources ( $\vec{J}^i, \vec{M}^i$ ) and the equivalent magnetic surface current  $\vec{M}_1$  (residing just above the PEC patch), are identical to ( $\vec{E}^a, \vec{H}^a$ ) in Fig. 1. That is,

$$\vec{E}^a = \vec{E}^a(\vec{J}^i, \vec{M}^i) + \vec{E}^a(\vec{M}_1), \quad (2)$$

$$\vec{H}^a = \vec{H}^a(\vec{J}^i, \vec{M}^i) + \vec{H}^a(\vec{M}_1). \quad (3)$$

The problem in Fig. 2 is a radiation problem of current sources over an infinite ground plane in a half-space filled with homogeneous dielectric medium. This type of problem can be solved by using image theory [10]. The ground plane is removed and the equivalent magnetic surface current is doubled. Impressed sources also have their images taken. The fields produced by these five sources, as they radiate in an unbounded homogeneous medium ( $\epsilon_a, \mu_a$ ) are the same as the fields of region  $a$  in Fig. 1. That is,

$$\vec{E}^a = \vec{E}^a(\vec{J}^i, \vec{M}^i) + \vec{E}^a(\vec{J}^{i, \text{img}}, \vec{M}^{i, \text{img}}) + \vec{E}^a(2\vec{M}_1), \quad (4)$$

$$\vec{H}^a = \vec{H}^a(\vec{J}^i, \vec{M}^i) + \vec{H}^a(\vec{J}^{i, \text{img}}, \vec{M}^{i, \text{img}}) + \vec{H}^a(2\vec{M}_1). \quad (5)$$

Equivalence for region  $b$  is shown in Fig. 3.

Lastly, the equivalent problem for the bottom region is set up. This is very similar to the top region equivalent problem with the major difference being not having impressed sources in the bottom region problem. The final form of the integral equations is obtained:

$$\begin{aligned} -2\vec{H}_{\text{tan}}^a(\vec{M}_1) - \vec{H}_{\text{tan}}^b(\vec{M}_1) - \vec{H}_{\text{tan}}^b(\vec{M}_2) + \vec{H}_{\text{tan}}^b(\vec{J}) &= 2\vec{H}_{\text{tan}}^{\text{inc}} \\ &\text{across top aperture,} \\ -\vec{H}_{\text{tan}}^b(\vec{M}_1) - \vec{H}_{\text{tan}}^b(\vec{M}_2) - 2\vec{H}_{\text{tan}}^c(\vec{M}_2) + \vec{H}_{\text{tan}}^c(\vec{J}) &= 0 \\ &\text{across bottom aperture,} \\ \vec{E}_{\text{tan}}^b(\vec{M}_1) + \vec{E}_{\text{tan}}^b(\vec{M}_2) - \vec{E}_{\text{tan}}^b(\vec{J}) &= 0 \text{ on } S_c, \end{aligned} \quad (6)$$

where  $\vec{H}^{\text{inc}}$  is the magnetic field of ( $\vec{J}, \vec{M}_i$ ) in unbounded homogeneous medium. These three equations are going to be used to solve for the three unknowns,  $\vec{M}_1$ ,  $\vec{M}_2$  and  $\vec{J}$  by the help of the method of moments.

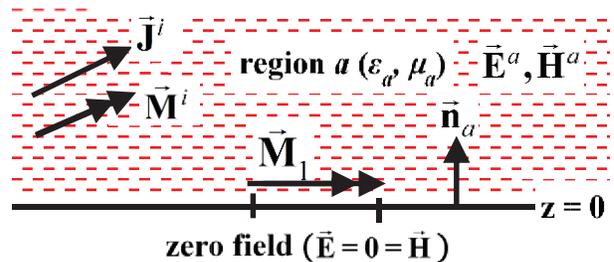


Fig. 2. The equivalent problem for region  $a$ .

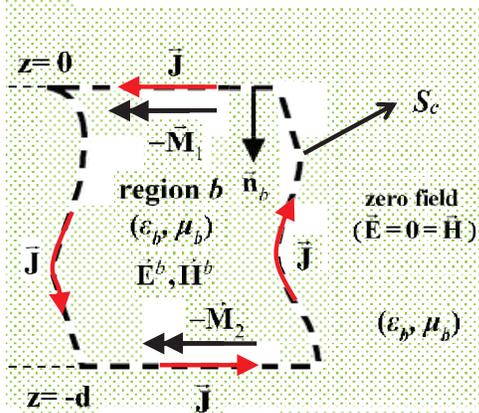


Fig. 3. The equivalence for cavity region.

### III. NUMERICAL RESULTS

The problem that is going to be analyzed is a thick conductor with square apertures on the top and the bottom. This problem is previously solved in [9] using the finite element method. The sides of the apertures are  $l=w=0.4\lambda$  and the conductor thickness is  $d=0.25\lambda$ . The triangular meshing is done in such a way that the triangles are more refined on the edges and on  $x$  and  $y$  axes on apertures. The top aperture is excited with a plane wave on normal incidence with polarization given as  $\vec{E}^{inc} = \vec{x}e^{jkz}$ . The electric fields on the apertures computed in [9] are in very good agreement with the electric fields in Fig. 4 calculated using the MoM formulation in this research.

Next, the backscattering cross section (RCS) and the transmission coefficient  $T$  of the structure are compared. The backscattering cross section is defined as [9]:

$$\sigma(\theta, \phi) = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|\vec{H}^s(\vec{r})|^2}{|\vec{H}^{inc}|^2}, \quad (7)$$

where  $\vec{H}^s(\vec{r})$  is the far zone scattered field, which is the scattered field in the backward direction minus the field scattered that would exist if the entire  $z=0$  plane was perfectly conducting. The transmission coefficient is defined as:

$$T = \frac{P_{trans}}{P_{inc}} = \frac{\text{Re} \left( \iint_{bot.aper.} (\vec{E}_c^* \times \vec{H}_c) \cdot \vec{n}_c ds \right)}{\eta_a |\vec{H}^{inc}|^2 A_1 \cos \theta^{inc}}, \quad (8)$$

where  $P_{inc}$  is the time average incident power that would be intercepted by the top aperture if all space was free space,  $P_{trans}$  is the time average power transmitted to region  $c$  through the bottom aperture,  $\eta_a$  is the impedance of region  $a$ , and  $A_1$  is the area of the top aperture. RCS and transmission coefficient plots computed by [9] and those in Fig. 5 computed by the MoM

formulation are very close to each other for square apertures on top and bottom in a thick conductor.

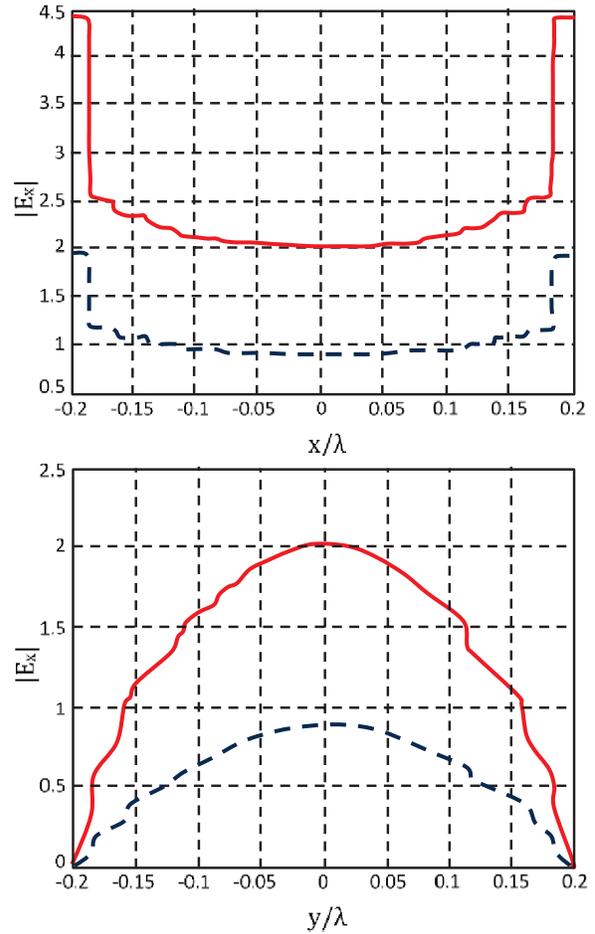
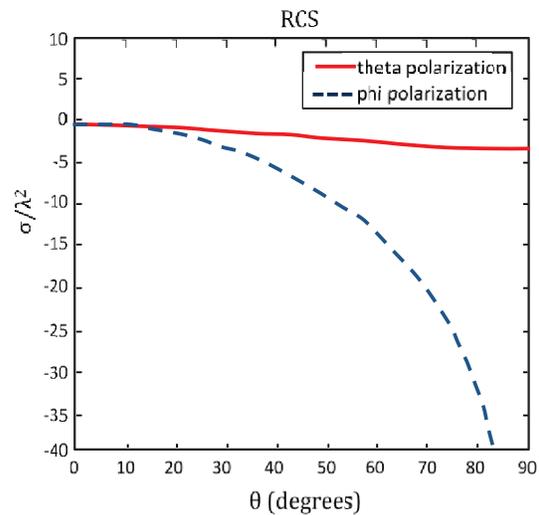


Fig. 4. (a) Electric field at the upper (solid line) and lower (dashed line) apertures, along  $x$ -axis, and (b) electric field at the upper (solid line) and lower (dashed line) apertures, along  $y$ -axis.



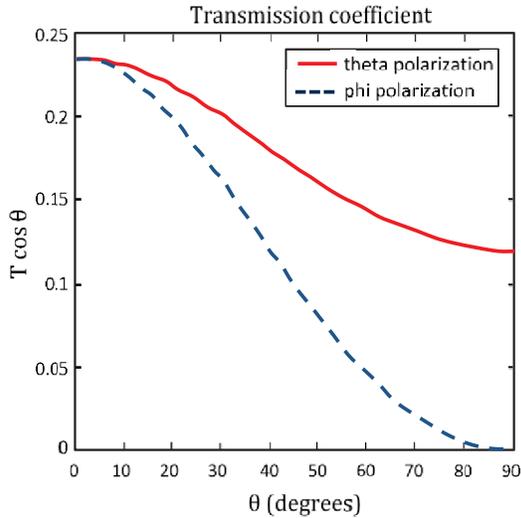


Fig. 5. (a) Backscatter RCS, and (b) transmission coefficient.

**A. Cross aperture**

A cross aperture on top and bottom is analyzed next. The cavity walls inside the conductor are the sides of a  $0.5\lambda$  by  $0.5\lambda$  square prism and the top and bottom walls are cross-shaped apertures. The geometry of mesh is shown in Fig. 6.

RCS and transmission coefficient plots given in Fig. 7 and Fig. 8 are very close to those given by [9].

PROBLEM GEOMETRY  $A = B = 0.5\lambda$ ,  $w = 0.125\lambda$ ,  $d = 0.25\lambda$

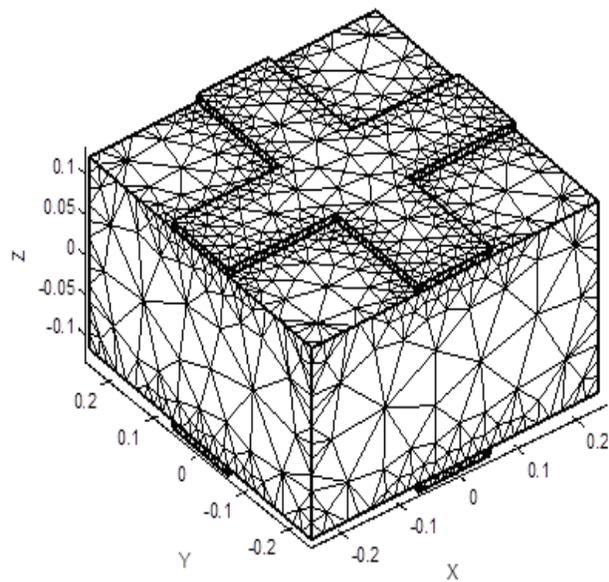


Fig. 6. Meshing of the cross aperture problem.

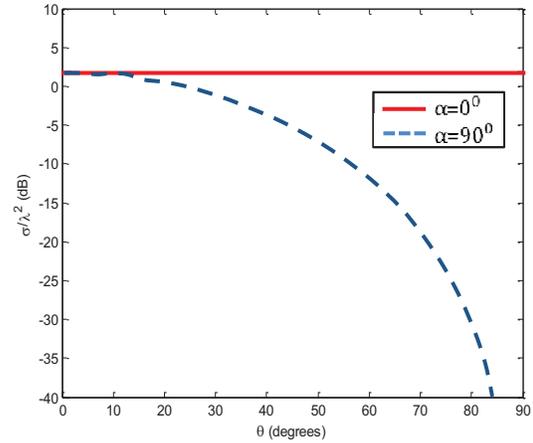


Fig. 7. Backscatter RCS as a function of incidence angle in the  $\phi = 0^\circ$  plane;  $\vec{E}^{inc}(\vec{r}) = (\vec{\theta}^{inc} \cos \alpha + \vec{\phi}^{inc} \sin \alpha)e^{-j\vec{k}^{inc} \cdot \vec{r}}$ .

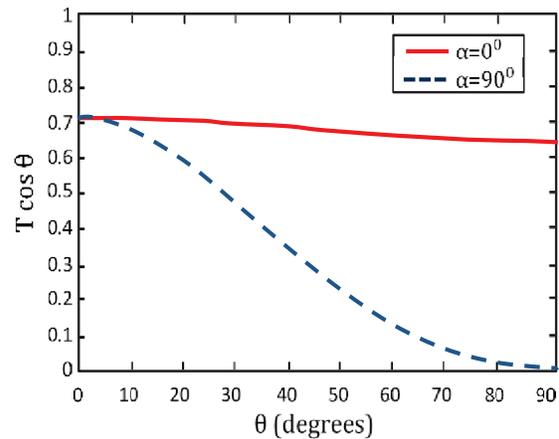


Fig. 8. Transmission coefficient plots of the structure in Fig. 6 as a function of incidence angle in the  $\phi = 0^\circ$  plane;  $\vec{E}^{inc}(\vec{r}) = (\vec{\theta}^{inc} \cos \alpha + \vec{\phi}^{inc} \sin \alpha)e^{-j\vec{k}^{inc} \cdot \vec{r}}$ .

**B. Circular aperture**

A cylindrical cavity with small circular apertures whose centers are on the  $z$ -axis is analyzed. The geometry of meshing is shown in Fig. 9. The radii of the small apertures are  $r=0.05\lambda$ . The radius of the cylindrical cavity is  $R=0.5\lambda$ . The thickness of the conductor is  $d=0.6\lambda$ . The flanges covering the cylindrical cavity and forming small circular apertures on top and bottom have a thickness of  $0.01\lambda$ . All regions are filled with  $(\epsilon_0, \mu_0)$ .

The incident field is  $\vec{E}^{inc} = \vec{y}e^{jkz}$ . The magnetic currents on the top and bottom apertures along  $x$  and  $y$  axes are in Fig. 10 and Fig. 11.

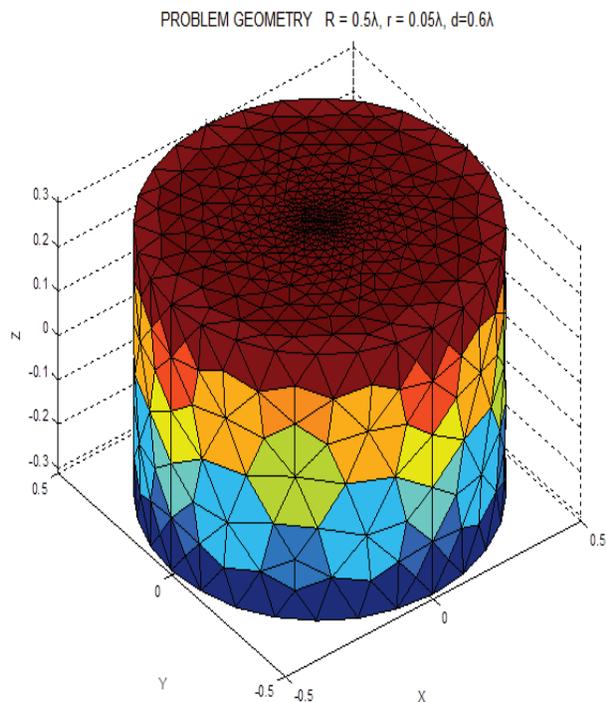


Fig. 9. The geometry of meshing of circular shape aperture.

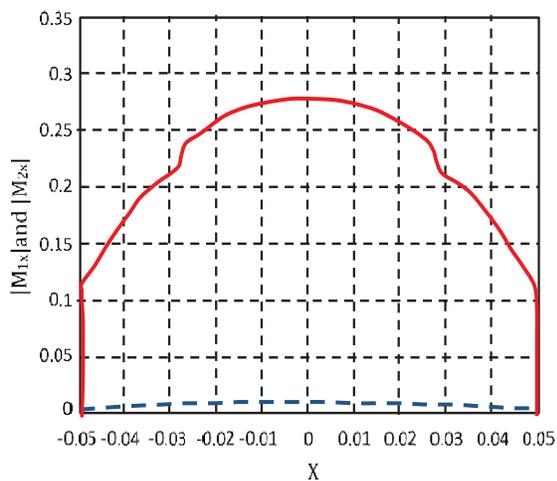


Fig. 10. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the x-axis for the problem.

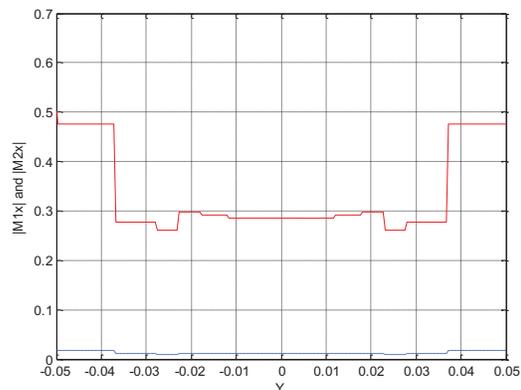


Fig. 11. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the y-axis for the problem.

**C. Effect of conductor thickness**

Square apertures with varying conductor thicknesses are analyzed to understand the effect of thickness on transmission of the plane wave through the cavity. The problem geometries and corresponding magnetic current plots for the top and bottom apertures are given in Fig. 12 through Fig. 17.

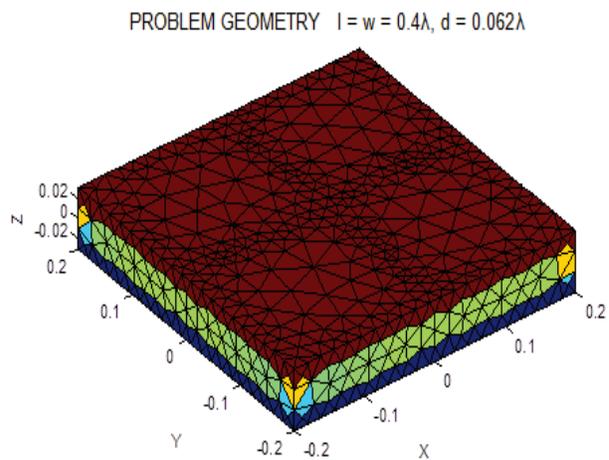


Fig. 12. Triangular meshing of the problem with square apertures (0.4λ-by-0.4λ) on top and bottom; conductor thickness  $d=0.062\lambda$ ,  $\epsilon_a = \epsilon_b = \epsilon_c = \epsilon_0$ ,  $\mu_a = \mu_b = \mu_c = \mu_0$ ,  $\vec{E}^{inc} = \vec{y}e^{jkz}$ .

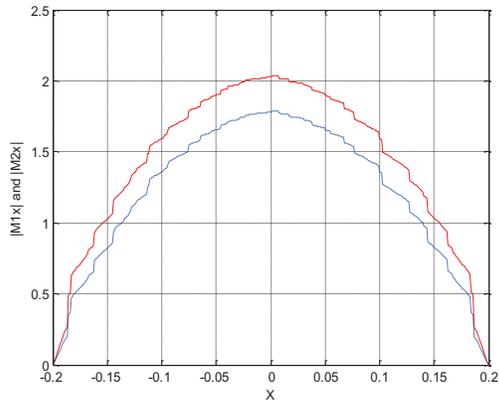


Fig. 13. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the x-axis for the problem in Fig. 12.

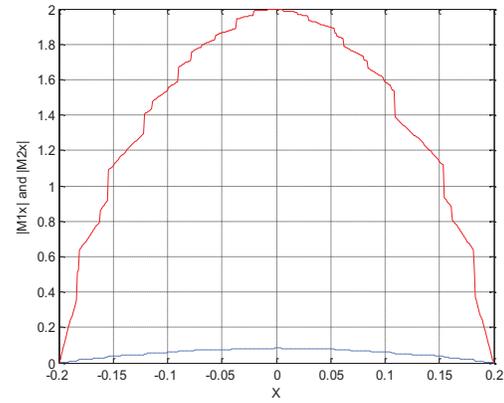


Fig. 16. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the x-axis for the problem in Fig. 15.

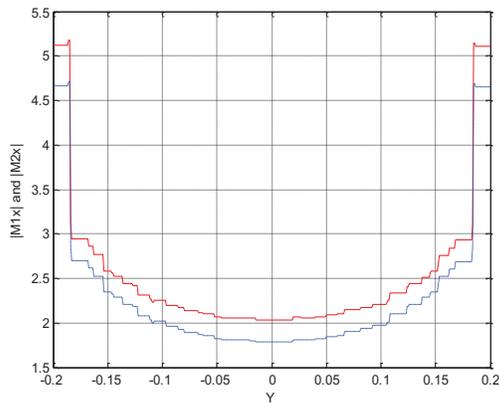


Fig. 14. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the y-axis for the problem in Fig. 12.

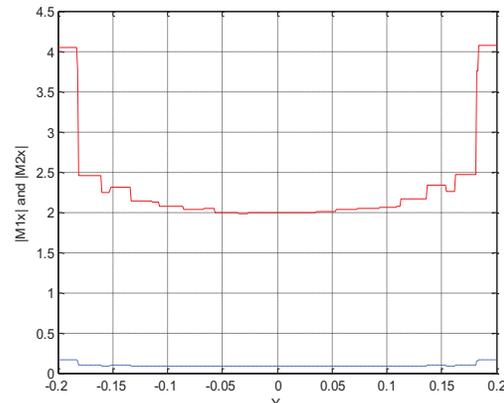


Fig. 17. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the y-axis for the problem in Fig. 15.

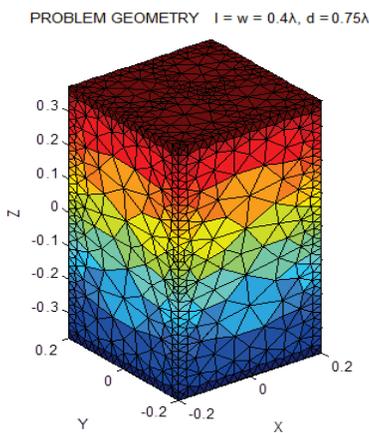


Fig. 15. Triangular meshing of the problem with square apertures ( $0.4\lambda$ -by- $0.4\lambda$ ) on top and bottom; conductor thickness  $d=0.75\lambda$ ,  $\epsilon_a = \epsilon_b = \epsilon_c = \epsilon_0$ ,  $\mu_a = \mu_b = \mu_c = \mu_0$ ,  $\vec{E}^{inc} = \vec{y}e^{jkz}$ .

As seen from the magnetic current plots, increasing the conductor thickness reduces the magnetic current on the bottom aperture and therefore reduces the tangential electric field on the bottom aperture. Reduction of the tangential electric field on bottom aperture is generally accompanied by reduction of the transmission coefficient.

The effect of meshing on convergence and many more numerical results are available in [11].

#### D. Results for circular aperture with a conic cavity

To prove the versatility of the method, a conic cavity in a thick conductor ( $d=0.5\lambda$ ) with circular apertures on top and bottom with radii of  $R=0.25\lambda$  and  $r=0.125\lambda$  respectively is analyzed. The problem geometry is shown in Fig. 18. The top and bottom magnetic currents along x and y axes are given in Fig. 19 and Fig. 20. RCS and transmission coefficients plots are shown in Fig. 21 and Fig. 22.

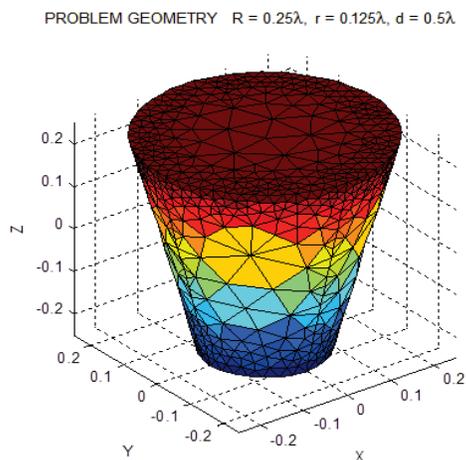


Fig. 18. Circular apertures with a conic cavity,  $\vec{E}^{inc} = \vec{y}e^{jkz}$ .

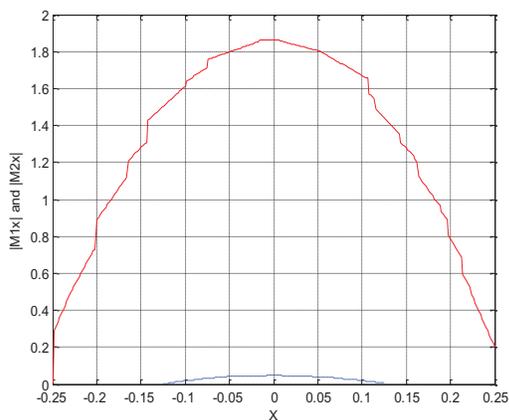


Fig. 19. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the  $x$ -axis for the problem in Fig. 18.

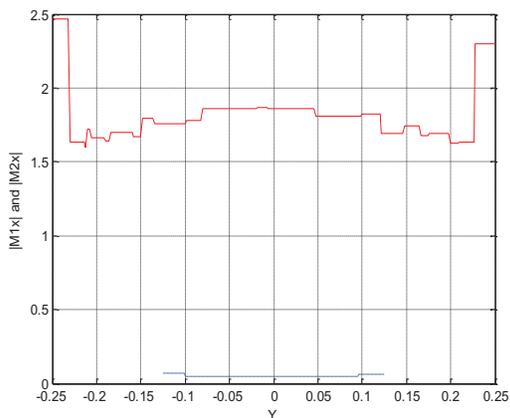


Fig. 20. Magnetic current on top aperture (solid line) and on bottom aperture (dashed line) along the  $y$ -axis for the problem in Fig. 18.

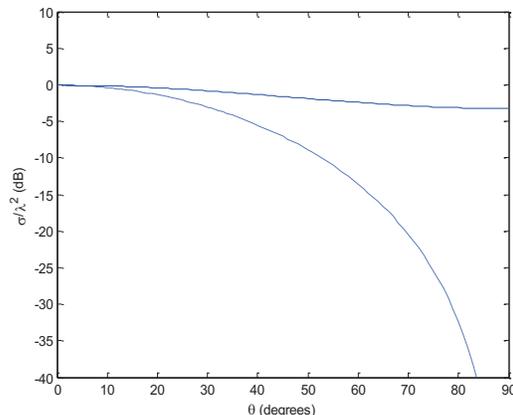


Fig. 21. Backscatter RCS of the structure in Fig. 18 as a function of incidence angle in the  $\phi = 0^\circ$  plane; solid line:  $\vec{E}^{inc}$  is phi polarized; dashed line:  $\vec{E}^{inc}$  is theta polarized.

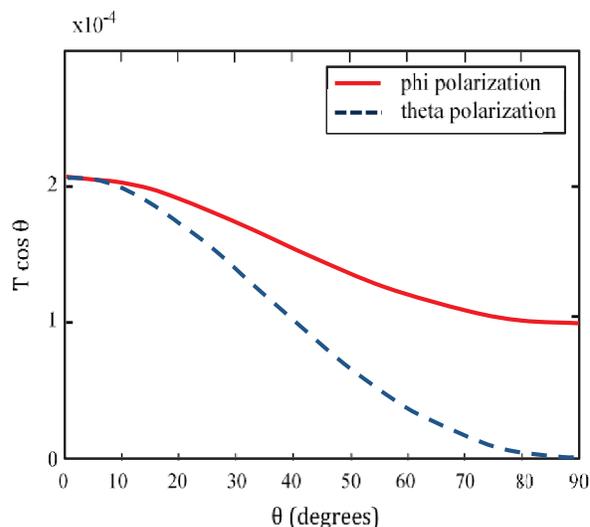


Fig. 22. Transmission coefficient plots of the structure in Fig. 18 as a function of incidence angle in the  $\phi = 0^\circ$  plane; solid line:  $\vec{E}^{inc}$  is phi polarized; dashed line:  $\vec{E}^{inc}$  is theta polarized.

#### IV. CONCLUSION

A simple moment solution is given to numerically analyze the problem of electromagnetic scattering from and transmission through an arbitrarily shaped aperture in a thick perfectly conducting ground plane. Computed results for square, cross and circular apertures are presented. Results for circular aperture with a conic cavity are also presented. The above computed results cannot be predicted by a simple theory. However, the results show that the fields on the shadow side of the aperture are much smaller than those on the lit side even when the thickness of the ground plane is small.

The method of moments formulation introduced in this paper can give the same results as the numerical methods that use volumetric meshing such as FEM. Since for homogeneous bodies MoM with only surface meshing is possible, it can give results faster than FEM, especially when problems with large volumes are to be solved.

It is known that, numerical results obtained by MOM and surface equivalence formulation may give erroneous results due to some spurious internal resonances [12]. Such spurious results were avoided by monitoring the condition number of the moment matrix and the stability of the computed results as the size, frequency or the number of triangular patches were changed slightly.

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